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Guaranteed Cost Control for Large-Scale Systems under Control Gain Perturbations

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SUMMARY

The guaranteed cost control problem of the decentralized robust control for large-scale systems with the norm-bounded time-varying parameter uncertainties and a given quadratic cost function is considered. Sufficient conditions for the existence of guaranteed cost controllers are given in terms of linear matrix inequality (LMI). It is shown that decentralized local state feedback controllers can be obtained by solving the LMI. The problem of guaranteed cost control for large-scale systems under the gain perturbations is also considered. © 2004 Wiley Periodicals, Inc. Electr Eng Jpn, 146(4): 43–57, 2004; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/eej.10265

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1. Introduction

In recent years, the decentralized robust control of large-scale systems with parameter uncertainties and disturbance has been widely studied [1–7]. In Refs. 1–3, for multimachine power systems, a decentralized stabilizing nonlinear state feedback controller using the algebraic Riccati equation (ARE) approach and Lyapunov function approach has been proposed. Furthermore, in Ref. 4, the results developed in Ref. 1 have been extended to the class of large-scale interconnected nonlinear systems via the robust decentralized linear control. On the other hand, in Ref. 7, the decentralized exciter stabilizing control for multimachine power systems by means of the H_∞ control has been considered.

In general, in the case of the control problem of large-scale systems with parameter uncertainties, it is desirable to design the control systems that guarantee not only the robust stability, but also an adequate level of performance. One approach to this problem is the so-called quadratic guaranteed cost control approach [8–13]. This approach has the advantage of providing an upper bound on a given performance index. It is well-known that there exist two different approaches for solving the quadratic guaranteed cost control approach. One is the ARE approach [8–10] and the other one is the LMI (Linear Matrix Inequality) [16–18] design method [11–13]. Recently, the quadratic guaranteed cost control for the class of large-scale interconnected nonlinear systems has been proposed via the state feedback control [12, 13]. Very recently, in the case where one applies the proposed controller to the practical system, it is shown that the robustness of the closed-loop system is not guaranteed without the margin of the controller gain perturbations because controller gain perturbations such as the modeling errors of the actuator/sensor or parameter perturbations arise [14]. Moreover, the following result is known. It is necessary that any controller that is part of a closed-loop system be able to tolerate some uncertainty in its coefficients [15]. There are at least two reasons for this. First, controller implementation is subject to the imprecision inherent in analog–digital and digital–analog conversion, finite word length, and finite resolution measuring instruments, and to roundoff errors in numerical computations. Thus, it is required that there exists a nonzero tolerance margin (although possibly small) around the controller designed. Second, every existing result for past years requires readjustment because no scalar index can capture all of the performance requirements of a control system. This means that any useful design procedure should generate a controller which also has sufficient room for readjustment of its coefficients. So far, the quadratic guaranteed cost control under controller gain additive perturbation for large-scale systems has never been studied except for our report [13].

In this paper, the decentralized quadratic guaranteed cost controller via state feedback is applied to a class of large-scale systems with norm-bounded parameter uncertainties. First, in order to understand the basic properties of the LMI, we will study the decentralized quadratic guaranteed cost control problem under the existing result [13]. Second, the decentralized quadratic guaranteed cost control problem under gain perturbations is solved. The main contribution of this paper is to construct the decentralized quadratic guaranteed cost controller by solving the parameter-dependent LMI. The crucial difference between the large-scale systems in Refs. 12 and 13 and our considered systems is that the controller gain perturbation is newly added. Moreover, in Ref. 7, the parameter uncertainties of the large-scale systems have not been studied, while we will assume that the norm-bounded parameter uncertainties are included in the large-scale systems. Therefore, we can construct the robust controller for more practical large-scale systems. Furthermore, it is possible to construct the robust controller independently from other interconnected subsystems by means of the proposed design method. Thus, the proposed controller design method is useful in the sense that the desired controller can be obtained as the similar technique in the optimal control problem of the large-scale systems [19, 20]. It should be noted that the problem of the quadratic guaranteed cost control for large-scale systems under gain perturbations is also considered in Ref. 13. However, although there exists a result, there is no proof of it. Moreover, the uncertainties of the interconnected systems are not considered and the conservative assumption called matching condition is imposed. Taking these drawbacks into account, the matching condition of the control gain perturbations could be relaxed.

The notations used in this paper are fairly standard. $\mathcal{S}^T$ denotes the transpose of matrix $\mathcal{S}$. **block – diag** denotes the block diagonal matrix. Trace $\mathcal{S}$ denotes the trace of matrix $\mathcal{S}$. $I_n \in \mathbf{R}^{n\times n}$ denote the identity matrices. $E[\cdot]$ denotes the expectation.

2. Case of No Existence of the Control Gain Perturbations

We first establish the sufficient condition for the existence of the quadratic guaranteed cost control for the large-scale systems without the uncertainties. Let us consider the following uncertain large-scale interconnected systems. It should be noted that there exist the uncertainties for the considered systems compared with the existing result [7]. Furthermore, the interconnected systems with the uncertainties are considered compared with Ref. 13:

$$\begin{aligned}\dot{x}_i(t) &= [A_i + \Delta A_i(t)]x_i(t) + [B_i + \Delta B_i(t)]u_i(t) \\ &\quad + \sum_{j=1,\ j\neq i}^{N} [A_{ij} + \Delta A_{ij}(t)]x_j(t) \qquad \text{(1a)}\\ x_i(t) &= x_i(0),\ i = 1,\ \cdots,\ N \qquad \text{(1b)}\end{aligned}$$

where $x_i \in \mathbf{R}^{n_i}$ and $u_i \in \mathbf{R}^{m_i}$ are the state and control of the i-th subsystems, respectively. A_i and B_i are constant matrices of appropriate dimensions and A_{ij} are interconnection constant matrices between the i-th subsystems and other subsystems. The parameter uncertainties considered here are assumed to be of the form

$$\begin{aligned}&\left[\ \Delta A_i(t) \quad \Delta B_i(t) \quad \Delta A_{ij}(t)\ \right] \\ &\quad = D_i F_i(t) \left[\ E_i^1 \quad E_i^2 \quad E_{ij}\ \right] \qquad (2)\end{aligned}$$

where $F_i(t) \in \boldsymbol{R}^{p_i\times r_i}$ are unknown matrix functions with Lebesgue measurable elements and satisfying

$$F_i^T(t)F_i(t) \le I_{r_i} \qquad (3)$$

Associated with system (1a) is the cost function

$$\begin{aligned}J &= \sum_{i=1}^{N} \int_0^{\infty} [x_i^T(t)Q_i x_i(t) + u_i^T(t)R_i u_i(t)]dt, \\ Q_i &= Q_i^T > 0,\ R_i = R_i^T > 0 \qquad (4)\end{aligned}$$

We give the definition of the quadratic guaranteed cost control for the large-scale systems (1) and the cost function (4).

Definition 1 *Consider the large-scale systems (1). A decentralized control law $u_i(t) = K_i x_i(t),\ i = 1, \ldots, N,$ is said to be a quadratic guaranteed cost control for the uncertain large-scale interconnected systems (1) and the cost function (4) if the closed-loop systems are quadratically stable and the closed-loop value of the cost function (4) satisfies the bound $J \le \mathcal{J}$ for all admissible uncertainties and $x_i(t) \neq 0$.*

The following theorem gives the sufficient condition for existence of the quadratic guaranteed cost control.

