

前回 3/11 体積形式、密度形式  
(第9回)

今日 3/12 入ト-7入の定理

## § 12 ズト-リラの定理



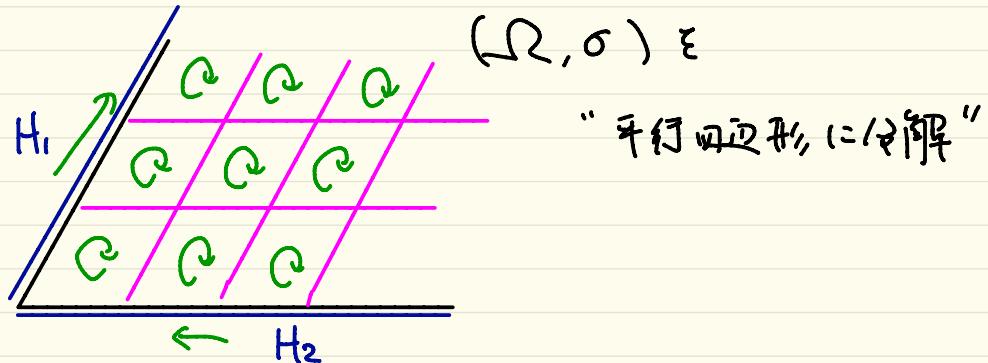
ズト-リラの定理 (微積分の基本定理の一般化)

$\forall g : \Omega$  上の  $C^\infty$ -級 k-1 form で support compact

$$\int_{\Omega} dg = \int_{\partial\Omega} g$$

外微分 境界の周辺

## 氣積分



$$\int_{(\Omega, \sigma)} dy \doteq \sum_a dy(a) \doteq \sum_a y(a) + y(b) + y(c) + y(d)$$

↗  
 a  
 d  
 ↙  
 b  
 ↘  
 c

外微分の条件

$$\int_{H_1} y + \int_{H_2} y$$

" "  
 $\int_{\partial\Omega} y$

## 内容

- 最高次形式の積分
- 境界条件の定義
- 境界条件の向き
- エト-ラスの定理

## § 12.1 最高次形式の積分

$(\Omega, \sigma)$ :  $k$ -dim'l  $C^\infty$ -mfld with corners.

$$\Lambda_c^k(\Omega) := \{ w \in \Lambda^k(\Omega) \mid \text{supp } w \text{ pl" compact} \}$$

$$\text{Def: } \text{supp } w := \overline{\{ p \in \Omega \mid w_p \neq 0 \}} \quad \leftarrow \text{Ω内の閉包} \text{ とす。}$$

Def 12.1: 若  $\omega \in \Lambda_c^k(\Omega)$  为  $\sigma$ -可积

$$\int_{(\Omega, \sigma)} \omega = \sum_{\lambda \in \Lambda} \sigma(U_\lambda, x_\lambda) I_{C_\lambda}((f_\lambda \cdot \beta_{U_\lambda}) \circ x_\lambda^{-1}) \in \mathbb{R}$$

↑  
 定义有限和

是  $\sigma$  可积.

$\{= \{x \in \Omega \mid f_\lambda(x) \neq 0\} \text{ if } PUS \text{ to } \bigcup \{(U, x) \in S_\Omega^{\text{con}} \mid U \subset$

$\forall \lambda \in \Lambda \Leftrightarrow (U_\lambda, x_\lambda) \in S_\Omega^{\text{con}} \text{ with } \text{supp } f_\lambda \subset U_\lambda$

$f_\lambda: \omega|_{U_\lambda} = \beta_{U_\lambda} \cdot d\sigma_{U_\lambda} - \lambda dx_\lambda \in \mathbb{R}^n$ .

是  $\sigma$  可积.

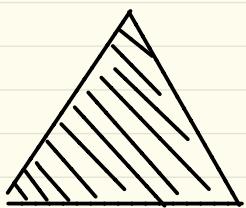
Theorem 12.2:  $(\Omega, \sigma) \in \text{fix } \{f_i\}_{i \in \mathbb{Z}}$

$$\begin{aligned} \Lambda_c^k(\Omega) &\rightarrow \mathbb{R} && \text{if well-defined } \Rightarrow \text{线型泛函} \\ \omega &\mapsto \int_{(\Omega, \sigma)} \omega && (\text{Hint: } \S 10 \text{ 例 17 同理}) \end{aligned}$$

## § 12.2 境界条件

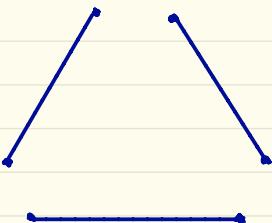
角形の物体  $\Omega$  の “境界”  $\partial\Omega$  を

1: えり



境界

$\partial\Omega$  (3つ、連結成分を構成)  
(一次元角形の集合)

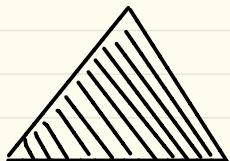


どうすれば定められる。

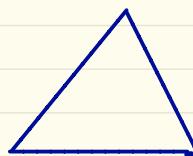
どうすれば定義可能か?

Remark:

52



の境界 =



(連結成分の一つ)

= 1つ (まとめて、

= いくつも個々の角形の複数体 = 1つ (< 1つ, 2つ以上).

