

## THE GUARANTEED COST CONTROL FOR UNCERTAIN LARGE-SCALE INTERCONNECTED SYSTEMS

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Abstract: The guaranteed cost control problem of the decentralized robust control for a class of large-scale interconnected systems with norm-bounded time-varying parameter uncertainties is considered. Based on the LMI design approach, a class of decentralized local state feedback controllers is proposed, and some sufficient conditions for the existence of guaranteed cost controllers are derived by making use of the LMI.

Keywords: Large-scale interconnected systems, Decentralized control, Guaranteed cost control, LMI, Parameter uncertainties

### 1. INTRODUCTION

The study of large-scale interconnected systems has received ever greater attention in the past few decades (see, for example, Siljak, 1978 and the references therein). In recent years, the problem of the decentralized robust control of large-scale systems with parameter uncertainties has been widely studied, and some solution approaches have been developed (Wang et al., 1997; Guo et al., 2000; Yan et al., 1998; Wang et al., 1998; Gong et al., 1996; Xie and Xie, 2000; Zhang et al., 1996). In Guo et al. (2000) the decentralized control problem for the uncertain interconnected time-varying systems which do not satisfy the so-called matching conditions has been studied. In Wang et al. (1997) and Guo et al. (2000), a decentralized stabilizing nonlinear state feedback controller has been proposed for the nonlinear multimachine power systems via the algebraic Riccati equation (ARE) approach. Furthermore, in Wang et al. (1998), the results developed in Wang et al. (1997) have been extended to the class of large-scale

interconnected nonlinear systems via the robust decentralized linear control. Compared with the nonlinear controllers, there exists an important feature that the linear controllers are of simpler structure and easier to be implemented. However, in case where we apply the ARE approach, if some additional requirements such as the minimization of the upper bound on the value of the cost function are added, there exists the drawback for solving such specific problems. Furthermore, how to select the optimal multiparameters which are included in the AREs has never been studied.

Although there have been numerous results on decentralized robust control of large-scale uncertain systems, much effort has been made towards finding a controller which guarantees robust stability. However, when controlling such systems, it is also desirable to design the control systems which guarantee not only the robust stability, but also an adequate level of performance. One approach to this problem is the so-called guaranteed cost control approach (Petersen and MacFarlane

1994). This approach has the advantage of providing an upper bound on a given performance index. Recent advance in theory of linear matrix inequality (LMI) has allowed a revisiting of the guaranteed cost control approach. In particular, the guaranteed cost control problem for a class of nonlinear large-scale interconnected systems which is based on the LMI design method was solved (Xie et al., 2000). The LMI design method is a very well-known and powerful tool, it can not only efficiently find feasible and global solutions, but also easily handle various kinds of additional linear constraints. However, so far the problem of guaranteed cost stabilization for the large-scale uncertain linear systems with respect to uncertainties in the controllers themselves has not been discussed.

In this paper, the guaranteed cost control problem (Petersen and MacFarlane 1994) which has received much attention recently of the decentralized robust control for a class of large-scale systems with the norm-bounded parameter uncertainties is considered. After defining the guaranteed cost control problem for the large-scale interconnected uncertain systems, a sufficient condition for the existence of the decentralized robust feedback controllers is derived by making use of the Lyapunov stability criterion such that uncertain large-scale interconnected systems can be asymptotically stabilized. The LMI design approach plays an important role to derive such sufficient conditions. The main contributions of this paper show that the guaranteed cost controllers can be constructed by solving the LMI. It is worth pointing out that the positive scaling parameters which are included in the LMI are chosen such that an upper bound on the quadratic cost performance is optimized. Consequently, the resulting controllers guarantee the adequate upper bound on a given performance. Finally, the problem of guaranteed cost control for large-scale uncertain systems under gain perturbations is considered.