Theorem 1 *Consider the large-scale interconnected systems (1) with the uncertainties (2) and (3). If there exist the symmetric positive definite matrices $P_i \in \mathbf{R}^{n_i\times n_i}$, such that for all uncertain matrices $F_i(t)$, the matrix inequality (5) is satisfied, the control laws $u_i(t) = K_i x_i(t),\ i = 1, \ldots, N,$ are said to be the guaranteed cost controller*:

$$\mathcal{M}_i = \begin{bmatrix} \Theta_i & P_i\tilde{A}_{i1} & \cdots & P_i\tilde{A}_{iN} \\ \tilde{A}_{i1}^T P_i & -I_{n_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{iN}^T P_i & 0 & \cdots & -I_{n_N} \end{bmatrix} < 0 \qquad (5)$$

where there exists no matrix $\mathcal{M}_i$ in $P_i\tilde{A}_{ii}$ and

$$\begin{aligned}&\mathcal{M}_i \in \mathbf{R}^{\tilde{N}\times\tilde{N}},\ \tilde{N} := \sum_{j=1}^{N} n_j \\ &\Theta_i := \tilde{A}_i^T P_i + P_i\tilde{A}_i + \bar{R}_i + (N-1)I_{n_i} \\ &\bar{A}_i := A_i + B_iK_i,\ \bar{E}_i := E_i^1 + E_i^2K_i\end{aligned}$$

$$\tilde{A}_i := \bar{A}_i + D_i F_i(t) \bar{E}_i, \; \bar{R}_i := Q_i + K_i^T R_i K_i$$
$$\tilde{A}_{ij} := A_{ij} + D_i F_i(t) E_{ij}$$

Proof: Combining the guaranteed cost controller $u_i(t) = K_i x_i(t)$ with (1) gives a closed-loop system of the form

$$\dot{x}_i(t) = \tilde{A}_i x_i(t) + \sum_{j=1,\; j\neq i}^{N} \tilde{A}_{ij} x_j(t) \tag{6}$$

Suppose now that there exist the symmetric positive definite matrices $P_i > 0$, $i = 1, \ldots, N$, such that the matrix inequality (5) holds for all admissible uncertainties. In order to prove the asymptotic stability of the closed-loop system (6), let us define the Lyapunov function candidate

$$V(x(t)) = \sum_{i=1}^{N} x_i^T(t) P_i x_i(t) \tag{7}$$

where $x(t) := [x_1^T(t) \cdots x_N^T(t)]^T \in \mathbf{R}^{\bar{N}}$.

Note that $V(x(t)) > 0$ whenever $x(t) \neq 0$. Then the time derivative of $V(x(t))$ along any trajectory of the closed-loop system (6) is given by

$$\begin{aligned}
&\frac{d}{dt}V(x(t)) \\
&= \sum_{i=1}^{N} \Bigg\{ x_i^T(t)[\tilde{A}_i^T P_i + P_i \tilde{A}_i] x_i(t) \\
&\qquad + 2x_i^T(t) P_i \sum_{j=1,\; j\neq i}^{N} \tilde{A}_{ij} x_j(t) \\
&\qquad + \sum_{j=1,\; j\neq i}^{N} [x_j^T(t) x_j(t) - x_i^T(t) x_i(t)] \Bigg\} \\
&= \sum_{i=1}^{N} \xi_i^T(t) \begin{bmatrix} \Theta_i - \bar{R}_i & P_i \tilde{A}_{i1} & \cdots & P_i \tilde{A}_{iN} \\ \tilde{A}_{i1}^T P_i & -I_{n_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{iN}^T P_i & 0 & \cdots & -I_{n_N} \end{bmatrix} \xi_i(t) \\
&= \sum_{i=1}^{N} \xi_i^T(t) \mathcal{M}_i \xi_i(t) - \sum_{i=1}^{N} x_i^T(t) \bar{R}_i x_i(t)
\end{aligned}$$

where

$$\xi_i(t) := \begin{bmatrix} x_i^T(t) \; x_1^T(t) \cdots x_N^T(t) \end{bmatrix}^T \in \mathbf{R}^{\bar{N}}$$

Taking into account the fact that the inequalities (5) hold, it follows immediately that

$$\frac{d}{dt}V(x(t)) < -\sum_{i=1}^{N} x_i^T(t) \bar{R}_i x_i(t) < 0 \tag{8}$$

Hence, $V(x(t))$ is a Lyapunov function for the closed-loop system (6). Therefore, the closed-loop system (6) is asymptotically stable and $u_i(t) = K_i x_i(t)$ is the guaranteed cost controller. Furthermore, by integrating both sides of the inequality (8) from 0 to T and using the initial conditions, we have

$$\begin{aligned}
&\sum_{i=1}^{N} \left(x_i^T(\infty) P_i x_i(\infty) - x_i^T(0) P_i x_i(0) \right) \\
&\quad < -\sum_{i=1}^{N} \int_0^{\infty} x_i^T(t) \bar{R}_i x_i(t) dt = -J
\end{aligned}$$

Since the closed-loop system (6) is asymptotically stable, that is, $x(\infty) \to 0$ or equivalently to $x_i(\infty) = 0$, $i = 1, \ldots\; N$, we obtain $V(x(T)) \to 0$. Thus, we get

$$\begin{aligned}
J &= \sum_{i=1}^{N} \int_0^{\infty} x_i^T(t) \bar{R}_i x_i(t) dt < \sum_{i=1}^{N} x_i^T(0) P_i x_i(0) \\
&= \mathcal{J}
\end{aligned} \tag{9}$$

Therefore, if there exist the positive definite matrices P_i such that the matrix inequalities (5) hold, $u_i(t) = K_i x_i(t)$ are said to be the quadratic guaranteed cost controllers with cost matrices P_i. The proof of Theorem 1 is completed. ■

We now give the LMI design approach for the large-scale systems for constructing the quadratic guaranteed cost controller.

Theorem 2 *Suppose there exist the constant parameters $\mu_i > 0$ such that for all $i = 1, \ldots, N$, the LMI (10) have the symmetric positive definite matrices $X_i > 0 \in \mathbf{R}^{n_i \times n_i}$ and a matrix $Y_i \in \mathbf{R}^{m_i \times n_i}$:*

$$\begin{bmatrix}
\Phi_i & (E_i^1 X_i + E_i^2 Y_i)^T & A_{i1} & 0 & \cdots & A_{iN} & 0 & X_i & Y_i^T & X_i & \cdots & X_i \\
E_i^1 X_i + E_i^2 Y_i & -\mu_i I_{r_i} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
A_{i1}^T & 0 & -I_{n_1} & E_{i1}^T & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & E_{i1} & -\mu_i I_{r_i} & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
A_{iN}^T & 0 & 0 & 0 & \cdots & -I_{n_N} & E_{iN}^T & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & E_{iN} & -\mu_i I_{r_i} & 0 & 0 & 0 & \cdots & 0 \\
X_i & 0 & 0 & 0 & \cdots & 0 & 0 & -Q_i^{-1} & 0 & 0 & \cdots & 0 \\
Y_i & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -R_i^{-1} & 0 & \cdots & 0 \\
X_i & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & -I_{n_i} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
X_i & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & -I_{n_i}
\end{bmatrix} < 0 \tag{10}$$

where $\Phi_i := A_iX_i + B_iY_i + (A_iX_i + B_iY_i)^T + \mu_i ND_iD_i^T$.

If such conditions are met, the control laws (11) are said to be the decentralized quadratic guaranteed cost for the closed-loop uncertain large-scale interconnected systems:

$$u_i(t) \;=\; K_ix_i(t) = Y_iX_i^{-1}x_i(t) \tag{11}$$

and the bound of the guaranteed cost is

$$J < \sum_{i=1}^{N} x_i^T(0)X_i^{-1}x_i(0) \tag{12}$$

In order to prove Theorem 2, we introduce the following useful lemma [8, 11].

Lemma 1 *Consider the appropriate matrix* $\mathcal{F}$ *which satisfies* $\mathcal{F}\mathcal{F}^T \le I_n$ *and for any matrices* $\mathcal{G}$ and $\mathcal{H}$ *there exists the positive parameter* $\varepsilon > 0$ *such that the following inequality holds*:

$$\mathcal{G}\mathcal{F}\mathcal{H} + \mathcal{H}^T\mathcal{F}^T\mathcal{G}^T \le \varepsilon\mathcal{G}\mathcal{G}^T + \varepsilon^{-1}\mathcal{H}^T\mathcal{H}$$

Proof: Let us introduce the matrix

$$\mathcal{T}_i := \textbf{block}-\textbf{diag}\begin{bmatrix} P_i & I_{r_i} & I_{n_i} & I_{r_i} & \cdots \\ I_{n_i} & I_{r_i} & I_{n_i} & I_{m_i} & I_{n_i} & \cdots & I_{n_i} \end{bmatrix}$$

