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LMI Based Neurocontroller for Guaranteed Cost Control of Discrete–Time Uncertain System

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***Abstract*— This paper investigates the application of neural networks to the guaranteed cost control problem of discrete–time uncertain system. Based on the Linear Matrix Inequality (LMI) design approach, a class of a state feedback controller is established, and sufficient conditions for the existence of guaranteed cost controller are derived by making use of the LMI. The novel contribution is that the neurocontroller is substituted for the additive gain perturbations. It is newly shown that although the neurocontroller is included in the discrete–time uncertain system, the robust stability for the closed-loop system and the reduction of the cost performance are attained.**

I. INTRODUCTION

In recent years, the problem of robust control for the discrete–time system with parameter uncertainties has been studied (see e.g., [1] and reference therein). In these studies, much effort has been made towards finding a controller that guarantees robust stability, but less attention has been paid to cost performance. In real control system applications, the ability to guarantee robust stability as well as an adequate cost performance should be taken into account for. One approach to this problem is the so–called guaranteed cost control approach [2]. This approach has the advantage of providing an upper bound on a given performance index. The guaranteed cost control for the uncertain discrete–time system by means of the output feedback control has been discussed in [3]. On the other hand, recent advance in theory of Linear Matrix Inequality (LMI) has allowed a revisiting of the guaranteed cost control approach. The guaranteed cost control problem for a class of the uncertain discrete–time system which is based on the LMI design approach was solved by using the state feedback [4]. However, due to the presence of the design parameter, it is well–known that the cost performance becomes quite large.

A neural network (NN) has been actively exploited to construct an intelligent control system because of its nonlinear mapping approximation for system uncertainties involved. Then some control methodologies utilizing NN have been proposed by combining with modern control techniques. For example, an adaptive controller using NN was designed within the framework of the adaptive control theory in the literature [5], and feedback control systems in which NN were placed instead of a conventional controller [6] or in parallel with [7] for identifying and canceling the plant uncertainties. As important studies in particular, the linear quadratic regulator (LQR) problem using multiple NN has been investigated [8, 9]. In these approaches, one neural network is dedicated to the forward model for identifying the uncertainties of the controlled plant, and the other network may compensate for the influence of the uncertainties based on the trained forward model. However, in these researches, there is a possibility that NN can not stabilize the system, because the stability of the closed–loop system which includes the neurocontroller has not been considered. For example, the system stability may not be guaranteed any longer when the degree of system nonlinearity is strong [8]. Moreover, it is shown that the robustness of the closed–loop system is not guaranteed without the margin of the controller gain perturbations by the neurocontroller [10]. Therefore, the neurocontroller in the closed–loop system is required to tolerate some uncertainties of the controlled system [11].

In this paper, the guaranteed cost control problem of the discrete–time uncertain system with the neurocontroller is discussed. Firstly, a class of the fixed state feedback controller of the discrete–time uncertain system with the gain perturbations is derived. Secondly, some sufficient conditions to design the guaranteed cost controller is newly established by means of the LMI. In order to reduce large cost performance caused by the guaranteed cost control, NN is used. The new idea is that the neurocontroller is substituted for the additive gain perturbations. The stability for the system with the neurocontroller is guaranteed. It should be noted that there is no result for the stability of the closed–loop system under the neurocontroller. The training data is extracted from the model difference between the practical plant and the nominal plant compared with the existing results. As a result, although the neurocontroller is included in the discrete–time uncertain system, the robust stability of the closed–loop system and the reduction of the cost are attained. Finally, in order to demonstrate the efficiency of our design approach, the numerical example is given.

