2006 IEEE International Conference on Fuzzy Systems
Sheraton Vancouver Wall Centre Hotel, Vancouver, BC, Canada
July 16-21, 2006

Decentralized Guaranteed Cost Control for Discrete-Time Uncertain Large-Scale Systems Using Fuzzy Control

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***Abstract*— This paper investigates an application of fuzzy control to the guaranteed cost control problem of decentralized robust control for a class of discrete-time uncertain large-scale systems. Based on Linear Matrix Inequality (LMI) design approach, a class of decentralized local fixed state feedback controllers with additive gain perturbations is established. The novel contribution of this paper is that in order to reduce the large cost caused by the LMI conditions, the fuzzy controllers are substituted for the additive gain perturbations. Although the fuzzy controllers are included in the uncertain large-scale systems, the closed-loop system is asymptotically stable. As another important feature, the control input matrices allow uncertainty and the conservative assumption is not needed compared with the existing result that is based on the neural networks. In order to demonstrate the efficiency of our proposed controller, the simple numerical example is given.**

I. Introduction

Large-scale interconnected systems are generally met in our modern society, such as transportation systems, power systems, communication network systems, economic systems, and so on. Such systems are generally characterized by a large number of variables representing the system, a strong interaction between the system variables, and a complex structure. The study of large-scale interconnected systems has received ever greater attention in the past few decades (see, for example, [1] and the references therein).

In recent years, the problem of the decentralized robust control of large-scale systems with parameter uncertainties has been widely studied, and some solution approaches have been developed. In [2], for the nonlinear multimachine power systems a decentralized stabilizing linear state feedback controller has been proposed by using the algebraic Riccati equation (ARE) approach. Furthermore, in [3], the results developed in [2] have been extended to the class of large-scale interconnected linear systems via the linear matrix inequality (LMI). Although there have been numerous useful results on decentralized robust control of uncertain large-scale systems, much effort has been made towards finding a controller that guarantees robust stability. However, when controlling such systems, it is also desirable to design the control systems which guarantee not only the robust stability, but also an adequate level of performance. One approach to this problem is the so-called guaranteed cost control approach [4]. This approach has the advantage of providing an upper bound on a given performance index.

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Recent advance in theory of LMI has allowed a revisiting of the guaranteed cost control approach [5]. The existence of the guaranteed cost control problem for a class of the interconnected systems have been established via the LMI conditions [6]. The LMI design method is a very powerful tool, it can not only efficiently find feasible and global solutions, but also easily handle various kinds of additional linear constraints. However, due to the presence of the design parameter of the LMI, it is well known that the cost performance becomes quite large. In order to avoid this drawback, the stability of the closed-loop system with the neurocontroller have been studied via the LMI-based design approach [9], [10]. However, in these researches, the conservative matching conditions for the neurocontroller have been made. Moreover, there are no uncertainties for the input matrix. Thus, it is not applicable to a wider class of the problems.

The fuzzy controller has been utilized for an intelligent control system. The fuzzy controller can decide the control algorithm without the strict mathematical model. As an example, the controller that guarantees the nonlinearity of the robot manipulator by means of fewer fuzzy rules has been proposed [7]. Using the fuzzy control, a design of the linear quadratic state feedback controller for the nonlinearity system has been proposed [8]. However, there is a possibility that fuzzy controller can not stabilize the system because the stability of the closed-loop system which includes the fuzzy controller has not been considered. For example, even if the fuzzy control is applied, the system stability may not be guaranteed without the consideration for the stability of the overall systems when the degree of system nonlinearity is strong.

In this paper, the decentralized guaranteed cost control problem of the discrete-time uncertain large-scale systems with the fuzzy control is discussed. The crucial difference between the method in [7] and the proposed method is that the decomposition of the optimization based on the LMI is newly considered and the fuzzy control is substituted for the additive gain perturbations. Our contributions are as follows. Firstly, a class of the fixed state feedback controller of the discrete-time uncertain large-scale systems with the gain perturbations is derived. Secondly, some sufficient conditions to design the decentralized guaranteed cost controller are newly established by means of the LMI. Finally, in order to reduce the large cost caused by the parameter uncertainties, fuzzy control are used. As a result, although the fuzzy control are included in the uncertain large-scale systems, it is newly shown that the robust stability of the closed-loop system and the reduction of the cost are both attained. As another important feature, the control input matrices allow uncertainty and the conservative assumption is not needed compared with the existing result that is based on the neural networks [10]. Thus, the proposed design method can be applied to the wide class for the uncertain large-scale systems. Finally, in order to verify the effectiveness of our design approach, the numerical example is given.