Pre- and post-multiplying both sides of the LMI (10) by $\mathcal{T}_i$, $\mathcal{T}_i^T$, respectively, we have LMI

$$\text{[See Eq. below]} \tag{13}$$

where $\Upsilon_i := \overline{A}_i^TP_i + P_i\overline{A}_i + \mu_i NP_iD_iD_i^TP_i$. Applying the Schur complement [17, 18] to the LMI (13) gives

$$\text{[See Eq. below]} \tag{14}$$

where $\Xi_i := \overline{A}_i^TP_i + P_i\overline{A}_i + \overline{R}_i + (N-1)I_{n_i}$. Using Lemma 1, for all admissible uncertainties (2) and (3), the following matrix inequality holds:

$$\begin{aligned}
&\mathcal{L}_i \\
\ge\; & \begin{bmatrix} \Xi_i & P_iA_{i1} & \cdots & P_iA_{iN} \\ A_{i1}^TP_i & -I_{n_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{iN}^TP_i & 0 & \cdots & -I_{n_N} \end{bmatrix} \\
&+ \begin{bmatrix} P_iD_i & \cdots & P_iD_i \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} F_i \begin{bmatrix} \bar{E}_i & 0 & \cdots & 0 \\ 0 & E_{i1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{iN} \end{bmatrix} \\
&+ \begin{bmatrix} \bar{E}_i & 0 & \cdots & 0 \\ 0 & E_{i1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{iN} \end{bmatrix}^T F_i^T \begin{bmatrix} P_iD_i & \cdots & P_iD_i \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}^T \\
=\; & \mathcal{M}_i
\end{aligned}$$

Thus, $\mathcal{M}_i < 0$ holds because of $\mathcal{L}_i < 0$. That is, feedback control (11) is the quadratic guaranteed cost controller. ■

$$\begin{bmatrix}
\Upsilon_i & \bar{E}_i^T & P_iA_{i1} & 0 & \cdots & P_iA_{iN} & 0 & I_{n_i} & K_i^T & I_{n_i} & \cdots & I_{n_i} \\
\bar{E}_i & -\mu_iI_{r_i} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
A_{i1}^TP_i & 0 & -I_{n_1} & E_{i1}^T & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & E_{i1} & -\mu_iI_{r_i} & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
A_{iN}^TP_i & 0 & 0 & 0 & \cdots & -I_{n_N} & E_{iN}^T & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & E_{iN} & -\mu_iI_{r_i} & 0 & 0 & 0 & \cdots & 0 \\
I_{n_i} & 0 & 0 & 0 & \cdots & 0 & 0 & -Q_i^{-1} & 0 & 0 & \cdots & 0 \\
K_i & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -R_i^{-1} & 0 & \cdots & 0 \\
I_{n_i} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & -I_{n_i} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
I_{n_i} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & -I_{n_i}
\end{bmatrix} < 0 \tag{13}$$

$$\mathcal{L}_i := \begin{bmatrix}
\Xi_i + \mu_iNP_iD_iD_i^TP_i + \mu_i^{-1}\bar{E}_i^T\bar{E}_i & P_iA_{i1} & \cdots & P_iA_{iN} \\
A_{i1}^TP_i & -I_{n_1} + \mu_i^{-1}E_{i1}^TE_{i1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
A_{iN}^TP_i & 0 & \cdots & -I_{n_N} + \mu_i^{-1}E_{iN}^TE_{iN}
\end{bmatrix} < 0 \tag{14}$$

Remark 1 *If the elements of interconnected matrices* A_{ij} and E_{ij} *are sufficiently small, it is easy to verify that the LMI (10) hold. However, in order to satisfy the LMI (10), the interconnections of the subsystems must be weak. Therefore, the decentralized controllers can be constructed for each subsystem because in the case where the interconnections of the subsystems are weak the properties of the subsystems are kept in the same way as the ordinary large-scale systems. As a result, it will be easy to design the decentralized controller. Finally, the obtained LMI (10) is suitable.*

Since the LMI (10) consists of a convex solution set of (μ_i, X_i, Y_i), various efficient convex optimization algorithms such as LMI Control Toolbox for MATLAB [16] can be applied. Consequently, let us consider the optimization problem that allows us to determine the optimal bound.

[Problem **A**] For all i, $i = 1, \ldots, N$, consider the LMI (10) and the following constrained conditions:

$$\begin{bmatrix} -\alpha_i & x_i^T(0) \\ x_i(0) & -X_i \end{bmatrix} < 0 \tag{15}$$

Moreover, also consider the convex set $\mathcal{X}_i \in (\mu_i, X_i, Y_i)$ such that $\mu_i > 0$ holds.

Find $K_i = Y_i X_i^{-1}$, $i = 1, \ldots, N$, such that the cost $\min_{\Sigma_{i=1}^N \mathcal{X}_i} \Sigma_{i=1}^N \alpha_i$ becomes as small as possible. That is, the problem addressed in this paper is

$$\mathcal{E}_0 : \min_{\Sigma_{i=1}^N \mathcal{X}_i} \sum_{i=1}^N \alpha_i, \ \mathcal{X}_i \in (\mu_i, \ X_i, \ Y_i) \tag{16}$$

s.t. LMI (10), (15), $\mu_i > 0$.

It is possible to replace the full-problem **A** with each optimization problem for all i by using the following result because the full-problem **A** can be decomposed.

Theorem 3 *If the above optimization problem* **A** *has the solution* μ_i, X_i, Y_i, and α_i, *then the control laws of the form (11) are the decentralized linear state feedback control laws which ensure the minimization of the guaranteed cost (12) for the uncertain large-scale interconnected systems. Moreover, the optimization problem (16) can be changed to the following problem*:

$$\min_{\Sigma_{i=1}^N \mathcal{X}_i} \sum_{i=1}^N \alpha_i = \sum_{i=1}^N \min_{\mathcal{X}_i} \ \alpha_i$$

Proof: Applying the Schur complement [17, 18] to the LMI (15), we have

$$(15) \Leftrightarrow x_i^T(0) X_i^{-1} x_i(0) < \alpha_i \tag{17}$$

Hence, the bound of the cost J of (12) satisfies $J < \Sigma_{i=1}^N \alpha_i$. Since the minimization of $\Sigma_{i=1}^N \alpha_i$ implies the solution of the optimization problem (16), the minimum value of the cost bound of (12) is given. Furthermore, the optimization problem for each subsystem can be done independently without other information of the interconnected systems. Therefore, it is commutable for the sequence of the optimization problem, that is,

$$\begin{aligned} J &< \sum_{i=1}^N x_i^T(0) X_i^{-1} x_i(0) < \sum_{i=1}^N \alpha_i \\ &< \min_{\Sigma_{i=1}^N \mathcal{X}_i} \sum_{i=1}^N \alpha_i = \sum_{i=1}^N \min_{\mathcal{X}_i} \ \alpha_i = J^* \end{aligned}$$

This is the desired result. ■

Remark 2 *It can be noted that the bound obtained in Theorem 3 depends on the initial condition* $x_i(0)$. *To remove this dependence on* $x_i(0)$, *we assume that* $x_i(0)$ *is a zero mean random variable satisfying* $E[x_i(0)x_i(0)^T] = I_{n_i}$ [8, 12]. *In this case, it is interesting to point out that the guaranteed cost becomes*

$$\begin{aligned} E[J] &< \sum_{i=1}^N E\left[x_i^T(0) X_i^{-1} x_i(0)\right] = \sum_{i=1}^N \text{Trace } [X_i^{-1}] \\ &< \sum_{i=1}^N \text{Trace } [V_i] < \sum_{i=1}^N \min_{\mathcal{Y}_i} \text{Trace } [V_i] = J^\dagger \end{aligned}$$

where

$$\begin{bmatrix} -V_i & I_{n_i} \\ I_{n_i} & -X_i \end{bmatrix} < 0, \ \mathcal{Y}_i \in (\mu_i, \ X_i, \ Y_i, \ V_i)$$

Moreover, for each subsystem the above LMIs can be solved for all i because μ_i, X_i, Y_i, and V_i *are independent of the other subsystems.*

$$\mathcal{E}_i : \ \min_{\mathcal{Y}_i} \text{Trace } [V_i], \ i = 1, \ldots, N$$

Thus, it is very reliable because one does not need to solve the large-scale optimization problem $\mathcal{E}_0$.