本講義では、境界は “面” である (=  $\partial D$  のこと)

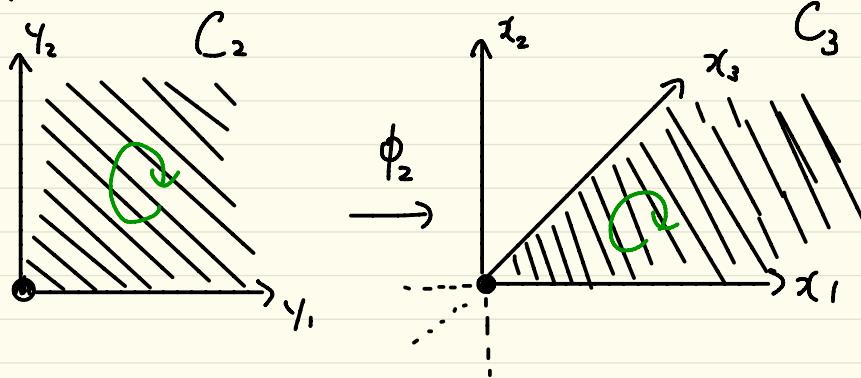
言葉の準備:  $k \geq 1$  を fix

今  $s = 1, \dots, k$  とする

$$\phi_s : C_{k-1} \rightarrow C_k$$

$$(y_1, \dots, y_{k-1}) \mapsto (y_1, \dots, y_{s-1}, 0, y_s, y_{s+1}, \dots, y_{k-1}) \in \mathbb{R}^k.$$

Ex  $k=2$

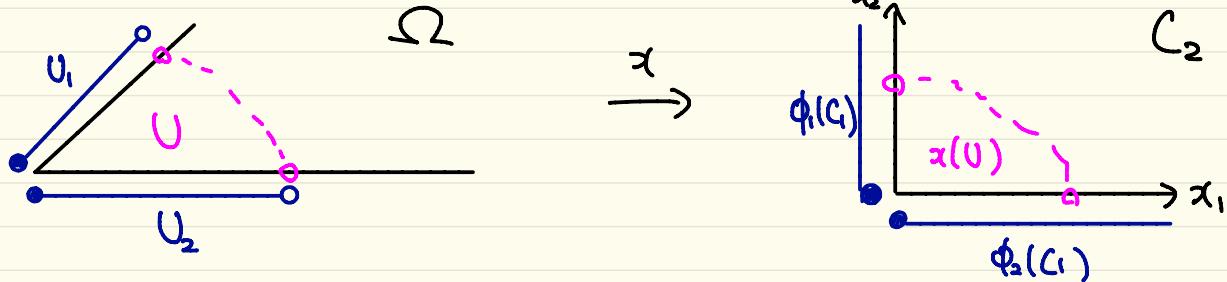


$\Omega$  : k-dim'l  $C^\infty$ -mfld w.c.  $\varepsilon \mathbb{R}^k$ .

Def 12.3  $\forall (U, \alpha) \in S_\Omega$ ,  $s = 1, \dots, k$   $\vdash \alpha|_U$

$$U_s := \alpha^{-1}(\phi_s(C_{k-s}) \cap \alpha(U)) \quad \text{et d's.}$$

Ex :



Def 12.4:  $S_{\Omega}^{\text{convex}} := \{(U, \chi) \in S_{\Omega} \mid \chi(U) \subset C_k \text{ is convex}$

*i.e.  $\forall a, b \in \chi(U), \forall t \in [0, 1] \quad ta + (1-t)b \in \chi(U)$*

[  $S_{\Omega}^{\text{convex}}$  ]

Prop 12.5  $S_{\Omega}^{\text{convex}}$  is  $C_k$ -atlas on  $\Omega$ .

[  $\exists I = \{U_i \mid (U_i, \chi_i) \in S_{\Omega}^{\text{convex}}\}$  is  $\Omega$  a open cover. ]

Observation 12.6

[  $(U, \chi) \in S_{\Omega}^{\text{convex}}, \forall s = 1, \dots, k$

$$\chi(U_s) = \chi(U) \cap \phi_s(C_{k-1}) \text{ is convex } \left( \begin{array}{l} \text{空集合不是} \\ \text{子集} \end{array} \right)$$

Def [2.7]  $(Y, \varphi)$  पर  $\Omega$  का सीमा मैट्रिक फील्ड (boundary mfd)

$\xrightarrow{\text{def}}$  (1)  $Y = (Y, S_Y)$  एक  $(k-1)$ -dimil  $C^\infty$ -mfd w.e.

प्रा.

(2)  $\varphi : Y \rightarrow \Omega$  एक  $C^\infty$ -द्वारा दिया गया फूल

प्रा.

(3)  $\forall (U, \chi) \in S_\Omega^{\text{convex}}$ ,  $\exists s = 1, \dots, k$  with  $U_s \neq \emptyset$

$\exists!$

$(V, \gamma) \in S_Y^{\text{convex}}$  s.t.  $\begin{cases} \varphi(V) = U_s \\ \gamma \circ \varphi = \phi_s \circ \gamma \text{ on } V. \end{cases}$

