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Robust H_∞ Control Problem for Nonstandard Singularly Perturbed Systems Under Perfect State Measurements

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Abstract

This paper considers the robust H_∞ control problem for nonstandard singularly perturbed systems with time-varying norm-bounded parameter uncertainties in state equations. In order to obtain the controller such that both robust stability and a disturbance attenuation level γ larger than a arbitrary boundary value are achieved, irrespective of uncertainties, we must solve algebraic Riccati equation with the singular perturbation parameter $\varepsilon > 0$. The main results in this paper are to propose a new recursive algorithm to solve the above equation and to find an ε independent sufficient conditions for the existence of the full-order dynamic controller. Using the algebraic Riccati equation approach, although the uncertain matrix $A_{22} + \Delta A_{22}(t)$ has unstable mode, our new results are applicable to both standard and nonstandard uncertain singularly perturbed systems. Furthermore, in order to show the effectiveness of the proposed algorithms, numerical examples are included.

1. Introduction

In recent years, some papers have considered the problem of stabilizing the singularly perturbed systems with uncertain parameters. Shi et *al.*(1998) have studied the robust H_∞ control problem for standard singularly perturbed systems by making use of singular perturbation methods. It is found that the basic assumption in Shi et *al.*(1998) that the state matrix $A_{22} + \Delta A_{22}(t)$ for the fast subsystem is stable play an important role in the study of the problem, where $\Delta A_{22}(t)$ is uncertainty. However, this assumption has often been found to be too conservative in applications for the practical systems because they contain uncertainties.

In this paper, we considers the robust H_∞ control problem for nonstandard singularly perturbed systems under perfect state measurements with time-varying norm-bounded parameter uncertainties in state equations. In order to obtain the controller such that both robust stability and a disturbance attenuation level γ larger than a arbitrary boundary value are achieved, irrespective of uncertainties, we must solve algebraic Riccati equation with the singular perturbation parameter $\varepsilon > 0$. The main results in this paper are to propose a new algorithm to solve the above equation and to find an ε independent sufficient conditions for the existence of the full-order controller. Therefore, our new results are applicable to the non-standard singularly perturbed systems such that the fast dynamics has unstable mode.

2. Problem Formulation and Primary Results

Consider a class of uncertain singularly perturbed systems

$$\dot{x}_1(t) = [A_{11} + \Delta A_{11}(t)]x_1(t) + [A_{12} + \Delta A_{12}(t)]x_2(t) + B_{11}w(t) + [B_{12} + \Delta B_{12}(t)]u(t), \quad x_1(0) = 0, \tag{1a}$$

$$\varepsilon\dot{x}_2(t) = [A_{21} + \Delta A_{21}(t)]x_1(t) + [A_{22} + \Delta A_{22}(t)]x_2(t) + B_{21}w(t) + [B_{22} + \Delta B_{22}(t)]u(t), \quad x_2(0) = 0, \tag{1b}$$

$$z(t) = C_{11}x_1(t) + C_{12}x_2(t) + D_{12}u(t), \tag{1c}$$

where ε is a small positive parameter, $x_1(t) \in \mathbf{R}^{n_1}$ and $x_2(t) \in \mathbf{R}^{n_2}$ are state vectors, $u(t) \in \mathbf{R}^{m_1}$ is the control input, $w(t) \in \mathbf{R}^{l_1}$ is the disturbance, $z(t) \in \mathbf{R}^{l_2}$ is the controlled output, $\Delta A_{ij}(t) \in \mathbf{R}^{k\times j}$ is a Lebesgue measurable matrix of uncertain parameters. All matrices above are of appropriate dimensions. The system (1) is said to be in the standard form if the matrix A_{22} is nonsingular. Otherwise, it is called the non–standard singularly perturbed systems.