3. Case of the Existence of Control Gain Perturbations

In this section, the existing result which has been studied [13] will be extended.That is, the uncertain interconnections are considered and the matching conditions for the gain perturbations will be relaxed. Consider the large-scale systems (18) with the gain perturbations:

$$\begin{aligned} \dot{x}_i(t) &= [A_i + \Delta A_i(t)] x_i(t) + B_i u_i(t) \\ &\quad + \sum_{j=1, \ j \neq i}^N [G_{ij} + \Delta G_{ij}(t)] g_{ij}(t, \ x_j) \end{aligned} \tag{18a}$$

$$u_i(t) = [K_i + \Delta K_i(t)]x_i(t) \tag{18b}$$
$$x_i(t) = x_i(0), \ i = 1, \cdots, N \tag{18c}$$

where $x_i \in \mathbf{R}^{n_i}$ and $u_i \in \mathbf{R}^{m_i}$ are the state and control of the i-th subsystems, respectively. A_i and B_i are constant matrices of appropriate dimensions and G_{ij} are interconnection matrices between the i-th subsystems and other subsystems. The unknown vector functions $g_{ij}(t, x_j) \in \mathbf{R}^{l_i}$ represent interconnections among the subsystems. It is assumed that the unknown vector functions $g_{ij}(t, x_j)$ are continuous and sufficiently smooth in x_j and piecewise continuous in t. The parameter uncertainties which are included in the controller gains are assumed to have the following form:

$$\begin{bmatrix} \Delta K_i(t) & \Delta G_{ij}(t) \end{bmatrix} = \begin{bmatrix} H_i F_i^k(t) E_i^k & D_{ij} F_{ij} E_{ij}^g \end{bmatrix} \tag{19}$$

where $F_i^k(t) \in \mathbf{R}^{q_i^k \times s_i^k}$, $F_{ij}(t) \in \mathbf{R}^{q_{ij} \times s_{ij}}$ such that

$$F_i^{kT}(t) F_i^k(t) \le I_{s_i^k}, \ F_{ij}^T(t) F_{ij}(t) \le I_{s_{ij}} \tag{20}$$

are the unknown matrix functions with Lebesgue measurable elements. Suppose also that $\Delta A_i(t)$ satisfies the inequality (2). Moreover, we make the assumption of the following conditions (21) concerning the unknown vector functions:

$$\|g_{ij}(t, \ x_j)\| \le \|W_{ij} x_j\| \tag{21}$$

where W_{ij} are the known constant matrices with appropriate dimensions. Compared with the existing result [13], it should be noted that the matching conditions are relaxed because $\Delta G_{ij}(t)$ are considered and $F_i^k(t) \ne F_i(t)$. On the other hand, the reason why the uncertainty of the matrices B_i is not considered is that the resulting closed-loop integrated systems represent the uncertainties of the input matrices [14].

The cost function associated with the systems (18) is given below. It should be noted that the cost function is different from the previous one (4) because of the gain perturbations:

$$\begin{aligned} J &= \sum_{i=1}^{N} \int_0^\infty [x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t)] dt \\ &= \sum_{i=1}^{N} \int_0^\infty \Big(x_i^T(t) Q_i x_i(t) \\ &\quad + x_i^T(t)[K_i + \Delta K_i(t)]^T R_i [K_i + \Delta K_i(t)] x_i(t) \Big) dt \end{aligned}$$

We give the sufficient condition for existence of the quadratic guaranteed cost control for the uncertain nonlinear systems (18) and the cost function (4).

Theorem 4 *Consider the large-scale interconnected nonlinear systems (18) with the uncertainties (19). If there exist symmetric positive definite matrices $P_i \in \mathbf{R}^{n_i \times n_i}$ such that for all uncertain matrices $\Delta A_i(t)$, $\Delta G_{ij}(t)$, $\Delta K_i(t)$, the matrix inequality (22) is satisfied, the control laws $u_i(t) = K_i x_i(t)$, $i = 1, \ldots, N$, are said to be the quadratic guaranteed cost control*:

$$\mathcal{N}_i = \begin{bmatrix} \Psi_i & P_i \hat{G}_{i1} & \cdots & P_i \hat{G}_{iN} \\ \hat{G}_{i1}^T P_i & -I_{l_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{G}_{iN}^T P_i & 0 & \cdots & -I_{l_N} \end{bmatrix} < 0 \tag{22}$$

where there exists no matrix $\mathcal{N}_i$ in $P_i \hat{G}_{ii}$ and

$$\mathcal{N}_i \in \mathbf{R}^{\tilde{N} \times \tilde{N}}, \ \hat{A}_i := \bar{A}_i + D_i F_i E_i^1 + B_i H_i F_i^k E_i^k$$

$$\Psi_i := \hat{A}_i^T P_i + P_i \hat{A}_i + \hat{R}_i + \sum_{j=1, \ j \ne i}^{N} W_{ji}^T W_{ji}$$

$$\hat{R}_i := Q_i + \hat{K}_i^T R_i \hat{K}_i, \ \hat{K}_i := K_i + H_i F_i^k E_i^k$$

$$\hat{G}_{ij} := G_{ij} + D_{ij} F_{ij} E_{ij}^g$$

Proof: Combining the guaranteed cost controller $u_i(t) = \hat{K}_i x_i(t)$ with (18) gives a closed-loop system of the form

$$\dot{x}_i = \hat{A}_i x_i + \sum_{j=1, \ j \ne i}^{N} \hat{G}_{ij} g_{ij}(t, \ x_j) \tag{23}$$

Suppose now there exist the symmetric positive definite matrices $P_i > 0$, $i = 1, \ldots, N$, such that the matrix inequalities (22) hold for all admissible uncertainties. In order to prove the asymptotic stability of the closed-loop system (23), let us define the following Lyapunov function candidate:

$$\begin{aligned} &\frac{d}{dt} V(x(t)) \\ &= \sum_{i=1}^{N} \Bigg\{ x_i^T [\hat{A}_i^T P_i + P_i \hat{A}_i] x_i \\ &\quad + \Bigg(\sum_{j=1, \ j \ne i}^{N} \hat{G}_{ij} g_{ij}(t, \ x_j) \Bigg)^T P_i x_i \\ &\quad + x_i^T P_i \Bigg(\sum_{j=1, \ j \ne i}^{N} \hat{G}_{ij} g_{ij}(t, \ x_j) \Bigg) \Bigg\} \end{aligned}$$

Taking into account the inequality

$$\sum_{i=1}^{N} \sum_{j=1, \ j \ne i}^{N} (x_i^T W_{ji}^T W_{ji} x_i - g_{ij}^T g_{ij})$$

$$= \sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} (x_j^T W_{ij}^T W_{ij} x_j - g_{ij}^T g_{ij})$$

we can change the form of $\frac{d}{dt} V(x(t))$:

$$\begin{aligned}
&\frac{d}{dt} V(x(t)) \\
&= \sum_{i=1}^{N} \eta_i^T \begin{bmatrix} \Psi_i - \hat{R}_i & P_i \hat{G}_{i1} & \cdots & P_i \hat{G}_{iN} \\ \hat{G}_{i1}^T P_i & -I_{l_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{G}_{iN}^T P_i & 0 & \cdots & -I_{l_N} \end{bmatrix} \eta_i \\
&\quad - \sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} (x_j^T W_{ij}^T W_{ij} x_j - g_{ij}^T g_{ij}) \\
&= \sum_{i=1}^{N} \eta_i^T \mathcal{N}_i \eta_i - \sum_{i=1}^{N} x_i^T \hat{R}_i x_i \\
&\quad - \sum_{i=1}^{N} \sum_{j=1,\ j\neq i}^{N} (x_j^T W_{ij}^T W_{ij} x_j - g_{ij}^T g_{ij})
\end{aligned}$$

where

$$\eta_i(t) := \begin{bmatrix} x_i^T & g_{i1}^T & \cdots & g_{iN}^T \end{bmatrix}^T \in \mathbf{R}^{\tilde{N}}$$

Taking into account the fact that the inequalities (21) and (22) hold, it follows immediately that

$$\frac{d}{dt} V(x(t)) < -\sum_{i=1}^{N} x_i^T(t) \hat{R}_i x_i(t) < 0 \qquad (24)$$

Hence, $V(x(t))$ is a Lyapunov function for the closed-loop system (23). Therefore, the closed-loop system (23) is asymptotically stable. Since the proof of the cost bound can be done by using a similar technique, it is omitted. ■

We give also the quadratic guaranteed cost controller for the nonlinear large-scale systems under the gain perturbations via the LMI.

