# Charmonium spectrum from lattice QCD

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## Introduction

- Charmonium spectrum
  - CKM matrix element

    $\Rightarrow$ nonperturbative calculations of

    heavy-heavy $(c\bar{c},\ b\bar{b})$

    heavy-light $(D,\ B)$ systems

    from lattice QCD simulations

  - Test for QCD and lattice formulation

    heavy quarks on the lattice

    charm-quark has some problem on the lattice

    $\Rightarrow$ page 7

  - Rich phenomenology of charmonium

- QCD phase transition

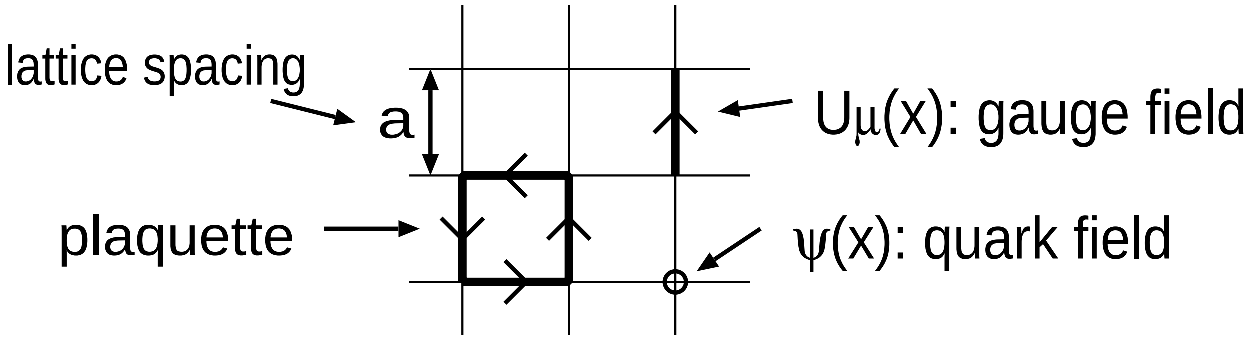
  Signals of QGP in heavy ion collision experiment

  at RHIC (BNL), SPS (CERN)

  $\Rightarrow J/\psi$ state at high temperature & density

# Lattice formulation of QCD

Discretization of space-time (Euclidean space)
with exact local gauge symmetry




$$S_g = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \Rightarrow \beta \sum ReTr\Box$$

$$\beta = 6/g^2 \text{ ( } g \text{ : QCD coupling const.)}$$

$$S_f = \bar{\psi}(x)(\not{D} + m)\psi(x) \Rightarrow \cdots$$

$$\mathrm{D}_\mu\psi(\mathrm{x}) = \frac{1}{2\mathrm{a}}[\mathrm{U}_\mu(\mathrm{x})\psi(\mathrm{x}+\hat{\mu}) - \mathrm{U}_\mu^\dagger(\mathrm{x}-\hat{\mu})\psi(\mathrm{x}-\hat{\mu})]$$

$\Longrightarrow$ Doubling problem (extra $2^4 - 1$ poles)

Nilsen-Ninomiya No-go theorem

$\Rightarrow$ several kinds of lattice fermion

- Wilson fermion (O(a) term )
- KS fermion ($2^4$ hyper cubic structure) etc...

# Path-integral with Monte Carlo method

- Partition function

$$Z = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_g - S_f}$$

$$S_f = \sum_{i=1}^{N_f} \int d^4x \bar{\psi}(x) Q_F^{(i)} \psi(x)$$

$$= \int \mathcal{D}U e^{-S_g} \prod_{i=1}^{N_f} \det Q_F^{(i)}$$

  - $\det Q_F = 1$ : quenched approximation

- Monte Carlo method

$$\langle O[U] \rangle = \frac{1}{Z} \int \mathcal{D}U O[U] e^{-S_g[U]} \prod_{i=1}^{N_f} \det Q_F^{(i)}[U]$$

$$\simeq \frac{1}{N_{\text{conf}}} \sum_{i=1}^{N_{\text{conf}}} O[U_i]$$

$\boxed{\text{Important sampling}}$

$U_i$ are generated with a weight

$$P(U_i) = e^{-S_g} \prod_{i=1}^{N_f} \det Q_F^{(i)}$$

## Physics from the lattice calculation

■ Scale determination

- Static quark potential $(r_0, \sqrt{\sigma})$
- Hadron mass ( $m_\rho$, $\Delta m$(1P-1S), $\cdots$)

