

The 23rd International Differential Geometry Workshop on Submanifolds in Homogeneous Spaces & Related Topics

- The 19th RIRCM-OCAMI Joint Differential Geometry Workshop -

- **Date:** July 2 (Fri) ~ July 3 (Sat), 2021
- **Place:** Zoom Meeting

- **Organizing Committee** (Republic of Korea)
Changhwa Woo (Pukyong Nat'l University, Republic of Korea)
Hyunjin Lee (Kyungpook Nat'l University & RIRCM, Republic of Korea)

- **Organizing Committee** (Japan)
Hiroshi Tamaru (Osaka City University & OCAMI, Japan)
Takayuki Okuda (Hiroshima University, Japan)
Kei Kondo (Okayama University, Japan)

- **Scientific Committee**
Young Jin Suh (Kyungpook Nat'l University & RIRCM, Republic of Korea)
Yoshihiro Ohnita (Osaka City University & OCAMI, Japan)

- **Supported by**
NRF (National Research Foundation of Korea)
JSPS (Japan Society for The Promotion of Science)

- **Organized by**
RIRCM (Research Institute of Real and Complex Manifolds)
OCAMI (Osaka City University Advanced Mathematical Institute)



July 2 (Friday), 2021

Place: Zoom

10:00~11:50 **Chair Prof. Young Jin Suh**

- 10:00~10:20 Opening Address
- 10:20~10:50 Jaigyoung Choe (Korea Institute for Advanced Study)
The zero determinant set and determinantal variety
- 10:50~11:20 Yong Seung Cho (Ewha Womans University)
Algebraic cycles in product spaces
- 11:20~11:50 Jong Taek Cho (Chonnam National University)
CR-symmetry of contact hypersurfaces and the generalized Tanaka-Webster connection

11:50~13:40 Lunch Time

13:40~15:20 **Chair Prof. Jaigyoung Choe**

- 13:40~14:20 Mukut Mani Tripathi (Banaras Hindu University, India)
Different kind of submanifolds in Hermitian and contact geometry
- 14:20~14:50 Juncheol Pyo (Pusan National University)
Translating solitons for the mean curvature flow
- 14:50~15:20 Seoung Dal Jung (Jeju National University)
Hard Lefschetz theorem on transversely symplectic foliations

15:20~15:40 Group Photo & Coffee Break

15:40~16:50 **Chair Prof. Byung Hak Kim**

- 15:40~16:10 Yoshihiro Ohnita (Osaka City University)
Parallel Kähler submanifolds and R-spaces
- 16:10~16:30 Kengo Nakahara (Okayama University)
Totally geodesic surfaces in $SO(n+2)/SO(2) \times SO(n)$
- 16:30~16:50 Masahiro Morimoto (Osaka City University)
Curvatures and austere property of orbits of path group actions induced by Hermann actions

16:50~17:20 Coffee Break

17:20~18:00 **Chair Prof. Juncheol Pyo**

- 17:20~17:40 Young-Jun Choi (Pusan National University)
Existence of a complete holomorphic vector field via the Kähler-Einstein metric
- 17:40~18:00 Luis Pedro Castellanos Moscoso (Osaka City University)
Moduli Spaces and left-invariant symplectic structures on Lie groups

