An analysis of the spectral function on finite temperature lattices

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Contents

Introduction I

QCD at high density and high temperature

$\Longrightarrow$ Quark-Gluon Plasma phase

- deconfinement
- chiral symmetry restoration

Study of QGP

- NJL model Hatsuda&Kunihiro(1994)

  soft mode : fluctuation of $\langle q\bar{q}\rangle \rightarrow \pi\ \rho$ mode

- DeTar's conjecture DeTar(1985)

  confining feature for scale $\geq 1/g^2T$

- QCD sum rule Hatsuda,Koike and Lee (1993)

fundamental degree of freedom

quark & gluon $\Longleftrightarrow$ hadron

$\Longrightarrow$ Non-perturbative treatment of QCD

Lattice QCD at $T > 0$

Charmonium at $T > 0$

- J/ψ (charmonium) states
  - important signal for formation of QGP

    heavy ion experiment at CERN(SPS),BNL(RHIC)

    c.f. NA50 Phys.Lett.B477(2000)28

- Theoretical understanding
  - Potential model

    Mass shift near T_c

    T. Hashimoto et al., Phys.Rev.Lett.57(1986)2123

    J/ψ suppression above T_c

    T.Matsui and H.Satz, Phys.Lett.B178(1986)416

    $\Longrightarrow$ no bound state at $T \gtrsim 1.1T_c$
  - Lattice QCD
    - ▷ Meson correlators

      T. Umeda et al., Int.J.Mod.Phys.A16(2000)2215

      $\Longrightarrow$ not free behavior at $T \sim 1.5T_c$

Study of Charmonium at $T > 0$ using lattice QCD

Our approach I

Temporal correlators in lattice simulations

$$C(t) = \sum_{\vec{x}} \langle O(\vec{x}, t) O^{\dagger}(0) \rangle \qquad [O(\vec{x}, t) : \text{meson op.}]$$

$\Longrightarrow$ Spectral function $A(\omega)$

Abrikosov et al. Sov. Phys. J. 36 (1959) 636

$$C(t) = \int d\omega K(t, \omega) A(\omega) \qquad K(t, \omega) = \frac{e^{-\omega t} + e^{-\omega(N_t - t)}}{1 - e^{-N_t \omega}}$$

- Anisotropic lattice
- Our analysis procedure
  - Maximum entropy method (MEM)

    Nakahara et al., Phys. Rev. D60 (99) 091503

    — no assumption for the form of $A(\omega)$
  - Fit with ansatz for spectral function

    — need information on the form of $A(\omega)$

    — with given form $A(\omega)$, more quantitative

    Constrained curve fitting

    Lepage et al., Nucl. Phys. B (PS) 106 (2002) 12

We use these two methods in complementary manner

- smearing operator

  enhancement of low frequency modes

Our approach II

☐ Reliability check for an analysis at $T > 0$

(1) Check of the stability;

stability against changes of input parameters
or model functions

(2) Check with T=0 correlators;

stability against restriction of d.o.f. used in analysis

(We use the data at $T = 0.9T_c$ for the latter check,
but it should be done with the data at $T = 0$.)

☐ Validity of smeared correlators

possibility of an artificial peak

I.Wetzorke et al., Nucl.Phys.B(PS)106(2002)513

$\Longrightarrow$ check with various smearing functions

Maximum Entropy Method (MEM)

Reconstruction of a spectral function
with Maximum Entropy Method

Y. Nakahara, et al., Phys. Rev. D60 (1999) 091503

$$C(\tau) = \int d\omega K(\tau,\omega) A(\omega)$$

kernel :

$$K(t,\omega) = \frac{e^{-\omega t} + e^{-\omega(N_t - t)}}{1 - e^{-N_t \omega}}$$

Standard χ^2-fit $\longrightarrow$ ill-posed problem

$\Longrightarrow$ MEM (based on Bayes' theorem)

Maximization of $Q = \alpha S - L$

$$S = \int d\omega \left[A(\omega) - m(\omega) - A(\omega) \ln \frac{A(\omega)}{m(\omega)} \right]$$

$m(\omega) = m_0 \omega^2$: model function

α : constant $\longrightarrow$ to be integrated out

L : Likelihood function (χ^2-term)

Constrained Curve Fit (CCF)

A simple modification of standard χ^2 fitting
based on the Bayesian statistics

G.Lepage et al.,Nucl. Phys.B(PS)106(2002)12

$$\chi^2 \rightarrow \chi^2_{arg} \equiv \chi^2 + \chi^2_{prior}, \quad \chi^2_{prior} \equiv \sum_i \frac{(c_i - \tilde{c}_i)^2}{\tilde{\sigma}^2_{c_i}}$$

c_i : fit parameters,

$\tilde{c}_i$, $\tilde{\sigma}_{c_i}$: input parameters as prior knowledge

- many parameters fitting become stable
- bias from the input parameters
  $\Longrightarrow$ MEM results are suitable for prior knowledge

Function form

We suppose the shape of spectral function.

(MEM gives rough estimate of shape of spectral function.)