## 2. PROBLEM FORMULATION

Consider a class of large-scale interconnected systems composed of  $N$  interconnected subsystems described by the following state equations:

$$\begin{aligned} \dot{x}_i(t) &= [A_i + \Delta A_i(t)]x_i(t) + [B_i + \Delta B_i(t)]u_i(t) \\ &+ \sum_{j=1, j \neq i}^N G_{ij}g_{ij}(t, x_j), \quad (1) \\ x_i(0) &= x_{i0}, \quad i = 1, 2, \dots, N, \end{aligned}$$

where  $x_i \in \mathbf{R}^{n_i}$  and  $u_i \in \mathbf{R}^{m_i}$  are the state and control of the  $i$ th subsystems, respectively.  $A_i$ ,  $B_i$  and  $G_{ij}$  are constant matrices of appropriate dimensions and  $G_{ij}$  are interconnection matrices

between the  $i$ th subsystems and other subsystems. The unknown vector functions  $g_{ij}(t, x_j) \in \mathbf{R}^{l_i}$  represent interconnections among the subsystems. It is assumed that the unknown vector functions  $g_{ij}(t, x_j)$  are continuous and sufficiently smooth in  $x_j$  and piecewise continuous in  $t$  (Gong et al., 1996). The parameter uncertainties considered here are assumed to be of the following form:

$$[\Delta A_i(t) \quad \Delta B_i(t)] = D_i F_i(t) [E_{1i} \quad E_{2i}], \quad (2)$$

where  $D_i$ ,  $E_{1i}$  and  $E_{2i}$  are known constant real matrices of appropriate dimensions.

$F_i(t) \in \mathbf{R}^{p_i \times q_i}$  are unknown matrix functions with Lebesgue measurable elements and satisfying  $F_i^T(t)F_i(t) \leq I_{q_i}$ , where  $I_l \in \mathbf{R}^{l \times l}$  denote the identity matrices.

We make the following assumptions concerning the unknown vector functions.

**Assumption 1:** There exists known constant matrix  $W_{ij}$  such that for all  $x_j \in \mathbf{R}^{n_j}$

$$\|g_{ij}(t, x_j)\| \leq \|W_{ij}x_j\|, \quad (3)$$

for all  $i, j$  and for all  $t \geq 0$ , where  $\|\cdot\|$  denotes the Euclidean norm.

**Assumption 2:** For all  $i$ ,  $\sum_{j=1}^N W_{ji}^T W_{ji} > 0$ .

**Remark 1:** The assumption 2 is made only for simplification of presentation.

Associated with system (1) is the cost function

$$J = \sum_{i=1}^N \int_0^{\infty} [x_i^T(t)Q_i x_i(t) + u_i^T(t)R_i u_i(t)] dt, \quad (4)$$

where  $Q_i$  and  $R_i$  are given positive definite symmetric matrices.

**Definition:** A control law  $u_i(t) = K_i x_i(t)$  is said to be a quadratic guaranteed cost control with cost matrix  $P_i > 0$  for uncertain large-scale interconnected systems (1) and cost function (4) if the closed-loop systems are quadratically stable and the closed-loop value of the cost function (4) satisfies the bound  $J \leq \bar{J}$  for all admissible uncertainties, that is,

$$\begin{aligned} \sum_{i=1}^N \left( \frac{d}{dt} x_i^T(t) P_i x_i(t) \right. \\ \left. + x_i^T(t) [Q_i + K_i^T R_i K_i] x_i(t) \right) < 0, \quad (5) \end{aligned}$$

for all nonzero  $x_i \in \mathbf{R}^{n_i}$  and all matrices  $F_i(t): F_i^T(t)F_i(t) \leq I_{q_i}$ .

The objective of this paper is to design a decentralized linear time-invariant guaranteed cost control law  $u_i(t) = K_i x_i(t)$ ,  $i = 1, 2, \dots, N$  for the large-scale interconnected systems (1) with uncertainties (2).

### 3. MAIN RESULTS

Now, we present a sufficient condition for existence of the state feedback guaranteed cost control laws for the uncertain system (1).