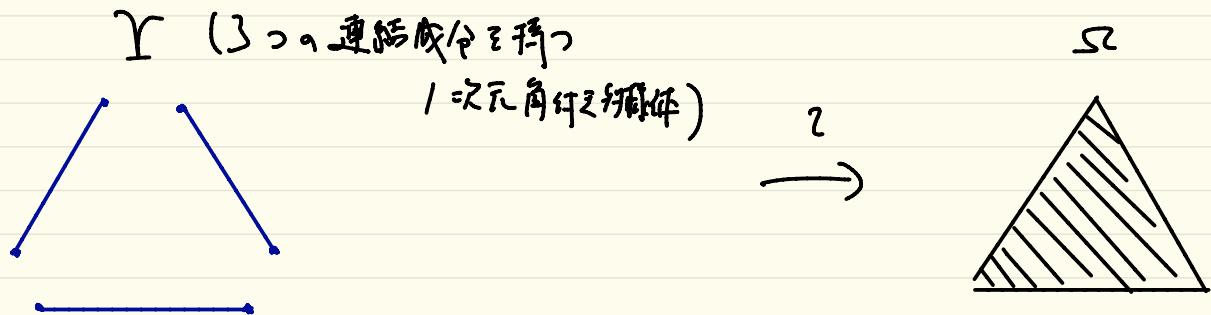
$$\begin{array}{ccc} V & \xrightarrow{\varphi} & U_s \\ \downarrow \gamma & & \downarrow \chi \\ C_{k+1} & \xrightarrow{\phi_s} & C_k \end{array}$$

प्रा

(4)  $\forall (V, \gamma) \in S_Y^{\text{convex}}$ ,  $\exists (U, \chi) \in S_\Omega^{\text{convex}}$  &  $\exists s = 1, \dots, k$

$$\text{s.t. } \begin{cases} \varphi(V) = U_s \\ \chi \circ \varphi = \phi_s \circ \gamma \text{ on } V \end{cases}$$

Ex.



2 次 単割では「事」に注意..

Remark: (3), (4) の条件は以下の通りです。

条件:

対応

$$\{(U, \chi), s\} \mid \begin{array}{l} (U, \chi) \in \mathcal{S}_X^{\text{convex}} \\ s = 1, \dots, k, U_s \neq \emptyset \end{array} \rightarrow \mathcal{S}_Y^{\text{convex}}$$

$$((U, \chi), s) \mapsto (V, \gamma)$$

$$\text{with } \gamma(V) = U_s \text{ & } \chi \circ \gamma = \phi_s \circ \psi \text{ on } V$$

は全射写像と定めます。

Theorem 12.8  $\Omega$ :  $k$ -dim'l  $C^\infty$ -mfld w.c. ( $k \geq 1$ )  $\Leftrightarrow$

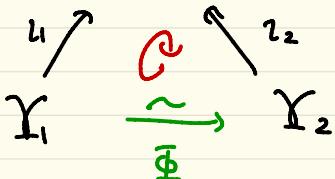
(1)  $\Omega$  a boundary mfld  $(X, \iota)$  が存在する.

(2)  $\Omega$  a boundary mfld は  $X$  の意味で唯一一意

@  $(X_1, \iota_1), (X_2, \iota_2)$  が  $\Omega$  a boundary mflds は

$\exists!$   $\bar{\Phi}: Y_1 \rightarrow X_2$  :  $C^\infty$ -diffeo s.t.  $\iota_2 \circ \bar{\Phi} = \iota_1$

$\Omega$



(証明は簡単ではないうえ、ここでは省略する)

以下、 $(\partial\Omega, \tau)$  と書く

$\Omega$  a boundary mfd (means  $\hookrightarrow$ )

を表すことを可

Thm 12.8 の意味で一意

② 何を a 誇る

$\sigma : S_{\Omega}^{\text{conv}} \rightarrow \{1, -1\} : \Omega \text{ 上の何れか} \rightarrow \text{は}.$

Def 12.9  $G_{\partial\Omega} : S_{\partial\Omega}^{\text{convex}} \rightarrow \{1, -1\}$  を以下で定め.

各  $(V, \gamma) \in S_{\partial\Omega}^{\text{convex}}$  に  $\sim$

$(U, \alpha) \in S_{\Omega}^{\text{convex}}, s = 1, \dots, k \in$

$i(V) = U_s$  かつ  $x_{s1} = \phi_s \circ \gamma$  のとき  $\begin{cases} 1 & \text{if } \gamma \text{ は} \\ -1 & \text{otherwise} \end{cases}$

(境界部分の符号)  
定義 (Def 12.7)  
より存在する  
証明.

したがって  $\sigma_{\partial\Omega}(V, \gamma) := (-1)^s \cdot \sigma(U, \alpha)$  とする.

Prop 12.10

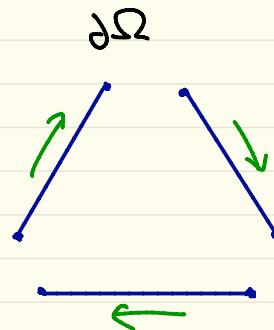
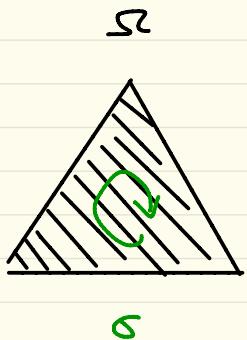
$\sigma_{\partial\Omega} : S_{\partial\Omega}^{\text{convex}} \rightarrow \{1, -1\}$  is well-defined and



Theorem 9.5 ⇒  $\sigma_{\partial\Omega}$  is well-defined.

$\sigma_{\partial\Omega}$  is "σ on the boundary  $\partial\Omega$  is well-defined" and

Ex :



### § 12.3 2+ - 7 人の定理

$\Omega$ :  $k$ -dim'l  $C^\infty$ -mfld with corners ( $k \geq 1$ )

$(\partial\Omega, \tau)$ :  $\Omega$  の boundary mfld.