∘ Breit-Wigner (BW) type :

$$A(\omega) = \omega^2 \rho(\omega), \quad \rho(\omega) = \frac{C\Gamma m}{(\omega^2 - m^2)^2 - \Gamma^2 m^2}$$

C : overlap, m : mass, Γ : width

$\Longrightarrow$ We apply multi-BW fits
and confirm number of terms (N_{term}) dependences.

The details of the quark action

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \;=\; & \delta_{x,y} - \kappa_t \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & -\kappa_s \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & -\kappa_s c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_s c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215
J. Harada et al., Phys. Rev. D 64 (2001) 074501

- Constructed following the Fermilab approach.

  El-Khadra et al., Phys. Rev. D 55 (1997) 3933

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
  (cf. another choice $r = 1$ was adopted in several works.)

- Tadpole improvement: $c_E = 1/u_s^2 u_t$, $c_B = 1/u_s^3$
  u_s, u_t: mean-field values of link variables

- Parameters varied in simulations: (κ, γ_F)

$$\gamma_F \equiv \frac{\kappa_t u_t}{\kappa_s u_s}, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_s u_s} - 2(\gamma_F + 3r - 4) \quad (= 2(m_0 \gamma_F + 4))$$

γ_F: bare anisotropy parameter
m_0: bare quark mass (in temporal lattice units)

Simulation parameters (for charmonia)

Gauge parameters:

— Anisotropic plaquette action in quenched approximation

□ $(\beta, \gamma_G) = (6.10, 3.2108)$

- $\xi = a_s/a_t = 4$

  Klassen, Nucl. Phys. B 533 (1998) 557

- $a_s^{-1} = 2.030(13)$ GeV

  Scale is set by hadronic radius r_0

□ Size : $20^3 \times N_t$

N_t	T/T_c	$N_{conf} \times N_{source}$	smearing function
160	~ 0	500×16	point, smeared
32	~ 0.9	1000×16	point, smeared
26	~ 1.1	1000×16	point, smeared, half-smeared

Quark parameters :

— $O(a)$ improved Wilson quark action

□ $(\kappa, \gamma_F) = (0.1120, 4.000)$

$\rightarrow$ roughly correspond to charm quark mass

H. Matsufuru et al., Phys. Rev. D 64 (2001) 114503

ground state & 1st excited state mass m_0 & m_1 in a_t units

m_0(PS)= 0.36856(9), m_1(PS)=0.500(22)

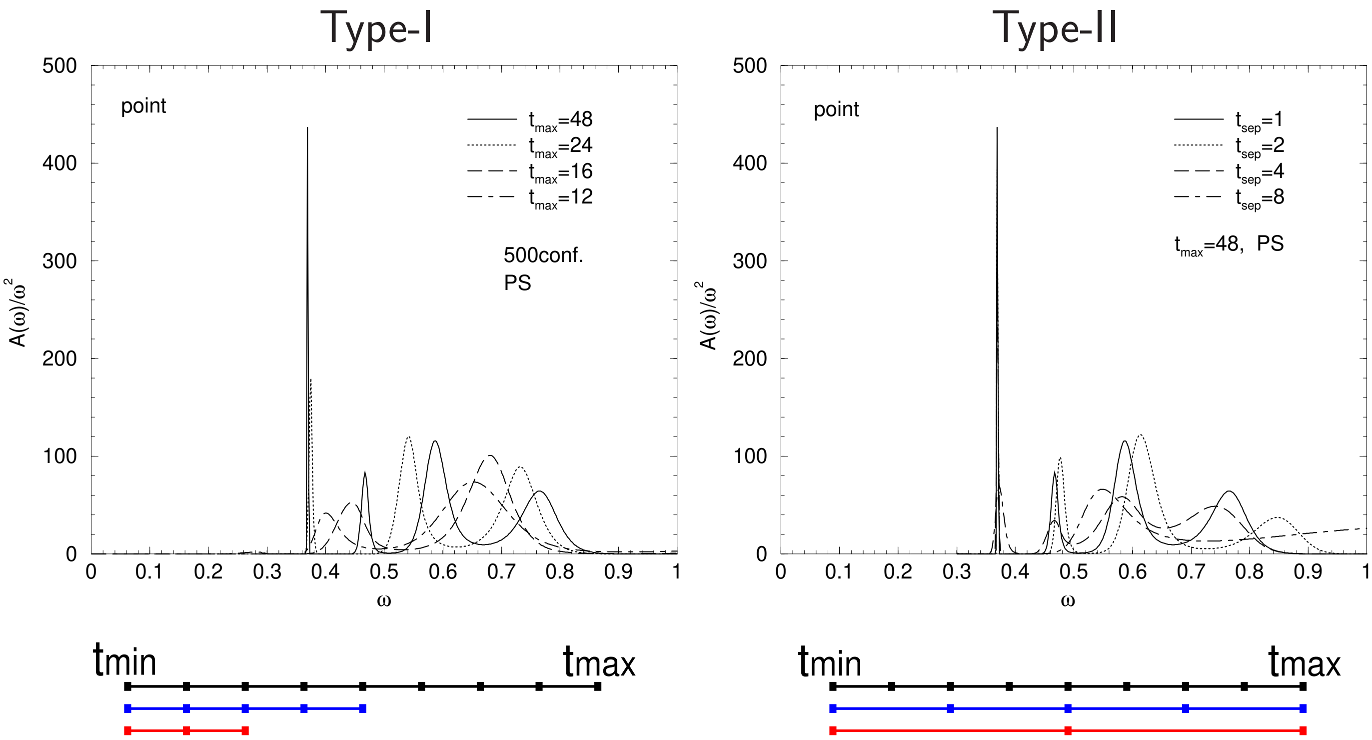
m_0(V) = 0.37769(12), m_1(V)=0.479(23)