The admissible parameter uncertainties are of the form

$$\begin{bmatrix} \Delta A_{11}(t) & \Delta A_{12}(t) & \Delta B_{12}(t) \\ \Delta A_{21}(t) & \Delta A_{22}(t) & \Delta B_{22}(t) \end{bmatrix} = \begin{bmatrix} H_{a1} \\ H_{a2} \end{bmatrix} F(t) \begin{bmatrix} E_{a1} & E_{a2} & E_b \end{bmatrix}, \tag{2a}$$

$$F^T(t)F(t) \le I_s, \quad F(t) \in \mathbf{R}^{p\times s}. \tag{2b}$$

Let us introduce the partitioned matrices

$$x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) \end{bmatrix}^T \in \mathbf{R}^n, \quad n = n_1 + n_2,$$

$$A_\varepsilon = \begin{bmatrix} A_{11} & A_{12} \\ \varepsilon^{-1}A_{21} & \varepsilon^{-1}A_{22} \end{bmatrix}, \quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

$$B_{1\varepsilon} = \begin{bmatrix} B_{11} \\ \varepsilon^{-1}B_{21} \end{bmatrix}, \quad B_1 = \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix},$$

$$B_{2\varepsilon} = \begin{bmatrix} B_{12} \\ \varepsilon^{-1}B_{22} \end{bmatrix}, \quad B_2 = \begin{bmatrix} B_{12} \\ B_{22} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix},$$

$$\Delta A_\varepsilon(t) = \begin{bmatrix} \Delta A_{11}(t) & \Delta A_{12}(t) \\ \varepsilon^{-1}\Delta A_{21}(t) & \varepsilon^{-1}\Delta A_{22}(t) \end{bmatrix},$$

$$\Delta B_{2\varepsilon}(t) = \begin{bmatrix} \Delta B_{21}(t) \\ \varepsilon^{-1}\Delta B_{22}(t) \end{bmatrix},$$

$$H_{a\varepsilon} = \begin{bmatrix} H_{a1} \\ \varepsilon^{-1}H_{a2} \end{bmatrix}, \quad H_a = \begin{bmatrix} H_{a1} \\ H_{a2} \end{bmatrix},$$

$$E_a = \begin{bmatrix} E_{a1} & E_{a2} \end{bmatrix}.$$

Then equations (1) can be rewritten as

$$\dot{x}(t) = [A_\varepsilon + \Delta A_\varepsilon(t)]x(t) + B_{1\varepsilon}w(t) + [B_{2\varepsilon} + \Delta B_{2\varepsilon}(t)]u(t), \quad x(0) = 0, \tag{3a}$$

$$z(t) = C_1x(t) + D_{12}u(t). \tag{3b}$$

For technical simplification, without loss of generality we shall make the following basic assumption (Xie *et al.* 1992).

Assumption 1
$D_{12}^T \begin{bmatrix} C_1 & D_{12} \end{bmatrix} = \begin{bmatrix} 0 & I_m \end{bmatrix}$

For a given $\gamma > 0$, if there exists a fixed linear time–invariant state feedback law $u = K_\varepsilon x$ such that for any admissible parameter uncertainty $\Delta A_\varepsilon(t)$ and $\Delta B_{2\varepsilon}(t)$ the following conditions are satisfied, then the uncertain singularly perturbed system (1) is said to be quadratically stable with an H_∞ norm bound γ (abbreviated as QS–H_∞–γ).

i) The closed–loop system is uniformly asymptotically stable.

ii) The uncertain singularly perturbed system (1) has an H_∞ norm bound $\gamma > 0$ in the sense

$$\|z(t)\|_2 < \gamma\|w(t)\|_2 \tag{4}$$

for any square integrable signal w with zero initial conditions.

The following lemma was shown by Gu (1994).

