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An LMI approach to decentralized guaranteed cost control for a class of uncertain nonlinear large-scale delay systems

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Abstract

The guaranteed cost control problem via the decentralized robust control for nonlinear uncertain large-scale systems that have delay in both state and control input is considered. Sufficient conditions for the existence of guaranteed cost controllers are given in terms of linear matrix inequality (LMI). It is shown that the decentralized local state feedback controllers can be obtained by solving the LMI.

1. Introduction

In recent years, the problem of the decentralized robust control of large-scale systems with parameter uncertainties has been widely studied (see, e.g., [1]). Although there have been numerous studies on the decentralized robust control of large-scale uncertain systems, much effort has been made toward finding a controller that guarantees robust stability. However, when controlling such systems, it is also desirable to design control systems that guarantee not only robust stability but also an adequate level of performance. One approach to this problem is the so-called guaranteed cost control approach [2]. This approach has the advantage of providing an upper bound on a given performance index.

Recent advances in the Linear Matrix Inequality (LMI) theory [11] have allowed a revisiting of the guaranteed cost control approach [3,6]. In [3], the guaranteed cost control

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technique for interconnected systems by means of the LMI approach has been discussed. In [6], the guaranteed cost control for nonlinear uncertain large-scale systems under gain perturbations has been considered. However, the time delays have not been considered in those reports. If the system does not have delays, the theoretical behavior would usually be more tractable. However, if delays are present, they may result in instability or serious deterioration in the performance of the resulting control systems. Therefore, the study of the control, considering these time delays on the guaranteed cost stability, is very important.

In this paper, the guaranteed cost control problem of the decentralized robust control for uncertain nonlinear large-scale systems that have delay in both state and control input is considered. It should be noted that although the robust control design method for parameter uncertain ordinary dynamic systems that have delay in both state and control input has been considered (see for example [4,5]), the guaranteed cost control for nonlinear uncertain large-scale systems that have delay in both state and control input has never been discussed. A sufficient condition for the existence of the decentralized robust feedback controllers is derived in terms of the LMI. The main result of this paper shows that the guaranteed cost controllers can be constructed by solving the LMI. The crucial difference between the existing results [3] and that of the present study is that the controller that guarantees the stability and the adequate level of performance of the large-scale delay systems is given. Thus, the applicability of the resulting controllers can be extended to more practical large-scale systems. Moreover, since the construction of the guaranteed cost controller consists of an LMI-based control design, the proposed method is computationally attractive and useful.

The notations used in this paper are fairly standard. The superscript T denotes the matrix transpose. $I_n \in \mathbf{R}^{n\times n}$ denotes the identity matrices. block-diag denotes the block diagonal matrix. $\|\cdot\|$ denotes the Euclidean norm. $\|\cdot\|_2$ denotes the largest singular value.

2. Analysis of robust performance

We consider continuous-time autonomous uncertain nonlinear large-scale interconnected delay systems, which consist of N subsystems of the form:

$$\dot{x}_i(t) = \left[\bar{A}_i + \Delta\bar{A}_i(t)\right]x_i(t) + \left[A_i^d + \Delta A_i^d(t)\right]x_i(t-\tau_i) + \left[H_i^d + \Delta H_i^d(t)\right]x_i(t-h_i) + \sum_{j=1, j\neq i}^{N}\left[G_{ij} + \Delta G_{ij}(t)\right]g_{ij}(x_i, x_j), \tag{1a}$$

$$x_i(t) = \phi_i(t), \quad t \in [-d_i, 0], \quad d_i = \max\{\tau_i, h_i\}, \quad i = 1, \ldots, N, \tag{1b}$$

where $x_i(t) \in \mathbf{R}^{n_i}$ are the states. $\tau_i > 0$ and $h_i > 0$ are the delay constants, and $\phi_i(t)$ are the given continuous vector valued initial functions. $\bar{A}_i$, A_i^d, and H_i^d are the constant matrices of appropriate dimensions. $G_{ij} \in \mathbf{R}^{n_i \times l_j}$ are the interconnection matrices between the ith subsystems and other subsystems. $g_{ij}(x_i, x_j) \in \mathbf{R}^{l_j}$ are unknown nonlinear vector functions that represent nonlinearity. The parameter uncertainties considered here are assumed to be of the following form:

$$\left[\Delta\bar{A}_i(t)\ \Delta A_i^d(t)\ \Delta H_i^d(t)\right] = D_i F_i(t)\left[\bar{E}_i^1\ E_i^{1d}\ \bar{E}_i^{d_h}\right], \tag{2a}$$