II. PRELIMINARY

Consider the following class of a uncertain discrete–time linear system:

$$x(k+1) = [A + D_1F(k)E_1]\,x(k) + Bu(k), \quad (1a)$$
$$u(k) = [K + D_2N(k)E_2]\,x(k), \quad (1b)$$

where $x(k) \in \Re^n$ is the state, $u(k) \in \Re^m$ is the control input, A, B, D_1, D_2, E_1 and E_2 are known constant

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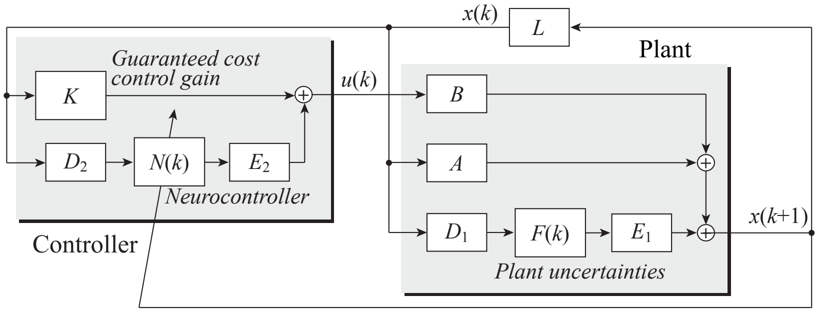

Fig. 1. Block diagram of proposed system.

matrices, K is the fixed gain matrix for the controller (1b), and $F(k) \in \Re^{p\times p}$ is unknown matrix function and $N(k) \in \Re^{q\times q}$ is the output of NN. It is assumed that $F(k)$ and $N(k)$ are satisfying

$$F^T(k)F(k) \le I_p, \quad N^T(k)N(k) \le I_q. \tag{2}$$

Block diagram of a new proposed method is shown in Fig. 1, where L is a time lag diagram. It should be noted that the controller (1b) has the neurocontroller as additive perturbations $D_2N(k)E_2$ compared with the existing results [1, 4].

Associated with the system (1) is the quadratic cost function

$$J = \sum_{k=0}^{\infty} \left[x^T(k)Qx(k) + u^T(k)Ru(k)\right], \tag{3}$$

where Q and R are given as positive definite symmetric matrices. In this situation, the definition of the guaranteed cost control with the additive gain perturbations is given below.

Definition 1: For the uncertain system (1) and cost function (3), if there exist a control gain matrix K and a positive scalar J^* such that for the admissible uncertainties and gain perturbations (2), the closed–loop system is asymptotically stable and the closed–loop value of the cost function (3) satisfies $J < J^*$, then J^* is said to be a guaranteed cost and K is said to be a guaranteed cost control gain matrix of the uncertain system (1) and cost function (3).

The above definition is very popular for dealing with time–varying uncertainties and is also used in [2].

It is noted that if the controller (1b) is the guaranteed cost control in the infinite horizon, then it is also the quadratically stabilizing controller. Conversely, it can be easily shown that a quadratically stabilizing controller will achieve the guaranteed cost. The following result shows that the guaranteed cost control for the system (1) has an upper bound on the cost function (3).

Lemma 1: Consider the following matrix inequality under the uncertain discrete–time system (1) with the cost function (3);

$$\begin{aligned} &x^T(k+1)Px(k+1) - x^T(k)Px(k) + x^T(k)\big[Q \\ &\quad +(K + D_2N(k)E_2)^T R(K + D_2N(k)E_2)\big]x(k) < 0, \end{aligned} \tag{4}$$

for all nonzero $x(k) \in \Re^n$ and the uncertain matrix $F(k)$, and the gain perturbation matrix $N(k)$.

Under such condition, the matrix K of the controller (1b) is the guaranteed cost control gain matrix associated with the cost function (3). That is, the closed–loop uncertain system

$$\begin{aligned} x(k+1) = &\ [(A + D_1F(k)E_1) \\ &+B(K + D_2N(k)E_2)]x(k) \end{aligned} \tag{5}$$

is stable and achieves

$$J < J^* = x^T(0)Px(0). \tag{6}$$

Proof: Let us define the following Lyapunov function candidate

$$V(x(k)) = x^T(k)Px(k), \tag{7}$$

where P is the positive definite matrix. Then, the proof can be done by using the similar technique in [10]. In this paper, it is omitted. ■

The objective of this section is to design the fixed guaranteed cost control gain matrix K for the discrete–time system (1).

