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An LMI Approach to Guaranteed Cost Control of Nonlinear Large–Scale Uncertain Delay Systems under Controller Gain Perturbations

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Abstract— The guaranteed cost control problem of the decentralized robust control for nonlinear large–scale uncertain delay systems under controller gain perturbations is considered. Sufficient condition for the existence of the guaranteed cost control is given in terms of the linear matrix inequality (LMI). It is shown that the decentralized local state feedback controllers can be obtained by solving the LMIs.

I. Introduction

The study of large–scale interconnected systems has received ever greater attention in the past few decades (see, for example, [1] and the references therein). In recent years, the problem of the decentralized robust control of large–scale systems with parameter uncertainties has been widely studied, and some solution approaches have been developed [2]–[7]. Although there have been numerous results on decentralized robust control of large–scale uncertain systems, much effort has been made towards finding a controller which guarantee robust stability. However, when controlling such systems, it is also desirable to design the control systems which guarantee not only the robust stability, but also an adequate level of performance. One approach to this problem is the so–called guaranteed cost control approach [8]. This approach has the advantage of providing an upper bound on a given performance index.

Recent advance in theory of linear matrix inequality (LMI) has allowed a revisiting of the guaranteed cost control approach [10]. The LMI design method is a very well–known and powerful tool, it can not only efficiently find feasible and global solutions, but also easily handle various kinds of additional linear constraints. In recent years, the guaranteed cost control problem for a class of nonlinear large–scale interconnected systems which is based on the LMI design method was solved [7]. However, the problem of guaranteed cost stabilization for large-scale uncertain nonlinear delay systems has not been discussed so far. Moreover, although the existing results [7] obtained the controllers that are robust with respect to the uncertainty in the controled plant for the ordinary dynamic systems, their robustness with respect to the uncertainty in the controllers for the large–scale uncertain nonlinear delay systems has not been studied.

In this paper, the guaranteed cost control problem of the decentralized robust control for such systems is considered. It should be noted that although in [11], a linear decentralized controller that guarantees the exponential stabilization of a class of interconnected uncertain delay systems has been proposed, the guaranteed cost control problem for such systems has not been considered. Moreover, our work is an extension of the previous one [12] in the sense that the large–scale systems are allowed to be the time delays and the controller gain perturbations. After defining the guaranteed cost control problem for the large–scale interconnected uncertain nonlinear delay systems under the additive controller gain perturbations, a sufficient condition for the existence of the decentralized robust feedback controllers which guarantees the adequate upper bound on a given performance is derived in term of the LMI. The main contribution of this paper is that the guaranteed cost controllers are constructed by using the LMI technique. The crucial difference between the existing results [7, 12] and our results is that the time delays and the gain perturbations are both included. Moreover, the considered systems are complicated and general compared with [12]. As a result, the proposed robust decentralized controller can be implemented for the practical system compared with the existing results [7, 12].

Notation: The notations used in this paper are fairly standard. The superscript T denotes matrix transpose. I_n denotes the $n \times n$ identity matrix. $\| \cdot \|$ denotes its Euclidean norm for a matrix.

II. Problem Formulation

The nonlinear uncertain large–scale interconnected delay system which consists of N subsystems is described by the following state equations:

$$\begin{aligned} \dot{x}_i(t) &= [A_i + \Delta A_i(t)]x_i(t) + B_i u_i(t) \\ &\quad + [A_i^d + \Delta A_i^d(t)]x_i(t - \tau_i) \end{aligned}$$

$$+ \sum_{j=1, j\neq i}^{N} [G_{ij} + \Delta G_{ij}(t)] g_{ij}(x_i,\ x_j), \quad (1a)$$

$$u_i(t) = [K_i + \Delta K_i(t)] x_i(t), \quad (1b)$$

$$x_i(t) = \phi_i(t),\ t \in [-\tau_i,\ 0],\ i = 1,\ 2,\ \cdots,\ N, \quad (1c)$$

where $x_i \in \mathbf{R}^{n_i}$ and $u_i \in \mathbf{R}^{m_i}$ are the state and the control of the ith subsystems, respectively. $\tau_i > 0$ is the delay constant, and $\phi_i(t)$ is a given continuous vector valued initial function. A_i, B_i and A_i^d are constant matrices of appropriate dimensions and G_{ij} are interconnection matrices between the ith subsystems and other subsystems. $g_{ij}(x_i,\ x_j) \in \mathbf{R}^{l_i}$ are unknown nonlinear vector functions that represent nonlinearity [4, 7].

For a given controllers $u_i(t) = K_i x_i(t)$, the actual controller implemented is assumed to be $u_i(t) = [K_i + \Delta K_i(t)] x_i(t)$, where K_i is the nominal controller gain, and $\Delta K_i(t)$ represents the gain perturbations. In fact, the controller gain perturbations can result from the actuator degradations, as well as from the requirement for re–adjustment of controller gains during the controller implementation stage [13].