■ Extrapolation to the continuum theory

- chiral extrapolation $m_q \longrightarrow$ physical quark mass
- continuum extrapolation $a \longrightarrow 0$

□ Improvement of lattice actions

– Standard gauge action has $O(a^2)$ error

– Standard quark action has $O(a)$ error

$O(a)$ improved Wilson (Clover) quark action

B.Sheikholeslami et al., Nucl. Phys.B259 (1985) 572

Tuning of clover coefficient $c_{sw}$ :

$*$ tree or mean-field improved tree

$*$ nonperturbative determination

## Difficulties of charm quarks on a lattice

Charm quark mass : $m_c \simeq 1.2$ GeV

— not so light, not so heavy

present typical lattice cutoff $a^{-1} = 1 - 3$ GeV

### ■ Standard lattice regularization

large lattice cutoff for $m_q a \ll 1$

$\Longrightarrow$ ($\triangle$) time consuming

### ■ Non-relativistic approach

- Nonrelativistic QCD (NRQCD)

  $1/m_q$ expansion : NRQCD needs $m_q a \gg 1$

  ($\triangle$) can NOT take a continuum limit

### ■ Relativistic approach

- Fermilab action

  mass dependent tuning of some coefficients

  $\Longrightarrow$ eliminate $O((m_q a)^n)$ error

- Anisotropic lattice ( $a_s > a_t$ )

  eliminate $O((m_q a_s)^n)$ error (at tree level)

## Lattice results (1)

- Anisotropic lattice in quenched QCD

M. Okamoto et al. (CP-PACS) Phys. Rev. D65 (2002) 094508

- quark action :

  $O(a)$ improved Wilson on anisotropic lattice

- improved coefficients $c_s, c_t$ :

  tree : mean-field tree

- tuning of anisotropy parameters :

  tree : mean-field tree : non-perturbative

- lattice cutoff :

  $a_s^{-1} = 1.0, 1.5, 2.0, 2.9$ GeV $\Rightarrow a \rightarrow 0$ limit

- scale input :

  $r_0$ : $\Delta m$(1P-1S) : $\Delta m$(2S-1S)

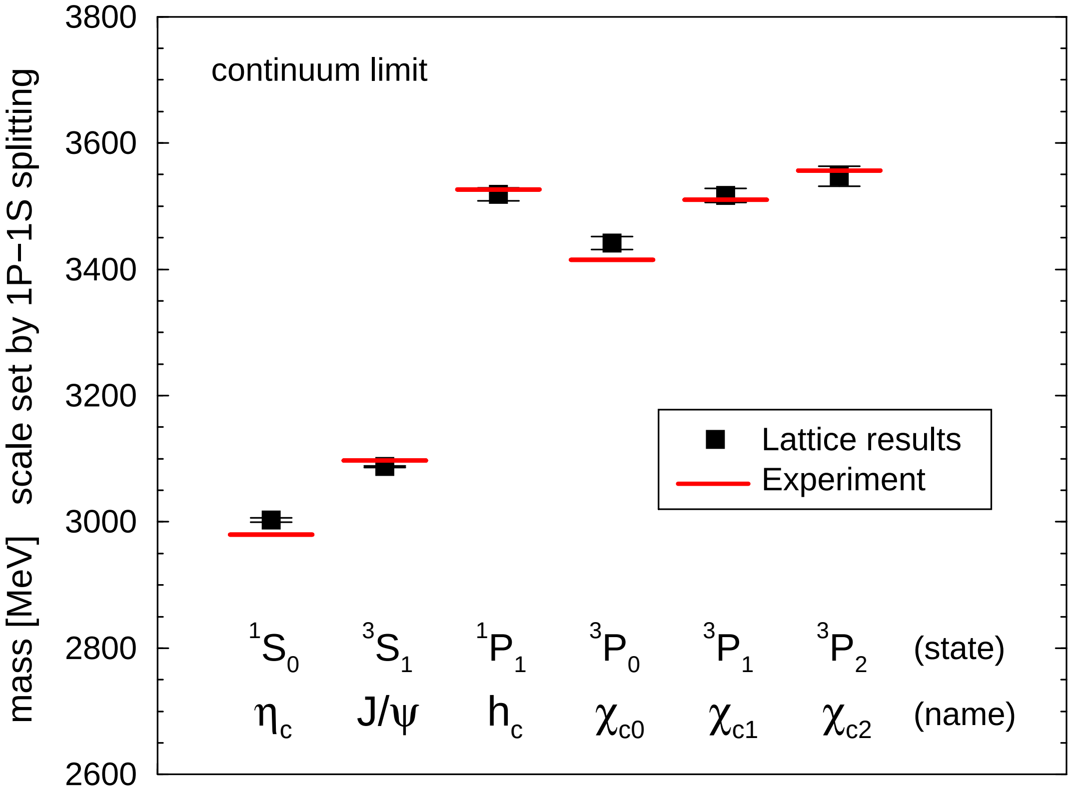
But ...