II. Problem Statement

Consider a class of large-scale interconnected systems composed of N subsystems described by the following state equations

$$x_i(k+1) = [A_i + \Delta A_i(k)]\, x_i(k) + [B_i + \Delta B_i(k)]\, u_i(k) + \sum_{j=1,\ j\neq i}^{N} [A_{ij} + \Delta A_{ij}(k)]\, x_j(k), \quad \text{(1a)}$$

$$u_i(k) = [K_i + \Delta K_i(k)]\, x_i(k), \quad \text{(1b)}$$

where $x_i \in \mathbf{R}^{n_i}$ and $u_i \in \mathbf{R}^{m_i}$ are the state and control of the ith subsystems, respectively. A_i, B_i and A_{ij} are constant matrices of appropriate dimensions. The parameter uncertainties considered here are assumed to be of the following form:

$$\begin{bmatrix} \Delta A_i(t) & \Delta B_i(t) & \Delta A_{ij}(t) \end{bmatrix} = \begin{bmatrix} D_{ai}F_{ai}(k) \begin{bmatrix} E_{ai} & E_{bi} \end{bmatrix} & D_{aij}F_{aij}(k)E_{aij} \end{bmatrix}, \quad (2)$$

where $F_{ai}(k) \in \mathbf{R}^{p_i \times r_i}$ and $F_{aij}(k) \in \mathbf{R}^{p_{ij} \times r_{ij}}$ are unknown matrix functions with Lebesgue measurable elements and satisfying

$$F_{ai}^T(k)F_{ai}(k) \le I_{r_i},\ F_{aij}^T(k)F_{aij}(k) \le I_{r_{ij}}.$$

Moreover, the matrix K_i of the gain matrix (1b) is the fixed gain that will be solved via the LMI, later. It is assumed that $\Delta K_i(k)$ has the following form

$$\Delta K_i(k) = D_{ki}F_{ki}(k)E_{ki}, \quad (3)$$

where $F_{ki}(k) \in \mathbf{R}^{q_{ki} \times s_{ki}}$ is the output of the fuzzy control satisfying

$$F_{ki}^T(k)F_{ki}(k) \le I_{s_{ki}}.$$

It should be noted that $\Delta K_i(k)$ is so-called additive gain perturbation [11]. Associated with system (1) is the cost function

$$J = \sum_{i=1}^{N}\sum_{k=0}^{\infty}\Big[x_i^T(k)Q_ix_i(k) + u_i^T(k)R_iu_i(k)\Big], \quad (4)$$

where Q_i and R_i are given as the positive definite symmetric matrices.

Definition 1: A decentralized control law $u_i(k) = [K_i + \Delta K_i(k)]\, x_i(k),\ i = 1,\ \dots, N$ is said to be a quadratic guaranteed cost control with cost matrix $P_i > 0$ for the uncertain large-scale interconnected systems (1a) and the cost function (4) if the closed-loop systems are quadratically stable and for some positive constant $\mathcal{J}$, the closed-loop value of the cost function (4) satisfies the bound $J \le \mathcal{J}$ for all admissible uncertainties, that is,

$$\sum_{i=1}^{N}\Bigg(x_i^T(k+1)P_ix_i(k+1) - x_i^T(k)P_ix_i(k) + \big[x_i^T(k)Q_ix_i(k) + u_i^T(k)R_iu_i(k)\big]\Bigg) < 0. \quad (5)$$

It should be noted that if the matrix inequality (5) holds, the closed-loop system is asymptotically stable and there exists an upper bound on the cost performance.

The objective of this paper is to design a decentralized linear time-variant guaranteed cost control law $u_i(k) = [K_i + \Delta K_i(k)]\, x_i(k)$ for the large-scale interconnected systems (1) with uncertainties (2) and fuzzy control input (3).

III. Preliminary Results

Now, a sufficient condition for existence of the guaranteed cost control for the uncertain large-scale systems (1) is established.

Theorem 1: Consider the large-scale interconnected systems (1) with the uncertainties (2) and the fuzzy control input (3). If there exist the symmetric positive definite matrices $P_i \in \mathbf{R}^{n_i \times n_i}$ such that the matrix inequality (6) is satisfied, the control laws $u_i(k) = [K_i + \Delta K_i(k)]\, x_i(k),\ i = 1, \cdots, N$ are the guaranteed cost controller.

$$\mathcal{M}_i := \begin{bmatrix} \tilde{A}_i^T P_i \tilde{A}_i + \Theta_i & \tilde{A}_i^T P_i \tilde{A}_{i1} & \cdots & \tilde{A}_i^T P_i \tilde{A}_{iN} \\ \tilde{A}_{i1}^T P_i \tilde{A}_i & \tilde{A}_{i1}^T P_i \tilde{A}_{i1} - I_{n_1} & \cdots & \tilde{A}_{i1}^T P_i \tilde{A}_{iN} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{iN}^T P_i \tilde{A}_i & \tilde{A}_{iN}^T P_i \tilde{A}_{i1} & \cdots & \tilde{A}_{iN}^T P_i \tilde{A}_{iN} - I_{n_N} \end{bmatrix} < 0, \quad (6)$$

where

$$\begin{aligned} \tilde{A}_i &= A_i + B_i\,[K_i + \Delta K_i(k)]\,, \\ &\quad + \Delta A_i(k) + \Delta B_i(k)\,[K_i + \Delta K_i(k)]\,, \\ \tilde{A}_{ij} &= A_{ij} + \Delta A_{ij}(k), \\ \Theta_i &= -P_i + (N-1)I_{n_i} + \tilde{Q}_i, \\ \tilde{Q}_i &= Q_i + [K_i + \Delta K_i(k)]^T R_i\,[K_i + \Delta K_i(k)]\,. \end{aligned}$$

Moreover, there exists no matrix $\tilde{A}_{i1}^T P_i \tilde{A}_{i1} - I_{n_i},\ i = 1,\ \dots, N$ in the matrix $\mathcal{M}_i$.