The parameter uncertainties considered here are assumed to be of the following form:

$$\left[\ \Delta A_i(t)\ \Delta A_i^d(t)\ \right] = D_i F_i(t) \left[\ E_{1i}\ E_i^d\ \right], \quad (2a)$$

$$\Delta G_{ij}(t) = D_{ij} F_{ij}(t) H_{ij}, \quad (2b)$$

$$\Delta K_i(t) = D_i^k F_i^k(t) E_i^k, \quad (2c)$$

where D_i, E_{1i}, E_i^d, D_{ij}, H_{ij} D_i^k and E_i^k are known constant real matrices of appropriate dimensions. $F_i(t) \in \mathbf{R}^{p_i \times q_i}$, $F_{ij}(t) \in \mathbf{R}^{r_i \times s_i}$ and $F_i^k(t) \in \mathbf{R}^{p_i^k \times q_i^k}$ are unknown matrix functions with Lebesgue measurable elements and satisfying

$$F_i^T(t) F_i(t) \le I_{q_i}, \quad (3a)$$

$$F_{ij}^T(t) F_{ij}(t) \le I_{s_i}, \quad (3b)$$

$$F_i^{kT}(t) F_i^k(t) \le I_{q_i^k}. \quad (3c)$$

Without loss of generality, the following assumptions concerning the unknown nonlinear vector functions are made.

Assumption 1 *There exist known constant matrices V_i and W_{ij} such that for all $x_i \in \mathbf{R}^{n_i}$ and $x_j \in \mathbf{R}^{n_j}$*

$$\|g_{ij}(x_i,\ x_j)\| \le \|V_i x_i\| + \|W_{ij} x_j\|, \quad (4)$$

for all i, j and for all $t \ge 0$.

Assumption 2 *For all i, $\sum_{j=1,\ j\neq i}^{N} W_{ji}^T W_{ji} > 0$.*

Remark 1 *Assumption 2 is made only for simplification of presentation.*

Associated with system (1) is the cost function

$$J = \sum_{i=1}^{N} \int_0^{\infty} [x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t)] dt, \quad (5)$$

where Q_i and R_i are given positive definite symmetric matrices.

Definition 1 *A decentralized control law $u_i(t) = [K_i + \Delta K_i(t)] x_i(t)$ is said to be a quadratic guaranteed cost control with associated cost matrix $P_i > 0$ for the uncertain large–scale interconnected delay system (1) and cost function (5) if the closed–loop system is quadratically stable and the closed–loop value of the cost function (5) satisfies the bound $J \le J^*$ for all admissible uncertainties, that is,*

$$\sum_{i=1}^{N} \left(\frac{d}{dt} x_i^T(t) P_i x_i(t) + x_i^T(t) \{ Q_i + [K_i + \Delta K_i(t)]^T R_i [K_i + \Delta K_i(t)] \} x_i(t) \right) < 0, \quad (6)$$

for all nonzero $x_i \in \mathbf{R}^{n_i}$ and all uncertain matrices (2).

The objective of this paper is to design the guaranteed cost control nominal gains K_i, $i = 1,\ 2,\ \cdots,\ N$ for the large–scale interconnected delay system (1) with uncertainties (2).

III. Main Results

Now, we present a sufficient condition for existence of the state feedback guaranteed cost control law for the uncertain delay systems (1).

Theorem 1 *Consider the large–scale interconnected delay systems (1) under Assumptions 1 and 2. If there exist symmetric positive definite matrices $P_i \in \mathbf{R}^{n_i \times n_i}$ and $S_i \in \mathbf{R}^{n_i \times n_i}$ such that for all uncertain matrices (2), the LMI (7) is satisfied, the control laws $u_i(t) = [K_i + \Delta K_i(t)] x_i(t)$, $i = 1, \cdots, N$ are the guaranteed cost controller,*

$$\Lambda_i = \begin{bmatrix} \Xi_i & P_i \tilde{A}_i^d & P_i \tilde{G}_{i1} & \cdots & P_i \tilde{G}_{iN} \\ \tilde{A}_i^{dT} P_i & -S_i & 0 & \cdots & 0 \\ \tilde{G}_{i1}^T P_i & 0 & -I_{l_1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{G}_{iN}^T P_i & 0 & 0 & \cdots & -I_{l_N} \end{bmatrix} < 0, \quad (7)$$

where $\Lambda_i \in \mathbf{R}^{\bar{N} \times \bar{N}}$, $\bar{N} = 2n_i + (N-1) l_i$ and