- No theoretical advantage in anisotropic lattice

  $O(m_q a_s)$ revives from 1-loop diagram

  S.Aoki et al. hep-lat/0107009

# Lattice results (2)




M.Okamoto et al. Phys. Rev. D65 (2002) 094508

## Other works

H.D.Trottier, Phys. Rev. D55 (1997) 6844. (NRQCD)

A. El-Khadra, Nucl.Phys.B(Proc.)26 (1992) 372. (Fermilab)

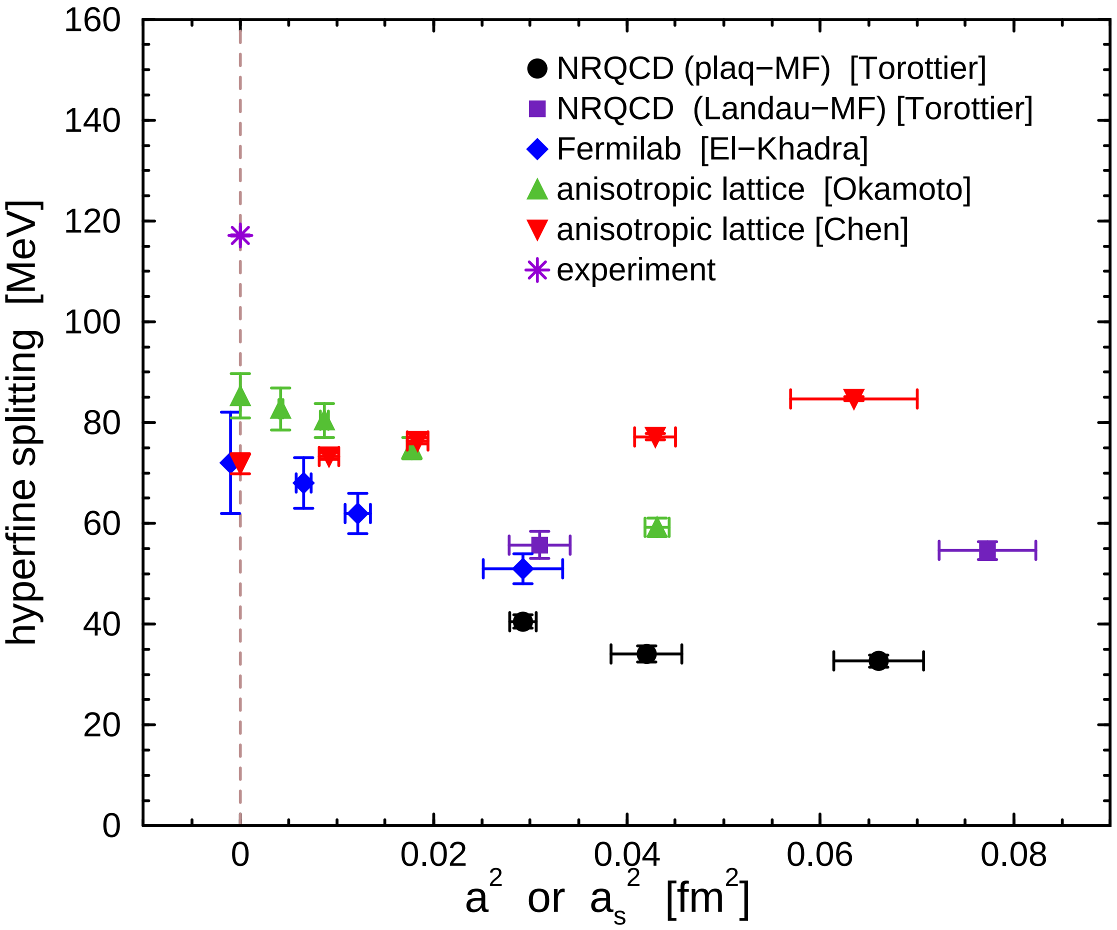
T. Klassen, hep-lat/9809174 (anisotropic)