Proof: Combining the guaranteed cost controller $u_i(k) = [K_i + \Delta K_i(k)]\, x_i(k)$ with (1) gives a closed-loop system (7).

$$x_i(k+1) = \tilde{A}_i x_i(k) + \sum_{j=1,\ j\neq i}^{N} \tilde{A}_{ij} x_j(k). \quad (7)$$

Suppose now there exist the symmetric positive definite matrices $P_i > 0,\ i = 1,\ \dots, N$ such that the matrix inequality (6) holds for all admissible uncertainties (2) and the fuzzy control input (3). In order to prove the asymptotic

$$
\Delta V(x(k)) = \sum_{i=1}^{N} \begin{bmatrix} x_i(k) \\ z(k) \end{bmatrix}^T \begin{bmatrix} \tilde{A}_i^T P_i \tilde{A}_i - P_i + (N+1) I_{n_i} & \tilde{A}_i^T P_i \tilde{A}_{i1} & \tilde{A}_i^T P_i \tilde{A}_{i2} & \cdots & \tilde{A}_i^T P_i \tilde{A}_{iN} \\ \tilde{A}_{i1}^T P_i \tilde{A}_i & \tilde{A}_{i1}^T P_i \tilde{A}_{i1} - I_{n_i} & \tilde{A}_{i1}^T P_i \tilde{A}_{i2} & \cdots & \tilde{A}_{i1}^T P_i \tilde{A}_{iN} \\ \tilde{A}_{i2}^T P_i \tilde{A}_i & \tilde{A}_{i2}^T P_i \tilde{A}_{i1} & \tilde{A}_{i2}^T P_i \tilde{A}_{i2} - I_{n_i} & \cdots & \tilde{A}_{i2}^T P_i \tilde{A}_{iN} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{iN}^T P_i \tilde{A}_i & \tilde{A}_{iN}^T P_i \tilde{A}_{i1} & \tilde{A}_{iN}^T P_i \tilde{A}_{i2} & \cdots & \tilde{A}_{iN}^T P_i \tilde{A}_{iN} - I_{n_i} \end{bmatrix} \begin{bmatrix} x_i(k) \\ z(k) \end{bmatrix}. \quad (11)
$$

stability of the closed-loop system (7), let us define the following Lyapunov function candidate

$$
V(x(k)) = \sum_{i=1}^{N} x_i^T(k) P_i x_i(k). \quad (8)
$$

The corresponding difference along any trajectory of the closed-loop system (7) is given by

$$
\begin{aligned}
&\Delta V(x(k)) := V(x(k+1)) - V(x(k)) \\
&= \sum_{i=1}^{N} \begin{bmatrix} x_i(k) \\ z(k) \end{bmatrix}^T \\
&\times \begin{bmatrix} \tilde{A}_i^T P_i \tilde{A}_i - P_i & \tilde{A}_i^T P_i \tilde{A}_{i1} & \tilde{A}_i^T P_i \tilde{A}_{i2} & \cdots & \tilde{A}_i^T P_i \tilde{A}_{iN} \\ \tilde{A}_{i1}^T P_i \tilde{A}_i & \tilde{A}_{i1}^T P_i \tilde{A}_{i1} & \tilde{A}_{i1}^T P_i \tilde{A}_{i2} & \cdots & \tilde{A}_{i1}^T P_i \tilde{A}_{iN} \\ \tilde{A}_{i2}^T P_i \tilde{A}_i & \tilde{A}_{i2}^T P_i \tilde{A}_{i1} & \tilde{A}_{i2}^T P_i \tilde{A}_{i2} & \cdots & \tilde{A}_{i2}^T P_i \tilde{A}_{iN} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{iN}^T P_i \tilde{A}_i & \tilde{A}_{iN}^T P_i \tilde{A}_{i1} & \tilde{A}_{iN}^T P_i \tilde{A}_{i2} & \cdots & \tilde{A}_{iN}^T P_i \tilde{A}_{iN} \end{bmatrix} \\
&\times \begin{bmatrix} x_i(k) \\ z(k) \end{bmatrix}, \quad (9)
\end{aligned}
$$

where

$$
z(k) := \begin{bmatrix} x_1^T(k) & x_2^T(k) & \cdots & x_N^T(k) \end{bmatrix}^T.
$$