Theorem 5 *Suppose there exist the constant parameters $\mu_i > 0$, $\varepsilon_i > 0$, $\nu_i > 0$ such that for all $i = 1, \ldots, N$ the LMI (25) have the symmetric positive definite matrices $X_i > 0 \in \mathbf{R}^{n_i \times n_i}$ and a matrix $Y_i \in \mathbf{R}^{m_i \times n_i}$:*

$$\begin{bmatrix}
\Gamma_i & X_i & Y_i^T + \varepsilon_i B_i H_i H_i^T & X_i & X_i E_i^{1T} & X_i E_i^{kT} & G_{i1} & 0 & \cdots & G_{iN} & 0 \\
X_i & -Q_i^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
Y_i + \varepsilon_i H_i H_i^T B_i^T & 0 & -R_i^{-1} + \varepsilon_i H_i H_i^T & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
X_i & 0 & 0 & -U_i^{-1} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
E_i^1 X_i & 0 & 0 & 0 & -\mu_i I_{r_i} & 0 & 0 & 0 & \cdots & 0 & 0 \\
E_i^k X_i & 0 & 0 & 0 & 0 & -\varepsilon_i I_{r_i} & 0 & 0 & \cdots & 0 & 0 \\
G_{i1}^T & 0 & 0 & 0 & 0 & 0 & -I_{l_1} & E_{i1}^{gT} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & E_{i1}^{g} & -\nu_i I_{r_i} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
G_{iN}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -I_{l_N} & E_{iN}^{gT} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & E_{iN}^{g} & -\nu_i I_{r_i}
\end{bmatrix} < 0 \qquad (25)$$

where

$$\begin{aligned}
\Gamma_i &:= A_i X_i + B_i Y_i + (A_i X_i + B_i Y_i)^T + \mu_i D_i D_i^T \\
&\quad + \varepsilon_i B_i H_i H_i^T B_i^T + \nu_i \sum_{j=1,\ j\neq i}^{N} D_{ij} D_{ij}^T \\
U_i &:= \sum_{j=1,\ j\neq i}^{N} W_{ji}^T W_{ji} > 0
\end{aligned}$$

If such conditions are met, the control gains $K_i = Y_i X_i^{-1}$, $i = 1, \ldots, N$, are said to be the decentralized quadratic guaranteed cost control gain. Moreover, the bound of the cost is given by (12).

Proof: Let us introduce the following block matrix:

$$\mathcal{S}_i := \mathbf{block-diag}$$

$$\begin{bmatrix} P_i & I_{n_i} & I_{m_i} & I_{n_i} & I_{r_i} & I_{r_i} & I_{l_1} & I_{r_i} & \cdots & I_{l_N} & I_{r_i} \end{bmatrix}$$

Pre- and post-multiplying both sides of the LMI (25) by $\mathcal{S}_i, \mathcal{S}_i^T$, respectively, we have LMI

[See Eq. on next page] (26)

where $\Omega_i := \overline{A}_i^T P_i + P_i \overline{A}_i + \mu_i P_i D_i D_i^T P_i + \varepsilon_i P_i B_i H_i H_i^T B_i^T P_i + \nu_i P_i (\Sigma_{j=1, j\neq i}^{N} D_{ij} D_{ij}^T) P_i$.

Applying the Schur complement [17, 18] to the LMI (26) gives

[See Eq. on next page] (27)

where $\Pi_i := \Omega_i + \mu_i^{-1} E_i^{1T} E_i^1 + \varepsilon_i^{-1} E_i^{kT} E_i^k$.

Applying Lemma 1 to the matrix inequality (27), we have

[See Eq. on next page] (28)

Applying the Schur complement [17, 18] to the matrix inequality $\mathcal{L}_i < 0$ as the matrix inequality (28), we get

$$\hat{A}_i^T P_i + P_i \hat{A}_i + P_i \left(\sum_{j=1,\ j\neq i}^{N} \hat{G}_{ij} \hat{G}_{ij}^T \right) P_i$$

$$
\begin{bmatrix}
\Omega_i & I_{n_i} & K_i^T + \varepsilon_i P_i B_i H_i H_i^T & I_{n_i} & E_i^{1T} & E_i^{kT} & P_i G_{i1} & 0 & \cdots & P_i G_{iN} & 0 \\
I_{n_i} & -Q_i^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
K_i + \varepsilon_i H_i H_i^T B_i^T P_i & 0 & -R_i^{-1} + \varepsilon_i H_i H_i^T & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
I_{n_i} & 0 & 0 & -U_i^{-1} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
E_i^1 & 0 & 0 & 0 & -\mu_i I_{r_i} & 0 & 0 & 0 & \cdots & 0 & 0 \\
E_i^k & 0 & 0 & 0 & 0 & -\varepsilon_i I_{r_i} & 0 & 0 & \cdots & 0 & 0 \\
G_{i1}^T P_i & 0 & 0 & 0 & 0 & 0 & -I_{l_1} & E_{i1}^{gT} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & E_{i1}^g & -\nu_i I_{r_i} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
G_{iN}^T P_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -I_{l_N} & E_{iN}^{gT} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & E_{iN}^g & -\nu_i I_{r_i}
\end{bmatrix} < 0 \quad (26)
$$

$$
\mathcal{L}_i := \begin{bmatrix}
\Pi_i & I_{n_i} & K_i^T + \varepsilon_i P_i B_i H_i H_i^T & I_{n_i} & P_i G_{i1} & \cdots & P_i G_{iN} \\
I_{n_i} & -Q_i^{-1} & 0 & 0 & 0 & \cdots & 0 \\
K_i + \varepsilon_i H_i H_i^T B_i^T P_i & 0 & -R_i^{-1} + \varepsilon_i H_i H_i^T & 0 & 0 & \cdots & 0 \\
I_{n_i} & 0 & 0 & -U_i^{-1} & 0 & \cdots & 0 \\
G_{i1}^T P_i & 0 & 0 & 0 & -I_{l_1} + \nu_i^{-1} E_{i1}^{gT} E_{i1}^g & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
G_{iN}^T P_i & 0 & 0 & 0 & 0 & \cdots & -I_{l_N} + \nu_i^{-1} E_{iN}^{gT} E_{iN}^g
\end{bmatrix} < 0 \quad (27)
$$

$$
\begin{aligned}
\mathcal{L}_i \geq{} & \begin{bmatrix}
\bar{A}_i^T P_i + P_i \bar{A}_i & I_{n_i} & K_i^T & I_{n_i} & P_i G_{i1} & \cdots & P_i G_{iN} \\
I_{n_i} & -Q_i^{-1} & 0 & 0 & 0 & \cdots & 0 \\
K_i & 0 & -R_i^{-1} & 0 & 0 & \cdots & 0 \\
I_{n_i} & 0 & 0 & -U_i^{-1} & 0 & \cdots & 0 \\
G_{i1}^T P_i & 0 & 0 & 0 & -I_{l_1} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
G_{iN}^T P_i & 0 & 0 & 0 & 0 & \cdots & -I_{l_N}
\end{bmatrix}
+ \begin{bmatrix} P_i D_i \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_i \begin{bmatrix} E_i^1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T
+ \begin{bmatrix} E_i^1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_i^T \begin{bmatrix} P_i D_i \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T \\
& + \begin{bmatrix} P_i B_i H_i \\ 0 \\ H_i \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_i^k \begin{bmatrix} E_i^k \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T
+ \begin{bmatrix} E_i^k \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} F_i^{kT} \begin{bmatrix} P_i B_i H_i \\ 0 \\ H_i \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T
+ \begin{bmatrix} O & P_i D_{i1} & \cdots & P_i D_{iN} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}
\begin{bmatrix} O & 0 & \cdots & 0 \\ 0 & F_{i1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_{iN} \end{bmatrix}
\begin{bmatrix} O & 0 & \cdots & 0 \\ 0 & E_{i1}^g & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{iN}^g \end{bmatrix} \\
& + \begin{bmatrix} O & 0 & \cdots & 0 \\ 0 & E_{i1}^g & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{iN}^g \end{bmatrix}^T
\begin{bmatrix} O & 0 & \cdots & 0 \\ 0 & F_{i1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_{iN} \end{bmatrix}^T
\begin{bmatrix} O & P_i D_{i1} & \cdots & P_i D_{iN} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}^T
\end{aligned} \quad (28)
$$

$$
(28) \Leftrightarrow 0 > \mathcal{L}_i \geq \begin{bmatrix} \hat{A}_i^T P_i + P_i \hat{A}_i & I_{n_i} & \hat{K}_i^T & I_{n_i} & P_i \hat{G}_{i1} & \cdots & P_i \hat{G}_{iN} \\ I_{n_i} & -Q_i^{-1} & 0 & 0 & 0 & \cdots & 0 \\ \hat{K}_i & 0 & -R_i^{-1} & 0 & 0 & \cdots & 0 \\ I_{n_i} & 0 & 0 & -U_i^{-1} & 0 & \cdots & 0 \\ \hat{G}_{i1}^T P_i & 0 & 0 & 0 & -I_{l_1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{G}_{iN}^T P_i & 0 & 0 & 0 & 0 & \cdots & -I_{l_N} \end{bmatrix} = \Lambda_i
$$

$$
+ \hat{R}_i + \sum_{j=1,\ j \neq i}^{N} W_{ji}^T W_{ji} < 0 \qquad (29)
$$

On the other hand, applying the Schur complement to the matrix inequality (22), we have also the matrix inequality (29). Therefore, the matrix inequality (22) holds. Thus, $K_i = Y_i X_i^{-1}$ is the decentralized quadratic guaranteed cost control gain matrices. Since the proof of the bound of the cost function is the same as the proof of Theorem 2, it is omitted briefly. ■