July 3 (Saturday), 2021

Place: Zoom

10:00~12:00		Chair Prof. Yoshihiro Ohnita
10:00~10:30	Qing-Ming Cheng (Fukuoka University) Complete self-shrinkers of mean curvature flow	
10:30~11:00	Seungsu Hwang (Chung-Ang University) Besse conjecture with positive isotropic curvature	
11:00~11:10 Coffee Break		
11:10~11:40	Hiroshi Tamaru (Osaka City University) Totally geodesic surfaces in symmetric spaces and applications	
11:40~12:00	Hyunjin Lee (Kyungpook National University) Killing tensor on Hopf real hypersurfaces in the complex quadric	
12:00~13:40 Lunch Time		
13:40~15:00		Chair Prof. Takayuki Okuda
13:40~14:20	Yaning Wang (Henan Normal University, P. R. China) Harmonicity of the curvature and Weyl tensors in real hypersurfaces in nonflat complex space forms	
14:20~14:40	Aeryeong Seo (Kyungpook National University) Proper holomorphic maps between bounded symmetric domains	
14:40~15:00	Yuichiro Sato (Tokyo Metropolitan University) Duality of hypersurfaces in pseudo-Riemannian space forms and lightcones	
15:00~15:20 Group Photo & Coffee Break		
15:20~16:20		Chair Prof. Changhwa Woo
15:20~15:40	Gyu Jong Kim (Woosuk University) Real hypersurfaces in the complex hyperbolic quadric with contact condition	
15:40~16:00	Yuya Ikeda (Hiroshima University) Designs on vector bundles	
16:00~16:20	Masahiro Kawamata (Hiroshima University) Left-invariant Ricci soliton metrics on some almost abelian Lie groups	
16:20~16:40		Chair Prof. Gyu Jong Kim
16:20~16:40	Changhwa Woo (Pukyong National University) Real hypersurfaces in the complex quadric with semi-symmetric structure Jacobi operators	
16:40~17:00 Closing Ceremony		
17:00~18:00 Discussions in Hermitian Symmetric Spaces		

Title: The zero determinant set and determinantal variety

Jaigyung Choe

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Abstract: It will be proved that the zero determinant set and the determinantal variety are minimal submanifolds in the Euclidean space.

Title: Algebraic Cycles in Product Spaces

Yong Seung Cho

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Abstract: We introduce the rational Hodge structure and morphisms between Hodge structures. As an example we show that the holomorphic maps on a Kähler manifold induce Hodge classes in its product space which are represented by their graphs in the product space, that is, algebraic cycles. On the Hodge manifolds, the Lefschetz isomorphism is represented by an algebraic class on its product space. We show that the inverse of the Lefschetz

isomorphism is an algebraic cycle on the product space, by studying the normal bundle of the diagonal in the product space.

Title: CR-symmetry of contact hypersurfaces and the generalized Tanaka-Webster connection

Jong Taek Cho

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Abstract: In this talk, we give a realization of some class of contact Riemannian manifolds by real hypersurfaces of the complex quadric and its non-compact dual space, which enables us to prove the classification theorem of CR-symmetric pseudo-Hermitian manifolds. On the other hand, we introduce the generalized Tanaka-Webster connection for real hypersurfaces of Kählerian manifolds and using it we have characterization theorems.

**Title: Different Kind of Submanifolds
in Hermitian and Contact Geometry**

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Abstract: In this presentation, a sincere attempt has been executed to give a brief review of different kind of submanifolds in Hermitian and contact geometry. Several classes of almost Hermitian structures have been recalled. These classes include Hermitian, Kaehler, almost Kaehler manifold, nearly Kaehler, and locally conformal Kaehler structure. In fact, according to Gray-Hervella classification, there are sixteen well-known classes of almost Hermitian structures. Different kind of submanifolds of almost Hermitian manifolds include J-invariant submanifold, complex submanifold, holomorphic submanifold, curvature invariant submanifold, anti-J-invariant submanifold, Lagrangian submanifold, CR-submanifold (different versions), generalized CR-submanifold, anti-holomorphic submanifold (different versions), totally real submanifold (different versions), f-submanifold, D-almost complex submanifold, semi-invariant submanifold, generalized semi-invariant submanifold (introduced by the author), purely real submanifold, θ -slant submanifold, quasi slant submanifold (or pointwise slant submanifold), semi-slant submanifold, a pointwise semi-slant submanifold, anti-slant submanifold (or hemi-slant submanifold), bi-slant submanifold, and generic submanifolds (different versions).