Lemma 1 *The singularly perturbed system (1) is $QS\text{-}H_\infty\text{-}\gamma$ if and only if there exists a $\lambda > 0$ such that the following system without uncertainties*

$$
\begin{aligned}
\dot{x}(t) &= A_\varepsilon x(t) \\
&\quad + \begin{bmatrix} B_{1\varepsilon} & \gamma\lambda H_{a\varepsilon} \end{bmatrix} \begin{bmatrix} w(t) \\ \hat{w}(t) \end{bmatrix} \\
&\quad + B_{2\varepsilon} u(t), \quad x(0) = 0, \qquad \text{(5a)} \\
\begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} &= \begin{bmatrix} C_1 \\ \frac{1}{\lambda} E_a \end{bmatrix} x(t) \\
&\quad + \begin{bmatrix} D_{12} \\ \frac{1}{\lambda} E_b \end{bmatrix} u(t) \qquad \text{(5b)}
\end{aligned}
$$

is *stable with H_∞ norm less than γ, that is, for any square integrable auxiliary signal w_e with zero initial conditions the uncertain singularly perturbed system (5) has an H_∞ norm bound $\gamma > 0$ in the sense*

$$\|z_e(t)\|_2 < \gamma \|w_e(t)\|_2 \qquad (6)$$

where

$$z_e = \begin{bmatrix} z \\ \hat{z} \end{bmatrix} \in \mathbf{R}^{l_2+s}, \; w_e = \begin{bmatrix} w \\ \hat{w} \end{bmatrix} \in \mathbf{R}^{l_1+p}$$

Define

$$
\begin{aligned}
B_{1\gamma\lambda\varepsilon} &= \begin{bmatrix} B_{1\varepsilon} & \gamma\lambda H_{a\varepsilon} \end{bmatrix}, \\
C_{1\lambda} &= \begin{bmatrix} C_1 \\ \frac{1}{\lambda} E_a \end{bmatrix}, \; D_{12\lambda} = \begin{bmatrix} D_{12} \\ \frac{1}{\lambda} E_b \end{bmatrix}.
\end{aligned}
$$

The equations (5) can be also rewritten as

$$
\begin{aligned}
\dot{x}(t) &= A_\varepsilon x(t) + B_{1\gamma\lambda\varepsilon} w_e(t) \qquad \text{(7a)} \\
&\quad + B_{2\varepsilon} u(t), \; x(0) = 0, \\
z_e(t) &= C_{1\lambda} x(t) + D_{12\lambda} u(t) \qquad \text{(7b)}
\end{aligned}
$$

We shall make the following basic structure assumptions for the full-order systems (7), which are typical in the H_∞ control problem.

Assumption 2 **A1.** *The pair $(A_\varepsilon, B_{2\varepsilon})$ is stabilizable and (C_1, A_ε) is detectable for $\varepsilon \in (0, \varepsilon^*]$ $(\varepsilon^* > 0)$.*

A2. $\operatorname{rank} \begin{bmatrix} D_{12} \\ E_b \end{bmatrix} = m_1$

A3. $\operatorname{rank} \begin{bmatrix} A - sI_n & B_2 \\ C_1 & D_{12} \\ E_a & E_b \end{bmatrix} = n + m_1$

where $s = j\omega$, $w \in \mathbf{R}^+$.

The $QS\text{-}H_\infty\text{-}\gamma$ condition can be written in a more convenient form (Shi *et al.* 1998).

Lemma 2 A *system (1) is $QS\text{-}H_\infty\text{-}\gamma$ if and only if there exists a symmetric positive semidefinite stabilizing solution X_ε such, that*

$$
\begin{aligned}
&A_\varepsilon^T X_\varepsilon + X_\varepsilon A_\varepsilon + \gamma^{-2} X_\varepsilon B_{1\gamma\lambda\varepsilon} B_{1\gamma\lambda\varepsilon}^T X_\varepsilon \\
&\quad - (X_\varepsilon B_{2\varepsilon} + C_{1\lambda}^T D_{12\lambda})(D_{12\lambda}^T D_{12\lambda})^{-1} \\
&\quad \cdot (B_{2\varepsilon}^T X_\varepsilon + D_{12\lambda}^T C_{1\lambda}) + C_{1\lambda}^T C_{1\lambda} = 0 \quad (8)
\end{aligned}
$$