$\sigma$ :  $\Omega$  上の向量

$\sigma_{\partial\Omega}$ :  $\sigma$  の誘導を  $\partial\Omega$  上の向量

とす。

Theorem 12.11 (Stokes's theorem)

$y \in L^1(\Omega) \cap C_c^k(\Omega)$

$$\boxed{\forall y \in \Lambda_c^{k-1}(\Omega), \int_{(\Omega, \sigma)} dy = \int_{(\partial\Omega, \sigma_{\partial\Omega})} i^*(y)}$$

$$j=j^*(\gamma) \in \Lambda_c^{k-1}(\partial\Omega) \text{ 且}$$

各  $g \in \partial\Omega$ ,  $w_1, \dots, w_{k-1} \in T_g \partial\Omega$  有

$T_{\gamma(g)}\Omega$  上 a 交代形式

$$(j^*(\gamma))_g(w_1, \dots, w_{k-1}) = \gamma_{\gamma(g)}(d_{\gamma(g)}(w_1), \dots, d_{\gamma(g)}(w_{k-1}))$$

是  $\times$   $\oplus$  a  $\times$   $\partial$ .

$T_{\gamma(g)}\Omega$  a 元

## ④ 2.1 - 1 次の定理の応用

### Cor 12.12

$\Omega$  が  $\mathbb{R}^n$  の境界  $\Gamma$  上で  $1 \in \Gamma_s$  の i.e.  $\partial\Omega = \emptyset$  (  $\Leftrightarrow \forall (U, \chi) \in \mathcal{S}_\Omega$  )  
とする。  
 $\forall s = 1, \dots, k$   
 $\cup_s = \emptyset$

$$\exists \alpha \in \Lambda_c^{k-1}(\Omega), \int_{(\Omega, \rho)} dy = 0$$

### Cor 12.13

$\gamma \in \Lambda_c^{k-1}(\Omega)$  として  $d\gamma = 0$  を満たすとする。

$$\exists \alpha \in \int_{(\Omega, \rho_\alpha)} \gamma^*(\gamma) = 0$$

2.1-7 介入の定理と証明 (= 2.1.7) :

主 - 亦しくは:

• 一変数関数についての微積分の基本定理

や、本質的に後割り可。

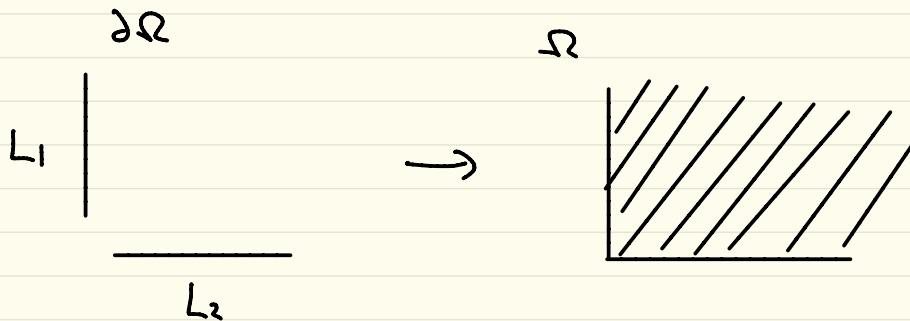
する練習として  $\Omega = C_k$  の場合を参考) .

(一般の場合については 1-1 の最後でやる) .

$$\Omega = C_k \cap \mathbb{R}^n,$$

$\therefore \Omega \approx \bigcup \partial \Omega = \underbrace{C_{k-1} \cup \dots \cup C_{k-1}}_{k_1} \cup \dots \cup \underbrace{C_{k-1}}_{k_k}$

$$\Omega = \phi_1 \cup \phi_2 \cup \dots \cup \phi_k \in \mathcal{G}_{\mathbb{R}^n},$$



$$f \in \Omega = C_k \text{ 上の } [0] \otimes \mathbb{R}$$

$$\sigma(C_k) := \sigma(C_k, id) = 1 \in H^k(\Omega; \mathbb{R})$$

$\uparrow$   
 $S_\Omega^{\text{conn}}$

$\cong \alpha \otimes 1$  かつ  $s_1 = -s_2$

$$\sigma_s(L_s) := \sigma_{\partial\Omega}(L_s, id) = (-1)^s \in H^k.$$

$${}^\Theta g \in \Lambda_c^{k-1}(\Omega) \in \mathcal{D}.$$

(ii)

$$\int_{(\Omega, \sigma)} dg = \sum_{s=1}^k \int_{(L_s, \sigma_s)} \phi_s^*(g)$$

??

$$dy = \xi \cdot dx_1 \wedge \dots \wedge dx_k \quad \text{on } \Omega = C_k$$

so  $\xi \in C^\infty(C_k) \subset \mathcal{J}$ .

( $x_1, \dots, x_k$  ist  $C_k$  a  $\mathbb{R}^k$ .)

=  $\alpha \circ \xi$

$$\text{左边} = \int_{(\Omega, \sigma)} dy = \underbrace{\sigma(\Omega, \text{id})}_{1} I_{C_k}(\xi \circ \text{id}) = I_{C_k}(\xi)$$

$$\oint \tau = \sum_{s=1}^k \tau_s$$

$$\phi_s^*(y) = \int_{\gamma_s} dy_1 \wedge \dots \wedge dy_{k-1} \quad (\tau_s \in \Omega^{k-1}(\gamma_1 \cup \dots \cup \gamma_{k-1}), C_{k-1} \text{ a } \mathbb{R}^n \text{ plane})$$

$\epsilon \gamma_s \in C^\infty(C_{k-1}) \otimes \mathcal{E}$ .