$$\Delta G_{ij}(t) = D_{ij} F_{ij}(t) E_{ij}, \tag{2b}$$

where D_i, $\bar{E}_i^1$, E_i^{1d}, $\bar{E}_i^{d_h}$, D_{ij}, and E_{ij} are known constant real matrices of appropriate dimensions. $F_i(t) \in \mathbf{R}^{p_i \times q_i}$ and $F_{ij}(t) \in \mathbf{R}^{r_{ij} \times s_{ij}}$ are unknown matrix functions with Lebesgue measurable elements and satisfy

$$F_i^T(t)F_i(t) \leqslant I_{q_i}, \qquad F_{ij}^T(t)F_{ij}(t) \leqslant I_{s_{ij}}. \tag{3}$$

We make the following assumptions concerning the unknown nonlinear vector functions.

(A1) There exist known constant matrices V_i and W_{ij} such that for all $i, j, t \geqslant 0$, $x_i \in \mathbf{R}^{n_i}$ and $x_j \in \mathbf{R}^{n_j}$,

$$\left\| g_{ij}(x_i, x_j) \right\| \leqslant \|V_i x_i\| + \|W_{ij} x_j\|.$$

(A2) For all i, j,

$$U_i := 2 \sum_{j=1,\, j \neq i}^{N} \left(V_i^T V_i + W_{ji}^T W_{ji} \right) > 0.$$

The cost function of the associated system (1) is given as

$$J = \sum_{i=1}^{N} \int_0^{\infty} x_i^T(t) \bar{Q}_i x_i(t)\, dt, \tag{4}$$

where $\bar{Q}_i$ is the given positive definite symmetric matrices.

The definition of the cost matrix for the uncertain nonlinear large-scale interconnected delay systems is given [2].

Definition 2.1. The set of matrices $P_i > 0$ is said to be the quadratic cost matrix for the uncertain nonlinear large-scale interconnected delay systems (1) if the following inequality holds

$$\sum_{i=1}^{N} \left(\frac{d}{dt} x_i^T(t) P_i x_i(t) + x_i^T(t) \bar{Q}_i x_i(t) \right) < 0, \tag{5}$$

for all nonzero $x_i \in \mathbf{R}^{n_i}$ and all uncertainties (2).

Theorem 2.1. *Under assumptions* (A1) *and* (A2), *suppose there exist the symmetric positive definite matrices $P_i > 0$, $S_i > 0$, $T_i > 0 \in \mathbf{R}^{n_i \times n_i}$ such that for all uncertain matrices* (2) *the following matrix inequality holds*:

$$\Lambda_i = \begin{bmatrix} \Xi_i & P_i \tilde{A}_i^d & P_i \tilde{H}_i^d & P_i \tilde{G}_{i1} & \cdots & P_i \tilde{G}_{iN} \\ \tilde{A}_i^{dT} P_i & -S_i & 0 & 0 & \cdots & 0 \\ \tilde{H}_i^{dT} P_i & 0 & -T_i & 0 & \cdots & 0 \\ \tilde{G}_{i1}^T P_i & 0 & 0 & -I_{l_1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{G}_{iN}^T P_i & 0 & 0 & 0 & \cdots & -I_{l_N} \end{bmatrix} < 0, \tag{6}$$

where $\Lambda_i \in \mathbf{R}^{\bar{N}\times\bar{N}}$, $\bar{N} = 3n_i + \sum_{j=1,\, j\neq i}^{N} l_j$ *and*

$$\Xi_i := \tilde{A}_i^T P_i + P_i\tilde{A}_i + U_i + \bar{Q}_i + S_i + T_i, \quad \tilde{A}_i := \bar{A}_i + \Delta\bar{A}_i(t),$$
$$\tilde{A}_i^d := A_i^d + \Delta A_i^d(t), \quad \tilde{H}_i^d := H_i^d + \Delta H_i^d(t), \quad \tilde{G}_{ij} := G_{ij} + \Delta G_{ij}(t).$$

Then the autonomous uncertain nonlinear large-scale interconnected delay systems (1) *are quadratically stable, and the corresponding value of the cost function* (4) *satisfies the following inequality*:

$$J < \sum_{i=1}^{N}\left[\phi_i^T(0)P_i\phi_i(0) + \int_{-\tau_i}^{0}\phi_i^T(s)S_i\phi_i(s)\,ds + \int_{-h_i}^{0}\phi_i^T(s)T_i\phi_i(s)\,ds\right]. \tag{7}$$

Remark 2.1. Note that there exists no matrix $P_i\tilde{G}_{ii}$, $i = 1,\ldots,N$, in the matrix Λ_i.