where

$$X_\varepsilon = \begin{bmatrix} X_{11} & \varepsilon X_{21}^T \\ \varepsilon X_{21} & \varepsilon X_{22} \end{bmatrix} \geq 0,$$

and

$$
\begin{aligned}
&A_\varepsilon + \gamma^{-2} B_{1\gamma\lambda\varepsilon} B_{1\gamma\lambda\varepsilon}^T X_\varepsilon \\
&\quad - B_{2\varepsilon}(D_{12\lambda}^T D_{12\lambda})^{-1}(B_{2\varepsilon}^T X_\varepsilon + D_{12\lambda}^T C_{1\lambda})
\end{aligned}
$$

is stable. Moreover, a suitable feedback control law is given by

$$
\begin{aligned}
u(t) &= K_\varepsilon x(t) \qquad (9) \\
K_\varepsilon &= -(D_{12\lambda}^T D_{12\lambda})^{-1}(B_{2\varepsilon}^T X_\varepsilon + D_{12\lambda}^T C_{1\lambda})
\end{aligned}
$$

In order to solve the algebraic Riccati equation (8), we introduce the following generalized algebraic Riccati equation (10).

$$
\begin{aligned}
&A^T X + X^T A + \gamma^{-2} X^T B_{1\gamma\lambda} B_{1\gamma\lambda}^T X \\
&\quad - (X^T B_2 + C_{1\lambda}^T D_{12\lambda})(D_{12\lambda}^T D_{12\lambda})^{-1} \\
&\quad . (B_2^T X + D_{12\lambda}^T C_{1\lambda}) + C_{1\lambda}^T C_{1\lambda} = 0 \quad (10)
\end{aligned}
$$

where

$$
\begin{aligned}
&X_\varepsilon = \Pi_\varepsilon^T X = X^T \Pi_\varepsilon, \; \Pi_\varepsilon = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \varepsilon I_{n_2} \end{bmatrix}, \\
&B_{1\gamma\lambda} = \begin{bmatrix} B_1 & \gamma\lambda H_a \end{bmatrix}, \\
&X = \begin{bmatrix} X_{11} & \varepsilon X_{21}^T \\ X_{21} & X_{22} \end{bmatrix}, \\
&X_{11} = X_{11}^T, \; X_{22} = X_{22}^T, \; A_\varepsilon = \Pi_\varepsilon^{-1} A, \\
&B_{1\gamma\lambda\varepsilon} = \Pi_\varepsilon^{-1} B_{1\gamma\lambda}, \; B_{2\varepsilon} = \Pi_\varepsilon^{-1} B_2.
\end{aligned}
$$

3. Main Results

In this section, we study the robust H_∞ control problem by using the state feedback control law for the linear time-invariant singularly perturbed system (5).

Let us define the following partition matrices

$$\begin{aligned} A_X &= A - B_2(D_{12\lambda}^T D_{12\lambda})^{-1} D_{12\lambda}^T C_{1\lambda} \\ &= \begin{bmatrix} A_{X11} & A_{X12} \\ A_{X21} & A_{X22} \end{bmatrix}, \qquad (11a) \\ R_X &= B_2(D_{12\lambda}^T D_{12\lambda})^{-1} B_2^T \\ &\quad -\gamma^{-2} B_{1\gamma\lambda} B_{1\gamma\lambda}^T \\ &= \begin{bmatrix} R_{X11} & R_{X12} \\ R_{X12}^T & R_{X22} \end{bmatrix}, \qquad (11b) \\ Q_X &= C_{1\lambda}^T [I_{l_2+s} \\ &\quad -D_{12\lambda}(D_{12\lambda}^T D_{12\lambda})^{-1} D_{12\lambda}^T] C_{1\lambda} \\ &= \begin{bmatrix} Q_{X11} & Q_{X12} \\ Q_{X12}^T & Q_{X22} \end{bmatrix}. \qquad (11c) \end{aligned}$$

Then, it follows that

$$X^T A_X + A_X^T X - X^T R_X X + Q_X = 0. \quad (12)$$

The generalized algebraic Riccati equation (12) will produce the unique positive definite stabilizing solution under the following conditions.