$\vdash \alpha \vdash \beta$

$$\begin{aligned} \underline{\tau} &= \sum_{s=1}^k \int_{(L_s, \gamma_s)} \phi_s^*(y) = \sum_{s=1}^k \sigma_s(L_s) I_{C_{k-1}}(\gamma_s) \\ &= \sum_{s=1}^k (-1)^s I_{C_{k-1}}(\gamma_s) \end{aligned}$$

従って以下を示せ。すな。

(示)  $I_{C_k}(\gamma) = \sum_{s=1}^k (-1)^s I_{C_{k-1}}(\gamma_s)$

$$\text{def } \mathfrak{J} \equiv dg\left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_k}\right)$$

$$= \sum_{s=1}^k (-1)^{s+1} \frac{\partial}{\partial x_s} g\left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_k}\right)$$

↗ 左側の  $\frac{\partial}{\partial x_s}$  は定義

$$[\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}] = 0$$

$$= \sum_{s=1}^k (-1)^{s+1} \frac{\partial}{\partial x_s} g_s$$

$$\text{def } \mathfrak{J}_s := g\left(\frac{\partial}{\partial x_1}, \dots, \overset{\wedge}{\frac{\partial}{\partial x_s}}, \dots, \frac{\partial}{\partial x_k}\right) \in \mathbb{R}$$

$$\text{def } J' \quad I_{C_k}(\mathfrak{J}) = \sum_{s=1}^k (-1)^{s+1} I_{C_k}\left(\frac{\partial}{\partial x_s} \mathfrak{J}_s\right) \in \mathbb{R}.$$

$$\text{Def} \quad \xi_s = (\phi_s^* \gamma) \left( \frac{\partial}{\partial y_1}, \dots, \frac{\partial}{\partial y_{k-1}} \right)$$

$$\begin{aligned}
 &= \gamma \left( d\phi_s \left( \frac{\partial}{\partial y_1}, \dots, \frac{\partial}{\partial y_{k-1}} \right) \right) \\
 &= \gamma \left( \frac{\partial}{\partial x_1}, \dots, \overset{\text{def}}{\underset{\partial}{\dots}}, \frac{\partial}{\partial x_k} \right) \circ \phi_s \equiv \gamma_s \circ \phi_s \quad \left( \because d\phi_s \left( \frac{\partial}{\partial y_j} \right) = \begin{cases} \frac{\partial}{\partial y_j} & (j \leq s-1) \\ \frac{\partial}{\partial x_{j+1}} & (j \geq s) \end{cases} \right) \\
 &\quad \text{on } C_{k-1} \\
 &\quad \text{def.}
 \end{aligned}$$

従って以下を示す

証.  $\forall s = 1, \dots, k : \text{示す}$

$$I_{C_k} \left( \frac{\partial}{\partial x_s} \gamma_s \right) = (-1) I_{C_k} (\gamma_s \circ \phi_s)$$

$$J_{C_k} \left( \frac{\partial}{\partial x_s} y_s \right) = \left( \int_0^{R_1} dx_1 \right) \cdots \left( \int_0^{R_k} dx_k \right) \frac{\partial y_s}{\partial x_s} (x_1, \dots, x_k) \quad (R_1 \cdots R_k > 0)$$

(是 T/P 公式)

$$= \left( \int_0^{R_1} dx_1 \right) \cdots \underset{s}{\underbrace{\left( \int_0^{R_k} dx_k \right)}} \left( \int_0^{R_s} \frac{\partial y_s}{\partial x_s} (x_1, \dots, x_k) dx_s \right)$$

微積分の基本定理

$$= \left( \int_0^{R_1} dx_1 \right) \cdots \underset{s}{\underbrace{\left( \int_0^{R_k} dx_k \right)}} \left( \overbrace{y_s(x_1, \dots, x_{s-1}, R_s, x_{s+1}, \dots, x_k)}^{\sim 0} - \overbrace{y_s(x_1, \dots, x_{s-1}, 0, x_{s+1}, \dots, x_k)}^{\sim 0} \right)$$

$$= (-1) \left( \int_0^{R_1} dx_1 \right) \cdots \underset{s}{\underbrace{\left( \int_0^{R_k} dx_k \right)}} y_s (x_1, \dots, x_{s-1}, 0, x_{s+1}, \dots, x_k)$$

變數變換

$$= (-1) \left( \int_0^{R_1} dy_1 \right) \cdots \left( \int_0^{R_{k-1}} dy_{k-1} \right) J_s (y_1, \dots, y_{s-1}, 0, y_s, \dots, y_{k-1})$$

$$\begin{aligned} &= (-1) \left( \int_0^{R_1} dy_1 \right) \dots \left( \int_0^{R_k} dy_{k-1} \right) (\gamma_s \circ \phi_s) \\ &= (-1)^k \gamma_s(\phi_s) \end{aligned}$$



## Theorem (2.11 の 証明 (-\delta \in \alpha の 場合))

予て 次の 補題を 認め て 主張を 示す。

Lemma (2.14)  $\psi \in C^\infty(\Omega)$  で

$\exists (U, \varphi) \in S_\Omega^{\text{convex}}$  s.t.  $\text{supp } \psi \subset U''$