Proof. Using the definitions $\tilde{A}_i$, $\tilde{A}_i^d$, $\tilde{H}_i^d$, and $\tilde{G}_{ij}$, we can change the form (1) similar to

$$\dot{x}_i(t) = \tilde{A}_i x_i(t) + \tilde{A}_i^d x_i(t-\tau_i) + \tilde{H}_i^d x_i(t-h_i) + \sum_{j=1,\, j\neq i}^{N}\tilde{G}_{ij}g_{ij}(x_i,x_j). \tag{8}$$

Let us assume that there exist the symmetric positive definite matrices P_i, S_i, and T_i, $i = 1,\ldots,N$, such that the matrix inequality (6) holds for all admissible uncertainties (2). In order to prove the asymptotic stability of the interconnected delay systems (8), let us define the following Lyapunov function candidate

$$V\big(x(t)\big) = \sum_{i=1}^{N}\left[x_i^T(t)P_i x_i(t) + \int_{t-\tau_i}^{t}x_i^T(s)S_i x_i(s)\,ds + \int_{t-h_i}^{t}x_i^T(s)T_i x_i(s)\,ds\right], \tag{9}$$

where $x(t) = [x_1^T(t)\ \cdots\ x_N^T(t)]^T$. Note that $V(x(t)) > 0$ whenever $x(t) \neq 0$. The time derivative of $V(x(t))$ along any trajectory of the interconnected delay systems (8) is given by

$$\frac{d}{dt}V\big(x(t)\big) = \sum_{i=1}^{N} z_i^T(t)\Lambda_i z_i(t) - \sum_{i=1}^{N} x_i^T(t)\bar{Q}_i x_i(t) - \sum_{i=1}^{N}\sum_{j=1,\, j\neq i}^{N}\big(2x_i^T V_i^T V_i x_i + 2x_j^T W_{ij}^T W_{ij}x_j - g_{ij}^T g_{ij}\big),$$

where

$$z_i = \big[x_i^T(t)\ x_i^T(t-\tau_i)\ x_i^T(t-h_i)\ g_{i1}^T\cdots g_{iN}^T\big]^T \in \mathbf{R}^{\bar{N}}$$

and Ξ_i and Λ_i are given in (6).

It is easy to verify whether the following inequality holds under assumption (A1):

$$2x_i^T V_i^T V_i x_i + 2x_j^T W_{ij}^T W_{ij} x_j \geqslant g_{ij}^T g_{ij}. \tag{10}$$

Given that the inequalities (6) and (10) hold, it immediately follows that

$$\frac{d}{dt} V\big(x(t)\big) < -\sum_{i=1}^{N} x_i^T(t)\bar{Q}_i x_i(t) < 0. \tag{11}$$

Hence, $V(x(t))$ is a Lyapunov function for the large-scale interconnected delay system (8). Therefore, the interconnected delay system (8) is asymptotically stable. Furthermore, by integrating both sides of the inequality (11) from 0 to T and using the initial conditions, we obtain

$$V\big(x(T)\big) - V\big(x(0)\big) < -\sum_{i=1}^{N} \int_0^T x_i^T(t)\bar{Q}_i x_i(t)\,dt. \tag{12}$$

Since the interconnected delay system (8) is asymptotically stable, that is, $x(T) \to 0$ when $T \to \infty$, we obtain $V(x(T)) \to 0$. Thus we obtain

$$\begin{aligned} J &= \sum_{i=1}^{N} \int_0^T x_i^T(t)\bar{Q}_i x_i(t)\,dt < V\big(x(0)\big) \\ &= \sum_{i=1}^{N} \left[\phi_i^T(0)P_i\phi_i(0) + \int_{-\tau_i}^{0} \phi_i^T(s)S_i\phi_i(s)\,ds + \int_{-h_i}^{0} \phi_i^T(s)T_i\phi_i(s)\,ds \right]. \end{aligned}$$