Assumption 3 A4. The pair $(A22, B22)$ is ***stabilizable and*** (C_{12}, A_{22}) ***is observable.***

A5. rank $\begin{bmatrix} A_{22} - sI_{n_2} & B_{22} \\ C_{12} & D_{12} \\ E_{a2} & E_b \end{bmatrix} = n_2 + m_1,$

A6. rank $\begin{bmatrix} \mathcal{A}(s) & B_2 \\ C_1 & D_{12} \\ E_a & E_b \end{bmatrix} = n + m_1,$

where $\mathcal{A}(s) = \begin{bmatrix} A_{11} - sI_{n_1} & A_{12} \\ A_{22} & A22 \end{bmatrix}$

By substituting X into (12) and letting $\varepsilon = 0$, we obtain the following O-order equations

$$\begin{aligned} &A_{X0}^T \bar{X}_{11} + \bar{X}_{11}^T A_{X0} \\ &\quad - \bar{X}_{11}^T R_{X0} \bar{X}_{11} + Q_{X0} = 0, \qquad (13a) \\ &A_{X22}^T \bar{X}_{22} + \bar{X}_{22}^T A_{X22} \\ &\quad - \bar{X}_{22}^T R_{X22} \bar{X}_{22} + Q_{X22} = 0, \qquad (13b) \\ &X_{21} = -N_2^T + N_1^T \bar{X}_{11}, \qquad (13c) \end{aligned}$$

where

$$\begin{aligned} A_{X0} &= A_{X11} + N_1 A_{X21} + R_{X12} N_2^T \\ &\quad + N_1 R_{X22} N_2^T, \\ R_{X0} &= R_{X11} + N_1 R_{X12}^T + R_{X12} N_1^T \\ &\quad + N_1 R_{X22} N_1^T, \\ Q_{X0} &= Q_{X11} - N_2 A_{X21} - A_{X21}^T N_2^T \\ &\quad - N_2 R_{X22} N_2^T, \\ N_2^T &= T_{X4}^{-T} L_{X12}^T, \quad N_1^T = -T_{X4}^{-T} T_{X2}^T, \\ T_{X1} &= A_{X11} - R_{X11} \bar{X}_{11} - R_{X12} \bar{X}_{21}, \\ T_{X3} &= A_{X21} - R_{X12}^T \bar{X}_{11} - R_{X22} \bar{X}_{21}, \\ T_{X2} &= A_{X12} - R_{X12} \bar{X}_{22}, \\ T_{X4} &= A_{X22} - R_{X22} \bar{X}_{22}, \\ L_{X12} &= Q_{X12} + A_{X21}^T \bar{X}_{22}, \\ T_{X0} &= T_{X1} - T_{X2} T_{X4}^{-1} T_{X3}. \end{aligned}$$

Now we are in a position to establish our main result in this paper.

Theorem 1 ***Consider the system (1) under Assumptions 1-3. Then, there exists sufficient small*** ε^* ***such that*** $\forall \varepsilon \in [0, \varepsilon^*)$, ***the algebraic Riccati equation (8) admits a positive semidefinite stabilizing solution if there exists a positive scalar*** λ ***such that the reduced-order Riccati equations (13a) and (13b) have positive semidefinite atabilizing solutions*** $\bar{X}_{11} \geq 0$ ***and*** $\bar{X}_{22} \geq 0$ ***respectively. Furthermore, minimal such solution of (8) can be approximated by***

$$X_\varepsilon = \begin{bmatrix} \bar{X}_{11} & \varepsilon \bar{X}_{21}^T \\ \varepsilon \bar{X}_{21} & \varepsilon \bar{X}_{22} \end{bmatrix} + \begin{bmatrix} \varepsilon E_{11}(\varepsilon) & \varepsilon^2 E_{21}^T(\varepsilon) \\ \varepsilon^2 E_{21}(\varepsilon) & \varepsilon^2 E_{22}(\varepsilon) \end{bmatrix}. \quad (14)$$