$$\Xi_i := \tilde{A}_i^T P_i + P_i \tilde{A}_i + U_i + \tilde{R}_i + S_i,$$

$$U_i := 2 \sum_{j=1,\ j\neq i}^{N} (V_i^T V_i + W_{ji}^T W_{ji}),$$

$$\tilde{A}_i := \bar{A}_i + D_i F_i(t) E_{1i} + B_i D_i^k F_i^k(t) E_i^k,$$

$$\tilde{A}_i^d := A_i^d + D_i F_i(t) E_i^d,$$

$$\tilde{G}_{ij} := G_{ij} + D_{ij} F_{ij}(t) H_{ij},\ \bar{A}_i := A_i + B_i K_i,$$

$$\tilde{R}_i := Q_i + [K_i + \Delta K_i(t)]^T R_i [K_i + \Delta K_i(t)].$$

Furthermore, the corresponding value of the cost function (5) satisfies the following inequality (8) for all admissible uncertainties (2).

$$J < \sum_{i=1}^{N} \left[\phi_i^T(0) P_i \phi_i(0) + \int_{-\tau_i}^{0} \phi_i^T(s) S_i \phi_i(s) ds \right]. \quad (8)$$

Remark 2 *Note that there exists no matrix $P_i\tilde{G}_{ii}$, $i = 1, \cdots, N$ in the matrix Λ_i.*

In order to prove Theorem 1, we need the following inequality

$$2x_i^T V_i^T V_i x_i + 2x_j^T W_{ij}^T W_{ij} x_j \geq g_{ij}^T g_{ij}. \quad (9)$$

Now, let us prove Theorem 1.

Proof: Combining the guaranteed cost controller $u_i(t) = [K_i + \Delta K_i(t)]x_i(t)$ with (1) gives a closed–loop system of the form

$$\begin{aligned} \dot{x}_i(t) &= \tilde{A}_i x_i(t) + \tilde{A}_i^d x_i(t-\tau_i) \\ &\quad + \sum_{j=1,\ j\neq i}^{N} \tilde{G}_{ij} g_{ij}(x_i,\ x_j). \end{aligned} \quad (10)$$

Suppose now there exist the symmetric positive definite matrices $P_i > 0,\ S_i = S_i^T > 0,\ i = 1, \cdots, N$ such that the LMI (7) holds for all admissible uncertainties. In order to prove the asymptotic stability of the closed–loop system (10), let us define the following Lyapunov function candidate

$$\begin{aligned} V(x(t)) &= \sum_{i=1}^{N} \Bigg[x_i^T(t) P_i x_i(t) \\ &\quad + \int_{t-\tau_i}^{t} x_i^T(s) S_i x_i(s) ds \Bigg], \end{aligned} \quad (11)$$

where $x(t) = \begin{bmatrix} x_1^T(t) & \cdots & x_N^T(t) \end{bmatrix}^T$. Note that $V(x(t)) > 0$ whenever $x(t) \neq 0$. Then the time derivative of $V(x(t))$ along any trajectory of the closed–loop system (10) is given by

$$\begin{aligned} &\frac{d}{dt}V(x(t)) \\ &= \sum_{i=1}^{N} \Bigg\{ x_i^T(t)(\tilde{A}_i^T P_i + P_i \tilde{A}_i) x_i(t) \\ &\quad +2x_i^T(t) P_i \tilde{A}_i^d x_i(t-\tau_i) \\ &\quad + \Bigg[\sum_{j=1,\ j\neq i}^{N} \tilde{G}_{ij} g_{ij}(x_i,\ x_j) \Bigg]^T P_i x_i(t) \\ &\quad + x_i^T(t) P_i \Bigg[\sum_{j=1,\ j\neq i}^{N} \tilde{G}_{ij} g_{ij}(x_i,\ x_j) \Bigg] \\ &\quad + x_i^T(t) S_i x_i(t) - x_i^T(t-\tau_i) S_i x_i(t-\tau_i) \Bigg\}. \end{aligned}$$

Since

$$\begin{aligned} &\sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} (2x_i^T V_i^T V_i x_i + 2x_i^T W_{ji}^T W_{ji} x_i - g_{ij}^T g_{ij}) \\ &= \sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} (2x_i^T V_i^T V_i x_i + 2x_j^T W_{ij}^T W_{ij} x_j - g_{ij}^T g_{ij}), \end{aligned}$$