Theorem 1: Consider the uncertain discrete–time system (1) and cost function (3). For the uncertain matrix $F(k)$ and the gain perturbation matrix $N(k)$, if the LMI (8) has a feasible solution such as symmetric positive definite matrix $X \in \Re^{n\times n}$ and $Y \in \Re^{m\times n}$, and positive scalar $\varepsilon_i > 0$ $(i = 1, 2)$, then $K = YX^{-1}$ is the guaranteed cost control gain matrix.

Furthermore, the corresponding value of the cost function (3) satisfies the following inequality (9) for all admissible uncertainties $F(k)$, and the gain perturbations $N(k)$:

$$J < J^* = x^T(0)X^{-1}x(0). \tag{9}$$

Proof: Let us introduce the matrices $X = P^{-1}$ and $Y = KP^{-1}$. Pre– and post–multiplying both sides of the inequality (8) by the positive definite matrix

$$\begin{bmatrix} P & 0 & 0 & 0 & 0 & 0 \\ 0 & I_p & 0 & 0 & 0 & 0 \\ 0 & 0 & I_q & 0 & 0 & 0 \\ 0 & 0 & 0 & I_n & 0 & 0 \\ 0 & 0 & 0 & 0 & I_m & 0 \\ 0 & 0 & 0 & 0 & 0 & I_n \end{bmatrix} > 0 \tag{10}$$

and applying the Schur complement [13] gives (11).

Using a standard matrix inequality [12] to the matrix inequality (11) for all admissible uncertainties and the gain perturbations (2), and applying the Schur complement to

$$\begin{bmatrix} -X & (E_1X)^T & (E_2X)^T & X & Y^T & (AX+BY)^T \\ E_1X & -\varepsilon_1 I_p & 0 & 0 & 0 & 0 \\ E_2X & 0 & -\varepsilon_2 I_q & 0 & 0 & 0 \\ X & 0 & 0 & -Q^{-1} & 0 & 0 \\ Y & 0 & 0 & 0 & -R^{-1}+\varepsilon_2 D_2 D_2^T & (\varepsilon_2 BD_2D_2^T)^T \\ AX+BY & 0 & 0 & 0 & \varepsilon_2 BD_2D_2^T & -X+\varepsilon_1 D_1D_1^T+\varepsilon_2 BD_2D_2^TB^T \end{bmatrix} < 0. \quad (8)$$

$$\begin{bmatrix} -P+\varepsilon_1^{-1}E_1^TE_1+\varepsilon_2^{-1}E_2^TE_2 & I_n & K^T & (A+BK)^T \\ I_n & -Q^{-1} & 0 & 0 \\ K & 0 & -R^{-1}+\varepsilon_2 D_2D_2^T & (\varepsilon_2 BD_2D_2^T)^T \\ A+BK & 0 & \varepsilon_2 BD_2D_2^T & -P^{-1}+\varepsilon_1 D_1D_1^T+\varepsilon_2 BD_2D_2^TB^T \end{bmatrix} < 0. \quad (11)$$

the matrix inequality, it is easy to verify that the matrix inequality (4) is satisfied.

Thus, K is the guaranteed cost control gain matrix. On the other hand, since the results of the cost bound (9) can be proved by using the similar argument for the proof of Theorem 1, it is omitted. ■

Since the LMI (8) consists of a convex solution set of $(\varepsilon_1,\ \varepsilon_2,\ X,\ Y)$, various efficient convex optimization algorithm can be applied. Moreover, its solutions represent a set of the guaranteed cost control gain matrix K. This parameterized representation can be exploited to design the guaranteed cost control gain which minimizes the value of the guaranteed cost for the closed–loop uncertain system. Consequently, solving the following optimization problem allows us to determine the optimal bound.

$$J < J^* < \min_{(\varepsilon_1,\ \varepsilon_2,\ X,\ Y)} \alpha, \quad (12)$$

such that (8) and

$$\begin{bmatrix} -\alpha & x^T(0) \\ x(0) & -X \end{bmatrix} < 0. \quad (13)$$

The problem addressed in this section is defined as follows:

Problem 1: Find the guaranteed cost control gain $K = YX^{-1}$ satisfying the LMIs (8) and (13) to make the cost α become as small as possible.