Proof: As a starting point we need to show the existence of a bounded solution of ***E*** in neighborhood of $\varepsilon = 0$. To prove that by the implicit function theorem, it is enough to show that the corresponding Jacobian is non-singular at $\varepsilon = 0$. For the equations (13), the Jacobian is given by

$$J|_{\varepsilon=0} = \begin{bmatrix} J_{11} & 0 & 0 \\ J_{21} & J_{22} & J_{23} \\ 0 & 0 & J_{33} \end{bmatrix} \qquad (15)$$

where, using the Kronecker products representation we have

$$\begin{aligned} J_{11} &= I \otimes T_{X0} + T_{X0}^T \otimes I, \\ J_{22} &= I \otimes T_{X4}, \\ J_{33} &= I \otimes T_{X4} + T_{X4}^T \otimes I. \end{aligned}$$

The matrix T_{X4} *is* non-singular because the reduced-order Riccati equation (13b) has positive semidefinite atabilizing solutions $\bar{X}_{22} \geq 0$. On the other hand, the matrix $A_{X0} - R_{X0}\bar{X}_{11}$ is also non-singular if the reduced-order Riccati equation (13a) has positive semidefinite atabilizing solutions $\bar{X}_{11} \geq 0$. Therefore, we obtain

$$A_{X0} - R_{X0}\bar{X}_{11} = T_{X0}. \qquad (16)$$

Since the matrix T_{X0} *is* stable too for a small parameter ε, we find that the corresponding Jacobian is non-singular at $\varepsilon = 0$. Then, by using the implicit function theorem, there exists ε^* such that the solutions $X_{11} = \bar{X}_{11} + O(\varepsilon)$, $X_{21} = \bar{X}_{21} + O(\varepsilon)$ and $X_{22} = \bar{X}_{22} + O(\varepsilon)$ are satisfied by the Riccati equation (8) for $\forall \varepsilon \in [0, \varepsilon^*)$. Thus, we have (14). □

Remark 1 ***It can be easily shown that if there exists positive scalar λ such that the Riccati equations (13a) and (13b) have positive semidefinite stabilizing solutions, then the uncertain singularly perturbed system with controller (9) is robust stabilizable and has a robust H_∞ performance γ.***

4. *New* Recursive Algorithm

In the rest of this section, we develop an elegant and simple algorithm which converges to the positive semidefinite stabilizing solution of (8). The proposed recursive algorithm is given in term of the standard algebraic Riccati equations, which have to be solved iteretively.

We propose the following new recursive algorithm for solving (8) with parameter ε.

$$\begin{aligned} &[A_X - R_X X^{(i)}]^T X^{(i+1)} \\ &\quad + X^{(i+1)T}[A_X - R_X X^{(i)}] \\ &\quad + X^{(i)T} R_X X^{(i)} + Q_X = 0, \end{aligned} \qquad (17)$$

with the initial condition obtained from

$$X^{(0)} = \begin{bmatrix} \bar{X}_{11} & \varepsilon \bar{X}_{21}^T \\ \bar{X}_{21} & \bar{X}_{22} \end{bmatrix} \qquad (18)$$

where $\bar{X}_{11}, \bar{X}_{21}$ and $\bar{X}_{22}$ are defined by (13).

The properties of this new recursive algorithm are stated in the next theorem.

Theorem 2 ***Under the stabilizability and detectability conditions, imposed in Assumptions 1-3, the new algorithm (17) converges to the exact solution of X^* with the rate of quadratic convergence. That is,***

$$\lim_{i\to\infty} \frac{\|X^{(i+1)} - X^*\|}{\|X^{(i)} - X^*\|} = 0, \qquad (19a)$$

$$\|X^{(i+1)} - X^*\| \leq \mathcal{M}\|X^{(i)} - X^*\|^2,$$

$$(0 < M < \infty),$$

$$\Leftrightarrow \|X^{(i)} - X^*\| = O(\varepsilon^{2^i}), \qquad (19b)$$

$$\|X^{(i)}\| \leq c < \infty, \qquad (19c)$$

where

$$X = X^* = \begin{bmatrix} X_{11} & \varepsilon X_{21}^T \\ X_{21} & X_{22} \end{bmatrix},$$

$$X^{(i)} = \begin{bmatrix} X_{11}^{(i)} & \varepsilon X_{21}^{T(i)} \\ X_{21}^{(i)} & X_{22}^{(i)} \end{bmatrix}.$$