Results for charmonia (1): point correlators

☐ Point-point correlators

	t_{min}	t_{max}	t_{sep}	N_{DF}		t_{min}	t_{max}	t_{sep}	N_{DF}
Type-I	1	48	1	48	Type-II	1	48	1	48
	1	24	1	24		2	48	2	24
	1	16	1	16		4	48	4	12
	1	12	1	12		8	48	8	6

Results of MEM:



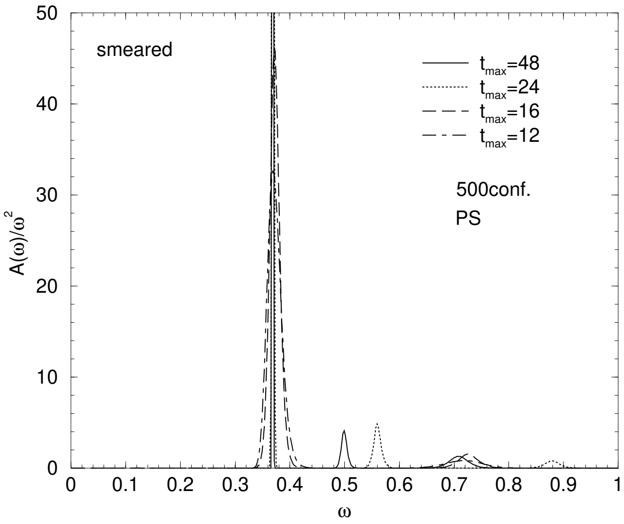
- $t_{max} \simeq 16$ is not acceptable
- $O(10)$ degree of freedom is necessary
- $t_{max} a_\tau$ of $O(1\ \text{fm})$ is necessary

$\Longrightarrow$ This requirement cannot be fulfilled at $T > 0$.