Since the bound in *Problem 1* depends on the initial condition $x(0)$, it is assumed to remove such condition that $x(0)$ is a zero mean random variable satisfying $E[x(0)x^T(0)] = I_n$.

Then, the LMI (13) yields

$$\begin{bmatrix} -M & I_n \\ I_n & -X \end{bmatrix} < 0, \quad (14)$$

where $E[\cdot]$ denotes the expectation, M is the expectation of α. In this paper, the condition (14) is used instead of (13) in the optimization problem and M would be gotten as small as possible.

The crucial difference between the uncertain discrete–time system in [1], [4] and the considered system of this paper is that the controller gain perturbations as the neurocontroller are newly added. Therefore, the obtaining results of this section are original.

III. NEURAL NETWORKS FOR ADDITIVE GAIN PERTURBATIONS

The LMI approach for the uncertain discrete–time systems usually results in the conservative controller design due to the existence of the uncertainties $F(k)$ and the gain perturbations $N(k)$, which lead the large cost J. The main purpose of this paper is to introduce NN as additive gain perturbations into the discrete-time uncertain system to improve the cost performance. Note that the proposed neurocontroller regulates its outputs in real-time under the robust stability guaranteed by the LMI approach.

A. On–line learning Algorithm of neurocontroller

It can be much expected that the reduction of the cost will be attain when a neurocontroller can manage the uncertain system as a nominal linear system while compensating for control errors by a conservative controller. That is, the neurocontroller is required to compensate the conservative controller to work as a LQR controller in the uncertain system.

Let us consider the following nominal system without uncertainties as:

$$\hat{x}(k+1) \;=\; A\hat{x}(k) + B\hat{u}(k), \quad (15)$$

where $\hat{x}(k) \in \Re^n$ is the state and $\hat{u}(k) \in \Re^m$ is the control input. For such linear system, it is well–known that the LQR control is an effective method to design a controller which can minimize the cost function (3). Based on LQR, the optimal control $\hat{u}^*(k)$ can be designed as

$$\hat{u}^*(k) \;=\; \hat{K}\hat{x}(k), \quad (16a)$$

$$\hat{K} \;=\; -(R + B^T\hat{P}B)^{-1}B^T\hat{P}A, \quad (16b)$$

where $\hat{K}$ is the optimal feedback gain, and the matrix $\hat{P}$ is the positive semidefinite symmetric solution of the following algebraic Riccati equation as:

$$\hat{P} = A^T\hat{P}A - A^T\hat{P}B(R + B^T\hat{P}B)^{-1}B^T\hat{P}A + Q. \quad (17)$$

The NN in the proposed system should be trained in real–time so that the state discrepancy $\left|\left|\hat{x}(k+1) - x(k+1)\right|\right|$ becomes as small as possible at each step k. $N(k)$, in equation (2), can be expressed as a nonlinear function of the state $x(k)$, the weight coefficient of NN $w(k)$, and the threshold $\theta(k)$ as follows

$$N(k) = f\big(x(k), w(k), \theta(k)\big). \quad (18)$$

An energy function $E(k)$ is defined as the discrepancy between the behavior of the nominal system according to the LQR method and the one of the uncertain discrete–time system of step k. At each step, the weight coefficients are modified so as to minimize $E(k)$ given as

$$E(k) \triangleq \frac{1}{2}(\hat{x}(k+1) - x(k+1))^T(\hat{x}(k+1) - x(k+1)) \quad (19)$$

$E(k)$ can be calculated by using the observed state value, $x(k+1)$. If $E(k)$ can be minimized as small as possible, the discrepancy $\left|\left|\hat{x}(k+1) - x(k+1)\right|\right|^2$ would also be minimized so that the cost of the uncertain discrete–time system is close to the cost of the nominal system based on the LQR control.