Taking into account the fact that

$$
\begin{aligned}
&\sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} x_j^T(k) x_j(k) - \sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} x_i^T(k) x_i(k) \\
&= 0, \quad (10)
\end{aligned}
$$

and summing this equality (10) to (9) results in (11). Moreover, using the assumption (6) and $\tilde{Q}_i > 0$, it follows immediately that

$$
\Delta V(x(k)) < -\sum_{i=1}^{N} x_i^T(k) \tilde{Q}_i x_i(k) < 0. \quad (12)
$$

Hence, $V(x(t))$ is a Lyapunov function for the closed-loop system (7). Therefore, the closed-loop system (7) is asymptotically stable and $u_i(t)$ is the guaranteed cost controller because the inequality (5) is satisfied. Furthermore, summing both sides of the inequality (9) from 0 to ∞ and using the initial conditions, the following inequality holds.

$$
\begin{aligned}
\sum_{k=0}^{\infty} \Delta V(x(k)) &= \sum_{k=0}^{\infty} [V(x(k+1)) - V(x(k))] \\
&= \sum_{i=1}^{N} \left[x_i^T(\infty) P_i x_i(\infty) - x_i^T(0) P_i x_i(0) \right] \\
&< -\sum_{i=1}^{N} \sum_{k=0}^{\infty} x_i^T(k) \tilde{Q}_i x_i(k) = -J. \quad (13)
\end{aligned}
$$

Since the closed-loop system (7) is asymptotically stable, $x_i(\infty) = 0\ i = 1,\ \ldots,\ N$. Finally, the following inequality holds.

$$
J = \sum_{i=1}^{N} \sum_{k=0}^{\infty} x_i^T(k) \tilde{Q}_i x_i(k) < \sum_{i=1}^{N} x_i^T(0) P_i x_i(0) = \mathcal{J}. \quad (14)
$$

The proof of Theorem 1 is completed. ■

Now, the LMI design approach to the construction of the guaranteed cost controller is given.

Theorem 2: Suppose there exists the constant parameters $\varepsilon_{ai} > 0$, $\varepsilon_{aij} > 0$ such that for all $i = 1,\ \ldots,\ N$ the LMI (15) have the symmetric positive definite matrices $X_i > 0 \in \mathbf{R}^{n_i \times n_i}$ and a matrix $Y_i \in \mathbf{R}^{m_i \times n_i}$, where

$$
\Xi_i = \varepsilon_{ai} D_{ai} D_{ai}^T + \sum_{j=1,\ j\neq i}^{N} \varepsilon_{aij} D_{aij} D_{aij}^T,
$$

and note that the matrix $\begin{bmatrix} -I_{n_i} & E_{aii}^T \\ E_{aii} & -\varepsilon_{aii} I_{r_{ii}} \end{bmatrix}$ is not included in $\mathcal{N}_i$.

If such conditions are met, the decentralized linear state feedback control laws (16)

$$
\begin{aligned}
u_i(k) &= [K_i + \Delta K_i(k)]\, x_i(k) \\
&= \left[Y_i X_i^{-1} + \Delta K_i(k) \right] x_i(k) \quad (16)
\end{aligned}
$$

are the guaranteed cost controllers with the additive fuzzy control input $\Delta K_i(k) = D_{ki} F_{ki}(k) E_{ki}$, where $K_i := Y_i X_i^{-1}$ is the fixed matrix gain. Moreover, the cost bound satisfies (17).

$$
J < \sum_{i=1}^{N} x_i^T(0) X_i^{-1} x_i(0). \quad (17)
$$

In order to prove Theorem 2, the following Lemma will be used [5].

$$
\mathcal{N}_i := \begin{bmatrix}
-X_i & X_iE_{ki}^T & X_i & X_i & Y_i^T & 0 & 0 & \cdots & 0 & 0 & \Lambda^T & 0 & \Psi^T \\
E_{ki}X_i & -I_{s_{ki}} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
X_i & 0 & -\dfrac{1}{N-1}I_{n_i} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
X_i & 0 & 0 & -Q_i^{-1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
Y_i & 0 & 0 & 0 & -R_i^{-1}+D_{ki}D_{ki}^T & 0 & 0 & \cdots & 0 & 0 & D_{ki}D_{ki}^TB_i^T & 0 & D_{ki}D_{ki}^TE_{bi}^T \\
0 & 0 & 0 & 0 & 0 & -I_{n_1} & E_{ai1}^T & \cdots & 0 & 0 & A_{i1}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & E_{ai1} & -\varepsilon_{ai1}I_{r_{i1}} & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -I_{n_N} & E_{aiN}^T & A_{iN}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & E_{aiN} & -\varepsilon_{aiN}I_{r_{iN}} & 0 & 0 & 0 \\
\Lambda & 0 & 0 & 0 & B_iD_{ki}D_{ki}^T & A_{i1} & 0 & \cdots & A_{iN} & 0 & -X_i+\Xi_i & B_iD_{ki} & D_{ki}^TE_{bi}^T \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & D_{ki}^TB_i^T & -I_{q_{ki}} & 0 \\
\Psi & 0 & 0 & 0 & E_{bi}D_{ki}D_{ki}^T & 0 & 0 & \cdots & 0 & 0 & E_{bi}D_{ki} & 0 & -\varepsilon_{ai}I_{r_i}
\end{bmatrix} < 0. \qquad (15)
$$

where $\Lambda := A_iX_i + B_iY_i$, $\Psi := E_{ai}X_i + E_{bi}Y_i$.