and using (9), it follows that

$$\begin{aligned} &\frac{d}{dt}V(x(t)) \\ &= \sum_{i=1}^{N} \Bigg\{ x_i^T(t)(\tilde{A}_i^T P_i + P_i \tilde{A}_i) x_i(t) \\ &\quad +2x_i^T(t) P_i \tilde{A}_i^d x_i(t-\tau_i) \\ &\quad + \Bigg[\sum_{j=1,\ j\neq i}^{N} \tilde{G}_{ij} g_{ij}(x_i,\ x_j) \Bigg]^T P_i x_i(t) \\ &\quad + x_i^T(t) P_i \Bigg[\sum_{j=1,\ j\neq i}^{N} \tilde{G}_{ij} g_{ij}(x_i,\ x_j) \Bigg] \\ &\quad + x_i^T(t) S_i x_i(t) - x_i^T(t-\tau_i) S_i x_i(t-\tau_i) \Bigg\} \\ &\quad + \sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} (2x_i^T V_i^T V_i x_i + 2x_i^T W_{ji}^T W_{ji} x_i \\ &\quad - g_{ij}^T g_{ij}) \\ &\quad - \sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} (2x_i^T V_i^T V_i x_i + 2x_j^T W_{ij}^T W_{ij} x_j \\ &\quad - g_{ij}^T g_{ij}) \\ &= \sum_{i=1}^{N} z_i^T \begin{bmatrix} \Xi_i - \tilde{R}_i & P_i \tilde{A}_i^d & P_i \tilde{G}_{i1} & \cdots & P_i \tilde{G}_{iN} \\ \tilde{A}_i^{dT} P_i & -S_i & 0 & \cdots & 0 \\ \tilde{G}_{i1}^T P_i & 0 & -I_{l_1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{G}_{iN}^T P_i & 0 & 0 & \cdots & -I_{l_N} \end{bmatrix} z_i \\ &\quad - \sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} (2x_i^T V_i^T V_i x_i + 2x_j^T W_{ij}^T W_{ij} x_j \\ &\quad - g_{ij}^T g_{ij}) \\ &< \sum_{i=1}^{N} z_i^T(t) \Lambda_i z_i(t) - \sum_{i=1}^{N} x_i^T(t) \tilde{R}_i x_i(t), \end{aligned}$$

where $z_i = \begin{bmatrix} x_i^T(t) & x_i^T(t-\tau_i) & g_{i1}^T & \cdots & g_{iN}^T \end{bmatrix}^T \in \mathbf{R}^{\bar{N}}$ and Ξ_i and Λ_i are given in (7). Taking (7) into account, it follows immediately that

$$\frac{d}{dt}V(x(t)) < -\sum_{i=1}^{N} x_i^T(t) \tilde{R}_i x_i(t) < 0. \quad (12)$$

Hence, $V(x(t))$ is a Lyapunov function for the closed–loop system (10). Therefore, the closed–loop system (10) is asymptotically stable and $u_i(t) = [K_i + \Delta K_i(t)]x_i(t)$ is the guaranteed cost controller because the inequality (6) is satisfied. Furthermore, by integrating both sides of the inequality (12) from 0 to T and using the initial conditions, we have

$$V(x(T)) - V(x(0)) < -\sum_{i=1}^{N} \int_0^T x_i^T(t) \tilde{R}_i x_i(t) dt. \quad (13)$$

Since the closed–loop system (10) is asymptotically stable, that is, $x(T) \to 0$, when $T \to \infty$, we obtain $V(x(T)) \to 0$. Thus we get

$$J = \sum_{i=1}^{N} \int_0^T x_i^T(t) \tilde{R}_i x_i(t) dt < V(x(0))$$

$$= \sum_{i=1}^{N} \left[\phi_i^T(0) P_i \phi_i(0) + \int_{-\tau_i}^{0} \phi_i^T(s) S_i \phi_i(s) ds \right].$$

The proof of Theorem 1 is completed. ■

We now give the LMI design approach to the construction of the guaranteed cost controller.

Theorem 2 *Under Assumptions 1 and 2, suppose there exist the constant parameters* $\mu_i > 0$, $\varepsilon_i > 0$ *and* $\nu_i > 0$ *such that for all* $i = 1, \cdots, N$ *the LMI* (14) *have the symmetric positive definite matrices* $X_i > 0 \in \mathbf{R}^{n_i \times n_i}$ *and* $\bar{S}_i > 0 \in \mathbf{R}^{n_i \times n_i}$ *and a matrix* $Y_i \in \mathbf{R}^{m_i \times n_i}$.