**Theorem 1:** Under the assumptions 1 and 2, let us consider the large-scale interconnected systems (1) with the uncertainties (2). If there exist symmetric positive definite matrices  $P_i \in \mathbf{R}^{n_i \times n_i}$  such that for all uncertain matrices  $F_i(t)$  the LMI (6) is satisfied, the control laws  $u_i(t) = K_i x_i(t)$ ,  $i = 1, 2, \dots, N$  are the guaranteed cost controller,

$$\Lambda_i = \begin{bmatrix} \Xi_i & P_i G_{i1} & \cdots & P_i G_{iN} \\ G_{i1}^T P_i & -I_{l_i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ G_{iN}^T P_i & 0 & \cdots & -I_{l_i} \end{bmatrix} < 0, \quad (6)$$

where  $\Lambda_i \in \mathbf{R}^{\bar{N} \times \bar{N}}$ ,  $\bar{N} = n_i + (N-1)l_i$  and

$$\begin{aligned} \Xi_i &:= \tilde{A}_i^T P_i + P_i \tilde{A}_i + \sum_{j=1, j \neq i}^N W_{ji}^T W_{ji} + \bar{R}_i, \\ \tilde{A}_i &:= \bar{A} + D_i F_i(t) \bar{E}_i, \quad \bar{A}_i := A_i + B_i K_i, \\ \bar{E}_i &:= E_{1i} + E_{2i} K_i, \quad \bar{R}_i := Q_i + K_i^T R_i K_i. \end{aligned}$$

Furthermore, the corresponding value of the cost function (4) satisfies the following inequality (7) for all admissible uncertainties  $F_i(t)$ .

$$J < \bar{J} = \sum_{i=1}^N x_i^T(0) P_i x_i(0). \quad (7)$$

*Remark 1.* Note that there exists no matrix  $P_i G_{ii}$ ,  $i = 1, \dots, N$  in the matrix  $\Lambda_i$ .

*Proof:* Combining the guaranteed cost controller  $u_i(t) = K_i x_i(t)$  with (1) gives a closed-loop system of the form

$$\dot{x}_i = \tilde{A}_i x_i + \sum_{j=1, j \neq i}^N G_{ij} g_{ij}(t, x_j). \quad (8)$$

Suppose now there exist symmetric positive definite matrices  $P_i > 0$  such that the LMI (6) holds for all admissible uncertainties. In order to prove the asymptotic stability of the closed-loop system (8), let the Lyapunov function candidate

$$V(x(t)) = \sum_{i=1}^N x_i^T(t) P_i x_i(t), \quad (9)$$

where  $x = [x_1^T \ x_2^T \ \cdots \ x_N^T]^T$ . Note that  $V(x) > 0$  whenever  $x \neq 0$ . Then the time derivative of  $V(x)$  along any trajectory of the closed-loop system (8) is given by

$$\begin{aligned} \frac{d}{dt} V(x) &= \sum_{i=1}^N z_i^T \begin{bmatrix} \Xi_i - \bar{R}_i & P_i G_{i1} & \cdots & P_i G_{iN} \\ G_{i1}^T P_i & -I_{l_i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ G_{iN}^T P_i & 0 & \cdots & -I_{l_i} \end{bmatrix} z_i \\ &\quad - \sum_{i=1}^N \sum_{j=1, j \neq i}^N (x_j^T W_{ij}^T W_{ij} x_j - g_{ij}^T g_{ij}) \\ &= \sum_{i=1}^N z_i^T \Lambda_i z_i - \sum_{i=1}^N x_i^T \bar{R}_i x_i \\ &\quad - \sum_{i=1}^N \sum_{j=1, j \neq i}^N (x_j^T W_{ij}^T W_{ij} x_j - g_{ij}^T g_{ij}), \end{aligned}$$

where  $z_i = [x_i^T \ g_{i1}^T \ g_{i2}^T \ \cdots \ g_{iN}^T]^T \in \mathbf{R}^{\bar{N}}$  and  $\Xi_i$  and  $\Lambda_i$  are given in (6). Taking into account the fact that the inequalities (6) hold, and using the Schur complement (Zhou, 1998), it follows immediately that

$$\frac{d}{dt} V(x) < - \sum_{i=1}^N x_i^T \bar{R}_i x_i < 0. \quad (10)$$

Hence,  $V(x)$  is a Lyapunov function for the closed-loop system (8). Therefore, the closed-loop system (8) is asymptotically stable and  $u_i(t) = K_i x_i(t)$  is the guaranteed cost controller because the inequality (5) is satisfied. Furthermore, by integrating both sides of the inequality (10) from 0 to  $T$  and using the initial conditions, we have

