微分積分学 II レポート (No.1) ・解答

(1) $\displaystyle\int x^{\frac{3}{2}}dx = \frac{2}{5}x^{\frac{5}{2}} + C.$

(2) $\displaystyle\int \frac{1}{\sqrt[3]{x}}dx = \int x^{-\frac{1}{3}} = \frac{3}{2}x^{\frac{2}{3}} + C = \frac{3}{2}\sqrt[3]{x^2} + C.$

(3) $\displaystyle\int (x^3 - 3x + 1)dx = \frac{1}{4}x^4 - \frac{3}{2}x^2 + x + C.$

(4) $\displaystyle\int \frac{x^3 - 3x + 1}{x^2}dx = \int \left(x - \frac{3}{x} + \frac{1}{x^2}\right)dx = \frac{1}{2}x^2 - 3\log|x| - \frac{1}{x} + C.$

(5) $\displaystyle\int \frac{1}{x^2 + 1}dx = \arctan x + C.$

(6) $\displaystyle\int \frac{x}{x^2 + 1}dx = \frac{1}{2}\int \frac{(x^2 + 1)'}{x^2 + 1}dx = \frac{1}{2}\log(x^2 + 1) + C.$

(7) $\displaystyle\int \tan x dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{(\cos x)'}{\cos x}dx = -\log|\cos x| + C.$

(8) $\displaystyle\int (2x + 1)^4 dx = \frac{1}{2} \cdot \frac{1}{5}(2x + 1)^5 + C = \frac{1}{10}(2x + 1)^5 + C.$

(9) $\displaystyle\int e^{3x}dx = \frac{1}{3}e^{3x} + C.$

(10) $\displaystyle\int \cos(2x + 1)dx = \frac{1}{2}\sin(2x + 1) + C.$

(11) $x^2+1=t$ とおくと, $2xdx=dt$.

$$\int 2x\sqrt{x^2+1}dx=\int \sqrt{t}dt=\frac{2}{3}\sqrt{t^3}+C=\frac{2}{3}\sqrt{(x^2+1)^3}+C.$$

(12) $\sin x=t$ とおくと, $\cos xdx=dt$.

$$\int \sin^2 x\cos xdx=\int t^2dt=\frac{1}{3}t^3+C=\frac{1}{3}\sin^3 x+C.$$

(13) $\log x=t$ とおくと, $\dfrac{1}{x}dx=dt$.

$$\int \frac{(\log x)^2}{x}dx=\int t^2dt=\frac{1}{3}t^3+C=\frac{1}{3}(\log x)^3+C.$$

(14) $x=\sqrt{2}t$ とおくと, $dx=\sqrt{2}dt$.

$$\int \frac{1}{\sqrt{2-x^2}}dx$$
$$=\int \frac{1}{\sqrt{2-2t^2}}\cdot\sqrt{2}dt=\int \frac{1}{\sqrt{1-t^2}}dt=\arcsin t+C=\arcsin\frac{x}{\sqrt{2}}+C.$$

(15) $x=2t$ とおくと, $dx=2dt$.

$$\int \frac{1}{4+x^2}dx$$
$$=\int \frac{1}{4+4t^2}\cdot 2dt=\frac{1}{2}\int \frac{1}{1+t^2}dt=\frac{1}{2}\arctan t+C=\frac{1}{2}\arctan\frac{x}{2}+C.$$

(16) $\int x\cos x dx$

$= x\sin x - \int 1\cdot \sin x dx = x\sin x - \int \sin x dx = x\sin x + \cos x + C.$

(17) $\int x\log x dx$

$= \frac{1}{2}x^2\log x - \int \frac{1}{2}x^2\cdot\frac{1}{x}dx = \frac{1}{2}x^2\log x - \frac{1}{2}\int x dx = \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + C.$

(18) $\int xe^{3x}dx$

$= x\cdot\frac{1}{3}e^{3x} - \int 1\cdot\frac{1}{3}e^{3x}dx = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C.$

(19) $\int x^2\sin x dx$

$= x^2(-\cos x) - \int 2x(-\cos x)dx = -x^2\cos x + 2\int x\cos x dx$

$= -x^2\cos x + 2(x\sin x + \cos x) + C = -x^2\cos x + 2x\sin x + 2\cos x + C.$

(20) $\int \log x dx$

$= x\log x - \int x\cdot\frac{1}{x}dx = x\log x - \int 1dx = x\log x - x + C.$