Period Analysis of Variable Stars using the Group LASSO

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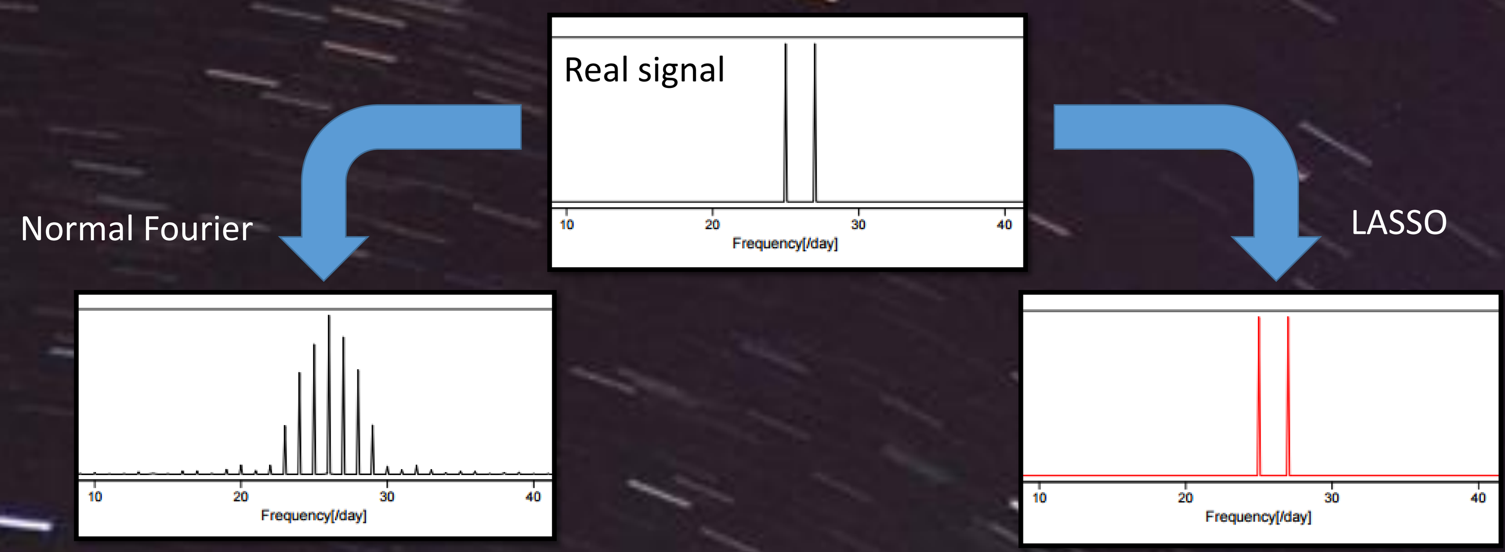
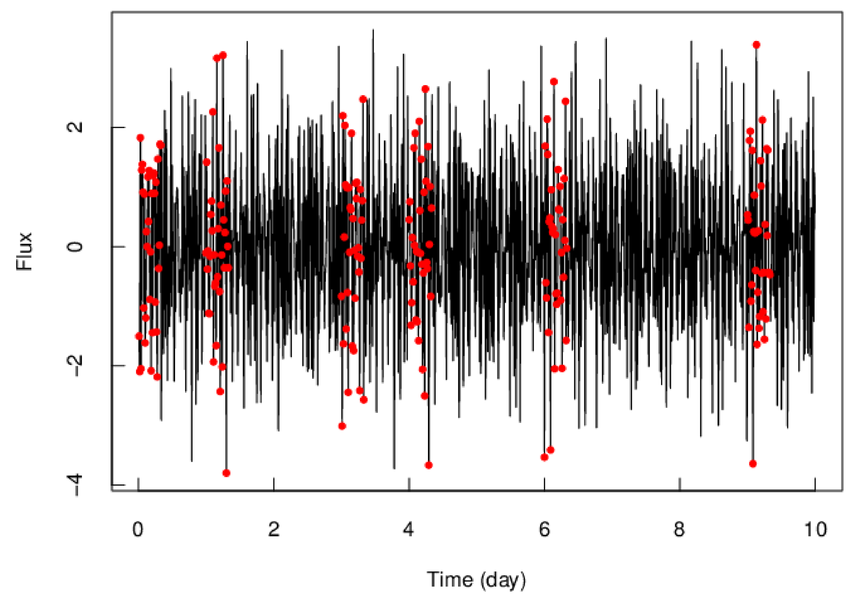
The period analysis is a fundamental tool to study variable stars in which we estimate one or a few periodic signals in the time-series data of stellar brightness (the so-called "light curve"). In the case of ground-based astronomical observations, the sampling pattern of the light curve is so irregular that aliases occasionally disturb the detection of real signals. Here, we report the capability and application of our period analysis methods using the LASSO and group-LASSO for variable stars. They are definitely useful for the reconstruction of power spectra because we can assume them sparse when we know the objects are periodic variables.



1. Background: Non-uniform sampling in astronomical ground-based observations



Sampling is disturbed by …
1) day-time
2) Seasonal condition
3) Bad weather

➔ Irregular, but some patterns.