$$V(x(T)) - V(x(0)) < - \sum_{i=1}^N \int_0^T x_i^T \bar{R}_i x_i dt.$$

Since the closed-loop system (8) is asymptotically stable, that is,  $x_i(T) \rightarrow 0$ , when  $T \rightarrow \infty$ , we obtain  $V(x(T)) \rightarrow 0$ . Thus we get

$$\begin{aligned} J &= \sum_{i=1}^N \int_0^T x_i^T \bar{R}_i x_i dt < V(x(0)) \\ &= \sum_{i=1}^N x_i^T(0) P_i x_i(0) = \bar{J}. \end{aligned}$$

It follows from the definition that the result of the theorem is true. The proof of Theorem 1 is completed.  $\square$

We now give the LMI design approach to the construction of a guaranteed cost controller.

**Theorem 2:** Under the assumptions 1 and 2, suppose there exist the constant parameters  $\mu_i > 0$

such that for all  $i = 1, 2, \dots, N$  the following LMI (11) have a symmetric positive definite matrix  $X_i > 0 \in \mathbf{R}^{n_i \times n_i}$  and a matrix  $Y_i \in \mathbf{R}^{m_i \times n_i}$

$$\begin{bmatrix} \Phi_i & \tilde{E}_i^T & X_i & Y_i^T & X_i \\ \tilde{E}_i & -\mu_i I_{n_i} & 0 & 0 & 0 \\ X_i & 0 & -Q_i^{-1} & 0 & 0 \\ Y_i & 0 & 0 & -R_i^{-1} & 0 \\ X_i & 0 & 0 & 0 & -U_i^{-1} \end{bmatrix} < 0, \quad (11)$$

where  $\tilde{E}_i := E_{1i}X_i + E_{2i}Y_i$ ,  $U_i := \sum_{j=1, j \neq i}^N W_{ji}^T W_{ji}$ ,

$$\begin{aligned} \Phi_i &:= A_i X_i + B_i Y_i + (A_i X_i + B_i Y_i)^T + \mu_i D_i D_i^T \\ &+ \sum_{j=1, j \neq i}^N G_{ij} G_{ij}^T. \end{aligned}$$

Then, the decentralized linear state feedback control laws

$$u_i(t) = K_i x_i(t) = Y_i X_i^{-1} x_i(t), \quad i = 1, \dots, N, \quad (12)$$

are the guaranteed cost control laws and

$$J < \sum_{i=1}^N x_i^T(0) X_i^{-1} x_i(0) = \bar{J} \quad (13)$$

is the guaranteed cost for the closed-loop uncertain large-scale interconnected systems.

*Proof:* Applying the Schur complement to the LMI (11) gives

$$\begin{aligned} &\Phi_i + \begin{bmatrix} (E_{1i}X_i + E_{2i}Y_i)^T & X_i & Y_i^T & X_i \end{bmatrix} \\ &\begin{bmatrix} \mu_i^{-1} I_{n_i} & 0 & 0 & 0 \\ 0 & Q_i & 0 & 0 \\ 0 & 0 & R_i & 0 \\ 0 & 0 & 0 & U_i \end{bmatrix} \begin{bmatrix} E_{1i}X_i + E_{2i}Y_i \\ X_i \\ Y_i \\ X_i \end{bmatrix} \\ &= A_i X_i + B_i Y_i + (A_i X_i + B_i Y_i)^T + \mu_i D_i D_i^T \\ &+ \sum_{j=1, j \neq i}^N G_{ij} G_{ij}^T \\ &+ \mu_i^{-1} (E_{1i}X_i + E_{2i}Y_i)^T (E_{1i}X_i + E_{2i}Y_i) \\ &+ X_i Q_i X_i + Y_i^T R_i Y_i + X_i U_i X_i < 0. \quad (14) \end{aligned}$$