This completes the proof of Theorem 2.1. □

3. Problem formulation

In this section, we consider the problem of the optimal guaranteed cost control via the state feedback for a class of nonlinear uncertain large-scale interconnected systems with delays. The uncertain delay systems under consideration are described by the state equations

$$\begin{aligned} \dot{x}_i(t) &= \big[A_i + \Delta A_i(t)\big]x_i(t) + \big[B_i + \Delta B_i(t)\big]u_i(t) \\ &\quad + \big[A_i^d + \Delta A_i^d(t)\big]x_i(t-\tau_i) + \big[B_i^d + \Delta B_i^d(t)\big]u_i(t-h_i) \\ &\quad + \sum_{j=1,\, j\neq i}^{N} [G_{ij} + \Delta G_{ij}]g_{ij}(x_i, x_j), \end{aligned} \tag{13a}$$

$$x_i(t) = \phi_i(t), \quad t \in [-d_i, 0], \quad d_i = \max\{\tau_i, h_i\}, \quad i = 1, \ldots, N, \tag{13b}$$

where $u_i(t) \in \mathbf{R}^{m_i}$ are the control inputs of the ith subsystems. The parameter uncertainties satisfy

$$\big[\Delta A_i(t)\ \Delta B_i(t)\ \Delta B_i^d(t)\big] = D_i F_i(t)\big[E_i^1\ E_i^2\ E_i^{2d}\big]. \tag{14}$$

A_i, B_i, B_i^d, E_i^1, E_i^2, and E_i^{2d} are the constant matrices of appropriate dimensions. The remainder constant real matrices and parameter uncertainties are the same as these in the large-scale delay systems (1). Moreover, it is assumed that (A1) and (A2) hold for the unknown nonlinear vector functions $g_{ij}(x_i, x_j) \in \mathbf{R}^{l_j}$. Associated with system (13) is the cost function

$$\mathcal{J} = \sum_{i=1}^{N} \int_0^{\infty} \big[x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t)\big]\,dt, \tag{15}$$

where Q_i and R_i are the given positive definite symmetric matrices.

Based on reference [2], the definition of the guaranteed cost control for uncertain nonlinear large-scale interconnected delay systems is given below.

Definition 3.1. A decentralized control law $u_i(t) = K_i x_i(t)$ is said to be a quadratic guaranteed cost control related to the set of matrices $P_i > 0$ for the uncertain large-scale interconnected system (13) and cost function (15) if the closed-loop system is quadratically stable and the closed-loop value of the cost function (15) satisfies the bound $\mathcal{J} \leqslant \mathcal{J}^*$ for all admissible uncertainties, that is,

$$\sum_{i=1}^{N} \left(\frac{d}{dt} x_i^T(t) P_i x_i(t) + x_i^T(t)\big[Q_i + K_i^T R_i K_i\big] x_i(t) \right) < 0, \tag{16}$$

for all nonzero $x_i \in \mathbf{R}^{n_i}$.

The objective of this paper is to design a decentralized guaranteed cost controller

$$u_i(t) = K_i x_i(t), \quad i = 1, \ldots, N,$$

for the uncertain large-scale interconnected delay system (13).

4. Main results

We now present the LMI design approach to the construction of a guaranteed cost controller.

Theorem 4.1. *Under assumptions* (A1) *and* (A2), *suppose there exist the constant positive parameters* $\mu_i > 0$ *and* $\varepsilon_i > 0$ *such that for all* $i = 1, \ldots, N$ *the following LMI* (17) *have the symmetric positive definite matrices* $X_i > 0$, $\bar{S}_i > 0$, $Z_i > 0 \in \mathbf{R}^{n_i \times n_i}$ *and a matrix* $Y_i \in \mathbf{R}^{m_i \times n_i}$:

$$\left[\begin{matrix} \Phi_i & A_i^d \bar{S}_i & B_i^d Y_i & (E_i^1 X_i + E_i^2 Y_i)^T & G_{i1} & 0 & \cdots \\ \bar{S}_i A_i^{dT} & -\bar{S}_i & 0 & \bar{S}_i E_i^{1dT} & 0 & 0 & \cdots \\ Y_i^T B_i^{dT} & 0 & -Z_i & Y_i^T E_i^{2dT} & 0 & 0 & \cdots \\ E_i^1 X_i + E_i^2 Y_i & E_i^{1d} \bar{S}_i & E_i^{2d} Y_i & -\mu_i I_{q_i} & 0 & 0 & \cdots \\ G_{i1}^T & 0 & 0 & 0 & -I_{l_1} & E_{i1}^T & \cdots \\ 0 & 0 & 0 & 0 & E_{i1} & -\varepsilon_i I_{s_{i1}} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ G_{iN}^T & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ X_i & 0 & 0 & 0 & 0 & 0 & \cdots \\ Y_i & 0 & 0 & 0 & 0 & 0 & \cdots \\ X_i & 0 & 0 & 0 & 0 & 0 & \cdots \\ X_i & 0 & 0 & 0 & 0 & 0 & \cdots \end{matrix}\right.$$