Lemma 1: Let G, H and F be real matrices of appropriate dimensions with $\mathtt{F}\mathtt{F}^T \le I_n$. Then, for any given $\varphi > 0$, the following inequality holds.

$$\mathtt{GFH} + (\mathtt{GFH})^T \le \varphi \mathtt{G}\mathtt{G}^T + \varphi^{-1}\mathtt{H}^T\mathtt{H}. \qquad (18)$$

Proof: Let us introduce the matrices $X_i = P_i^{-1}$ and $Y_i = K_iP_i^{-1}$. Pre- and post-multiplying both sides of the LMI (15) by

$$\mathcal{T}_i := \textbf{block diag}\begin{bmatrix} P_i & I_{s_{ki}} & I_{n_i} & I_{n_i} & I_{m_i} \\ I_{n_i} & I_{r_{i1}} & \cdots & I_{n_N} & I_{r_{iN}} & I_{n_i} & I_{q_{ki}} & I_{r_i} \end{bmatrix}$$

yields (19). Applying the Schur complement [12] to the matrix inequality (19) gives (20). Using Lemma 1 for all admissible uncertainties (2) and the fuzzy control input (3) and after a direct but tedious matrix manipulation, the matrix inequality (6) holds. On the other hand, since the results of the cost bound (17) can be proved by using the similar argument for the proof of Theorem 1, it is omitted. ■

Since the LMI (15) consists of a convex solution set of $\mathcal{X}_i \in (\varepsilon_{ai},\ \varepsilon_{ai1},\ \dots,\varepsilon_{aiN},\ X_i,\ Y_i)$, various efficient convex optimization algorithms can be applied. Moreover, its solutions represent the set of the guaranteed cost controllers. This parameterized representation can be exploited to design the guaranteed cost controllers which minimizes the value of the guaranteed cost for the closed-loop uncertain large-scale interconnected systems. Consequently, solving the following optimization problem allows us to determine the optimal bound.

$$
\begin{aligned}
&\mathcal{E}_0 : \min_{\Sigma_{i=1}^{N}\mathcal{X}_i} \sum_{i=1}^{N} \alpha_i, \\
&\mathcal{X}_i \in (\varepsilon_{ai},\ \varepsilon_{ai1},\ \dots,\varepsilon_{aiN},\ X_i,\ Y_i). \qquad (21)
\end{aligned}
$$

s.t. LMI (15), (22), $\varepsilon_{ai} > 0$, $\varepsilon_{aij} > 0$.

$$\begin{bmatrix} -\alpha_i & x_i^T(0) \\ x_i(0) & -X_i \end{bmatrix} < 0. \qquad (22)$$

That is, the problem addressed in this paper is as follows: "Find $K_i = Y_iX_i^{-1},\ i = 1,\ \dots,\ N$ such that LMI (15) and (22) are satisfied and for all i, the cost J becomes as small as possible."

Finally, we are in a position to establish the main result of this section.

Theorem 3: If the above optimization problem has the feasible solution set of ε_{ai}, ε_{aij}, X_i and Y_i, then the control laws of the form (16) are the decentralized linear state feedback control laws which ensure the minimization of the guaranteed cost (17) for the uncertain large-scale interconnected systems.

Proof: Using Theorem 2, the control laws (16) that consist of the feasible solution set of ε_{ai}, ε_{aij}, X_i and Y_i are the guaranteed cost controllers of the uncertain large-scale interconnected systems (1). Using the Schur complement to the LMI (22) results in

$$(22) \Leftrightarrow x_i^T(0)X_i^{-1}x_i(0) < \alpha_i. \qquad (23)$$

It follows that

$$
\begin{aligned}
J &< \sum_{i=1}^{N} x_i^T(0)X_i^{-1}x_i(0) < \sum_{i=1}^{N}\alpha_i \\
&< \min_{\Sigma_{i=1}^{N}\mathcal{X}_i} \sum_{i=1}^{N}\ \alpha_i = \sum_{i=1}^{N}\min_{\mathcal{X}_i}\ \alpha_i = J^*. \qquad (24)
\end{aligned}
$$

Thus, the minimization of α_i implies the minimum value J^* of the guaranteed cost for the interconnected uncertain systems (1). The optimality of the solution of the optimization problem follows from the convexity of the objective function under the LMI constraints. This is the desired result. ■