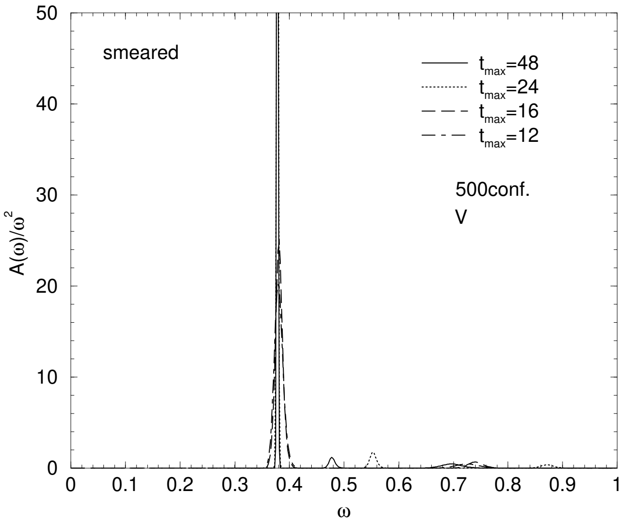
Results for charmonia (2): $T = 0$

☐ Result of MEM

Pseudoscalar

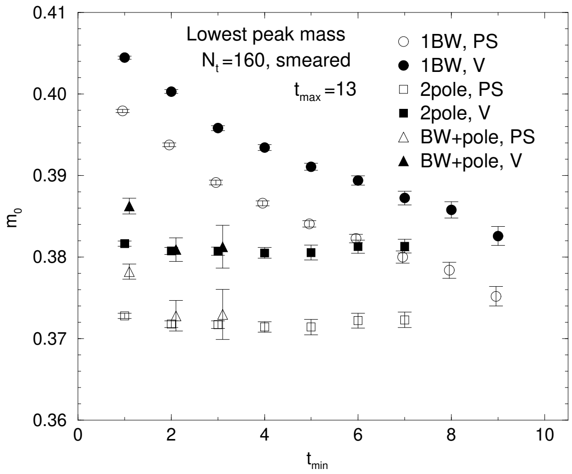


Vector

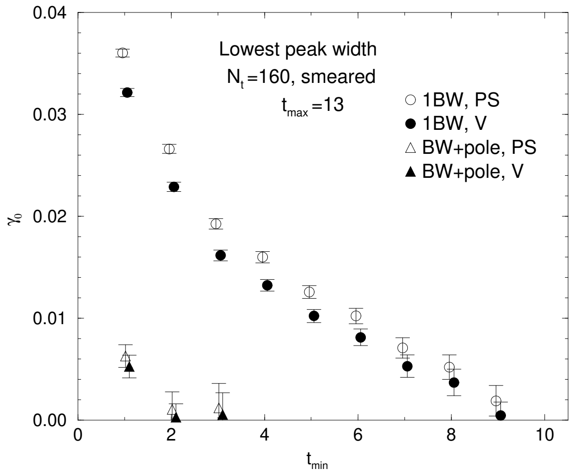


☐ Result of Fits — 2-pole, 1-BW, BW+pole forms

Ground state: mass



width

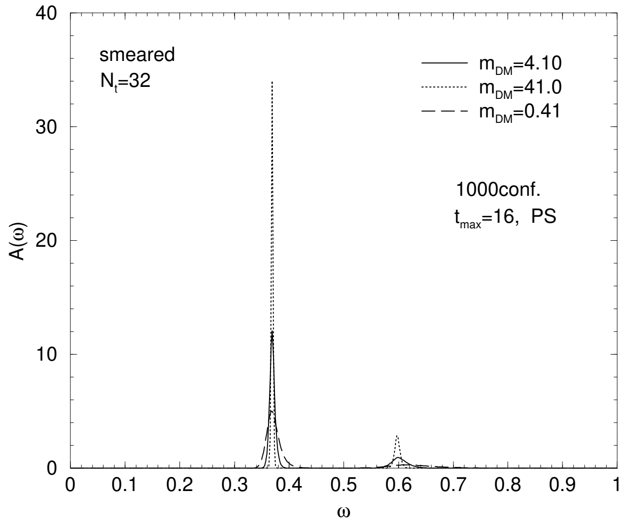


- In MEM, ground state peak is stable with change of t_{max}
- Consistency of methods
- No indication of finite width for ground state, as expected