$$\left.\begin{matrix} G_{iN} & 0 & X_i & Y_i^T & X_i & X_i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ Y_i^T B_i^{dT} & 0 & -Z_i & Y_i^T E_i^{2dT} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -I_{l_N} & E_{iN}^T & 0 & 0 & 0 & 0 \\ E_{iN} & -\varepsilon_i I_{s_{iN}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -Q_i^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -R_i^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\bar{S}_i & 0 \\ 0 & 0 & 0 & 0 & 0 & -U_i^{-1} \end{matrix}\right] < 0, \tag{17}$$

where $\Phi_i := A_i X_i + B_i Y_i + (A_i X_i + B_i Y_i)^T + Z_i + \mu_i D_i D_i^T + H_i$, $H_i := \sum_{j=1,\, j\neq i}^{N} \varepsilon_i \cdot D_{ij} D_{ij}^T$. *If such conditions are met, the decentralized linear state feedback control laws*

$$u_i(t) = K_i x_i(t) = Y_i X_i^{-1} x_i(t), \quad i = 1, \ldots, N, \tag{18}$$

are the guaranteed cost controllers and

$$\mathcal{J} < \sum_{i=1}^{N} \Bigg[\phi_i^T(0) X_i^{-1} \phi_i(0) + \int_{-\tau_i}^{0} \phi_i^T(s) \bar{S}_i^{-1} \phi_i(s)\, ds + \int_{-h_i}^{0} \phi_i^T(s) X_i^{-1} Z_i X_i^{-1} \phi_i(s)\, ds \Bigg] \tag{19}$$

is the guaranteed cost for the closed-loop uncertain large-scale interconnected delay systems.

Proof. Let us introduce the matrices $X_i := P_i^{-1}$, $Y_i := K_i P_i^{-1}$, $\bar{S}_i := S_i^{-1}$ and $Z_i := P_i^{-1} T_i P_i^{-1}$. Pre- and post-multiplying both sides of the inequality (17) by

$$\text{block-diag}[P_i \; S_i \; P_i \; I_{q_i} \; I_{l_1} \; I_{s_{i1}} \; \cdots \; I_{l_N} \; I_{s_{iN}} \; I_{n_i} \; I_{m_i} \; I_{n_i} \; I_{n_i}]$$

yields (20):

$$\left[\begin{array}{cccccccccccc}
\Psi_i & P_i A_i^d & P_i B_i^d K_i & \bar{E}_i^T & P_i G_{i1} & 0 & P_i G_{iN} & 0 & I_{n_i} & K_i^T & I_{n_i} & I_{n_i} \\
A_i^{dT} P_i & -S_i & 0 & E_i^{1dT} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K_i^T B_i^{dT} P_i & 0 & -T_i & K_i^T E_i^{2dT} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{E}_i & E_i^{1d} & E_i^{2d} K_i & -\mu_i I_{q_i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
G_{i1}^T P_i & 0 & 0 & 0 & -I_{l_1} & E_{i1}^T & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & E_{i1} & -\varepsilon_i I_{s_{i1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
G_{iN}^T P_i & 0 & 0 & 0 & 0 & 0 & -I_{l_N} & E_{iN}^T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & E_{iN} & -\varepsilon_i I_{s_{iN}} & 0 & 0 & 0 & 0 \\
I_{n_i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_i^{-1} & 0 & 0 & 0 \\
K_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_i^{-1} & 0 & 0 \\
I_{n_i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_i^{-1} & 0 \\
I_{n_i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -U_i^{-1}
\end{array}\right] < 0, \tag{20}$$

where $\Psi_i := \bar{A}_i^T P_i + P_i \bar{A}_i + T_i + \mu_i P_i D_i D_i^T P_i + P_i H_i P_i$, $\bar{A}_i := A_i + B_i K_i$, $\bar{E}_i := E_i^1 + E_i^2 K_i$.