If such conditions are met, the decentralized linear state feedback nominal gains are given by (15)

$$K_i = Y_i X_i^{-1}, \ i = 1, \cdots, \ N. \tag{15}$$

Moreover, the guaranteed cost bound for the closed–loop uncertain large–scale interconnected delay systems is given below

$$J < \sum_{i=1}^{N} \left[\phi_i^T(0) X_i^{-1} \phi_i(0) + \int_{-\tau_i}^{0} \phi_i^T(s) \bar{S}_i^{-1} \phi_i(s) ds \right]. \tag{16}$$

Proof: Let us introduce the matrices $X_i = P_i^{-1}$, $Y_i = K_i P_i^{-1}$ and $\bar{S}_i := S_i^{-1}$. Pre– and post–multiplying both sides of the inequality (14) by

$$\text{block}-\text{diag}\left[\begin{array}{cccccc} P_i & S_i & I_{m_i} & I_{l_1} & I_{s_1} & \cdots \\ I_{l_N} & I_{s_N} & I_{n_i} & I_{m_i} & I_{n_i} & I_{n_i} & I_{r_i^k} \end{array} \right]$$

yields (17).

Using the Schur complement [14], the LMI (17) holds if, and only if, the LMI (18) holds. Furthermore, applying the Schur complement to the LMI (18) gives (19).

Using a standard matrix inequality [4, 9], for all admissible uncertainties (2), the matrix inequality (7) holds. This is the required result. On the other hand, since the results of the cost bound (16) can be proved by using the similar argument for the proof of Theorem 1, it is omitted. ■

Since the LMI (14) consists of a convex solution set of ($\mu_i, \ \varepsilon_i, \ \nu_i, \ X_i, \ Y_i, \ \bar{S}_i$), various efficient convex optimization algorithm can be applied. Moreover, its solutions represent the set of the guaranteed cost controllers. This parameterized representation can be exploited to design the guaranteed cost controllers which minimizes the value of the guaranteed cost for the closed–loop uncertain large–scale interconnected delay systems. Consequently, solving the following optimization problem allows us to determine the optimal bound.

$$\begin{aligned} \mathcal{D}_0 : \ & \sum_{i=1}^{N} \min_{\mathcal{X}_i} J_i = J^*, \\ & J_i := \alpha_i + \text{Trace}\,[\mathcal{M}_i], \\ & \mathcal{X}_i \in (\mu_i, \ \varepsilon_i, \ \nu_i, \ X_i, \ Y_i, \ \bar{S}_i, \ \alpha_i, \ \mathcal{M}_i), \end{aligned} \tag{20}$$

such that (14) and

$$\left[\begin{array}{cc} -\alpha_i & \phi_i^T(0) \\ \phi_i(0) & -X_i \end{array} \right] < 0, \ \left[\begin{array}{cc} -\mathcal{M}_i & N_i^T \\ N_i & -\bar{S}_i \end{array} \right] < 0, \tag{21}$$

where $N_i N_i^T := \int_{-\tau_i}^{0} \phi_i(s) \phi_i^T(s) ds$.

That is, the problem addressed in this paper is as follows: "Find $K_i = Y_i X_i^{-1}, \ i = 1, \cdots, \ N$ such that LMI (14) and (21) are satisfied and for all i, the cost $J_i, \ i = 1, \cdots, N$ becomes as small as possible."

Finally, we are in a position to establish the main result of this section.

Theorem 3 *If the above optimization problem has the solution* $\mu_i, \ \varepsilon_i, \ \nu_i, \ X_i, \ Y_i, \ \bar{S}_i, \ \alpha_i$ *and* $\mathcal{M}_i$, *then the control gains of the form* (15) *are the decentralized linear state feedback control gains which ensure the minimization of the guaranteed cost* (16) *for the uncertain large–scale interconnected delay systems.*

Proof: By Theorem 2, the nominal control gains (15) constructed from the feasible solutions μ_i, ε_i, ν_i, X_i, Y_i, $\bar{S}_i$, α_i and $\mathcal{M}_i$ are the guaranteed cost controllers of the uncertain large–scale interconnected delay systems (1). Using the Schur complement to the LMI (21), we have

$$\begin{aligned} & \phi_i^T(0) X_i^{-1} \phi_i(0) < \alpha_i, \\ & \int_{-\tau_i}^{0} \phi_i^T(s) \bar{S}_i^{-1} \phi_i(s) ds < \text{Trace}\,[\mathcal{M}_i]. \end{aligned}$$

It follows that

$$\begin{aligned} J & < \sum_{i=1}^{N} \left[\phi_i^T(0) X_i^{-1} \phi_i(0) + \int_{-\tau_i}^{0} \phi_i^T(s) \bar{S}_i^{-1} \phi_i(s) ds \right] \\ & < \sum_{i=1}^{N} \min_{\mathcal{X}_i} (\alpha_i + \text{Trace}\,[\mathcal{M}_i]) = \sum_{i=1}^{N} \min_{\mathcal{X}_i} J_i = J^*. \end{aligned} \tag{22}$$

Thus, the minimization of J_i implies the minimum value J^* of the guaranteed cost for the interconnected uncertain delay systems (1). The optimality of the solution of the optimization problem follows from the convexity of the objective function under the LMI constraints. This is the required result. ■

