

# 微分積分学 II 自習用問題・解答

## 1 色々な関数の不定積分

以下, 積分定数  $C$  は省略します.

問 1.1.

(1)  $\log|x|$       (2)  $-\frac{1}{2}x^{-2}$       (3)  $\frac{2}{5}x^{\frac{5}{2}}$       (4)  $\frac{3}{4}\sqrt[3]{x^4}$       (5)  $2\sqrt{x}$       (6)  $-\frac{2}{\sqrt{x}}$

問 1.2.

(1)  $\frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x$       (2)  $\frac{1}{5}x^5 - 4x$       (3)  $\frac{1}{3}x^3 - 2\cos x$   
(4)  $\frac{1}{2}x^2 - 3\log|x| - \frac{2}{x}$       (5)  $\frac{1}{2}x^2 + 4\sqrt{x} - \frac{1}{x}$       (6)  $\frac{2}{5}\sqrt[2]{x^5} + \frac{8}{3}\sqrt[2]{x^3} + 8\sqrt{x}$

問 1.3.

(1)  $\log(3x^2 + 2)$       (2)  $\frac{1}{2}\log|x^2 - 4|$       (3)  $\frac{1}{2}\log(x^2 + 2x + 2)$   
(4)  $\log(\sin x + 2)$       (5)  $-\log|\sin x + \cos x|$       (6)  $-\log|\cos x|$   
(7)  $\frac{1}{2}\log(e^{2x} + 1)$       (8)  $\log|\log x|$       (9)  $\log(x^2 + 1) + \arctan x$

問 1.4.

(1)  $\frac{1}{10}(2x + 3)^5$       (2)  $-\frac{1}{4x + 6}$       (3)  $\frac{1}{5}\sqrt{(2x + 3)^5}$       (4)  $\frac{1}{3}\log|3x - 1|$   
(5)  $\frac{1}{3}e^{3x-1}$       (6)  $\frac{1}{3}\sin(3x - 1)$       (7)  $2\sin\frac{x}{2}$       (8)  $\frac{1}{2} - \frac{1}{4}\sin 2x$   
(9)  $\frac{1}{2}\tan 2x$       (10)  $\arcsin(x - 1)$       (11)  $\arctan(x + 1)$       (12)  $\frac{1}{2}\arctan 2x$   
(13)  $\arcsin(x - 1)$       (14)  $\arctan(x + 1)$       (15)  $\frac{1}{2}\log(x^2 + 2x + 2) - \arctan(x + 1)$

問 1.5.

- $\int \frac{\sin x}{\sin x + \cos x} dx = \frac{1}{2}(x - \log|\sin x + \cos x|),$
- $\int \frac{\cos x}{\sin x + \cos x} dx = \frac{1}{2}(x + \log|\sin x + \cos x|).$

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## 2 置換積分

### 問 2.1.

$$(1) \frac{2}{3} \sqrt{(x^2 + 1)^3}$$

$$(2) \frac{3}{4} \sqrt[3]{(x^2 + 2)^2}$$

$$(3) \frac{1}{2} \arctan x^2$$

$$(4) \frac{1}{5} (x^3 + 1)^5$$

$$(5) \frac{1}{3} \sqrt{2x^3 + 1}$$

$$(6) \frac{1}{3} \arcsin x^3$$

$$(7) -\cos x^2$$

$$(8) \frac{1}{2} e^{x^2}$$

$$(9) \frac{1}{3} e^{x^3}$$

### 問 2.2.

$$(1) \frac{1}{3} \sin^3 x$$

$$(2) -\frac{1}{6} \cos^6 x$$

$$(3) \sin x - \frac{2 \sin^3 x}{3} + \frac{\sin^5 x}{5}$$

$$(4) -\frac{3}{4} \sqrt[3]{(\cos x + 2)^4}$$

$$(5) e^{\sin x}$$

$$(6) -\frac{1}{2} \arctan(\cos 2x)$$

$$(7) -\log |\cos x|$$

$$(8) -\cos x + \frac{1}{3} \cos^3 x$$

$$(9) \sin x - \frac{2 \sin^3 x}{3} + \frac{\sin^5 x}{5}$$

### 問 2.3.

$$(1) \frac{1}{2} (\log x)^2$$

$$(2) \frac{1}{3} (\log x)^3$$

$$(3) \log |\log x|$$

$$(4) \frac{1}{3} (e^x + 1)^3$$

$$(5) \frac{3}{4} \sqrt[3]{(e^x + 2)^4}$$

$$(6) \arctan e^x$$

### 問 2.4.

$$(1) \arcsin \frac{x}{2}$$

$$(2) \arcsin \frac{x}{3}$$

$$(3) \arcsin \frac{x}{\sqrt{5}}$$

$$(4) \arcsin \frac{x-2}{2}$$

$$(5) \arcsin \frac{x-2}{2}$$

$$(6) \arcsin \frac{x+2}{\sqrt{5}}$$

### 問 2.5.

$$(1) \frac{1}{2} \arctan \frac{x}{2}$$

$$(2) \frac{1}{3} \arctan \frac{x}{3}$$

$$(3) \frac{1}{\sqrt{5}} \arctan \frac{x}{\sqrt{5}}$$

$$(4) \frac{1}{2} \arctan \frac{x+1}{2}$$

$$(5) \frac{1}{2} \arctan \frac{x+1}{2}$$

$$(6) \frac{1}{\sqrt{5}} \arctan \frac{x+1}{\sqrt{5}}$$