Using the Schur complement [9], the matrix inequality (20) holds if, and only if, the following inequality (21) holds:

$$\mathcal{F}_i := \begin{bmatrix} \Gamma_i & P_i A_i^d + \mu_i^{-1} \bar{E}_i^T E_i^{1d} & P_i B_i^d K_i + \mu_i^{-1} \bar{E}_i^T E_i^{2d} K_i & P_i G_{i1} & \cdots & P_i G_{iN} \\ A_i^{dT} P_i + \mu_i^{-1} E_i^{1dT} \bar{E}_i & \mu_i^{-1} E_i^{1dT} E_i^{1d} - S_i & \mu_i^{-1} E_i^{1dT} E_i^{2d} K_i & 0 & \cdots & 0 \\ K_i^T B_i^{dT} P_i + \mu_i^{-1} K_i^T E_i^{2dT} \bar{E}_i & \mu_i^{-1} K_i^T E_i^{2dT} E_i^{1d} & \mu_i^{-1} K_i^T E_i^{2dT} E_i^{2d} K_i - T_i & 0 & \cdots & 0 \\ G_{i1}^T P_i & 0 & 0 & \Theta_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{iN}^T P_i & 0 & 0 & 0 & \cdots & \Theta_N \end{bmatrix} < 0, \tag{21}$$

where $\Gamma_i := \bar{A}_i^T P_i + P_i \bar{A}_i + U_i + \bar{R}_i + S_i + T_i + \mu_i P_i D_i D_i^T P_i + P_i H_i P_i + \mu_i^{-1} \bar{E}_i^T \bar{E}_i$, $\bar{R}_i := Q_i + K_i^T R_i K_i$, $\Theta_j := \varepsilon_i^{-1} E_{ij}^T E_{ij} - I_{l_j}$.

Using a standard matrix inequality [8] for all admissible uncertainties (2) and (14), the matrix inequality (22) holds:

$$\begin{aligned} 0 &> \mathcal{F}_i \\ &\geqslant \begin{bmatrix} \bar{A}_i^T P_i + P_i \bar{A}_i + U_i + \bar{R}_i + S_i + T_i & P_i A_i^d & P_i B_i^d K_i & P_i G_{i1} & \cdots & P_i G_{iN} \\ A_i^{dT} P_i & -S_i & 0 & 0 & \cdots & 0 \\ K_i^T B_i^{dT} P_i & 0 & -T_i & 0 & \cdots & 0 \\ G_{i1}^T P_i & 0 & 0 & -I_{l_1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{iN}^T P_i & 0 & 0 & 0 & \cdots & -I_{l_N} \end{bmatrix} \\ &\quad + \begin{bmatrix} P_i D_i \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} \bar{E}_i^T \\ E_i^{1dT} \\ K_i^T E_i^{2dT} \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T + \begin{bmatrix} \bar{E}_i^T \\ E_i^{1dT} \\ K_i^T E_i^{2dT} \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_i^T(t) \begin{bmatrix} P_i D_i \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T \\ &\quad + \begin{bmatrix} 0 & 0 & 0 & P_i D_{i1} & \cdots & P_i D_{iN} \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & F_{i1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & F_{iN} \end{bmatrix} \end{aligned}$$

$$\cdot \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & E_{i1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & E_{iN} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & E_{i1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & E_{iN} \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & F_{i1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & F_{iN} \end{bmatrix}^T$$

$$\cdot \begin{bmatrix} 0 & 0 & 0 & P_i D_{i1} & \cdots & P_i D_{iN} \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}^T = \mathcal{L}_i. \tag{22}$$

Noting $A_i^d + D_i F_i(t) E_i^{1d} = \bar{A}_i^d$ and $G_{ij} + D_{ij} F_{ij}(t) E_{ij} = \tilde{G}_{ij}$ and setting $\bar{A}_i + D_i F_i(t) \bar{E}_i \to \tilde{A}_i = \bar{A}_i + \Delta\bar{A}_i(t)$, $[B_i^d + \Delta B_i^d(t)]K_i \to H_i^d + \Delta H_i^d(t)$ and $Q_i + K_i^T R_i K_i = \bar{R}_i \to \bar{Q}_i$, we have $\mathcal{L}_i = \Lambda_i$. Hence, the closed-loop systems are asymptotically stable under Theorem 2.1. On the other hand, since the results of the cost bound (19) can be proved by using similar arguments for the proof of Theorem 2.1, it is omitted. □