を 確認せよ。

$\simeq \alpha \otimes \bar{j}$

$$\int_{(\Omega, \sigma)} \psi d\bar{y} = \int_{(\partial\Omega, \sigma|_{\partial\Omega})} i^*(\psi \bar{y}) - \int_{(\Omega, \sigma)} d\psi \wedge \bar{y}$$

$k-1$  form on  $\Omega$

$k$  form on  $\Omega$

Theorem (2.11) の 証明の 問題は 何ですか。

①  $\int_{(\Omega, \sigma)} dy = \int_{(\partial\Omega, \sigma_{\partial\Omega})} z^*(y)$

左辺の  $f_\lambda$  ( $\lambda \in \Lambda$ ) が PUS と  $\{U | (U, x) \in \mathcal{L}_{\Omega}^{\text{convex}}\}$  の  
 $f_\lambda \in C^\infty(\Omega)$  ( $\forall \lambda \in \Lambda$ ) を意味する事

(Thm 8.8 (= 定理 8.8 の 保障))

証明  $\int_{(\Omega, \sigma)} dy = \sum_{\lambda \in \Lambda} \int_{(\Omega, \sigma)} f_\lambda dy$  を 考えます。  
↓  
右辺の 和

Lemma (2.14 F')

$$\int_{(\Omega, \sigma)} dg = \sum_{\lambda \in \Lambda} \int_{(\Omega, \sigma)} f_\lambda dg$$
$$= \sum_{\lambda \in \Lambda} \left( \int_{(\partial\Omega, \sigma_{\partial\Omega})} i^*(f_\lambda) - \int_{(\Omega, \sigma)} df_\lambda \wedge g \right)$$

$$= \sum_{\lambda \in \Lambda} \int_{(\partial\Omega, \sigma_{\partial\Omega})} i^*(f_\lambda) - \sum_{\lambda \in \Lambda} \int_{(\Omega, \sigma)} df_\lambda \wedge g$$

↑  
左の有限和  
↑  
右の有限和

有限和の処理 →

$$= \int_{(\partial\Omega, \sigma_{\partial\Omega})} i^*\left(\left(\sum_{\lambda \in \Lambda} f_\lambda\right) \cdot g\right) - \int_{(\Omega, \sigma)} d\left(\sum_{\lambda \in \Lambda} f_\lambda\right) \wedge g$$

1  
↑  
左の有限和  
1  
↑  
右の有限和

$i = \pi \circ j$  の  
延長は厳密には  
ゼロではない  
なぜ

$$= \int_{(\partial\Omega, \sigma_{\partial\Omega})} i^*(g) - \int_{(\Omega, \sigma)} (d1) \wedge g = \int_{(\partial\Omega, \sigma_{\partial\Omega})} i^*(g)$$

□

Lemma (2.14 と 3.2).

$(U, x) \in S_{\Sigma}^{\text{convex}}$  with  $\text{supp } \gamma \subset U$  は 固定する.

$\gamma = \gamma^*$  で  $\xi = d\gamma\left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_k}\right) \in C^\infty(U)$  とする,

$$\int_{(2\pi)} \psi dy = \sigma(U, x) I_{C_k}((\psi \cdot \xi) \circ x^{-1})$$

と書ける.  
①

$\exists \in \mathbb{N} \quad \forall s = 1, \dots, k \quad 1 \leq s \leq k$

$$y_s \in C^\infty(U) \quad \text{et} \quad y_s = y\left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_k}\right) \quad \text{et} \quad \frac{\partial}{\partial x_s}$$

$$\text{supp}(d\psi \wedge y) \subset \text{supp } d\psi \subset \text{supp } \psi \subset U \quad \text{et} \quad \delta \geq \varepsilon$$

$$(d\psi \wedge y)\left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_k}\right) = \sum_{s=1}^k (-1)^{s+1} d\psi\left(\frac{\partial}{\partial x_s}\right) \cdot y\left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_k}\right)$$

$$= \sum_{s=1}^k (-1)^{s+1} \frac{\partial}{\partial x_s} \psi \cdot y_s \quad (\text{cf. Prop 4.11})$$

(cf. Prop 5.8)

et de plus

$$\int_{(U,x)} d\psi \wedge y = \sigma(U,x) \sum_{s=1}^k (-1)^{s+1} I_{C_k}\left(\left(\frac{\partial}{\partial x_s} \psi \cdot y_s\right) \circ x^{-1}\right) \quad \text{et}$$

③

$\Sigma \subset \mathbb{Z}^n$   $\Sigma := \{ s \in \{1, \dots, k\} \mid U_s \neq \emptyset \} \subseteq \mathcal{A}^c.$