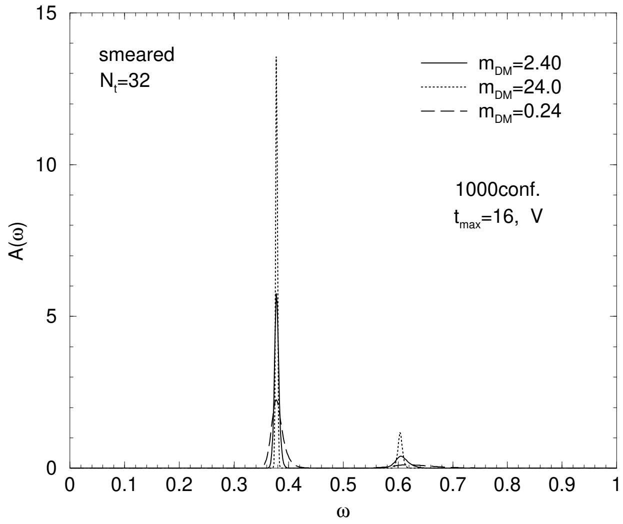
Results for charmonia (3): $T = 0.9T_c$

☐ Result of MEM

Pseudoscalar

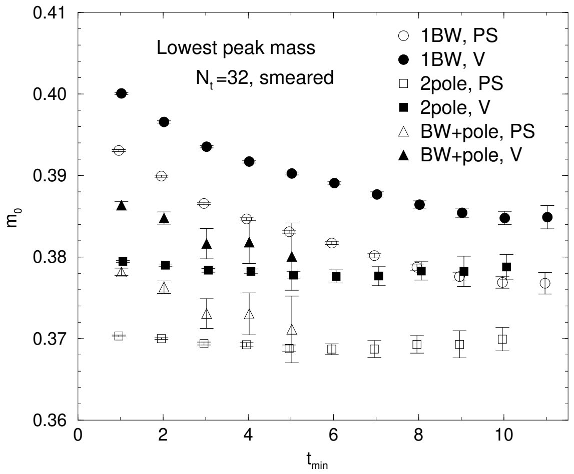


Vector

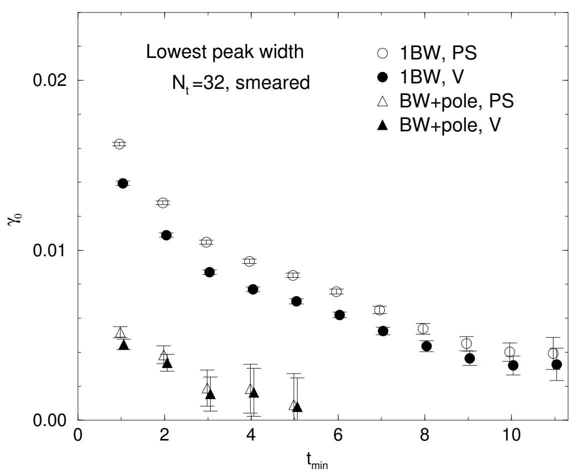


☐ Result of Fits — 2-pole, 1-BW, BW+pole forms

Ground state: mass



width

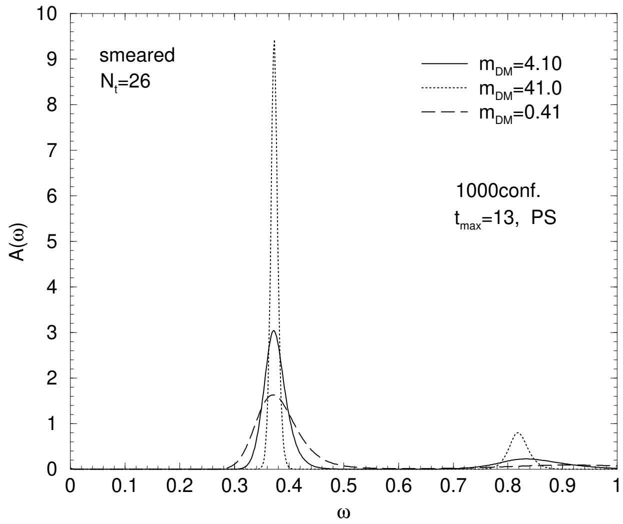


- Ground state peaks locate at almost the same as $T = 0$
- Width from fit is consistent with zero.
  – No indication of finite width for ground state

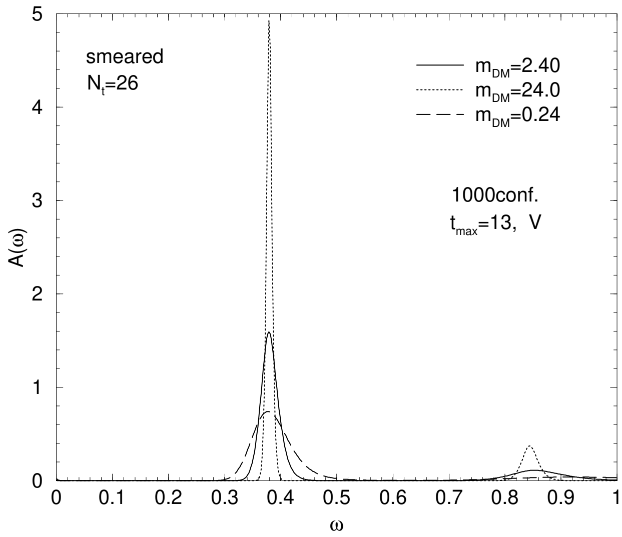
Results for charmonia (4): $T = 1.1T_c$

☐ Result of MEM

Pseudoscalar

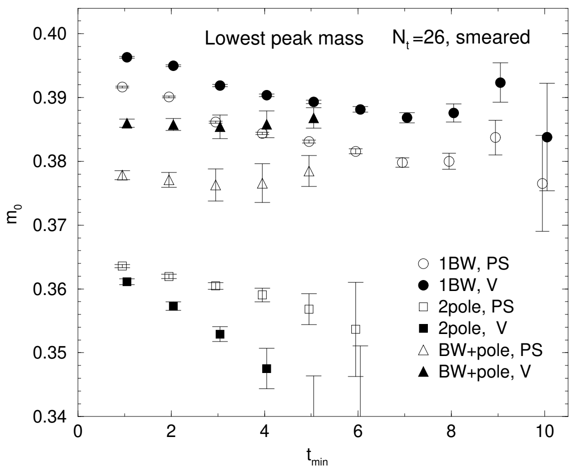


Vector

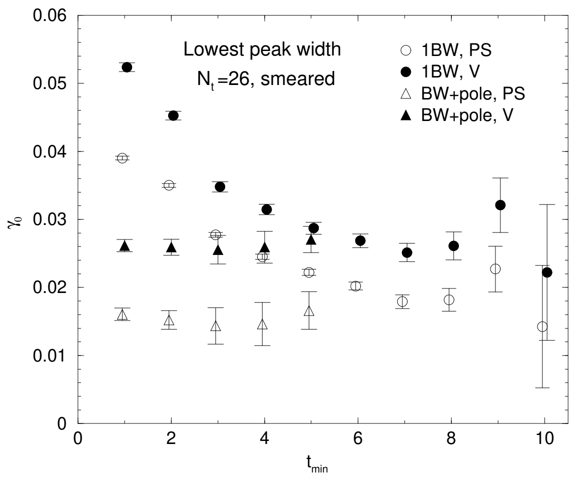


☐ Result of Fits — 2-pole, 1-BW, BW+pole forms

Ground state: mass



width



- Smeared and half-smeared correlators give *consistent* results
  $\Longrightarrow$ We conclude that physical peak is observed
- Finite width is observed ($\Gamma \sim$ 200 MeV)
- Peak positions are almost same as $T < T_c$

Results for charmonia (5): conclusion and outlook

□ $T \simeq 1.1T_c$

- Spectral function: still strong peak structure at almost
  the same position as $T < T_c$

  ⇒Existence of quasi-stable bound-state-like structure
  Finite width of 100-200 MeV

- Cf. spatial $c\bar{c}$ correlation

  ⇒ Strong spatial correlation between $c\bar{c}$ still at $T \sim 1.5T_c$

□ $T \simeq 0.9T_c$

- No indication of finite width
- No indication of mass shift

□ Procedures:

- MEM (as well as fit analysis) should be applied carefully
- For point correlators, naive application may fail at $T > 0$
- Better to combine with other analysis methods

Outlook

○ Analysis in wider range of temperature

○ Dynamical quark effects

Simulation parameters (for light mesons)

Gauge parameters:
— Anisotropic plaquette action in quenched approximation

□ $(\beta, \gamma_G) = (6.10, 3.2108)$
- $\xi = a_s/a_t = 4$

  Klassen, Nucl. Phys. B 533 (1998) 557
- $a_s^{-1} = 2.030(13)$ GeV

  Scale is set by hadronic radius r_0

□ Size : $20^3 \times N_t$

N_t	T/T_c	$N_{conf} \times N_{source}$	smearing function
160	~ 0	500 $\times$ 4	point
32	~ 0.9	1000 $\times$ 16	point, half-smeared,smeared
26	~ 1.1	1000 $\times$ 16	point, smeared

Quark parameters :
— $O(a)$ improved Wilson quark action

□ $(\kappa, \gamma_F) = (0.1235, 4.034)$

$\longrightarrow$ roughly correspond to strange quark mass

H. Matsufuru et al., Phys. Rev. D 64 (2001) 114503

ground state & 1st excited state mass m_0 & m_1 in a_t units

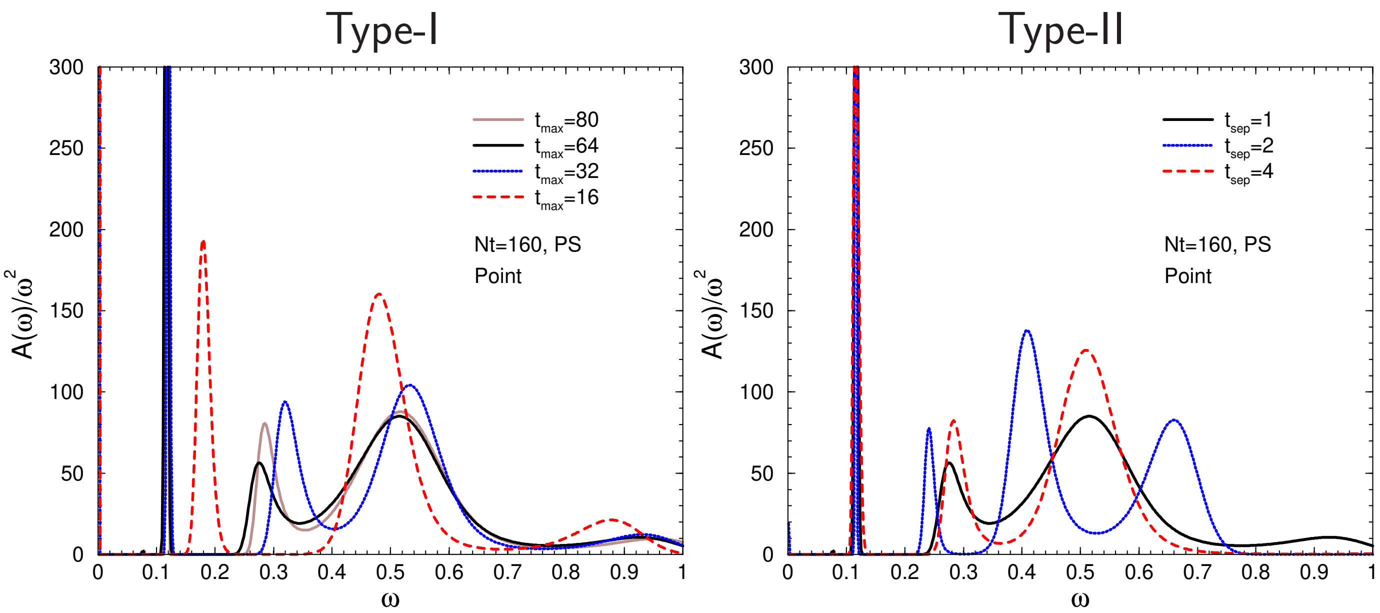
m_0(PS)$=$ 0.1160(3), m_1(PS)$\sim$ 0.27

m_0(V) $=$ 0.1489(7), m_1(V) $\sim$ 0.29

Results for light mesons (1): point correlators

☐ Point-Point correlators

	t_{min}	t_{max}	t_{sep}	N_{DF}
Type-I	1	80	1	80
	1	64	1	64
	1	32	1	32
	1	16	1	16
Type-II	1	64	1	64
	2	64	2	32
	3	64	4	16

