Jose Angel Frías and Enrique Ramírez-Losada pointed out that the proof of Lemma 5.9 in my paper [1] is false. This is caused by the improper application of the argument in the proof of Lemma 5.3. The purpose of this note is to amend the proof.

Proof of Lemma 5.9. The proof remains valid until the end of the second paragraph. Assume \( t > 4 \). For simplicity, let \( \sigma(1) = r \). Then \( G_S \) has two \( S \)-cycles with label pairs \( \{t/2, t/2 + 1\} \) and \( \{t/2 + r - 1, t/2 + r\} \). Since \( r \geq 3 \), these pairs are disjoint. Thus the edges of these \( S \)-cycles form two essential cycles on \( \hat{T} \), which split \( \hat{T} \) into two annuli \( F_1 \) and \( F_2 \).

Although the families \( A \), \( B \) and \( D \) correspond to the same permutation \( \sigma \), \( C \) corresponds to another permutation \( \sigma' \) such that an edge in \( C \) has label \( j \) at \( u_1 \) and label \( \sigma'(j) \equiv j + r - 3 \pmod{t} \) at \( u_2 \). Note that \( A \) contains a \( \{t/2, t/2 + r - 1\} \)-edge \( a_1 \) and a \( \{t/2 + 1, t/2 + r\} \)-edge \( a_2 \). They lie in the same annulus, \( F_1 \) say, or distinct annuli on \( \hat{T} \). In either case, it implies that \( \sigma \) has exactly two orbits. Thus the edges of \( A \) form two disjoint cycles \( L_1 \) and \( L_2 \) on \( \hat{T} \), where \( L_1 \) contains \( v_{t/2} \) and \( L_2 \) contains \( v_{t/2+1} \). If \( a_1 \) and \( a_2 \) lie in \( F_1 \), then there is no vertex inside \( F_2 \), so we have a contradiction as in the proof of Lemma 2.7(2) by constructing a twice-punctured Klein bottle. Hence we may assume that \( a_i \) lies in \( F_i \) for \( i = 1, 2 \). Then all vertices on \( L_1 \) but \( v_{t/2} \) lie inside \( F_2 \), and all vertices on \( L_2 \) but \( v_{t/2+1} \) lie inside \( F_1 \).

Assume \( t \geq 8 \). Among the positive loops at \( u_1 \), there is an edge \( e \) whose labels are not the labels of those \( S \)-cycles. In fact, if \( r \neq 3, t - 1 \), then choose the \( \{t/2 - 1, t/2 + 2\} \)-edge as \( e \). Otherwise, choose the \( \{t/2 + 4, t/2 - 3\} \)-edge as \( e \). However, we cannot locate the edge \( e \) on \( \hat{T} \).

Assume \( t = 6 \). Then \( r = 3 \) or \( 5 \). Let \( a, b \) and \( d \) be the edges of \( A, B \) and \( D \) with label 1 at \( u_1 \). Then these three edges, connecting \( v_1 \) with \( v_r \), are mutually parallel on \( \hat{T} \). Hence there are at least five parallel edges between \( v_1 \) and \( v_r \). This means that \( G_S \) contains at least two edges with label \( r \) at \( u_1 \) and label \( 1 \) at \( u_2 \). However, there is no such edge when \( r = 3 \), or only one such edge when \( r = 5 \).

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References


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