P.Chen, Phys. Rev. D64 (2001) 034509. (anisotropic)

X.Liao and T.Manke, hep-lat/0210030. (anisotropic)

Other works are almost consistent with this result.

# Hyperfine splitting




HFS at $a \rightarrow 0$ limit $\sim 80$ MeV

30% smaller than exp. value.

## possible uncertainties

- quenching effects
- tuning of improvement coefficient $c_{sw}$

# Dynamical quark effects

■ full QCD result

M. di Pierro et al. hep-lat/0210051

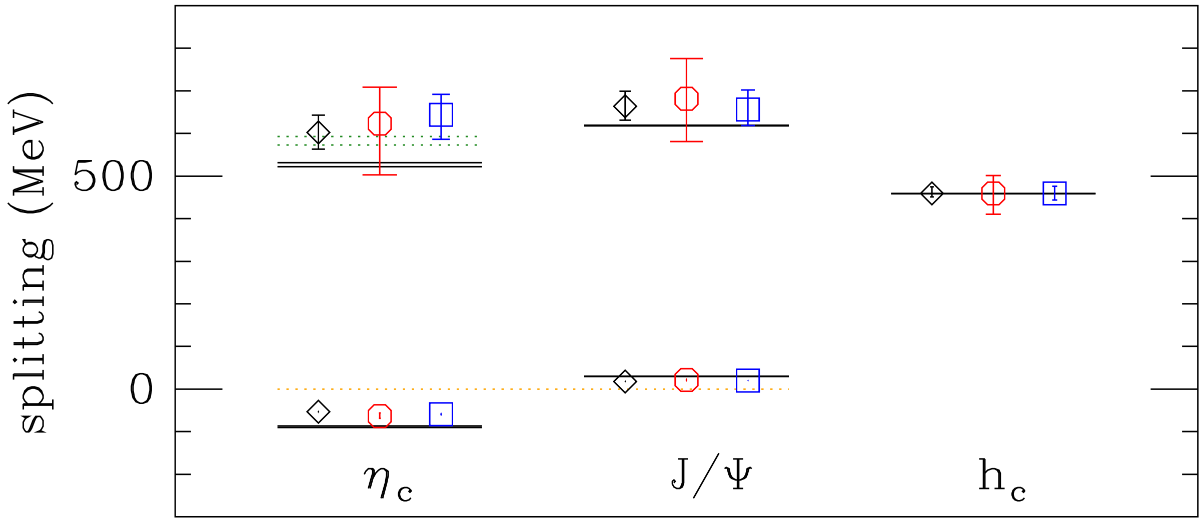
valence quark : Fermilab action

sea quark : KS action (MILC Collab.)

□ estimation of dynamical quark effects

at $a \simeq 0.13$ fm ( no continuum limit )

Nf = 0 (◇), 3 (○), 2+1 (□)




□ other full QCD works

T.Manke et al., Phys. Rev. D62 (2000) 114508.

C.Stewart and R.Koniuk, Phys. Rev. D63 (2001) 054503.

$\Longrightarrow$ dynamical quark effects ∼10%

# Lattice results (3)

■ Isotropic lattice in quenched QCD (preliminary)