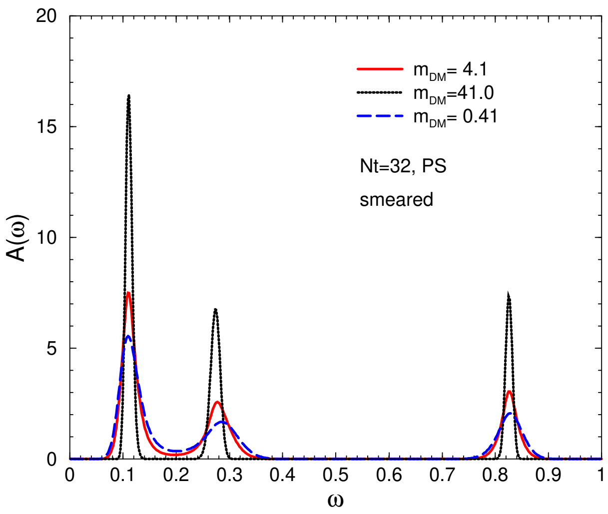


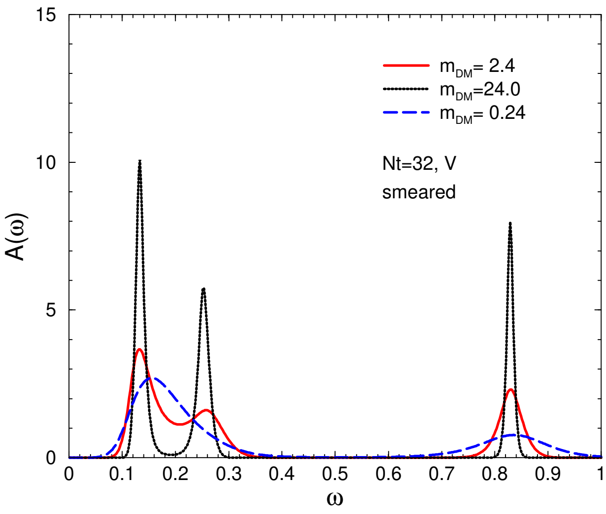
- $t_{max} \simeq 16$ is not acceptable
- lowest peaks are well reproduced at $t_{sep} = 4$ $(N_{DF} = 16)$
- $t_{max}a_t$ of the order of 1 fm is necessary

$\Longrightarrow$ This requirement can not be fulfilled at $T > 0$.

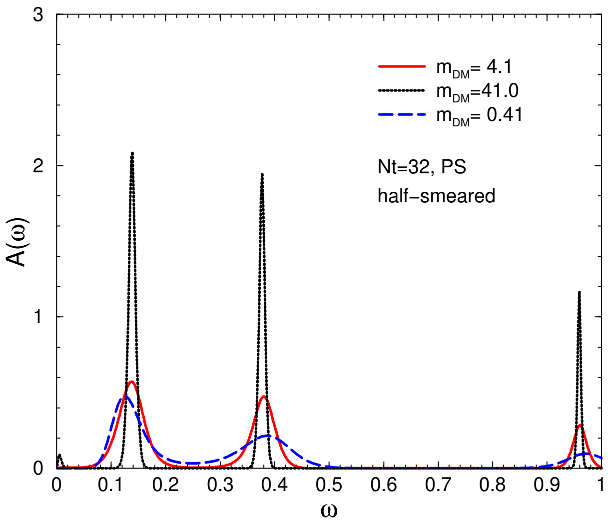
Results for light mesons (2): MEM analysis at $T = 0.9T_c$