In the learning of NN, the modification of weight coefficient, $\Delta w(k)$, is given as

$$w(k+1) = w(k) + \Delta w(k), \tag{20a}$$

$$\Delta w(k) = -\eta \frac{\partial E(k)}{\partial w(k)}, \tag{20b}$$

$$\frac{\partial E(k)}{\partial w(k)} = \frac{\partial E(k)}{\partial N(k)} \frac{\partial N(k)}{\partial w(k)}, \tag{20c}$$

where η is the learning ratio. The term $\dfrac{\partial E(t)}{\partial N(k)}$ can be calculated from the energy function (19) as follows:

$$\frac{\partial E(k)}{\partial N(k)} = -\big(\hat{x}(k+1) - x(k+1)\big) B D_2 E_2 x(k) \tag{21}$$

and $\dfrac{\partial N(k)}{\partial w(k)}$ can be calculated using the chain rule on the NN. From (18)$\sim$(21), NN can be trained so as to decrease the cost J on–line.

B. Multilayered Neural networks

The utilized NN are of a three–layer feed–forward network as shown in Fig. 2. A linear function is utilized in the neurons of the input and the hidden layers, and a sigmoid function in the output layer. Inputs and outputs of each layer can be described as follows

$$s_g^i(k) = \begin{cases} U_i(k) & \{g = 1(\text{input layer})\} \\ \sum w_1^{(i,j)}(k) o_1^j(k) & \{g = 2(\text{hidden layer})\} \\ \sum w_2^{(i,j)}(k) o_2^j(k) & \{g = 3(\text{output layer})\}, \end{cases}$$

$$o_g^i(k) = \begin{cases} s_1^i(k) & \{g = 1(\text{input layer})\} \\ s_2^i(k) + \theta_1^{(i)}(k) & \{g = 2(\text{hidden layer})\} \\ \dfrac{1 - e^{(-s_3^i(k) + \theta_2^{(i)}(k))}}{1 + e^{(-s_3^i(k) + \theta_2^{(i)}(k))}} & \{g = 3(\text{output layer})\}, \end{cases}$$

where $s_g^i(k)$ and $o_g^i(k)$ are the input and output of neuron i in the g–th layer at step k, $w_g^{(i,j)}(k)$ indicates the weight coefficient from neuron j in the g–th layer to neuron i in the $(g+1)$–th layer, $U_i(k)$ is the input of NN, $\theta_g^{(i)}(k)$ is a positive constant for the threshold of neuron i in the $(g+1)$–th layer. As the additive gain perturbations defined in the formula (2), the outputs of NN are set in the range of $[-1.0,\ 1.0]$.

IV. NUMERICAL EXAMPLE

In this section, the effectiveness of the proposed method is verified with the discrete–time uncertain system given by

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.2 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.15 & 0.1 \end{bmatrix}, \quad E_2 = 1,$$

$$F(k) = 1.0, \quad N(k) = \begin{bmatrix} N_1(k) & 0 \\ 0 & N_2(k) \end{bmatrix},$$

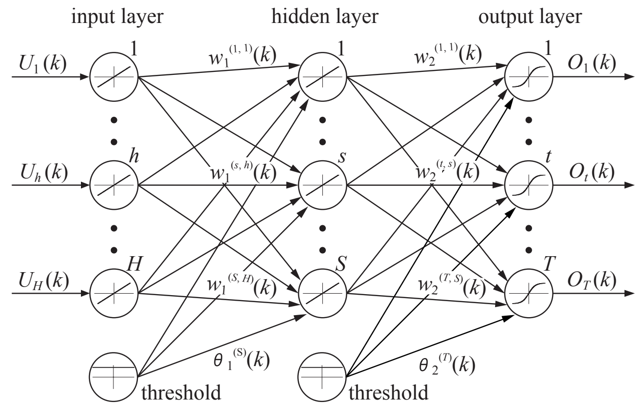

Fig. 2. Structure of the multilayered neural networks.

where $N_1(k)$ and $N_2(k)$ are the outputs of NN. The initial system condition is $x(0) = [4\ \ -4]^T$, and the weighting matrices are chosen as $Q = \text{diag}(1, 2)$ and $R = 1.0$, respectively.