Remark 3 *Taking into account the fact that the LMI (25) consist of the set* (μ_i, ε_i, ν_i, X_i, Y_i) *of the convex solution, it is possible to optimize via the various efficient convex optimization algorithms such as LMI Control Toolbox of MATLAB* [16].

Remark 4 *In fact, it should be noted that the control inputs are not* $u_i(t) = [K_i + \Delta K_i(t)]x_i(t)$ *of Eq. (18b) but* $u_i(t) = K_i x_i(t)$. *That is, due to the consideration of controller gain perturbations, it means that the uncertainties* $\Delta K_i(t)$ *are not included in the gain matrices. If there exist the actuators with time-variant uncertainties, it is possible to implement the quadratic guaranteed cost control because the gain perturbations* $\Delta K_i(t)$ *are considered. Therefore, our attention is focused on the practical systems compared with the existing result* [12, 13].

4. Numerical Example

In order to demonstrate the efficiency of our proposed quadratic guaranteed cost control, we have run two simple numerical examples.

4.1 Case of no existence of the control gain perturbations

Consider the interconnected uncertain large-scale systems composed of three four-dimensional subsystems. We assume that the system structures are based on the uncertain power systems in Ref. 7. The system matrices and the uncertainties of Eqs. (1) are given as follows:

$$
A_1 = \begin{bmatrix} -0.922 & 1 & -0.266 & -0.009 \\ -2.75 & -2.78 & -1.36 & -0.37 \\ 0 & 0 & 0 & 1 \\ -4.95 & 0 & -55.5 & -0.39 \end{bmatrix}
$$

$$
A_{12} = \begin{bmatrix} 0.024 & 0 & -0.087 & -0.002 \\ -0.0158 & 0 & 0.11 & -0.011 \\ 0 & 0 & 0 & 0 \\ 0.222 & 0 & 0.0817 & 0.004 \end{bmatrix}
$$

$$
A_{13} = \begin{bmatrix} 0.072 & 0 & -0.025 & -0.003 \\ -0.046 & 0 & 0.28 & -0.02 \\ 0 & 0 & 0 & 0 \\ 0.0924 & 0 & 0.175 & 0.02 \end{bmatrix}
$$

$$
A_2 = \begin{bmatrix} -0.21 & 1 & -1.6 & -0.005 \\ -1.9 & -1.8 & 9.3 & -0.12 \\ 0 & 0 & 0 & 1 \\ -3.1 & 0 & -5.6 & 0.032 \end{bmatrix}
$$

$$
A_{21} = \begin{bmatrix} 0.021 & 0 & 0.0121 & 0.003 \\ -0.011 & 0 & -0.162 & -0.015 \\ 0 & 0 & 0 & 0 \\ -0.0243 & 0 & 0.0137 & -0.034 \end{bmatrix}
$$

$$
A_{23} = \begin{bmatrix} 0.06 & 0 & 0.046 & 0.002 \\ -1 & 0 & 0.00149 & -0.04 \\ 0 & 0 & 0 & 0 \\ 0.012 & 0 & 0.0298 & -0.028 \end{bmatrix}
$$

$$
A_3 = \begin{bmatrix} -0.197 & 1 & -1.2 & -0.003 \\ -54.4 & 20 & 70.1 & -2.37 \\ 0 & 0 & 0 & 1 \\ -3.4 & 0 & -21 & -0.017 \end{bmatrix}
$$

$$
A_{31} = \begin{bmatrix} -0.002 & 0 & 0.083 & 0 \\ -0.0678 & 0 & -0.101 & -0.09 \\ 0 & 0 & 0 & 0 \\ -0.124 & 0 & 0.0498 & -0.017 \end{bmatrix}
$$

$$A_{32} = \begin{bmatrix} 0.011 & 0 & 0.022 & 0 \\ -0.021 & 0 & 0.017 & -0.0123 \\ 0 & 0 & 0 & 0 \\ -0.007 & 0 & 0.0637 & -0.011 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 3.61 \\ 0 \\ 0 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ 7.89 \\ 0 \\ 0 \end{bmatrix}, \; B_3 = \begin{bmatrix} 0 \\ 5.63 \\ 0 \\ 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 \\ 0.01 \\ 0 \\ 0.01 \end{bmatrix}, \; D_2 = D_1, \; D_3 = D_1$$

$$E_1^1 = \begin{bmatrix} 0 & 0 & 0.01 & 0.02 \end{bmatrix}, \; E_1^2 = \begin{bmatrix} 0.015 \end{bmatrix}$$

$$E_{12} = \begin{bmatrix} 0 & 0 & 0.015 & 0.005 \end{bmatrix}$$

$$E_{13} = \begin{bmatrix} 0 & 0 & 0.03 & 0.0001 \end{bmatrix}$$

$$E_2^1 = \begin{bmatrix} 0 & 0 & 0.01 & 0.001 \end{bmatrix}, \; E_2^2 = \begin{bmatrix} 0.01 \end{bmatrix}$$

$$E_{23} = \begin{bmatrix} 0 & 0 & 0.02 & 0.001 \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 0 & 0 & 0.01 & 0.0005 \end{bmatrix}$$

$$E_3^1 = \begin{bmatrix} 0 & 0 & 0.001 & 0.02 \end{bmatrix}, \; E_3^2 = \begin{bmatrix} 0.02 \end{bmatrix}$$

$$E_{31} = \begin{bmatrix} 0 & 0 & 0.005 & 0.01 \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 0 & 0 & 0.05 & 0.001 \end{bmatrix}$$

$$R_i = 0.1, \; Q_i = \text{diag}\begin{bmatrix} 1 & 1 & 2 & 2 \end{bmatrix}, \; i = 1, \; 2, \; 3$$

By applying Theorem 3 and solving the corresponding optimization problem, we obtain the decentralized linear state feedback control gains K_i, $i = 1, 2, 3$, of Eq. (11):

$$K_1 \;=$$

$$\begin{bmatrix} -4.8173 & -4.9833 & -3.5753 \times 10 & -6.1535 \times 10^{-1} \end{bmatrix}$$

$$K_2 \;=$$

$$\begin{bmatrix} -1.3976 \times 10 & -5.5612 & -6.0286 & 5.5556 \end{bmatrix}$$

$$K_3 \;=$$

$$\begin{bmatrix} -7.4229 & -1.0271 \times 10 & -3.5804 \times 10 & 4.3695 \end{bmatrix}$$

In this case, the bound of the quadratic guaranteed cost is $J^{\dagger} = 3.9653 \times 10^2$, where

$$\min_{\mathcal{Y}_1} \; \text{Trace}\,[V_1] = 2.5732 \times 10^2$$

$$\min_{\mathcal{Y}_2} \; \text{Trace}\,[V_2] = 2.3863 \times 10$$

$$\min_{\mathcal{Y}_3} \; \text{Trace}\,[V_3] = 1.1535 \times 10^2$$

The results of the time histories of the closed-loop systems via the decentralized quadratic guaranteed cost control are depicted in Figs. 1 to 3. The related uncertainties $F_i(t)$ are

$$F_1(t) = \sin(60\pi t), \; F_2(t) = 1 - \exp(-0.01t)$$
$$F_3(t) = \frac{1}{2}\sin(120\pi t)$$