$\forall s \in \Sigma \quad \vdash \rightarrow u \tau$

$(V_s, \gamma^s) \in S_{\partial \Omega}^{\text{convex}}$

$$\varphi(V_s) = U_s \quad \& \quad x \circ \varphi = \phi_s \circ \gamma^s$$

条件もあれば

(Def 12.7 由)  $(V_s, \gamma^s)$  は - 意見  
存在する

$\forall s \in \Sigma \quad V_s \cap V_{s'} = \emptyset \quad (s \neq s')$

$\bigsqcup_{s \in \Sigma} V_s = \varphi^{-1}(U)$  も成り立つことを示せ

$\sigma_{\partial \Omega}(V_s, \gamma^s) = (-1)^s \sigma(U, x)$  (理由: 2 面用の非対称性)

$$\text{supp } i^*(f \cdot g) = \text{supp } (i^*(f) \cdot i^*(g)) \subset \text{supp } i^*(f) \subset \bigcup_{S \in \mathcal{B}} V_S$$

$$f \circ z = \sum_{i=1}^r S_i \in \bigcup_{i=1}^r U_i \quad \text{for } i=1..r.$$

$$i^*(f \cdot g) \left( \frac{\partial}{\partial y_1}, \dots, \frac{\partial}{\partial y_{k+1}} \right) = (f \circ z) \cdot (g \left( d_z \left( \frac{\partial}{\partial y_1}, \dots, \frac{\partial}{\partial y_s} \right) \right) \circ z)$$

$\mathcal{C}^\infty(V_S)$      $\mathcal{C}^\infty(U_z)$

$$= (f \circ z) \cdot \left( g \left( \frac{\partial}{\partial x_1}, \dots, \overset{\text{↑}}{\underset{\text{↑}}{\frac{\partial}{\partial x_s}}}, \dots, \frac{\partial}{\partial x_{k+1}} \right) \circ z \right)$$

$$= (f \circ z) \cdot (g_s \circ z)$$

$$= (f \cdot g_s) \circ z$$

$$\left( \oplus d_z \left( \frac{\partial}{\partial y_i} \right) \cdot \begin{cases} \frac{\partial}{\partial x_i} & (i < s) \\ \frac{\partial}{\partial x_{i+1}} & (i \geq s) \end{cases} \right)$$

$\vdash f \circ z = f \circ g_s \circ z$

$$\int_{(\partial\Omega, \gamma_{\partial\Omega})} q^k(\chi \cdot \gamma) = \sum_{s \in S} \sigma_{\partial\Omega}(v_s, \gamma_s) I_{C_{k-1}}((\chi \cdot \gamma_s) \circ \tau \circ (\gamma_s)^{-1})$$



$$\tau \circ (\gamma_s)^{-1} = \tilde{\tau}^{-1} \circ \phi_s \text{ on } \gamma_s(v_s)$$



$$= \sum_{s \in S} \sigma_{\partial\Omega}(v_s, \gamma_s) I_{C_{k-1}}((\chi \cdot \gamma_s) \circ (\tilde{\tau}^{-1} \circ \phi_s))$$

$$= \sigma(0, \chi) \sum_{s \in S} (-1)^s I_{C_{k-1}}((\chi \cdot \gamma_s) \circ (\tilde{\tau}^{-1} \circ \phi_s))$$


---

(3)

①, ②, ③ 以下은 그림과 같다.

④

$$I_{C_k}((\varphi \cdot \tilde{y}) \circ x^{-1}) = \sum_{s=1}^k (-1)^s \left( I_{C_{k-1}}((\varphi \cdot y_s) \circ (x^{-1} \circ \phi_s)) + I_{C_k}(\frac{\partial}{\partial x_s} (\varphi \cdot y_s) \circ x^{-1}) \right)$$

$$\begin{cases} I_{C_{k-1}}(U_s \neq \emptyset \text{ and } I_{C_{k-1}}((\varphi \cdot y_s) \circ (x^{-1} \circ \phi_s)) = 0 \text{ and } \\ (s \notin B) \end{cases}$$

그리고 각 원소  $\frac{\partial}{\partial x_i}$ 는  $\varphi$ 에 대한 미분 (cf. Section 6.2)  $\in [\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}] = 0$  on  $\cup U_i$

$$\tilde{y} = d\varphi \left( \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_k} \right) \quad (i, j = 1, \dots, k)$$

$$= \sum_{s=1}^k (-1)^{s+1} \frac{\partial}{\partial x_s} \left( \varphi \left( \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_k} \right) \right)_s$$

$$= \sum_{s=1}^k (-1)^{s+1} \frac{\partial}{\partial x_s} y_s \in \mathbb{R}.$$

従って

$$\text{左辺} = I_{C_k}((\varphi \cdot \xi) \circ x^{-1}) = \sum_{s=1}^k (-1)^{s+1} I_{C_k}((\varphi \cdot (\frac{\partial}{\partial x_s} g_s)) \circ x^{-1})$$

以下を示せば十分

④ 各  $s = 1, \dots, k$  について

$$\begin{aligned} & I_{C_k}((\varphi \cdot \frac{\partial}{\partial x_s} g_s) \circ x^{-1}) \\ &= (-1) \left( I_{C_{k-1}}((\varphi \cdot g_s) \circ (\bar{x}^s \circ \phi_s)) + I_{C_k}((\frac{\partial}{\partial x_s} \varphi \cdot g_s) \circ x^{-1}) \right) \end{aligned}$$

$S = 1, \dots, k$  は 固定可.