QCD-TARO, M. García Pérez et al., cf. hep-lat/0110104

□ quark action : Wilson : Clover

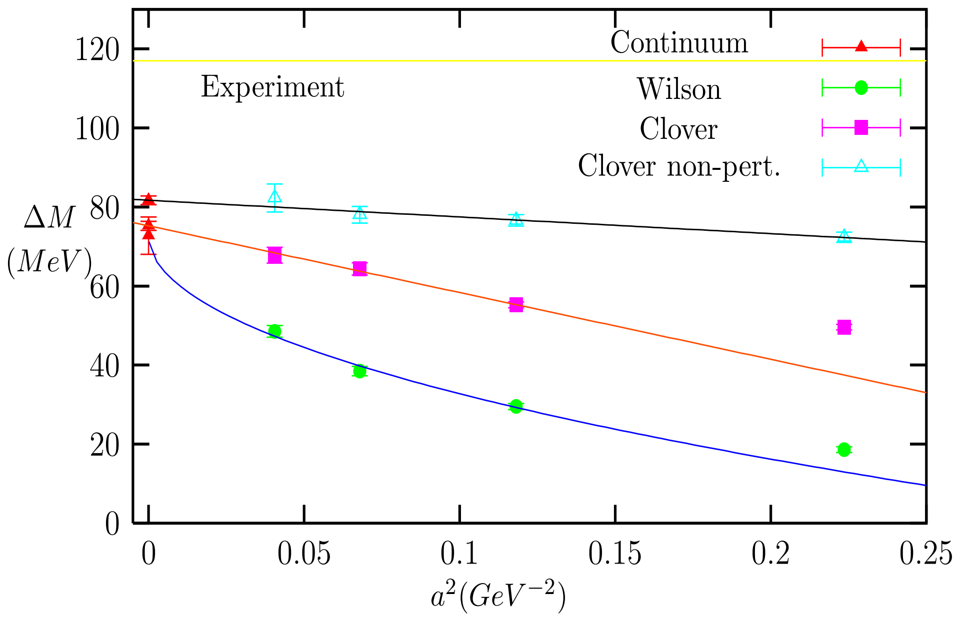
□ improved coefficients $c_{sw}$ :

tree : non-perturbative*

*M. Lüscher et al., Nucl.Phys.B491(1997)323

□ lattice cutoff :

$a^{-1} = 2.1, 2.9, 3.8, 5.0$ GeV $\Rightarrow a \rightarrow 0$ limit




$\Delta M_{J/\psi - \eta_c} =$ 82(1)(4)MeV (scale from $r_0$)

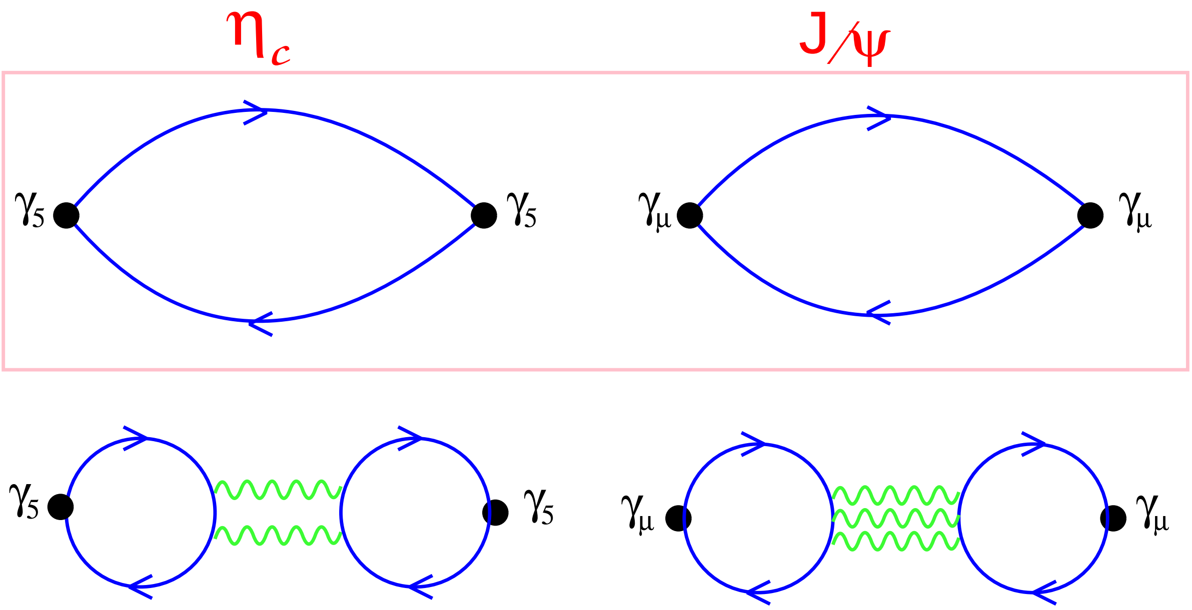
## OZI–forbidden diagram

■ Possible uncertainties

difference of HFS $\sim$ 30%

- continuum extrapolation $\longrightarrow$ well controlled
- dynamical quark effects $\sim$ 10%
- OZI-suppressed "disconnected" diagrams