Let us introduce the matrices  $X_i = P_i^{-1}$  and  $Y_i = K_i P_i^{-1}$ . Substituting these matrices into the above inequality (14) and pre- and post-multiplying both sides of the inequality (14) by  $P_i$  yield

$$\begin{aligned} \Gamma_i &:= \bar{A}_i^T P_i + P_i \bar{A}_i + \mu_i P_i D_i D_i^T P_i \\ &+ \mu_i^{-1} \bar{E}_i^T \bar{E}_i + P_i \left( \sum_{j=1, j \neq i}^N G_{ij} G_{ij}^T \right) P_i \\ &+ \sum_{j=1, j \neq i}^N W_{ji}^T W_{ji} + \bar{R}_i < 0. \quad (15) \end{aligned}$$

We will use these inequalities in order to establish (6). For any admissible uncertainties (2), it follows from (15) and a standard matrix inequality (Petersen and MacFarlane 1994) that

$$\begin{aligned} &\tilde{A}_i^T P_i + P_i \tilde{A}_i + P_i \left( \sum_{j=1, j \neq i}^N G_{ij} G_{ij}^T \right) P_i \\ &+ \sum_{j=1, j \neq i}^N W_{ji}^T W_{ji} + \bar{R}_i \leq \Gamma_i. \end{aligned}$$

On the other hand, by applying the Schur complement, it is easy to verify that the above inequality is equivalent to (6).  $\square$

Since the LMI (11) consists of a convex solution set of  $(\mu_i, X_i, Y_i)$ , various efficient convex optimization algorithm can be applied. Moreover, its solutions parameterize the set of the guaranteed cost controllers. This parameterized representation can be exploited to design the guaranteed cost controllers which minimizes the value of the guaranteed cost for the closed-loop uncertain large-scale systems. Consequently, solving the following optimization problem allows us to determine the optimal bound.

$$\Sigma_0 : \min_{\mathcal{X}_i} \sum_{i=1}^N \alpha_i = J^*, \quad \mathcal{X}_i \in (\mu_i, X_i, Y_i, \alpha_i) \quad (16)$$

such that (11) and

$$\begin{bmatrix} -\alpha_i & x_i^T(0) \\ x_i(0) & -X_i \end{bmatrix} < 0. \quad (17)$$

That is, the problem addressed in this paper is as follows: ‘‘Find  $K_i = Y_i X_i^{-1}$ ,  $i = 1, 2, \dots, N$  such that LMI (11) and (17) are satisfied and the cost  $\bar{J} < \sum_{i=1}^N \alpha_i$  becomes as small as possible.’’

Thus, the minimization of  $\bar{J}$  implies the minimization of the guaranteed cost for the uncertain large-scale systems (1). Finally, we are in a position to establish the main result of this section.

**Theorem 3:** If the above optimization problem has the solution  $\mu_i, X_i, Y_i$  and  $\alpha_i$ , then the control laws of the form (12) are the decentralized linear optimal state feedback control laws which ensure the minimization of the guaranteed cost  $\bar{J}$  for the uncertain large-scale interconnected systems.

*Proof:* By Theorem 2, the control laws (12) constructed from the feasible solutions  $\mu_i, X_i, Y_i$  and  $\alpha_i$  are the guaranteed cost controllers of the uncertain large-scale interconnected systems (1). Using the Schur complement to the LMI (17), we have

$$\begin{bmatrix} -\alpha_i & x_i^T(0) \\ x_i(0) & -X_i \end{bmatrix} < 0 \Leftrightarrow x_i^T(0)X_i^{-1}x_i(0) < \alpha_i.$$

It follows that

$$J < \sum_{i=1}^N x_i^T(0)X_i^{-1}x_i(0) < \min_{\mathcal{X}_i} \sum_{i=1}^N \alpha_i = J^*.$$

Thus, the minimization of  $\sum_{i=1}^N \alpha_i$  implies the minimization of the guaranteed cost  $\bar{J}$  for the inter-connected uncertain systems (1). The optimality of the solution of the optimization problem follows from the convexity of the objective function under the LMI constraints. This is the required result.  $\square$