Moreover, the initial conditions are chosen randomly:

$$x_1(0) = \begin{bmatrix} x_{11}(0) \\ x_{12}(0) \\ x_{13}(0) \\ x_{14}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0.5 \end{bmatrix}$$

$$x_2(0) = \begin{bmatrix} x_{21}(0) \\ x_{22}(0) \\ x_{23}(0) \\ x_{24}(0) \end{bmatrix} = \begin{bmatrix} 1.2 \\ 1.5 \\ 0 \\ -0.5 \end{bmatrix}$$

$$x_3(0) = \begin{bmatrix} x_{31}(0) \\ x_{32}(0) \\ x_{33}(0) \\ x_{34}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 0 \\ 0.5 \end{bmatrix}$$

It is shown from Figs. 1 to 3 that the closed-loop systems are asymptotically stable. On the other hand, the time histories of the closed-loop systems via the Linear Quadratic Regulator (LQR) control are depicted in Figs. 4 to 6

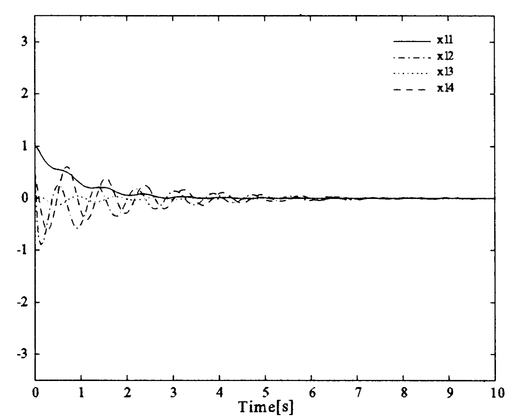

Fig. 1. Response of the closed-loop system 1 with the proposed control method.

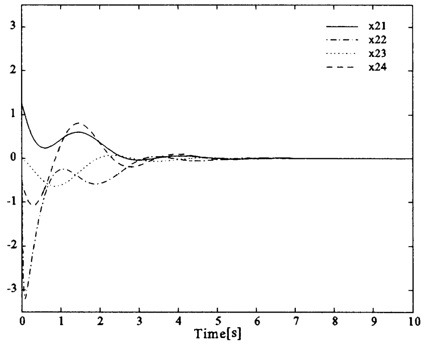

Fig. 2. Response of the closed-loop system 2 with the proposed control method.

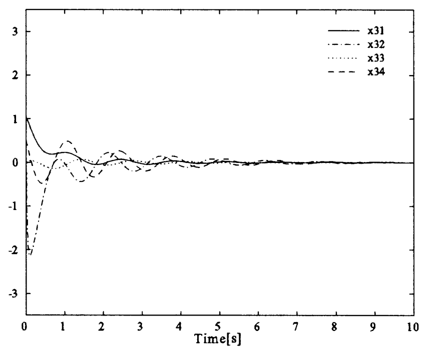

Fig. 3. Response of the closed-loop system 3 with the proposed control method.

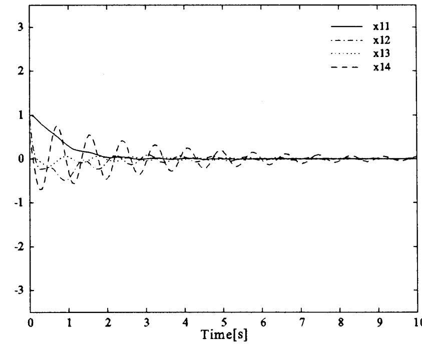

Fig. 4. Response of the closed-loop system 1 with the LQR.

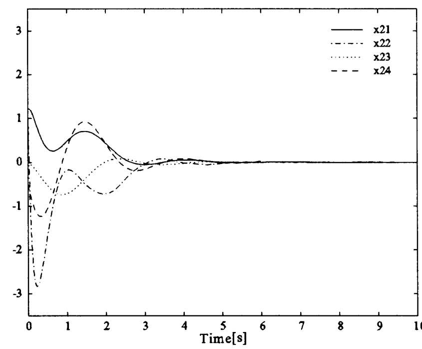

Fig. 5. Response of the closed-loop system 2 with the LQR.

compared with our proposed controllers. The matrix gains which are based on the LQR controllers are given below:

$$K_{11} = \begin{bmatrix} -2.3488 & -2.6787 & -1.8729 \times 10 & -7.8836 \times 10^{-1} \\ -1.9019 \times 10^{-2} & -2.4626 \times 10^{-3} & 1.1910 & -1.7495 \times 10^{-2} \\ -1.0979 \times 10^{-1} & -8.6706 \times 10^{-3} & 3.5433 \times 10^{-1} & -1.5428 \times 10^{-2} \end{bmatrix}$$

$$K_{12} = \begin{bmatrix} -3.6575 \times 10^{-2} & -1.1267 \times 10^{-3} & 1.9080 \times 10^{-2} & 5.3747 \times 10^{-2} \\ -8.9413 & -3.2816 & -5.1435 & 3.8390 \\ -7.4113 \times 10^{-2} & -1.5123 \times 10^{-3} & 5.6544 \times 10^{-2} & -3.2155 \times 10^{-2} \end{bmatrix}$$

$$K_{13} = \begin{bmatrix} -1.5338 \times 10^{-1} & -5.5596 \times 10^{-3} & -5.6913 \times 10^{-1} & 3.0857 \times 10^{-2} \\ -1.3786 \times 10^{-2} & -2.1193 \times 10^{-3} & 2.4132 \times 10^{-1} & 6.1275 \times 10^{-3} \\ -4.9484 & -8.4897 & -2.7941 \times 10 & 4.2131 \end{bmatrix}$$

where

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \end{bmatrix} = -R^{-1}B^TP$$

$$PA + A^TP - PBR^{-1}B^TP + Q = 0$$

$$A = \begin{bmatrix} A_1 & A_{12} & A_{13} \\ A_{21} & A_2 & A_{23} \\ A_{31} & A_{32} & A_3 \end{bmatrix}$$

$$B = \mathbf{block-diag} \begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix}$$

$$Q = \mathbf{block-diag}\begin{bmatrix} Q_1 & Q_2 & Q_3 \end{bmatrix}$$
$$R = \mathbf{block-diag}\begin{bmatrix} R_1 & R_2 & R_3 \end{bmatrix}$$

The minimum cost bound is $J^{\dagger}$ = Trace $P = 1.2084 \times 10^2$. From Figs. 1 to 3 and 4 to 6, the trajectories of the time histories are similar. However, in the case of the quadratic guaranteed cost control, the convergence speed becomes a little fast due to the increase in the cost. Although we have also run the simulation via the decentralized LQR control [19, 20], since its time histories are the same as the optimal control (LQR), it is omitted. The decentralized LQR gains are given as follows:

$$u(t) = \begin{bmatrix} \bar{u}_1(t) \\ \bar{u}_2(t) \\ \bar{u}_3(t) \end{bmatrix} = \begin{bmatrix} \bar{K}_1 & 0 & 0 \\ 0 & \bar{K}_2 & 0 \\ 0 & 0 & \bar{K}_3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$
$$\bar{K}_i = -R_i^{-1}B_i^T P_{ii},\ i = 1,\ 2,\ 3,$$
$$P_{ii}A_i + A_i^T P_{ii} - P_{ii}B_iR_i^{-1}B_i^T P_{ii} + Q_i = 0$$
$$\bar{K}_1 = \begin{bmatrix} -2.3455 & -2.6785 & -1.8747 \times 10 & -7.8908 \times 10^{-1} \end{bmatrix}$$
$$\bar{K}_2 = \begin{bmatrix} -8.9350 & -3.2814 & -5.1449 & 3.8377 \end{bmatrix}$$
$$\bar{K}_3 = \begin{bmatrix} -4.9441 & -8.4896 & -2.7946 \times 10 & 4.2138 \end{bmatrix}$$

Although the quadratic guaranteed cost control has the drawback such as increase in the cost bound compared with the LQR control, the robust stability is guaranteed for the uncertain systems. Moreover, even if the simulation is not carried out, we will realize the desired transient response by adjusting the weight matrices of the cost function in the same way as the LQR control. As a result, the quadratic guaranteed cost control is very reliable.