227'

$$\tilde{Y} \cdot \tilde{\gamma}_s$$

$$(Y \cdot \gamma_s) \circ x^{-1} \in C^\infty(U)$$

$$\tilde{Y} \cdot \frac{\partial}{\partial x_s} \gamma_s \in \mathcal{C}_k \text{ で } (Y \cdot \frac{\partial}{\partial x_s} \gamma_s) \circ x^{-1} \in C^\infty(U) \in C_k \text{ 上の } C^{\infty \text{-級数}}$$

$$\tilde{\frac{\partial}{\partial x_s}} Y \cdot \gamma_s$$

$$(\frac{\partial}{\partial x_s} Y \cdot \gamma_s) \circ x^{-1} \in C^\infty(U)$$

関数  $= \frac{\partial F}{\partial x}$

( $\frac{\partial}{\partial x_s}$  が  $\alpha$  の  $\bar{\alpha}$  )

$$\left( 
 \begin{array}{l}
 \text{支集}(=) \supp(Y \cdot \gamma_s) \\
 \supp(Y \cdot \frac{\partial}{\partial x_s} \gamma_s) \text{ の } \# = 1 \text{ つ } \text{ かつ } \text{ 便} \rightarrow \text{ 1 口} = \text{1 本の } \overline{\text{支集}} \\
 \supp(\frac{\partial}{\partial x_s} Y \cdot \gamma_s)
 \end{array}
 \right)$$

$$\text{Def} \quad \underbrace{\frac{\partial}{\partial x_s} (\widehat{Y} \cdot \vec{y}_s)}_{C^0(C_k)} = \widehat{Y} \cdot \underbrace{\frac{\partial}{\partial x_s} \vec{y}_s}_{\text{Def}} + \underbrace{\frac{\partial}{\partial x_s} Y \cdot \vec{y}_s}_{\text{Def}}$$

Def von Def.

$\approx$  def

$$I_{C_k} \left( (Y \cdot \frac{\partial}{\partial x_s} \vec{y}_s) \cdot x^{-1} \right)$$

$$= I_{C_k} \left( \widehat{Y} \cdot \underbrace{\frac{\partial}{\partial x_s} \vec{y}_s}_{\text{Def}} \right)$$

$$= I_{C_k} \left( \frac{\partial}{\partial x_s} (\widehat{Y} \cdot \vec{y}_s) \right) - I_{C_k} \left( \frac{\partial}{\partial x_s} Y \cdot \vec{y}_s \right)$$

Def.

$R_1, \dots, R_k > 0 \in \mathbb{R}$

$$I_{Ck} \left( \frac{\partial}{\partial x_s} (\widehat{Y \cdot \gamma}_s) \right)$$

$$= \left( \int_0^{R_1} dx_1 \right) \cdots \left( \int_0^{R_k} dx_k \right) \left( \frac{\partial}{\partial x_s} (\widehat{Y \cdot \gamma}_s) \right) (x_1, \dots, x_k)$$

$\gamma_t =$  →  $= \left( \int_0^{R_1} dx_1 \right) \cdots \underset{s}{\overset{\wedge}{\left( \int_0^{R_k} dx_k \right)}} \int_0^{R_s} \frac{\partial}{\partial x_s} (\widehat{Y \cdot \gamma}_s) (x_1, \dots, x_k) dx_s$

$\alpha$  這裡 = 0 ( $= R_s \cdot 0 + 1/2 \pi \cdot 0$ )

$\text{微積分} \Rightarrow$  →  $= \left( \int_0^{R_1} dx_1 \right) \cdots \underset{s}{\overset{\wedge}{\left( \int_0^{R_k} dx_k \right)}} \left( \begin{array}{l} \widehat{Y \cdot \gamma}_s (x_1, \dots, x_{s-1}, R_s, x_{s+1}, \dots, x_k) \\ - \widehat{Y \cdot \gamma}_s (x_1, \dots, x_{s-1}, 0, x_{s+1}, \dots, x_k) \end{array} \right)$

$\text{基底定理}$

$$= (-1) \left( \int_0^{R_1} dx_1 \right) \cdots \underset{s}{\overset{\wedge}{\left( \int_0^{R_k} dx_k \right)}} \widehat{Y \cdot \gamma}_s (x_1, \dots, x_{s-1}, 0, x_{s+1}, \dots, x_k)$$

$$= (-1) \left( \int_0^{R_1} dy_1 \right) \cdots \left( \int_0^{R_k} dy_{k+1} \right) \widehat{(\psi \cdot \eta)_s}(y_1, \dots, y_{s+1}, 0, y_s, \dots, y_{k+1})$$

变量替换

$$= (-1) \left( \int_0^{R_1} dy_1 \right) \cdots \left( \int_0^{R_k} dy_{k+1} \right) ((\widehat{\psi \cdot \eta}_s) \circ \phi_s)(y_1, \dots, y_{k+1})$$

$$= (-1) I_{C_{k+1}} (\widehat{\psi \cdot \eta}_s \circ \phi_s)$$

$$= (-1) I_{C_{k+1}} ((\psi \cdot \eta)_s \circ (\pi' \circ \phi_s))$$

 ①

よし

$$I_{C_k} \left( \underbrace{\frac{\partial}{\partial x_s} \psi \cdot \gamma_s}_{\text{②}} \right) = I_{C_k} \left( \left( \frac{\partial}{\partial x_s} \psi \cdot \gamma_s \right) \circ x^{-1} \right) \quad \text{③}$$

①, ② より

$$\begin{aligned} & I_{C_k} \left( (\psi \cdot \frac{\partial}{\partial x_s} \gamma_s) \circ x^{-1} \right) \\ &= (-1) \left( I_{C_{k-1}} ((\psi \cdot \gamma_s) \circ (x^{-1} \circ \phi_r)) + I_{C_k} \left( \left( \frac{\partial}{\partial x_s} \psi \cdot \gamma_s \right) \circ x^{-1} \right) \right) \end{aligned}$$

以上 ふさわしい。