Remark 3 *It should be noted that the original optimization problem for the guaranteed cost* (20) *can be decomposed to the following reduced optimization problems* (23) *because each optimization problem* (23) *is independent of other LMI. Hence, we have only to solve the optimization problems* (23) *for each independent subsystem.*

$$\begin{aligned} & \mathcal{D}_i : \ \min_{\mathcal{X}_i} J_i = \min_{\mathcal{X}_i} (\alpha_i + \text{Trace}\,[\mathcal{M}_i]), \ i = 1, \cdots, N, \\ & \mathcal{X}_i \in (\mu_i, \ \varepsilon_i, \ \nu_i, X_i, \ Y_i, \ \bar{S}_i, \ \alpha_i, \ \mathcal{M}_i). \end{aligned} \tag{23}$$

IV. Numerical Example

In order to demonstrate the efficiency of our proposed control, we have run a simple numerical example. Consider the interconnected uncertain system (1) composed of three two–dimensional subsystems. The system matrices and the

$$
\begin{bmatrix}
\Phi_i & A_i^d\bar{S}_i & X_iE_{1i}^T & G_{i1} & 0 & \cdots & G_{iN} & 0 & X_i & Y_i^T+\nu_iB_iD_i^kD_i^{kT} & X_i & X_i & X_iE_i^{kT} \\
\bar{S}_iA_i^{dT} & -\bar{S}_i & \bar{S}_iE_i^{dT} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
E_{1i}X_i & E_i^d\bar{S}_i & -\mu_iI_{m_i} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
G_{i1}^T & 0 & 0 & -I_{l_1} & H_{i1}^T & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & H_{i1} & -\varepsilon_iI_{s_1} & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
G_{iN}^T & 0 & 0 & 0 & 0 & \cdots & -I_{l_N} & H_{iN}^T & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & H_{iN} & -\varepsilon_iI_{s_N} & 0 & 0 & 0 & 0 & 0 \\
X_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -Q_i^{-1} & 0 & 0 & 0 & 0 \\
Y_i+\nu_iD_i^kD_i^{kT}B_i^T & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \nu_iD_i^kD_i^{kT}-R_i^{-1} & 0 & 0 & 0 \\
X_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & -\bar{S}_i & 0 & 0 \\
X_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & -U_i^{-1} & 0 \\
E_i^kX_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & -\nu_iI_{q_i^k}
\end{bmatrix} < 0, \qquad (14)
$$

where $\Phi_i := A_iX_i + B_iY_i + (A_iX_i + B_iY_i)^T + \mu_iD_iD_i^T + T_i + \nu_iB_iD_i^kD_i^{kT}B_i^T,\ T_i := \sum_{j=1,\ j\neq i}^{N} \varepsilon_iD_{ij}D_{ij}^T.$

$$
\begin{bmatrix}
\Psi_i & P_iA_i^d & E_{1i}^T & P_iG_{i1} & 0 & \cdots & P_iG_{iN} & 0 & I_{n_i} & K_i^T+\nu_iP_iB_iD_i^kD_i^{kT} & I_{n_i} & I_{n_i} & E_i^{kT} \\
A_i^{dT}P_i & -S_i & E_i^{dT} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
E_{1i} & E_i^d & -\mu_iI_{m_i} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
G_{i1}^TP_i & 0 & 0 & -I_{l_1} & H_{i1}^T & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & H_{i1} & -\varepsilon_iI_{s_1} & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
G_{iN}^TP_i & 0 & 0 & 0 & 0 & \cdots & -I_{l_N} & H_{iN}^T & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & H_{iN} & -\varepsilon_iI_{s_N} & 0 & 0 & 0 & 0 & 0 \\
I_{n_i} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -Q_i^{-1} & 0 & 0 & 0 & 0 \\
K_i+\nu_iD_i^kD_i^{kT}B_i^TP_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \nu_iD_i^kD_i^{kT}-R_i^{-1} & 0 & 0 & 0 \\
I_{n_i} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & -S_i^{-1} & 0 & 0 \\
I_{n_i} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & -U_i^{-1} & 0 \\
E_i^k & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & -\nu_iI_{q_i^k}
\end{bmatrix} < 0, \quad (17)
$$

where $\Psi_i := \bar{A}_i^TP_i + P_i\bar{A}_i + \mu_iP_iD_iD_i^TP_i + P_iT_iP_i + \nu_iP_iB_iD_i^kD_i^{kT}B_i^TP_i.$

nonlinear functions with the uncertainties are given as follows.

