微分積分学 II レポート (No.3) ・解答

(1) $\displaystyle\int_1^2 3x^2dx = \Big[x^3\Big]_1^2 = 7.$

(2) $\displaystyle\int_1^{e^2} \frac{1}{x}dx = \Big[\log|x|\Big]_1^{e^2} = 2.$

(3) $\displaystyle\int_1^2 \frac{1}{\sqrt{x}}dx = \Big[2\sqrt{x}\Big]_1^2 = 2\sqrt{2} - 2.$

(4) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}dx = \left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}.$

(5) $\displaystyle\int_{-1}^{\sqrt{3}} \frac{1}{x^2+1}dx = \Big[\arctan x\Big]_{-1}^{\sqrt{3}} = \frac{\pi}{3} - \left(-\frac{\pi}{4}\right) = \frac{7}{12}\pi.$

(6) $\displaystyle\int_0^1 \frac{x}{x^2+1}dx = \left[\frac{1}{2}\log(x^2+1)\right]_0^1 = \frac{1}{2}\log 2.$

(7) $x^2+1=t$ とおくと, $2xdx=dt$, $\begin{array}{c|ccc} x & 0 & \to & 1 \\ \hline t & 1 & \to & 2 \end{array}$ なので,

$$\int_0^1 x(x^2+1)^3dx = \int_1^2 \frac{1}{2}t^3dt = \left[\frac{1}{8}t^4\right]_1^2 = \frac{15}{8}.$$

(8) $\sin x = t$ とおくと, $\cos x dx = dt$, $\begin{array}{c|ccc} x & 0 & \to & \frac{\pi}{2} \\ \hline t & 0 & \to & 1 \end{array}$ なので,

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx = \int_0^1 t^2dt = \left[\frac{1}{3}t^3\right]_0^1 = \frac{1}{3}.$$

(9) $\log x = t$ とおくと, $\frac{1}{x}dx = dt$, $\begin{array}{c|ccc} x & 1 & \to & e \\ \hline t & 0 & \to & 1 \end{array}$ なので,

$$\int_1^e \frac{\log x}{x}dx = \int_0^1 tdt = \left[\frac{1}{2}t^2\right]_0^1 = \frac{1}{2}.$$

(10) $x=2t$ とおくと, $dx=2dt$, $\begin{array}{c|ccc} x & -1 & \to & 1 \\ \hline t & -\frac{1}{2} & \to & \frac{1}{2} \end{array}$ なので,

$$\begin{aligned}\int_{-1}^1 \frac{1}{\sqrt{4-x^2}}dx &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{4-4t^2}} \cdot 2dt \\ &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-t^2}}dt = \left[\arcsin t\right]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}.\end{aligned}$$

(11) $\displaystyle \int_0^{\frac{\pi}{2}} x\cos x dx = \Big[x\sin x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x dx = \frac{\pi}{2} - \Big[-\cos x\Big]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1.$

(12) $\displaystyle \int_1^{e} x\log x dx = \left[\frac{1}{2}x^2\log x\right]_1^{e} - \int_1^{e} \frac{1}{2}x dx$

$$= \frac{1}{2}e^2 - \left[\frac{1}{4}x^2\right]_1^{e} = \frac{1}{2}e^2 - \left(\frac{1}{4}e^2 - \frac{1}{4}\right) = \frac{1}{4}e^2 + \frac{1}{4}.$$

(13) $\displaystyle \int_1^{e} \log x dx = \Big[x\log x\Big]_1^{e} - \int_1^{e} 1 dx = e - \Big[x\Big]_1^{e} = e - (e-1) = 1.$

(14) $\displaystyle \int_0^{1} x^2 e^x dx = \Big[x^2 e^x\Big]_0^{1} - \int_0^{1} 2xe^x dx = e - \Big[2xe^x\Big]_0^{1} + \int_0^{1} 2e^x dx$

$$= e - 2e + \Big[2e^x\Big]_0^{1} = e - 2e + (2e-2) = e - 2.$$

(15) $\displaystyle \int_{-1}^{2} (x+1)(x-2) dx = -\int_{-1}^{2} (x+1)(2-x) dx = -\frac{1}{6}\left\{2-(-1)\right\}^3 = -\frac{9}{2}.$

(16) $$\int_0^2 \frac{1}{\sqrt{x}}dx = \lim_{a\to+0}\int_a^2 \frac{1}{\sqrt{x}}dx$$

$$= \lim_{a\to+0}\left[2\sqrt{x}\right]_a^2 = \lim_{a\to+0}\left(2\sqrt{2}-2\sqrt{a}\right) = 2\sqrt{2}.$$

(17) $$\int_0^1 \frac{1}{\sqrt{1-x^2}}dx = \lim_{b\to1-0}\int_0^b \frac{1}{\sqrt{1-x^2}}dx$$

$$= \lim_{b\to1-0}\left[\arcsin x\right]_0^b = \lim_{b\to1-0}\arcsin b = \frac{\pi}{2}.$$

(18) $$\int_2^\infty \frac{1}{x^2}dx = \lim_{b\to\infty}\int_2^b \frac{1}{x^2}dx$$

$$= \lim_{b\to\infty}\left[-\frac{1}{x}\right]_2^b = \lim_{b\to\infty}\left(-\frac{1}{b}+\frac{1}{2}\right) = \frac{1}{2}.$$

(19) $$\int_0^\infty \frac{1}{1+x^2}dx = \lim_{b\to\infty}\int_0^b \frac{1}{1+x^2}dx$$

$$= \lim_{b\to\infty}\left[\arctan x\right]_0^b = \lim_{b\to\infty}\arctan b = \frac{\pi}{2}.$$

(20) $$\int_0^\infty e^{-3x}dx = \lim_{b\to\infty}\int_0^b e^{-3x}dx$$

$$= \lim_{b\to\infty}\left[-\frac{1}{3}e^{-3x}\right]_0^b = \lim_{b\to\infty}\left(-\frac{1}{3}e^{-3b}+\frac{1}{3}\right) = \frac{1}{3}.$$