It can be noted that the original optimization problem for the guaranteed cost (24) can be decomposed to the reduced optimization problems $\min\limits_{\mathcal{X}_i}\ \alpha_i$ because each optimization problem is independent of other LMI. Hence, this optimization problems for each independent subsystem would be solved.

$$
\begin{bmatrix}
-P_i & E_{ki}^T & I_{n_i} & I_{n_i} & K_i^T & 0 & 0 & \cdots & 0 & 0 & (A_i+B_iK_i)^T & 0 & (E_{ai}+E_{bi}K_i)^T \\
E_{ki} & -I_{s_{ki}} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
I_{n_i} & 0 & -\dfrac{1}{N-1}I_{n_i} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
I_{n_i} & 0 & 0 & -Q_i^{-1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
K_i & 0 & 0 & 0 & -R_i^{-1}+D_{ki}D_{ki}^T & 0 & 0 & \cdots & 0 & 0 & D_{ki}D_{ki}^TB_i^T & 0 & D_{ki}D_{ki}^TE_{bi}^T \\
0 & 0 & 0 & 0 & 0 & -I_{n_1} & E_{ai1}^T & \cdots & 0 & 0 & A_{i1}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & E_{ai1} & -\varepsilon_{ai1}I_{r_{i1}} & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -I_{n_N} & E_{aiN}^T & A_{iN}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & E_{aiN} & -\varepsilon_{aiN}I_{r_{iN}} & 0 & 0 & 0 \\
A_i+B_iK_i & 0 & 0 & 0 & B_iD_{ki}D_{ki}^T & A_{i1} & 0 & \cdots & A_{iN} & 0 & -X_i+\Xi_i & B_iD_{ki} & D_{ki}^TE_{bi}^T \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & D_{ki}^TB_i^T & -I_{q_{ki}} & 0 \\
E_{ai}+E_{bi}K_i & 0 & 0 & 0 & E_{bi}D_{ki}D_{ki}^T & 0 & 0 & \cdots & 0 & 0 & E_{bi}D_{ki} & 0 & -\varepsilon_{ai}I_{r_i}
\end{bmatrix}
< 0. \tag{19}
$$

$$
\begin{bmatrix}
-P_i+E_{ki}^TE_{ki} & I_{n_i} & I_{n_i} & K_i^T & 0 & \cdots & 0 & (A_i+B_iK_i)^T & 0 \\
I_{n_i} & -\dfrac{1}{N-1}I_{n_i} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
I_{n_i} & 0 & -Q_i^{-1} & 0 & 0 & \cdots & 0 & 0 & 0 \\
K_i & 0 & 0 & -R_i^{-1}+D_{ki}D_{ki}^T & 0 & \cdots & 0 & D_{ki}D_{ki}^TB_i^T & 0 \\
0 & 0 & 0 & 0 & -I_{n_1} & \cdots & 0 & A_{i1}^T & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & -I_{n_N} & A_{iN}^T & 0 \\
A_i+B_iK_i & 0 & 0 & B_iD_{ki}D_{ki}^T & A_{i1} & \cdots & A_{iN} & -P_i^{-1} & B_iD_{ki} \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & D_{ki}^TB_i^T & -I_{q_{ki}}
\end{bmatrix}
$$

$$
+\begin{bmatrix} 0 \\ \vdots \\ 0 \\ D_{ai} \\ 0 \end{bmatrix} F_{ai}
\begin{bmatrix} (E_{ai}+E_{bi}K_i)^T \\ 0 \\ 0 \\ 0 \\ D_{ki}D_{ki}^TE_{bi}^T \\ 0 \\ \vdots \\ 0 \\ D_{ki}^TE_{bi}^T \\ 0 \end{bmatrix}^T
+\begin{bmatrix} (E_{ai}+E_{bi}K_i)^T \\ 0 \\ 0 \\ 0 \\ D_{ki}D_{ki}^TE_{bi}^T \\ 0 \\ \vdots \\ 0 \\ D_{ki}^TE_{bi}^T \\ 0 \end{bmatrix} F_{ai}^T
\begin{bmatrix} 0 \\ \vdots \\ 0 \\ D_{ai} \\ 0 \end{bmatrix}^T
$$

$$
+\sum_{j=1,\ j\neq i}^{N}\begin{bmatrix} 0 \\ \vdots \\ 0 \\ D_{aij} \\ 0 \end{bmatrix} F_{aij}
\begin{bmatrix} 0 \\ \vdots \\ 0 \\ E_{aij}^T \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T
+\sum_{j=1,\ j\neq i}^{N}\begin{bmatrix} 0 \\ \vdots \\ 0 \\ E_{aij}^T \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_{aiN}^T
\begin{bmatrix} 0 \\ \vdots \\ 0 \\ D_{aij} \\ 0 \end{bmatrix}^T < 0. \tag{20}
$$