2. Period analysis using LASSO[1][2][3]

The l1 minimization of the Fourier coefficients
➔ Sparse power spectra

Data (light curve) / Fourier coefficients

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{pmatrix} = \begin{pmatrix} \cos(2\pi t_1\nu_1) & \dots & \cos(2\pi t_1\nu_N) & \sin(2\pi t_1\nu_1) & \dots & \sin(2\pi t_1\nu_N) \\ \cos(2\pi t_2\nu_1) & \dots & \cos(2\pi t_2\nu_N) & \sin(2\pi t_2\nu_1) & \dots & \sin(2\pi t_2\nu_N) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \cos(2\pi t_N\nu_1) & \dots & \cos(2\pi t_N\nu_N) & \sin(2\pi t_N\nu_1) & \dots & \sin(2\pi t_N\nu_N) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \\ b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix}$$

✓ How much data is required?
➔ Section 3

✓ Does it efficiently work?
➔ Section 4

LASSO $\hat{x} = \arg\min_{\boldsymbol{x}} (\|\boldsymbol{y} - A\boldsymbol{x}\|_2^2 + \lambda\|\boldsymbol{x}\|_1)$

Group-LASSO $\hat{x} = \arg\min_{\boldsymbol{x}} \left(\|\boldsymbol{y} - A\boldsymbol{x}\|_2^2 + \lambda \sum_{i=1}^{N} \sqrt{a_i^2 + b_i^2}\right)$

3. Conditions for the l1 reconstruction of the power spectra for pulsating stars

3-1 Experiments

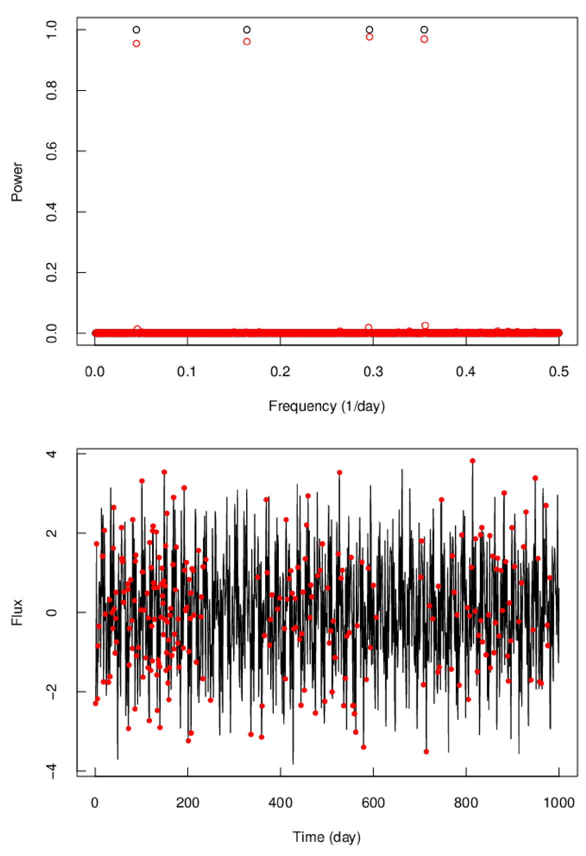
Step 1: Making a power spectrum
- K non-zero signals for 500 coefficients
- All signals have an amplitude of 1.0

Step 2: Making a light curve
- Full data: 1000 data points
- Making a sub-sample
- Noise: Gaussian, 10% of power
- Phase: random

Step 3: Estimating the power spectrum using LASSO and group-LASSO
- We used AIC for model selection

Step 4: Decision
- Successful reconstructions, if…
- Real signals are detected with amplitudes of > 0.1,
- And false positive signals have amplitudes of < 0.1.

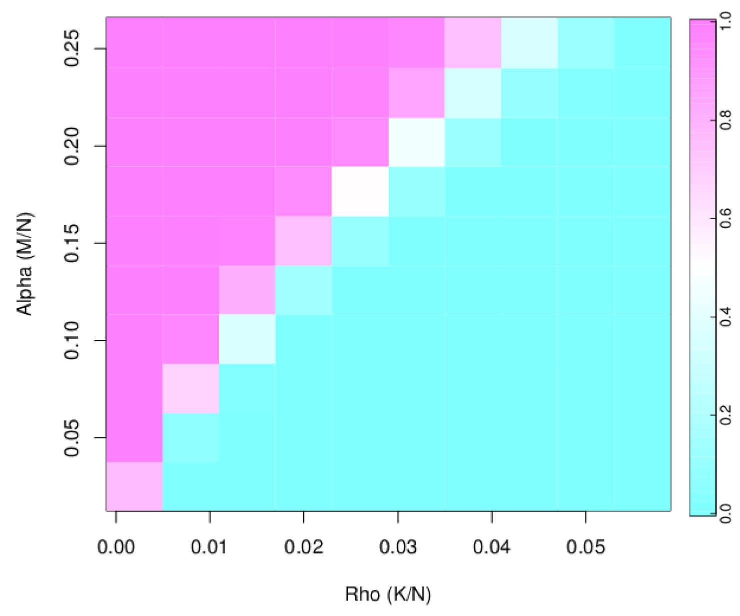
10^4 trials of the above steps were performed for each condition.
➔ Calculating the reconstruction probability