The state feedback control gain K based on the proposed LMI design method with a neurocontroller is given by

$$\begin{aligned} K &= \begin{bmatrix} K_1 & K_2 \end{bmatrix} \\ &= \begin{bmatrix} -4.596 \times 10^{-1} & -7.718 \times 10^{-2} \end{bmatrix}. \end{aligned} \tag{22}$$

Based on the LQR control (see (16) and (17)), the state feedback control gain $\hat{K}$ is calculated as follows:

$$\begin{aligned} \hat{K} &= \begin{bmatrix} \hat{K}_1 & \hat{K}_2 \end{bmatrix} \\ &= \begin{bmatrix} -4.641 \times 10^{-1} & -7.971 \times 10^{-17} \end{bmatrix}. \end{aligned} \tag{23}$$

For the system without the proposed neurocontroller, that is $N(k) \equiv 0$, the control input of the uncertain system is described as

$$u(k) = \bar{K} x(k), \tag{24}$$

where the state feedback control gain $\bar{K}$ is designed based on the LMI approach [4] as

$$\begin{aligned} \bar{K} &= \begin{bmatrix} \bar{K}_1 & \bar{K}_2 \end{bmatrix} \\ &= \begin{bmatrix} -4.289 \times 10^{-1} & -1.423 \times 10^{-1} \end{bmatrix}. \end{aligned} \tag{25}$$

The neurocontroller is composed of 30 neurons in the hidden layer, and two neurons in the input and the output layers, respectively. The state variables are used as the NN' inputs and the learning ratio $\eta = 0.1$. The initial weights are randomly set in the range of $[-0.05, 0.05]$.

The cost J with the gain matrix K is 107.8402, while the cost without the neurocontroller $\bar{J}$ with $\bar{K}$ is 178.0061. Various uncertain systems were examined, by changing $F(k)$ is 0, $\exp(-0.5k)$, and $\sin(0.5\pi k)$. Table I shows that the cost of the proposed system is smaller than that of the system without the neurocontroller in all cases.

The simulation results ($F(k) = 1$) are shown in Fig. 3. The response of the proposed neurocontroller is stabilized faster than that without the neurocontroller (Fig. 3 (a)$\sim$(c)). Fig. 3(d) shows the feedback gain with the additive gain $K + \tilde{K}$, i.e., $K + D_2 N(k) E_2$. As other examples, the simulation results are shown in Figs. 4, 5, 6. The response of the proposed one is also stabilized faster than that of the

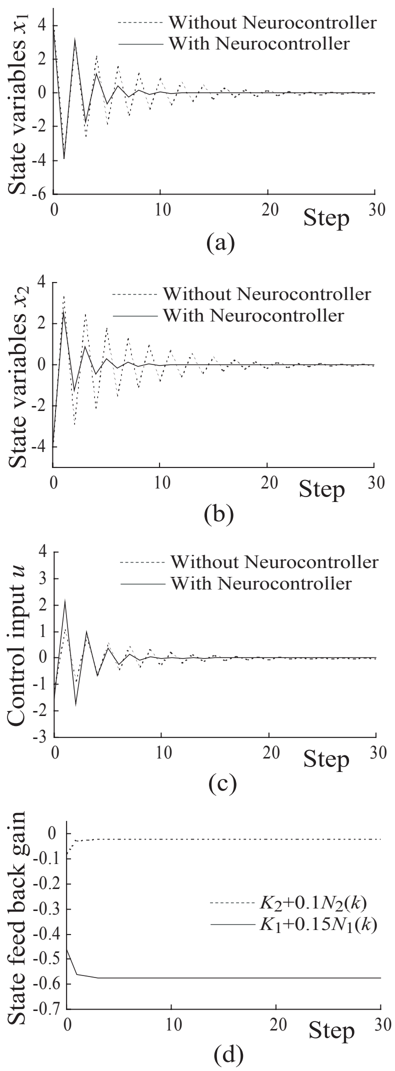

Fig. 3. Simulation results by using the neurocontroller ($F = 1$). (a), (b) State variables. (c) Control input. (d) State feedback gain with additive gain.