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, A_1^d = \begin{bmatrix} 0 & 0 \\ -0.1 & -0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, G_{12} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, G_{13} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, A_2^d = \begin{bmatrix} 0 & 0 \\ -0.2 & -0.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, G_{23} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, G_{21} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, A_3^d = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, G_{31} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, G_{32} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},$$

$$E_{1j} = \begin{bmatrix} 0 & 0.1 \end{bmatrix},\ E_j^d = \begin{bmatrix} 0 & 0.01 \end{bmatrix},\ j = 1, 2, 3,$$

$$E_j^k = \begin{bmatrix} 0.01 \end{bmatrix},\ D_j^k = \begin{bmatrix} 0 & 0.1 \end{bmatrix},\ j = 1, 2, 3,$$

$$D_{12} = D_{13} = D_{23} = D_{21} = D_{31} = D_{32} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$H_{12} = H_{13} = [0.015],\ H_{23} = H_{21} = [0.01],$$

$$H_{31} = H_{31} = [0.02],\ \tau_1 = \tau_2 = \tau_3 = 1,$$

$$\phi_1(t) = \phi_2(t) = \phi_3(t) = \begin{bmatrix} \exp(t+1) \\ 0 \end{bmatrix},$$

$$g_{1j} = [0.1 + \delta_1(t)]\Big(\sin\begin{bmatrix} 1 & 0 \end{bmatrix} x_1 - \sin\begin{bmatrix} 1 & 0 \end{bmatrix} x_j\Big),$$

$$|\delta_1(t)| \le 0.1,\ j = 2,\ 3,$$

$$g_{2j} = [0.1 + \delta_2(t)]\Big(\sin\begin{bmatrix} 1 & 0 \end{bmatrix} x_2 - \sin\begin{bmatrix} 1 & 0 \end{bmatrix} x_j\Big),$$

$$|\delta_2(t)| \le 0.1,\ j = 3,\ 1,$$

$$g_{3j} = [0.1 + \delta_3(t)]\Big(\sin\begin{bmatrix} 1 & 0 \end{bmatrix} x_3 - \sin\begin{bmatrix} 1 & 0 \end{bmatrix} x_j\Big),$$

$$|\delta_3(t)| \le 0.1,\ j = 1,\ 2.$$

In that case the unknown functions $g_{ij}(x_i,\ x_j)$ satisfy

$$\|g_{ij}(t)\| \le 0.2\Big(\|x_i\| + \|x_j\|\Big).$$

Therefore, we choose as $V_1 = V_2 = V_3 = W_{12} = W_{13} = W_{23} = W_{21} = W_{31} = W_{32} = 0.2I_2$.

Now, we choose as $R_i = 0.1$ and $Q_i = \mathrm{diag}\begin{bmatrix} 0.2 & 0.1 \end{bmatrix}$, $i = 1,\ 2,\ 3$. By applying Theorem 3 and solving the corresponding optimization problem (23), we obtain the decentralized feedback control gains

$$\begin{aligned} K_1 &= \begin{bmatrix} -1.0490 & -1.3701 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -1.0462 & -8.4753 \times 10^{-1} \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -5.7931 & -5.6983 \end{bmatrix}. \end{aligned}$$

Consequently,
the optimal guaranteed cost of the uncertain closed–loop delay system is $J^* = 15.768881$, where $\min_{\mathcal{X}_1} J_1 = 2.501906$, $\min_{\mathcal{X}_2} J_2 = 2.194812$ and $\min_{\mathcal{X}_3} J_3 = 11.072163$.

$$\begin{bmatrix} \Gamma_i & K_i^T + \nu_i P_i B_i D_i^k D_i^{kT} & P_i A_i^d + \mu_i^{-1} E_{1i}^T E_i^d & P_i G_{i1} & 0 & \cdots & P_i G_{iN} & 0 \\ K_i + \nu_i D_i^k D_i^{kT} B_i P_i & \nu_i D_i^{kT} D_i^k - R_i^{-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ A_i^{dT} P_i + \mu_i^{-1} E_i^{dT} E_{1i} & 0 & \mu_i^{-1} E_i^{dT} E_i^d - S_i & 0 & 0 & \cdots & 0 & 0 \\ G_{i1}^T P_i & 0 & 0 & -I_{l_1} & H_{i1}^T & \cdots & 0 & 0 \\ 0 & 0 & 0 & H_{i1} & -\varepsilon_i I_{s_1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ G_{iN}^T P_i & 0 & 0 & 0 & 0 & \cdots & -I_{l_N} & H_{iN}^T \\ 0 & 0 & 0 & 0 & 0 & \cdots & H_{iN} & -\varepsilon_i I_{s_N} \end{bmatrix} < 0. \tag{18}$$