suggested by S. Sint




disconnected diagrams are neglected

$\Longleftarrow$ high cost & very small contribution

however, it may contribute to HFS $\sim O(10)$MeV

## Summary of charmonium spectrum on the lattice

### Test for the QCD and the lattice regularization

- good agreement with experimental values
- HFS of charmonium is open problem
  in the lattice QCD

### Outlook

- full QCD calculation
  - $N_f = 2$ (u- & d-quark) + 1 (s-quark)
  - continuum limit
  - improvement of quark action
- evaluation of OZI-forbidden diagrams

## Charmonium at $T > 0$

- $J/\psi$ (charmonium) states
  - important signal for formation of QGP

    heavy ion experiment at CERN(SPS),BNL(RHIC)

    c.f. NA50 Phys.Lett.B477(2000)28

- Theoretical understanding
  - Potential model

    Mass shift near $T_c$

    T. Hashimoto et al., Phys.Rev.Lett.57(1986)2123

    $J/\psi$ suppression above $T_c$

    T.Matsui and H.Satz, Phys.Lett.B178(1986)416

    $\Longrightarrow$ no bound state at $T \gtrsim 1.1 T_c$
  - Lattice QCD
    - Spectral function, Wave function

      T. Umeda et al., Int.J.Mod.Phys.A16(2000)2215

      T. Umeda et al., hep-lat/0211003

      S. Datta et al., hep-lat/0208012

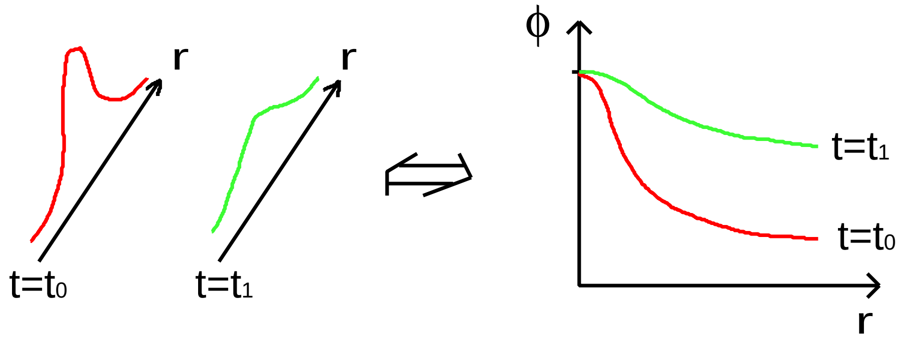
      $\Longrightarrow$ not free behavior at $T \sim 1.5 T_c$