Remark 1: It can be noted that the cost bound (22) depends on the initial condition $x_i(0)$. To remove this dependence on $x_i(0)$, it is assumed that $x_i(0)$ is a zero mean random variable satisfying $E[x_i(0)x_i(0)^T] = I_{n_i}$ [4], [6]. In this case, it is interesting to point out that the guaranteed cost becomes

$$
\begin{aligned}
E[J] &< \sum_{i=1}^{N} E\left[x_i^T(0)X_i^{-1}x_i(0)\right] \\
&= \sum_{i=1}^{N} \text{Trace }[X_i^{-1}] \\
&< \sum_{i=1}^{N} \text{Trace }[V_i] < \sum_{i=1}^{N} \min_{\mathcal{Y}_i} \text{Trace }[V_i] = J^{\dagger},
\end{aligned} \tag{25}
$$

where

$$\begin{bmatrix} -V_i & I_{n_i} \\ I_{n_i} & -X_i \end{bmatrix} < 0,$$
$$\mathcal{Y}_i \in (\varepsilon_{ai},\ \varepsilon_{ai1}, \cdots, \varepsilon_{aiN},\ X_i,\ Y_i,\ V_i). \quad (26)$$

Moreover, since the solutions set of ε_{ai}, ε_{aij}, X_i, Y_i and V_i are independent from the other subsystems, each suboptimization problem can be solved.

$$\mathcal{E}_i : \min_{\mathcal{Y}_i} \text{Trace } [V_i],\ i = 1,\ \dots, N. \quad (27)$$

Finally, since the global optimization problem $\mathcal{E}_0$ does not need to be solved, the proposed reduced-order optimization technique is very useful and reliable.

IV. Additive Gain as Fuzzy Control

The LMI approach for the uncertain large-scale systems usually results in the conservative controller design due to the existence of the uncertainties ΔA_i, ΔA_{ij}, ΔB_i and the additive gain perturbations ΔK_i. As a result, the cost J becomes large. The main contribution of this paper is to apply the fuzzy control as the additive gain perturbations to improve the cost. It should be noted that the proposed fuzzy controller regulate its outputs in real-time under the robust stability by the LMI approach.

In this paper, the error $E_{ri}(k)$ and the difference of error $\Delta E_{ri}(k)$ can be defined as

$$E_{ri}(k) = x_i(k), \quad (28a)$$
$$\Delta E_{ri}(k) = E_{ri}(k) - E_{ri}(k-1). \quad (28b)$$

In this paper, fuzzy subsets of the output $F_{ki}(k)$ are given by the following form

If $E_{ri}(k)$ is $L_{1j}(k)$ and $\Delta E_{ri}(k)$ is $L_{2j}(k)$,
then $F_{ki}(k)$ is $H_j(k)$ $j = 1,\ \dots, M$, (29)

where M is the total rules number, $L_{1j}(k)$, $L_{2j}(k)$, and $H_j(k)$ are fuzzy subsets of the input at step k. OR operation is applied to the fuzzy subsets $H_j(k)$, $j = 1,\ \dots, M$, and $F_{ki}(k)$ can be obtained by calculating its center of gravity.

$$F_{ki}(k) := \frac{\sum_{j=1}^{M} \phi_j\ S(\phi_j)}{\sum_{j=1}^{M} S(\phi_j)}. \quad (30)$$

where $S(\phi_j)$ is OR operation set of $H_j(k)$, ϕ_j is the horizontal axis of the membership function for the output of the fuzzy controller.

$E_{ri}(k)$ and $\Delta E_{ri}(k)$ are defined as the input of the fuzzy controller. Hence, the fuzzy controller outputs arbitrary function $F_{ki}(k)$. The membership functions and their ranges are shown in Fig. 1 and Fig. 2. As a symbol that denotes degree and sign of $E_{ri}(k)$, $\Delta E_{ri}(k)$ and $F_{ki}(k)$, Negative Big (NB), Negative Small (NS), Zero (ZO), Positive Small (PS), Positive Big (PB) are defined. The range of the membership functions is selected according to the maximum error and the difference of the error values when $F_{ki}(k) = 0$ for the proposed system (1).

TABLE I
The fuzzy rule for uncertain system.

		$\Delta E_{ri}(k)$				
		NB	NS	ZO	PS	PB
$E_{ri}(k)$	NB			PB		
	NS			PS		NS
	ZO	PB	PS	ZO	NS	NB
	PS	PS		NS		
	PB			NB		

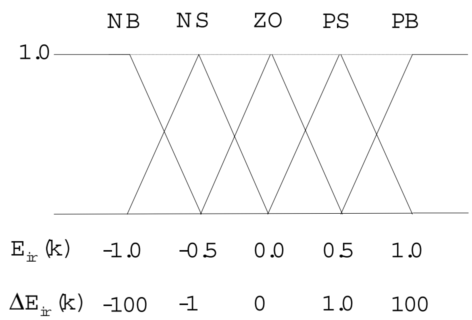

Fig. 1. Input of membership function.