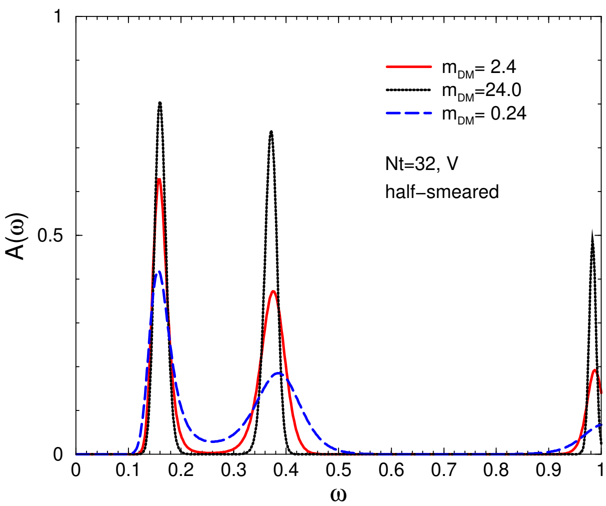
Nt=32 ($T \simeq 0.9T_c$) : smeared





Nt=32 ($T \simeq 0.9T_c$) : half-smeared



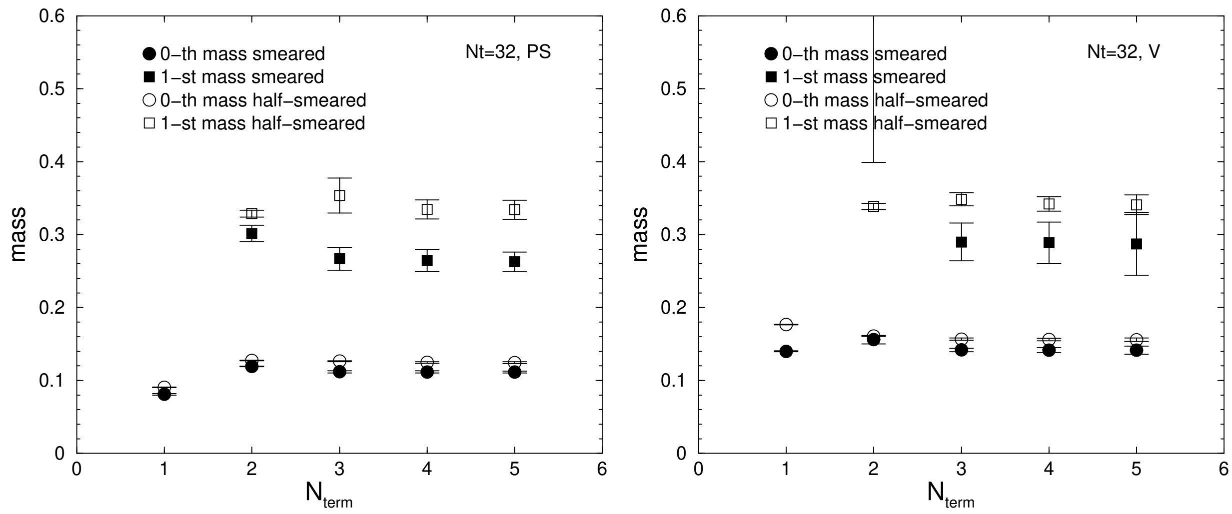


- ☐ most cases indicate pole-like structure
- ☐ lower masses are consistent with T=0 in smeared cases
- ■ masses are slightly different from T=0 in half-smeared case
- ■ width significantly depends on the model function

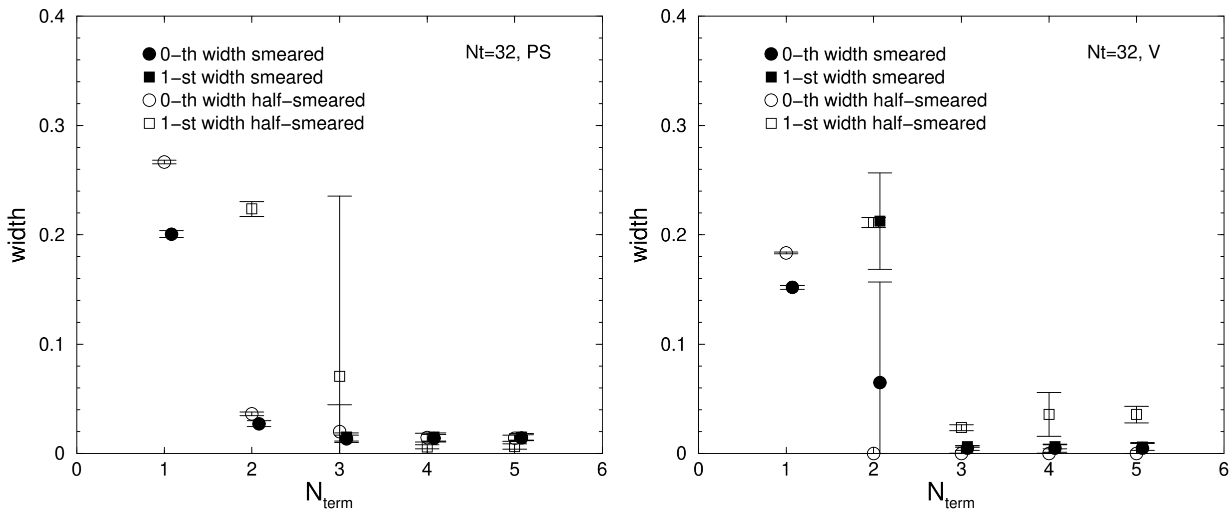
Results for light mesons (3): Fit analysis at $T = 0.9T_c$

☐ input parameters are chosen based on the results of MEM

Nt=32 ($T \simeq 0.9T_c$) : ground state & 1st excited state mass



Nt=32 ($T \simeq 0.9T_c$) : ground state & 1st excited state width



☐ fit parameters become stable as N_{term} increases

■ differences between smeared and half-smeared are beyond statistical error

■ result is largely dependent on the input parameters
$\Longrightarrow$ systematic error is larger than statistical one

Results for light mesons (4): Discussion

☐ Reliablility check of our procedure

(1) Check of the stability;

— stability against changes of input parameters
or medel functions —

MEM and CCF results largely depends on input parameters or model functions.

- MEM can work only for rough estimation of SFs

- Systematic error of CCF results is larger than statistical error

⇒ The results of smeared and half-smeared may be consistent.

(2) Check with T=0 correlators;

—stability against restriction of d.o.f. used in analysis—

(Here we suppose
the system at $T = 0.9T_c$ is similar to that at T=0)

Results with smeared operators are consistent with the results at T=0.

⇒ need more checks at T=0

Results for light mesons (5): Summary

- ☐ Light quark systems are analyzed with the method used in the study of charmonium.

  (Smeared operator + MEM&Fit analysis)

- ☐ We discuss the reliability conditions as a necessary condition of analysis method.

- ☐ We apply the reliability check to our analysis procedure.

Outlook

- ☐ reliability check with smeared correlators at T=0
- ☐ improvement of smearing functions
- ☐ application to systems at $T > T_c$
- ☐ calculations with dynamical quarks