Next, several classes of almost contact metric structures have also been recalled, including contact, K-contact, Sasakian, Kenmotsu, and cosymplectic structures. Different kind of submanifolds of almost contact metric manifolds include invariant submanifold, anti-invariant submanifold, semi-invariant submanifold, contact CR-submanifold, θ -slant sub-manifold, pointwise slant submanifold, semi-slant submanifold, pointwise semi-slant sub-manifold, anti-slant submanifold (or pseudo-slant submanifold or hemi-slant submanifold), bi-slant submanifold, quasi hemi-slant submanifold. Interestingly, these submanifolds are particular cases of an almost semi-invariant submanifold introduced by the author in 1996. Some basic results will be presented.

Title: Translating solitons for the mean curvature flow

Juncheol Pyo

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Abstract: Translating solitons are not only special solutions of the mean curvature flow but also blow-up limit at a singularity. They are minimal submanifolds in a Riemannian manifold which is conformal with Euclidean space and so they share properties with minimal surfaces in Euclidean space.

In this talk, we introduce translating solitons and some of their geometric properties. Moreover, we determine which half-space contains a complete translating soliton of the mean curvature flow and it is related to the well-known half-space theorem for minimal surfaces.

Title: Hard Lefschetz theorem on transversely symplectic foliations

Seoung Dal Jung

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Abstract: On a compact Riemannian manifolds, any cohomology class contains just one harmonic form. But on symplectic manifolds, we can not define the harmonic form as in the Riemannian case. In 1988, J.-L. Brylinski introduced the notion of symplectic harmonic form on a symplectic manifold and in 1955, O. Mathieu proved the hard Lefschetz theorem on a symplectic manifold. In this talk, we discuss the hard Lefschetz theorem on transversely symplectic foliations.

Title: Parallel Kähler submanifolds and R-spaces

Yoshihiro Ohnita

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& Osaka City University
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Abstract: Parallel Kähler submanifolds here mean complex submanifolds immersed in complex projective spaces with parallel second fundamental form.

Such submanifolds were classified by Hisao Nakagawa and Ryoichi Takagi (1976), Masaru Takeuchi (1978, 1984) by two different methods of unitary representation theory and Jordan triple systems.

In this talk we briefly survey such related submanifold theory and give the third proof for their classification theorem, based on the differential geometric characterization of R -spaces due to Carlos Olmos and Cristian Utreras (1991).

Title: Totally geodesic surfaces in $SO(n+2)/SO(2)\times SO(n)$

Kengo Nakahara

Okayama University
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Japan

Abstract: In symmetric spaces it is a well-known fact that, if there is a Lie triple system in the tangent space at a point, then the totally geodesic submanifold tangent to the system at the point is obtained as an image of the exponential map of the system. In this talk we explicitly give totally geodesic surfaces corresponding to some Lie triple systems in the tangent space of $SO(4)/SO(2)\times SO(2)$.

Title: Curvatures and austere property of orbits of path group actions induced by Hermann actions

Masahiro Morimoto

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Japan

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Abstract: It is known that an isometric action of a Lie group on a compact symmetric space induces a proper Fredholm action of a path group on a path space via the gauge transformations. In this talk, supposing that the isometric action is a Hermann action, we

study the principal curvatures and the austere property of orbits of the induced path group action.

Title: Existence of a complete holomorphic vector field via the Kähler-Einstein metric

Youngjun Choi

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Abstract: A fundamental problem in Several Complex Variables is to classify bounded pseudoconvex domains in the complex Euclidean space with a noncompact automorphism group, especially with a compact quotient.

In the results of Wong-Rosay and Frankel, they make use of the "Scaling method" for obtaining an 1-parameter family of automorphisms, which generates a holomorphic vector field. In this talk, we discuss the existence of a nowhere vanishing complete holomorphic vector field on a strongly pseudoconvex manifold admitting a negatively curved Kähler-Einstein metric and discrete sequence of automorphisms by introducing the scaling method on potentials of the Kähler-Einstein metric. This is a joint work with Kang-Hyurk Lee in Gyeongsang National University.