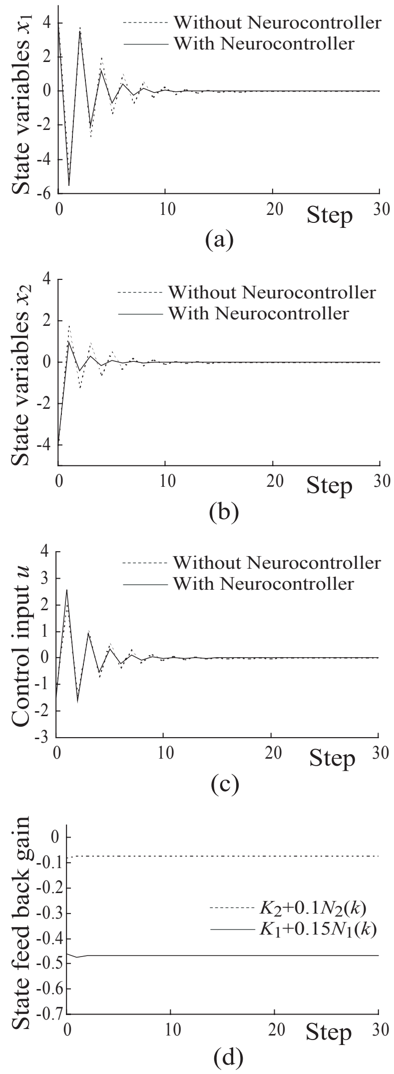

Fig. 4. Simulation results by using the neurocontroller ($F = 0$). (a), (b) State variables. (c) Control input. (d) State feedback gain with additive gain.

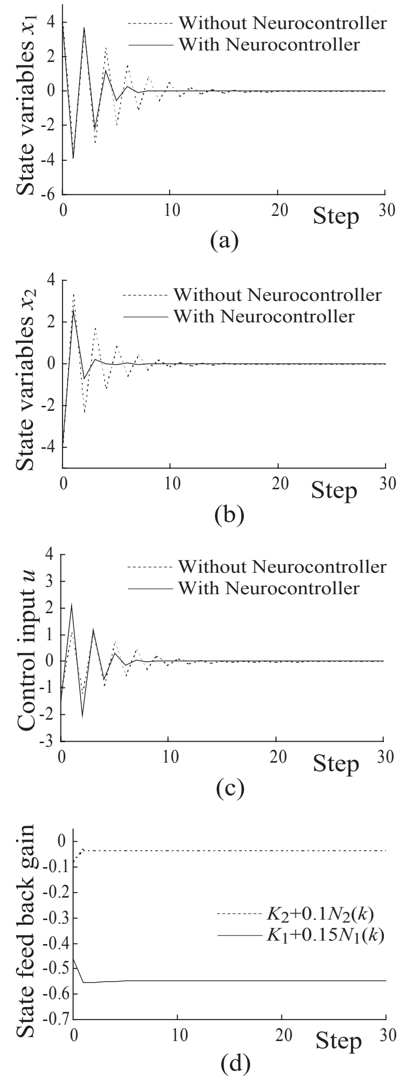

Fig. 5. Simulation results by using the neurocontroller ($F = \exp(-0.5k)$). (a), (b) State variables. (c) Control input. (d) State feedback gain with additive gain.

controller without one. The proposed neurocontroller could reduce the cost and compensate for the uncertainties of the system.

Fig. 7 shows the response with the proposed neurocontroller and LQR control ($F(k) = 1$). The state variables x_1, x_2 can trace the state variables $\hat{x}_1$, $\hat{x}_2$ well as shown in Fig. 7(a), (b). Therefore, the proposed neurocontroller can reduce the cost. Thus, $K + \tilde{K}$ changes in order to compensate for the system uncertainties, and its response can be close to the nominal response via the LQR design method. Therefore, the energy function $E(k)$ is adequate for the learning algorithm.

TABLE I

A COMPARISON OF THE COST IN EACH CONDITION.