**Remark 2:** It can be noted that the bound obtained in Theorem 3 depends on the initial condition  $x_i(0)$ . To remove this dependence on  $x_i(0)$ , we assume that  $x_i(0)$  is a zero mean random variable satisfying  $E[x_i(0)x_i(0)^T] = I_{n_i}$ , where  $E[\cdot]$  denotes the expectation. In this case, it should be pointed out that the guaranteed cost becomes

$$E[J] = \sum_{i=1}^N E[x_i^T(0)X_i^{-1}x_i(0)] = \sum_{i=1}^N \text{Trace } X_i^{-1} < \sum_{i=1}^N \text{Trace } \mathcal{A}_i \text{ such that (11) and}$$

$$\begin{bmatrix} -\mathcal{A}_i & I_{n_i} \\ I_{n_i} & -X_i \end{bmatrix} < 0. \quad (18)$$

Furthermore, it is worth pointing out that the original optimization problem for the guaranteed cost (16) can be decomposed to the reduced optimization problems (19) because each optimization problem (19) is independent of other LMI. Hence, we have only to solve the optimization problems (19) for each independent subsystem.

$$\Sigma_i : \min_{\mathcal{Y}_i} \text{Trace } \mathcal{A}_i, \mathcal{Y}_i \in (\mu_i, X_i, Y_i, \mathcal{A}_i). \quad (19)$$

#### 4. GUARANTEED COST CONTROL UNDER ADDITIVE GAIN PERTURBATIONS

Consider the class of large-scale interconnected systems (1), where  $E_{2i} \equiv 0$ . For a given controllers  $u_i(t) = K_i x_i(t)$ , the actual controller implemented is assumed to be  $u_i(t) = [K_i + H_i F_i(t) E_i^k] x_i(t)$ , where  $K_i$  is the nominal controller gain, and  $H_i F_i(t) E_i^k$  represents the gain perturbations. In fact, the controller gain perturbations can result from the actuator degradations, as well as from the requirement for re-adjustment of controller gains during the controller implementation stage (see e.g. Yang et al., 2000).

By using the similar algebraic technique used in Theorem 2, we give the following theorem.

**Theorem 4:** Under the assumptions 1 and 2, suppose there exist the constant parameters  $\mu_i > 0$  such that for all  $i = 1, 2, \dots, N$  the following LMI (20) have a symmetric positive definite matrix  $X_i > 0 \in \mathbf{R}^{n_i \times n_i}$  and a matrix  $Y_i \in \mathbf{R}^{m_i \times n_i}$

$$\begin{bmatrix} \Psi_i & X_i & X_i & Y_i^T + B_i \tilde{H}_i^\mu & X_i \hat{E}_i^T \\ X_i & -Q_i^{-1} & 0 & 0 & 0 \\ X_i & 0 & -U_i^{-1} & 0 & 0 \\ Y_i + \tilde{H}_i^\mu B_i^T & 0 & 0 & -R_i^{-1} + \tilde{H}_i^\mu & 0 \\ \hat{E}_i X_i & 0 & 0 & 0 & -\mu_i I_{n_i} \end{bmatrix} < 0, \quad (20)$$

where  $\hat{E}_i := [E_{1i}^T \ E_i^{kT}]^T$ ,  $\tilde{H}_i^\mu := \mu_i H_i H_i^T$

$$\Psi_i := A_i X_i + B_i Y_i + (A_i X_i + B_i Y_i)^T + \mu_i (D_i D_i^T + B_i H_i H_i^T B_i^T) + \sum_{j=1, j \neq i}^N G_{ij} G_{ij}^T$$

$$U_i := \sum_{j=1, j \neq i}^N W_{ji}^T W_{ji} > 0.$$

Then, the decentralized linear state feedback control laws (12) are the guaranteed cost control laws and the cost bounds (13) is the guaranteed cost for the closed-loop uncertain large-scale interconnected systems.

*Proof:* Since it is easy to prove this theorem by applying the Schur complement and the standard matrix inequality (Petersen and MacFarlane 1994), the proof is omitted.  $\square$

#### 5. NUMERICAL EXAMPLE

In order to demonstrate the efficiency of our proposed control, we have run a simple numerical examples. Consider the interconnected uncertain systems (1) composed of three two-dimensional subsystems. The system matrices and the unknown functions with the uncertainties are given as follows.