Since the LMI (17) consists of a solution set of $(\mu_i, \varepsilon_i X_i, Y_i, \bar{S}_i, Z_i)$, various efficient convex optimization algorithms can be applied. Moreover, its solutions represent the set of guaranteed cost controllers. This parameterized representation can be exploited to design the guaranteed cost controllers, which minimizes the value of the guaranteed cost for the closed-loop uncertain large-scale interconnected delay systems. Consequently, solving the following optimization problem allows us to determine the optimal bound:

$$\begin{aligned} &\mathcal{D}_0\colon \min_{\mathcal{X}_i} \sum_{i=1}^{N} \bar{\mathcal{J}}_i = \mathcal{J}^*, \\ &\bar{\mathcal{J}}_i := \alpha_i + \mathrm{Trace}[\mathcal{M}_i] + c_i^2 \left\| N_i N_i^T \right\|_2 \cdot \mathrm{Trace}[Z_i], \\ &\mathcal{X}_i \in \left(\mu_i, \varepsilon_i X_i, Y_i, \bar{S}_i, Z_i, \alpha_i, \mathcal{M}_i\right), \end{aligned} \tag{23}$$

such that (17) and

$$\begin{bmatrix} -\alpha_i & \phi_i^T(0) \\ \phi_i(0) & -X_i \end{bmatrix} < 0, \tag{24a}$$

$$\begin{bmatrix} -\mathcal{M}_i & M_i^T \\ M_i & -\bar{S}_i \end{bmatrix} < 0, \tag{24b}$$

$$\begin{bmatrix} -c_i I_{n_i} & I_{n_i} \\ I_{n_i} & -X_i \end{bmatrix} < 0, \tag{24c}$$

where c_i are the given positive constants,

$$M_i M_i^T := \int_{-\tau_i}^{0} \phi_i(s)\phi_i^T(s)\,ds, \qquad N_i N_i^T := \int_{-h_i}^{0} \phi_i(s)\phi_i^T(s)\,ds.$$

That is, the problem addressed in this paper is as follows: "Find $K_i = Y_i X_i^{-1}$, $i = 1, \ldots, N$, such that LMI (17) and (24) are satisfied, and the cost $\sum_{i=1}^{N} \bar{\mathcal{J}}_i$ becomes as small as possible".

Finally, we are in a position to establish the main result of this section.

Theorem 4.2. *If the above optimization problem has the solution μ_i, ε_i, X_i, Y_i, $\bar{S}_i$, Z_i, α_i, and $\mathcal{M}_i$, then the control laws of the form* (18) *are the decentralized linear state feedback control laws, which ensure the minimization of the guaranteed cost* (19) *for the uncertain large-scale interconnected delay systems.*

Proof. By Theorem 4.1, the control laws (18) constructed from the feasible solutions μ_i, ε_i, X_i, Y_i, $\bar{S}_i$, Z_i, α_i, and $\mathcal{M}_i$ are the guaranteed cost controllers of the uncertain large-scale interconnected delay systems (13). Applying the Schur complement to the LMI (24) and using the following inequality [10]:

$$\mathrm{Trace}[\mathcal{X}\mathcal{Y}] \leqslant \|\mathcal{X}\|_2 \mathrm{Trace}[\mathcal{Y}], \quad \mathcal{Y} = \mathcal{Y}^T \geqslant 0,\ \mathcal{X} = \mathcal{X}^T,$$

we have

$$\text{(24a)} \quad \Leftrightarrow \quad \phi_i^T(0) X_i^{-1} \phi_i(0) < \alpha_i,$$

$$\begin{aligned}\text{(24b)} \quad \Rightarrow \quad \int_{-\tau_i}^{0} \phi_i^T(s) \bar{S}_i^{-1} \phi_i(s)\,ds &= \int_{-\tau_i}^{0} \mathrm{Trace}\big[\phi_i^T(s) \bar{S}_i^{-1} \phi_i(s)\big]\,ds \\ &= \mathrm{Trace}\big[M_i^T \bar{S}_i^{-1} M_i\big] < \mathrm{Trace}[\mathcal{M}_i],\end{aligned}$$

$$\begin{aligned}\text{(24c)} \quad \Rightarrow \quad & \int_{-h_i}^{0} \phi_i^T(s) X_i^{-1} Z_i X_i^{-1} \phi_i(s)\,ds \\ &= \int_{-h_i}^{0} \mathrm{Trace}\big[\phi_i^T(s) X_i^{-1} Z_i X_i^{-1} \phi_i(s)\big]\,ds \\ &= \mathrm{Trace}\big[N_i^T X_i^{-1} Z_i X_i^{-1} N_i\big] \leqslant \big\|N_i N_i^T\big\|_2 \cdot \big\|X_i^{-1}\big\|_2^2 \cdot \mathrm{Trace}[Z_i] \\ &< c_i^2 \big\|N_i N_i^T\big\|_2 \cdot \mathrm{Trace}[Z_i].\end{aligned}$$