Title: Real hypersurfaces in the complex quadric with semi-symmetric structure Jacobi operators

Changhwa Woo*, Young Jin Suh, Gyu Jong Kim

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Abstract: In this paper, we introduce new notions of semi-symmetric structure Jacobi operator in the complex quadric $Q_m = SO_{m+2}/SO_mSO_2$. Next we prove that there does not exist a Hopf real hypersurface in the complex quadric $Q_m = SO_{m+2}/SO_mSO_2$ with such a notion. Naturally, in the ambient space, we find out new results of non existence of real hypersurfaces with the notion of symmetric parallel, recurrent for the structure Jacobi operators as a corollary.

Title: Moduli Spaces and left-invariant symplectic structures on Lie groups

Luis Pedro Castellanos Moscoso

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Abstract: We are interested in the classification or finding conditions for the existence of left-invariant symplectic structures on Lie groups. Some classifications are known, especially in low dimensions. We approach this problem by studying the "moduli space of left-invariant nondegenerate 2 -forms", which is a certain orbit space in the set of all nondegenerate 2 -forms on a Lie algebra. We

present some of the results obtained so far with our approach, including a classification of left-invariant symplectic structures on some almost abelian Lie algebras

Title: Complete self-shrinkers of mean curvature flow

Qing-Ming Cheng

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Abstract: In this talk, we consider classifications of complete self-shrinkers in R^{n+1} with constant squared norm of the second fundamental form. The following conjecture is well-known and very important:

Conjecture. *A n -dimensional complete self-shrinker $X: M \rightarrow R^{n+1}$ in R^{n+1} with constant squared norm of the second fundamental form is isometric to one of the following:*

- (1) R^n
- (2) $S^k(\sqrt{k}) \times R^{n-k}, k = 1, 2, \dots, n-1,$
- (3) $S^n(\sqrt{n})$

For $n = 2$, Cheng and Ogata have solved this conjecture affirmatively.

For the higher dimension n , it is hard to classify complete self-shrinkers in Euclidean space with constant squared norm S .

Cheng, Li and Wei ([to appear in Math. Z.](#)), under the assumption that f_4 is constant, have solved this conjecture for $n = 3$, affirmatively.

Theorem. Let $X: M^3 \rightarrow R^4$ be a 3-dimensional complete self-shrinker in R^4 . If the squared norm S of the second fundamental form and f_4 are constant, then $X: M^3 \rightarrow R^4$ is isometric to one of

- (1) R^3
- (2) $S^1(1) \times R^2$
- (3) $S^2(\sqrt{2}) \times R^1$
- (4) $S^3(\sqrt{3})$

Title: Besse conjecture with Positive isotropic curvature

Seungsu Hwang

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Abstract: The critical point equation(CPE) arises as a critical point of the total scalar curvature functional defined on the space of constant scalar curvature metrics of a unit volume on a compact manifold. It has been conjectured that if (g, f) is a solution of the critical point equation, then g is Einstein and so (M, g) is isometric to a standard sphere. In this talk, we show that this conjecture is true if the given Riemannian metric has positive isotropic curvature.

Title: Totally geodesic surfaces in symmetric spaces and applications

Hiroshi Tamaru

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Abstract: In this talk we discuss totally geodesic surfaces in symmetric spaces, especially on the classification problems. As its

applications, we also mention some results on totally geodesic complex curves in Hermitian symmetric spaces, and totally geodesic submanifolds in symmetric spaces of type AI.