$$\mathcal{F}_i := \begin{bmatrix} \Gamma_i & K_i^T + \nu_i P_i B_i D_i^k D_i^{kT} & P_i A_i^d + \mu_i^{-1} E_{1i}^T E_i^d & P_i G_{i1} & \cdots & P_i G_{iN} \\ K_i + \nu_i D_i^k D_i^{kT} B_i P_i & \nu_i D_i^{kT} D_i^k - R_i^{-1} & 0 & 0 & \cdots & 0 \\ A_i^{dT} P_i + \mu_i^{-1} E_i^{dT} E_{1i} & 0 & \mu_i^{-1} E_i^{dT} E_i^d - S_i & 0 & \cdots & 0 \\ G_{i1}^T P_i & 0 & 0 & \varepsilon_i^{-1} H_{i1}^T H_{i1} - I_{l_1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{iN}^T P_i & 0 & 0 & 0 & \cdots & \varepsilon_i^{-1} H_{iN}^T H_{iN} - I_{l_N} \end{bmatrix} < 0. \tag{19}$$

where $\Gamma_i := \bar{A}_i^T P_i + P_i \bar{A}_i + U_i + Q_i + S_i + P_i T_i P_i + \mu_i P_i D_i D_i^T P_i + \mu_i^{-1} E_{1i}^T E_{1i} + \nu_i P_i B_i D_i^k D_i^{kT} B_i^T P_i + \nu_i^{-1} E_i^{kT} E_i^k$.

It should be noted that although there exist both the time delays and the gain perturbations compared with the existing results [7, 12], we can construct the decentralized robust controller. Therefore, the proposed design method is useful in the sense that the resulting decentralized robust controller can be implemented to more practical large–scale systems.

V. Conclusions

In this paper, a solution of the guaranteed cost control problem for nonlinear large–scale uncertain delay systems have been presented. The decentralized robust optimal guaranteed cost controller which minimizes the value of the guaranteed cost for the closed–loop uncertain system can be solved by using software such as MATLAB's LMI control Toolbox. Thus, the resulting decentralized linear feedback controller can guarantee the quadratic stability and the optimal cost bound for the uncertain large–scale delay systems.

References

[1] Siljak, D. D., *Large–Scale Dynamic Systems: Stability and Structure*, Amsterdam: North Holland, 1978.

[2] Wang, Y., G. Guo and D. J. Hill, "Robust decentralized non–linear controller design for multimachine power systems," *Automatica*, vol.33, pp. 1725–1733, 1997.

[3] Guo, Y., D. J. Hill and Y. Wang, "Nonlinear decentralized control of large–scale power systems," *Automatica*, vol.36, pp. 1275–1289, 2000.

[4] Wang, Y., D. J. Hill and G. Guo, "Robust decentralized control for multimachine power systems," *IEEE Trans. Circuits and Systems*, vol.45, pp. 271–279, 1998.

[5] Ni, M.–L. and Y. Chen, "Decentralized stabilization and output tracking of large–scale uncertain systems," *Automatica*, vol.32, pp. 1077–1080, 1996.

[6] Xie, S. and L. Xie, "Stabilization of a class of uncertain large–scale stochastic systems with time delays," *Automatica*, vol.36, pp. 161–167, 2000.

[7] Xie, S., L. Xie, T. Wang and G. Guo, "Decentralized control of multimachine power systems with guaranteed performance," *IEE Proc. Control Theory Appl.*, vol.147, pp. 355–365, 2000.

[8] Petersen, I. R. and D. C. McFarlane, "Optimal guaranteed cost control and filtering for uncertain linear systems," *IEEE Trans. Automat. Contr.*, vol.39, pp. 1971–1977, 1994.

[9] Petersen, I. R., "A stabilization algorithm for a class of uncertain systems," *Syst. Contr. Lett.*, vol.8, pp. 181–188, 1987.

[10] Yu, L., G. Chen and J. Chu, "Optimal guaranteed cost control of linear systems: LMI approach," in *Proc. 14th IFAC World Congress*, Beijing, P. R. Chaina, vol.G, pp. 541–546, 1999.

[11] Mahmoud, M. S. and S. Bingulac, "Robust design of stabilizing controllers for interconnected time–delay systems," *Automatica*, vol. 34, pp. 795–800, 1998.

[12] Mukaidani, H., Y. Takato, Y. Tanaka and K. Mizukami, "The guaranteed cost control for uncertain large–scale interconnected systems," *in Proc.* 15th IFAC World Congress, Barcelona, 2002 CD–Rom.

[13] Haddad, W. M. and J. R. Corrado, "Robust resilient dynamic controolers for systems with parametric uncertainty and controller gain variations," *in Proc.* American Control Conference, pp. 2837–2841, 1998.

[14] Zhou, K., *Essentials of Robust Control*, New Jersey: Prentice–Hall, 1998.