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \\ -1 & 0.01 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ G_{12} &= \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, G_{13} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, D_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \\ G_{23} &= \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, G_{21} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, x_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, D_3 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \\ G_{31} &= \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, G_{32} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, x_3 = \begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix}, \\ E_{11} &= E_{12} = E_{13} = [0 \ 0.1], \\ E_{21} &= E_{22} = E_{23} = [0.01], \\ g_{1j} &= [0.1 + \delta_1(t)] \sin [1 \ 0] x_j, \quad j = 2, 3, \\ g_{2j} &= [0.1 + \delta_2(t)] \sin [1 \ 0] x_j, \quad j = 3, 1, \\ g_{3j} &= [0.1 + \delta_3(t)] \sin [1 \ 0] x_j, \quad j = 1, 2, \end{aligned}$$

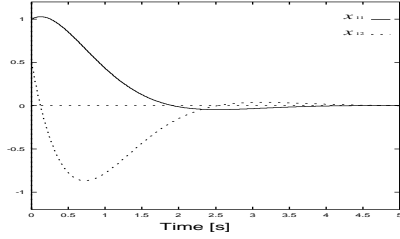


Fig. 1. Response of the closed-loop system 1.

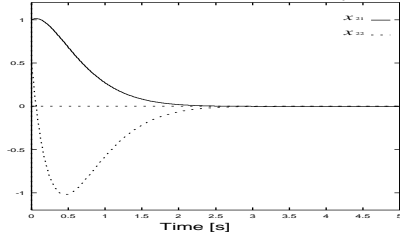


Fig. 2. Response of the closed-loop system 2.

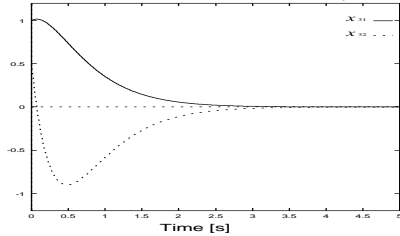


Fig. 3. Response of the closed-loop system 3.

$$|\delta_i(t)| \leq 0.1, \quad i = 1, 2, 3,$$

In that case the unknown functions  $g_{ij}(t, x_j)$  satisfy  $\|g_{ij}(t, x_j)\| \leq 0.2\|\sin[1 \ 0]x_j\| \leq 0.2\|x_j\|$ . Therefore, we choose as  $W_{12} = W_{13} = W_{23} = W_{21} = W_{31} = W_{32} = 0.2I_2$ . The uncertain parameters  $F_i(t)$ ,  $i = 1, 2, 3$  are time-variant and satisfy  $\|F_i(t)\| \leq 1$ . Now, we choose as  $R_i = 1$  and  $Q_i = \text{diag}[10.0 \ 1.0]$ ,  $i = 1, 2, 3$ . By applying Theorem 3 and solving the corresponding optimization problem (19), we obtain the decentralized linear optimal state feedback control laws

$$\begin{aligned} K_1 &= [-2.4071 \ -2.6079], \\ K_2 &= [-2.4177 \ -9.4040 \times 10^{-1}], \\ K_3 &= [-1.0667 \times 10 \ -9.7170]. \end{aligned}$$

Consequently, the guaranteed cost of the uncertain closed-loop system is  $J^* = 66.71748$ , where  $\min_{\mathcal{Y}_1} \text{Trace } \mathcal{A}_1 = 10.857576$ ,  $\min_{\mathcal{Y}_2} \text{Trace } \mathcal{A}_2 = 7.068615$  and  $\min_{\mathcal{Y}_3} \text{Trace } \mathcal{A}_3 = 48.791289$ . Finally, the results of the simulation of this example are depicted in Fig. 1–3. The initial state is set as  $x_i(0) = [1 \ 0.5]^T$ ,  $i = 1, 2, 3$ . It is shown from Fig. 1–3 that the closed-loop system is asymptotically stable.

## 6. CONCLUSIONS

In this paper, a solution of the guaranteed cost control problem for large-scale interconnected

systems with the norm-bounded parameter uncertainties has been presented. The decentralized robust optimal guaranteed cost controller which minimizes the value of the guaranteed cost for the closed-loop uncertain large-scale systems can be solved by using software such as MATLAB's LMI control Toolbox. Thus, the resulting decentralized linear feedback controller can guarantee the quadratic stability and the optimal cost bound for the uncertain large-scale systems.

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