It follows that

$$\begin{aligned}
\mathcal{J} &< \sum_{i=1}^{N} \Bigg[\phi_i^T(0) X_i^{-1} \phi_i(0) + \int_{-\tau_i}^{0} \phi_i^T(s) \bar{S}_i^{-1} \phi_i(s)\, ds \\
&\quad + \int_{-h_i}^{0} \phi_i^T(s) X_i^{-1} Z_i X_i^{-1} \phi_i(s)\, ds \Bigg] \\
&< \sum_{i=1}^{N} \big(\alpha_i + \mathrm{Trace}[\mathcal{M}_i] + c_i^2 \big\| N_i N_i^T \big\|_2 \cdot \mathrm{Trace}[Z_i] \big) \\
&\leqslant \min_{\mathcal{X}_i} \sum_{i=1}^{N} \bar{\mathcal{J}}_i = \mathcal{J}^*.
\end{aligned} \tag{25}$$

Thus, the minimization of $\sum_{i=1}^{N} \bar{\mathcal{J}}_i$ implies the minimum value $\mathcal{J}^*$ of the guaranteed cost for the interconnected uncertain delay systems (13). The optimality of the solution of the optimization problem follows from the convexity of the objective function under the LMI constraints. This is the required result. □

Remark 4.1. It can be noted that the original optimization problem for the guaranteed cost (23) can be decomposed to the following reduced optimization problems (26) because each optimization problem (26) is independent of other LMI. Hence, we only have to solve the optimization problems (26) for each independent subsystem:

$$\begin{aligned}
&\min_{\mathcal{X}_i} \sum_{i=1}^{N} \bar{\mathcal{J}}_i = \sum_{i=1}^{N} \min_{\mathcal{X}_i} \bar{\mathcal{J}}_i, \\
&\mathcal{X}_i \in \big(\mu_i, \varepsilon_i X_i, Y_i, \bar{S}_i, Z_i, \alpha_i, \mathcal{M}_i\big), \quad \mathcal{D}_i\colon \min_{\mathcal{X}_i} \bar{\mathcal{J}}_i,\ i = 1, \ldots, N, \\
&\bar{\mathcal{J}}_i := \alpha_i + \mathrm{Trace}[\mathcal{M}_i] + c_i^2 \big\| N_i N_i^T \big\|_2 \cdot \mathrm{Trace}[Z_i].
\end{aligned} \tag{26}$$

Remark 4.2. The constant parameter c_i, which is included in the inequality (24c), needs to be optimized as the LMI constraints. In this case, it is hard to obtain the optimum guaranteed cost, because the resulting problem is nonconvex optimization problem. Hence, we propose the above suboptimal guaranteed cost control instead of solving the nonconvex optimization problem. As a result, the decentralized robust suboptimal guaranteed cost controller, which minimizes the value of the guaranteed cost for the closed-loop uncertain delay systems, can be easily solved by using the LMI.

The chosen constant parameter c_i needs to be as small as possible. However, if there is no solution to the considered optimization problem, we need to consider the large parameter c_i. On the other hand, it should be noted that the parameter c_i cannot become large, because the matrix X_i is constrained by the inequality (24a).

5. Conclusions

In this paper, a solution to the guaranteed cost control problem for the nonlinear large-scale uncertain systems that have delay in both state and control input has been presented. The decentralized robust optimal guaranteed cost controller, which minimizes the value of the guaranteed cost for the closed-loop uncertain delay systems, can be solved by using software such as MATLAB's LMI control toolbox. Thus, the resulting decentralized linear feedback controller can guarantee the quadratic stability and the optimal cost bound for these uncertain large-scale delay systems. On the other hand, there exist drawbacks that cannot be ignored. In view of the practical systems, since the considered problem has to be solvable, some of the bounds for the uncertainties will turn out to be quite conservative. That is, in order to guarantee the existence of the LMI solution, the bounds for the uncertainties have to be small. Consequently, we need to relax these conservative conditions. Furthermore, in order to obtain the control gain matrix, all information for the subsystems is needed. These conditions have to be removed because there is no guarantee that we can always obtain the subsystems information. However, it is worth pointing out that although similar problems have recently been solved, the guaranteed cost control problem for the nonlinear large-scale uncertain delay systems that have delay in both state and control input via the LMI technique has not been investigated so far.

In future research, it is expected that the LMI approach will also be applied to the output feedback case [7]. This problem is more realistic than that of the state feedback case and will be addressed in future investigations.

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