The cost of the nominal system $\hat{J}$ = 110.8512

$F(k)$	Learning ratio η	With NN	Without NN
1	0.1	107.8402	178.0061
0	0.05	112.3453	122.9732
$\exp(-0.5k)$	0.09	110.1992	145.8423
$\sin(0.5\pi k)$	0.8	119.8763	150.6293

V. CONCLUSIONS

The application of neural networks to the guaranteed cost control problem of the discrete–time uncertain system has been investigated. Using the LMI technique, the class of the state feedback gain has been derived. Substituting the neurocontroller into the gain perturbations, the robust stability of the closed–loop system is guaranteed even if the systems include NN. Moreover, the reduction of the cost is attained by using neurocontroller. The numerical example have shown the excellent result that the NN have succeeded in reducing the large cost caused by the LMI.

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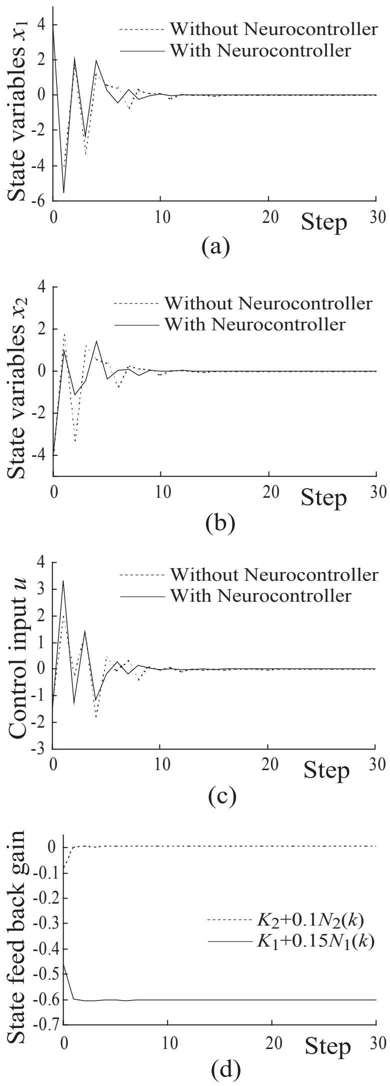

Fig. 6. Simulation results by using the neurocontroller ($F = \sin(0.5\pi k)$). (a), (b) State variables. (c) Control input. (d) State feedback gain with additive gain.

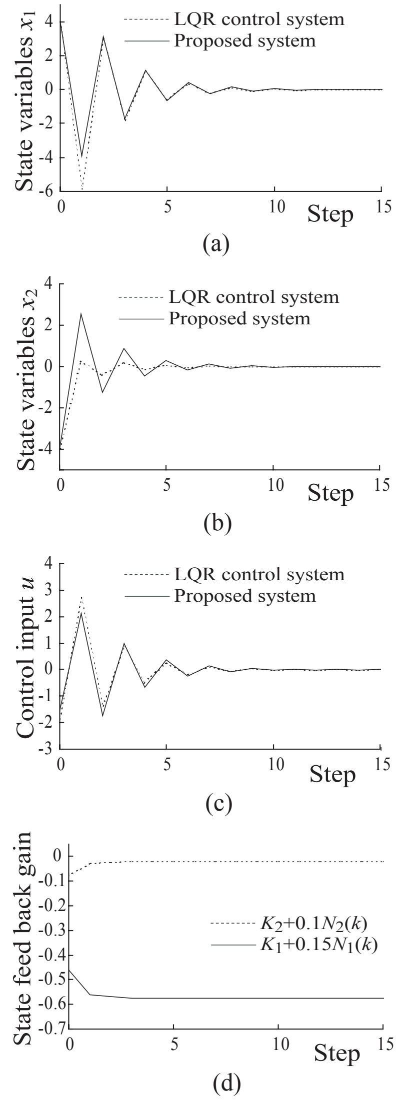

Fig. 7. Simulation results of the contrast between proposed system and LQR control system ($F = 1$). (a), (b) State variables. (c) Control input. (d) State feedback gain with additive gain.

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