Proof: We show that under Assumption 1 the algorithm (17) converges to the desired solution of (8). Let us define

$$\begin{aligned} &F(X_\varepsilon) \\ &= A_\varepsilon^{\mu T} X_\varepsilon + X_\varepsilon A_{X\varepsilon} - X_\varepsilon R_{X\varepsilon} X_\varepsilon + Q_X. \end{aligned}$$

If Assumption 1 hold, then $F(X_\varepsilon) = 0$, i.e., *(8)* has positive definite stabilizing solution

$X_\varepsilon = X_\varepsilon^* = \Pi_\varepsilon^T X^*$. Taking the partial differentiation of the function $F(X_\varepsilon)$ with respect to X_ε yields

$$\nabla F(X_\varepsilon)|_{X_\varepsilon = X_\varepsilon^*} = \frac{\partial F(X_\varepsilon)}{\partial X_\varepsilon}|_{X_\varepsilon = X_\varepsilon^*}$$
$$= (A_{X\varepsilon} - R_{X\varepsilon} X_\varepsilon^*)^T \otimes I_n$$
$$+ I_n \otimes (A_{X\varepsilon} - R_{X\varepsilon} X_\varepsilon^*)^T.$$

It is obvious that $\nabla F(X_\varepsilon)$ is continuous at X_ε^*, and that $\nabla F(X_\varepsilon^*)$ is nonsingular since (8) has stabilizing solution X_ε^*. Therefore; X_ε^* is the point of attraction of the iteration (17) since $X_\varepsilon^{(0)}$ is sufficiently close to X_ε^*. Moreover, it is obtained immediately from above equation that

$$\|\nabla F(X_\varepsilon) - \nabla F(X_\varepsilon^*)\| \le \mathcal{L} \|X_\varepsilon - X_\varepsilon^*\|,$$
$$0 < \mathcal{L} < \infty.$$

Taking into account for $\|X_\varepsilon^{(i)} - X_\varepsilon^*\| = O(\varepsilon)$ and $\mathcal{A}_\varepsilon(i)$ is stable for all $i \in$ N, there exists a constant $M < \infty$ such that $\|X^{(i+1)} - X^*\| \le \mathcal{M} \|X^{(i)} - X^*\|^2$. This is equivalent to the following equation.

$$\|X^{(i)} - X^*\| \le \mathcal{M}^{2^i - 1} \|X^{(0)} - X^*\|^{2^i}$$
$$= O(\varepsilon^{2^i}).$$

As a result, we found that the sequence $\{X_\varepsilon^{(i)}\} = \{\Pi_\varepsilon X^{(i)}\}, (i = 1, 2, 3, \cdots)$ is quadratic convergence by using Lemma 81.10 in Ortega (1990) even though the matrix $R_{X\varepsilon}$ is in general indefinite. cl

Comparing with Mukaidani et al. (1999), since the proposed algorithm is quadratic convergence, the required solution can be easily obtained up to an arbitrary order of accuracy, that is, $O(\varepsilon^{2^i})$ where i is a iteration number.

5. Conclusion

In this paper: we have considered the robust H_∞ control problem for nonstandard singularly perturbed systems under prefect state measurements. We have provided a controller such that both robust stability and a disturbance attenuation level γ larger than a arbitrary boundary value are achieved, irrespective of uncertainties. It has been also shown that a new recursive algorithm to solve the generalized algebraic Riccati equations was proposed and the ε-independent sufficient condition for the existence of the full-order dynamic controller was derived.

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