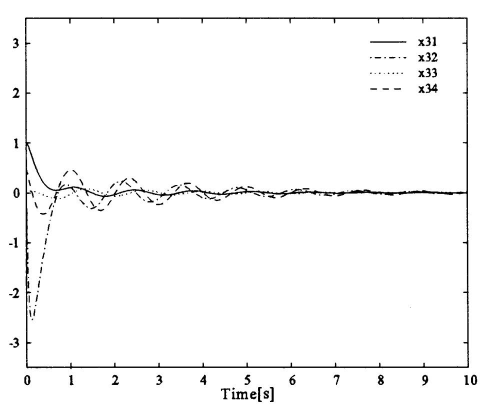

Fig. 6. Response of the closed-loop system 3 with the LQR.

4.2 Case of existence of the control gain perturbations

Consider the interconnected uncertain large-scale systems composed of three two-dimensional subsystems. We assume that the gain perturbations for the considered systems are included.The system matrices and the uncertainties of Eqs. (18) are given as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix},\ G_{12} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},\ G_{13} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ D_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix},\ G_{23} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix},\ G_{21} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix},\ D_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ G_{31} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},\ G_{32} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$$
$$B_3 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},\ D_3 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$$

$$E_1^1 = E_2^1 = E_3^1 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}$$
$$H_1 = H_2 = H_3 = \begin{bmatrix} 0.01 \end{bmatrix}$$
$$D_{ij} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ E_{ij}^g = \begin{bmatrix} 0.01 \end{bmatrix}$$
$$E_1^k = E_2^k = E_3^k = \begin{bmatrix} 0 & 0.1 \end{bmatrix}$$
$$R_1 = R_2 = R_3 = 1,\ Q_1 = Q_2 = Q_3 = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.01 \end{bmatrix}$$
$$g_{12}(t,\ x_2) = \sin([1\ 0]x_2),\ g_{13}(t,\ x_3) = \sin([1\ 0]x_3)$$
$$g_{23}(t,\ x_3) = \sin([1\ 0]x_3),\ g_{21}(t,\ x_1) = \sin([1\ 0]x_1)$$
$$g_{31}(t,\ x_1) = \sin([1\ 0]x_1),\ g_{32}(t,\ x_2) = \sin([1\ 0]x_2)$$

Since the unknown functions $g_{ij}(t, x_j)$ satisfy $|g_{ij}(t, x_j)| \le \|x_j\|$, we choose $W_{12} = W_{13} = W_{23} = W_{21} = W_{31} = W_{32} = I_2$. By applying Theorem 5 and solving the corresponding optimization problem, we obtain the following decentralized linear state feedback controls K_i, $i = 1, 2, 3$:

$$K_1 = \begin{bmatrix} -4.0697 \times 10^{-2} & -9.0891 \times 10^{-2} \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -4.1200 \times 10^{-2} & -4.5351 \times 10^{-2} \end{bmatrix}$$

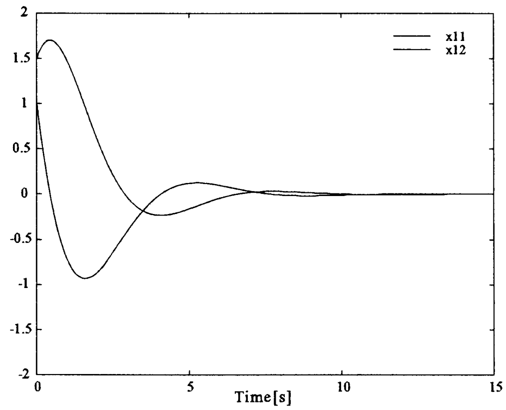

Fig. 7. Response of the closed-loop system 1 with the proposed control method.

$$K_3 = \begin{bmatrix} -1.4869 & -1.5208 \end{bmatrix}$$

In this case, the bound of the quadratic guaranteed cost is $J^\dagger = 2.426062$, where

$$\min_{\mathcal{Y}_1} \text{ Trace } [V_1] = 2.21600 \times 10^{-1}$$
$$\min_{\mathcal{Y}_2} \text{ Trace } [V_2] = 1.28286 \times 10^{-1}$$
$$\min_{\mathcal{Y}_3} \text{ Trace } [V_3] = 2.076176$$

Thus, we can obtain the decentralized controller which minimizes the cost bound by solving the LMI (25). The time histories of the closed-loop systems via the decentralized quadratic guaranteed cost control which is based on Theorem 5 are shown in Figs. 7 to 9. The functions which represent the uncertainties (19) are given below:

$$F_1(t) = \sin(60\pi t), \; F_1^k(t) = 1 - \exp(-0.01t)$$
$$F_{12}(t) = F_{13}(t) = \cos^2(60\pi t)$$

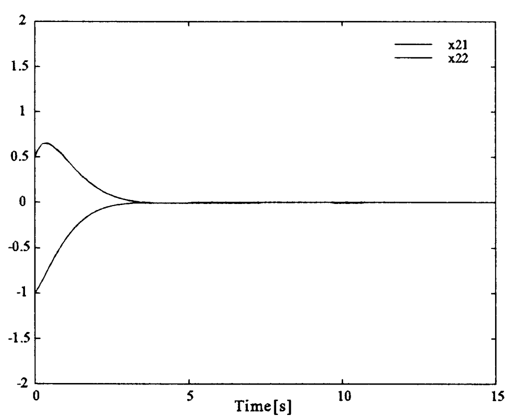

Fig. 8. Response of the closed-loop system 2 with the proposed control method.

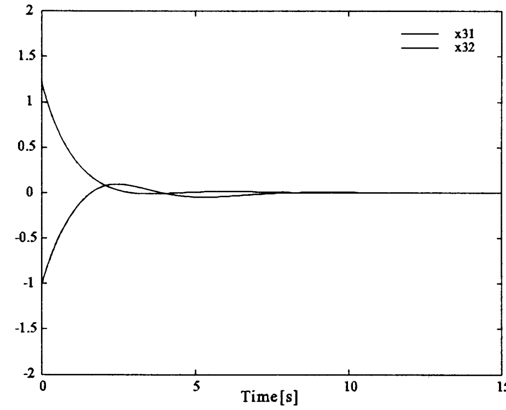

Fig. 9. Response of the closed-loop system 3 with the proposed control method.

$$F_2(t) = \sin(120\pi t), \; F_2^k(t) = 1 - \exp(-0.02t)$$
$$F_{23}(t) = F_{21}(t) = \cos^2(60\pi t)$$
$$F_3(t) = \sin(180\pi t), \; F_3^k(t) = 1 - \exp(-0.03t)$$
$$F_{31}(t) = F_{32}(t) = \cos^2(60\pi t)$$

Moreover, the initial conditions are chosen randomly:

$$x_1(0) = \begin{bmatrix} x_{11}(0) \\ x_{12}(0) \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$$
$$x_2(0) = \begin{bmatrix} x_{21}(0) \\ x_{22}(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}$$
$$x_3(0) = \begin{bmatrix} x_{31}(0) \\ x_{32}(0) \end{bmatrix} = \begin{bmatrix} 1.2 \\ -1 \end{bmatrix}$$

It is easy to verify that the resulting closed-loop uncertain large-scale systems are asymptotically stable.

5. Conclusion

In this paper, a solution of the quadratic guaranteed cost control problem for uncertain large-scale system has been presented. The main contribution is that the decentralized guaranteed cost controller can be constructed by solving the parameter-dependent LMIs for each subsystem. Using the proposed design method, the decentralized controllers can be calculated by catching each subsystem information only. Therefore, it is very useful in the same way as the existing decentralized control for large-scale systems [19, 20]. Furthermore, although the controller gain perturbations are included, our proposed method enables us to also construct the quadratic guaranteed cost controllers. Thus, it is possible to design the quadratic guaranteed cost controller for a wider class of large-scale systems compared with the existing results [12, 13]. As another important feature, the necessary optimization problem to get the decentralized controllers can be easily solved by using software such as MATLAB's LMI Control Toolbox [17].

Finally, it is expected that the LMI approach is also applied to the output feedback case. That problem is more realistic than the state feedback case because it is possible to implement for practical systems. In Ref. 9, it has been shown that the quadratic guaranteed cost control which is based on the output feedback can be obtained by solving three matrix coupled algebraic equations. However, to the best of our knowledge, the quadratic guaranteed cost control of the output feedback control via the LMI has not been studied. This problem will be addressed in future investigations.

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