Title: Killing tensor on Hopf real hypersurfaces in the complex quadric

Hyunjin Lee
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Abstract: In this talk, we will give some characterization for Hopf real hypersurfaces in the complex quadric $Q^m = SO_{m+2}/SO_m SO_2$. The complex quadric Q^m is a kind of real Grassmannians with rank 2 of compact type, which is given by a complex hypersurface in the complex projective space. Accordingly, Q^m admits both a real structure and a complex structure with anti-commutes with each other. From these geometric structures of Q^m we can also induce the geometric structures of a real hypersurface M in Q^m . By using these structures, in this talk we will give some classification problems for Hopf real hypersurfaces in Q^m in terms of Killing with respect to $(1,1)$ type symmetric tensors of M .

Title: Harmonicity of the curvature and Weyl tensors in real hypersurfaces in nonflat complex space forms

Yaning Wang
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Abstract: The local classification problem for real hypersurfaces in nonflat complex space forms under certain geometric condition is one of the most interesting problems in geometry of real hypersurfaces. In this talk, we introduce some latest results in this field regarding the hamonicity of the curvature and Weyl tensors.

Title: Proper holomorphic maps between bounded symmetric domains

Aeryeong Seo

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Abstract: If the inverse images of a map of compact subsets of the target manifold are compact, then we say that the map is proper. Proper holomorphic maps between bounded domains in the Euclidean spaces have been studied quite intensively since Alexander proved that any proper holomorphic self-map of the unit ball is an automorphism. In this talk, I will introduce some background work on the rigidity of proper holomorphic maps between bounded domains and present my recent work with Sung-Yeon Kim about that between bounded symmetric domains.

**Title: Duality of hypersurfaces in pseudo-Riemannian space
forms and lightcones**

Yuichiro Sato

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Abstract: When we consider hypersurfaces in pseudo-spheres, pseudo-hyperbolic spaces and lightcones, there exist three types of duality, which we call the duality of hypersurfaces. In this talk, we will introduce the notion of palindromic hypersurfaces and show that infinitesimal symmetries can be shifted by the duality of hypersurfaces. Moreover, we will give some applications.

**Title: Real hypersurfaces in the complex hyperbolic quadric
with contact condition**

Gyu Jong Kim
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Abstract: We give a new proof of the classification of contact real hypersurfaces in the complex hyperbolic quadric $Q^{m,*} = SO_{m,2}^o / SO_{m,2}$, where $m \geq 3$, without the assumption of constant mean curvature. We show that a contact real

hypersurface M in Q^{m+1} for $m \geq 3$ is locally congruent to a tube of radius $r \in \mathbb{R}^+$ around the complex hyperbolic quadric Q^{m-1} , or to a tube of radius $r \in \mathbb{R}^+$ around the A -principal m -dimensional real hyperbolic space $\mathbb{R}H^m$ in Q^{m+1} , or to a horosphere in Q^{m+1} induced by a class of $\frac{A}{m}$ -principal geodesics in Q^{m+1} .

Title: Designs on vector bundles

Yuya Ikeda

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Japan

Abstract: Let M be a homogenous G -space. Fix H a G -representation consisting of continuous functions on M and denote by S the G -invariant averaging operator on M . A finite subset X of M is called a "design" for (H, S) if the averaging operator S on M is same as the averaging operator on X for H . Designs on spheres, unitary groups and some other compact symmetric spaces have been studied by many researchers in the area of combinatorics.

In this talk, we consider G -equivariant vector bundles E on M , and generalize the concept of designs for G -representations H consisting of continuous sections of E , and G -intertwining operators S on H .

Title: Left-invariant Ricci soliton metrics on some almost abelian Lie groups

Masahiro Kawamata

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Japan

Abstract: For a given Lie group, it is known that the group which is generated by its automorphisms and non-zero scalar acts on the space of all left-invariant Riemannian metrics on that Lie group. In terms of this group action, each left-invariant Riemannian metric corresponds to each orbit of that action. Then, a special left-invariant Riemannian metric can be expected to correspond to a special orbit.

In this talk, for almost abelian Lie groups which this group action is of cohomogeneity one, we introduce that a left-invariant Riemannian metric is a left-invariant Ricci soliton metric if and only if the corresponding orbit is an isolated orbit.