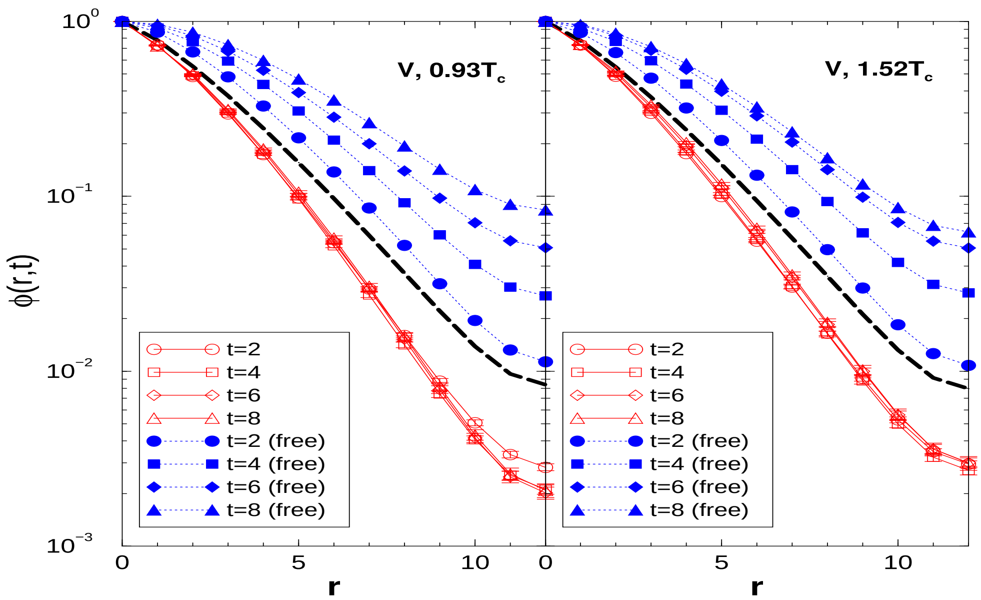
# Spatial $c\bar{c}$ correlation

"Wave function" on the Coulomb gauge

$$w_\Gamma(r,t) = \sum_{\vec{x}} \langle \bar{c}(\vec{x}+\vec{r},t)\Gamma c(\vec{x},t) O^\dagger(0)\rangle$$

$\phi(r,t)$ : normalized at the spatial origin




T. Umeda et al., Int.J.Mod.Phys.A16(2000)2215

► $c$-$\bar{c}$ strongly correlate even at $T \simeq 1.5T_c$

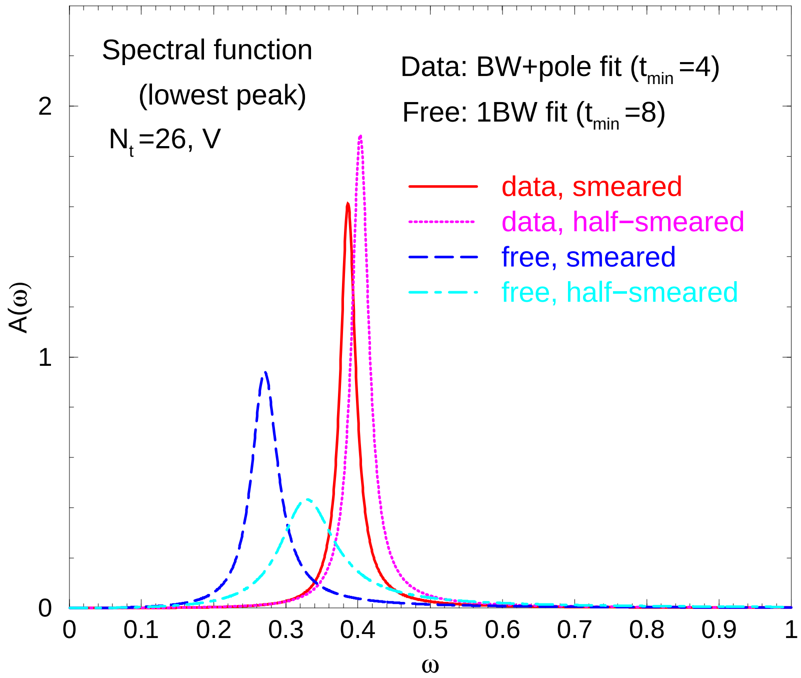
# Spectral function

■ Meson correlator $C(t)$ & Spectral function $A(\omega)$

$$C(t) = \int d\vec{x} \langle O(\vec{x}, t) O^\dagger(0,0) \rangle$$
$$= \int d\omega A(\omega) \frac{\cosh(\omega(N_t/2 - t))}{2\pi \sinh(\omega N_t/2)}$$

□ Maximum Entropy Method * and Fit with ansatz

* M. Asakawa et al. Phys. Rev. D60 (1999) 091503




* T. Umeda et al. hep-lat/0211003

▶ SPFs has strong peaks at $T \sim 1.1 T_c$

## Summary at $T > 0$

- Potential model analysis

  No bound state at $T \gtrsim 1.05T_c$
- Spatial $c\bar{c}$ correlation

  Strong spatial correlation of $c\bar{c}$ at $T \sim 1.5T_c$
- Spectral function

  $(0.9T_c)$ : Almost same results as $T = 0$

  $(1.1T_c)$ : Still strong peaks at same position and finite width

$$\Downarrow$$

The existence of quasi-stable bound state-like structure above $T_c$ (at least $1.1T_c$)

## Outlook

MEM & Fit analysis in wider range of temperature
Dynamical quark effects