The relationship between the input and the output of the fuzzy controller is the most important part. This relationship must be obtained correctly to improve the performance of the fuzzy logic control system, which is called If-Then rules. The fuzzy logic is determined by not a strict value but a vague expression. Therefore, the proposed fuzzy rules can be achieved by expression such as "Big" or "Small". The process for determining the rules is whether the arbitrary function $F_{ki}(k)$ should be increased or decreased by the error $E_{ri}(k)$ and the difference of the error $\Delta E_{ri}(k)$. The control rules when the initial condition of the proposed system changes from the positive to the origin are considered.

In order to determine the amount of increment or decrement for the arbitrary function $F_{ki}(k)$, If-Then rules are used. These rules are converted into a table as given by Table 1. For example, when $\Delta E_{ri}(k)$ is Zero (ZO) and $E_{ri}(k)$ is Negative Big (NB), then $F_{ki}(k)$ is should be Positive Big (PB) to increase the absolute value of $\| K_i + \Delta K_i(k) \|$. As a result, the convergence (change) will be fast (great). In other case, when $\Delta E_{ri}(k)$ is Positive Big (PB) and $E_{ri}(k)$ is Zero (ZO), then $F_{ki}(k)$ is should be Negative Big (NB) to decrease the absolute value of $\| K_i + \Delta K_i(k) \|$. Thus, the convergence (change) will be slow (small). In this way, the control rules are set by considering how $E_{ri}(k)$ and $\Delta E_{ri}(k)$ change.

V. Numerical Example

In order to demonstrate the effectiveness of proposed fuzzy controller, a numerical example is given. The system matrices

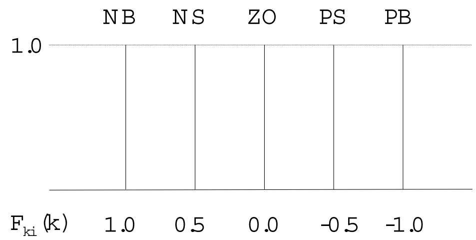

Fig. 2. Output of membership function.

Table 2. The actual costs.

$F_{ai}(k) = F_{aij}(k)$	$\hat{J}$ (Without fuzzy)	J (With fuzzy)
1.0	7.2610	6.5425
0.0	7.2609	6.5423
-0.1	7.2609	6.5421
$0.5\sin(120\pi k)$	7.2609	6.5423
$1-\exp(-k)$	7.2610	6.5424

are given below.

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix},\ A_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},\ A_{13} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix},\ A_{21} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.1 \end{bmatrix},\ A_{23} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ A_{31} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix},\ A_{32} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix},\ B_3 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},$$
$$D_{ai} = \begin{bmatrix} 0 \\ 0.01 \times i \end{bmatrix},$$
$$E_{ai} = \begin{bmatrix} 0 & 0.001 \times i \end{bmatrix},\ E_{bi} = \begin{bmatrix} 0.001 \times i \end{bmatrix},$$
$$D_{aij} = \begin{bmatrix} 0 \\ 0.01 \times i \end{bmatrix},\ E_{aij} = \begin{bmatrix} 0 & 0.001 \times i \end{bmatrix},$$
$$D_{ki} = \begin{bmatrix} 0 & 0.1 \end{bmatrix},\ E_{ki} = \begin{bmatrix} 2.0 \end{bmatrix},$$
$$R_i = i,\ Q_i = \begin{bmatrix} 0.001 \times i & 0 \\ 0 & 0.01 \times i \end{bmatrix},\ i = 1,\ 2,\ 3.$$

The fixed state feedback control gains K_i which is based on the proposed LMIs (15) are given by

$$\begin{aligned} K_1 &= \begin{bmatrix} 9.5354e-1 & 8.2313e-1 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 9.7967e-1 & 1.3495 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} 1.1787e-1 & -9.4046e-1 \end{bmatrix}. \end{aligned}$$

The results of the cost for the proposed system (1) with the fuzzy controller and uncertain system without the additive gain perturbations are shown in Table 2. In all cases, the cost J with the fuzzy controller is smaller than the cost $\hat{J}$ without the fuzzy controller. Therefore, it is also shown from Table 2 that it is possible to improve the cost by applying the new proposed fuzzy controller. It should be noted that although the simple fuzzy rule such as Table 1 is applied to the uncertain large-scale systems, the reduction of the cost is attained.

VI. Conclusions

The application of the fuzzy control for the guaranteed cost control problem of the large-scale system that has uncertainties in both state and input matrices has been investigated. Compared with the existing results the new LMI condition have been derived. In order to reduce the cost, the fuzzy control has been newly introduced. Substituting the fuzzy control into the additive gain perturbations, the robust stability and the adequate cost of the closed-loop system are both guaranteed even if such systems include these artificial controllers. Moreover, since the conservative assumption that is related to the neural networks [10] is not needed, the proposed design method can be applied to the wide class of the uncertain large-scale systems. The numerical example has shown that the fuzzy control has succeeded in reducing the large cost caused by the LMI technique.

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