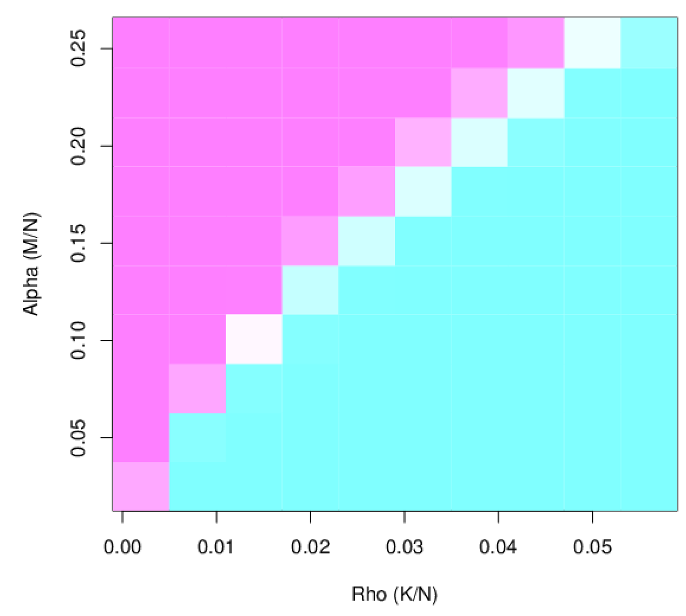
Example of the assumed (black in the upper panel), and estimated (red) power spectra, and light curve (the lower panel. black: full data, red: simulated data).

3-2 Case for long (>1 day) periods

- Condition for reconstruction in the case of the astronomical sampling (left panel) is almost same as that of random sampling (right panel) if the assumed periods are longer than 1 day.
- This suggests that the LASSO-based period analysis is quite useful for the study of long period variables.



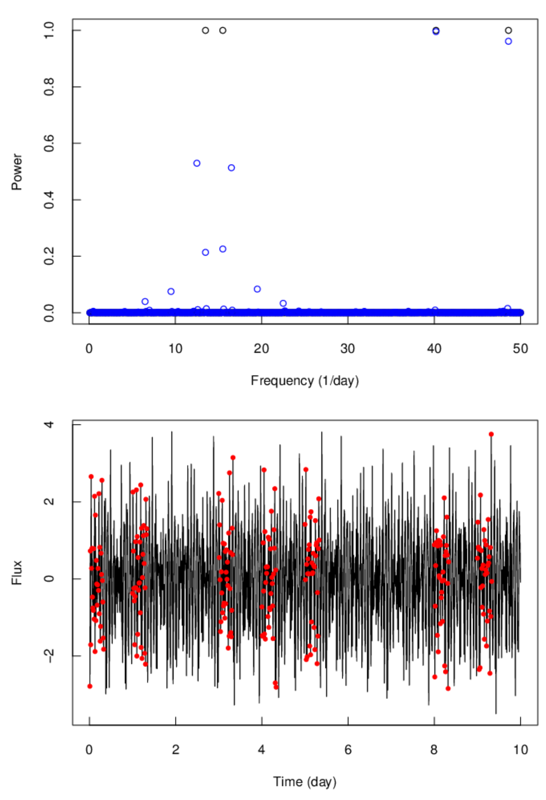
The data size (alpha) v.s. sparsity of spectra (rho) for long period variables and astronomical sampling.



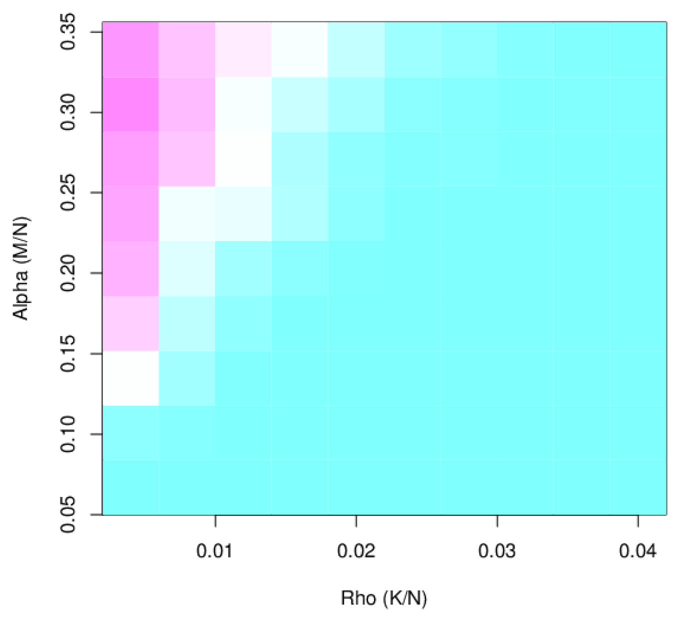
The data size (alpha) v.s. sparsity of spectra (rho) for long period variables and random sampling.

3-3 Case for short (< 1 day) periods

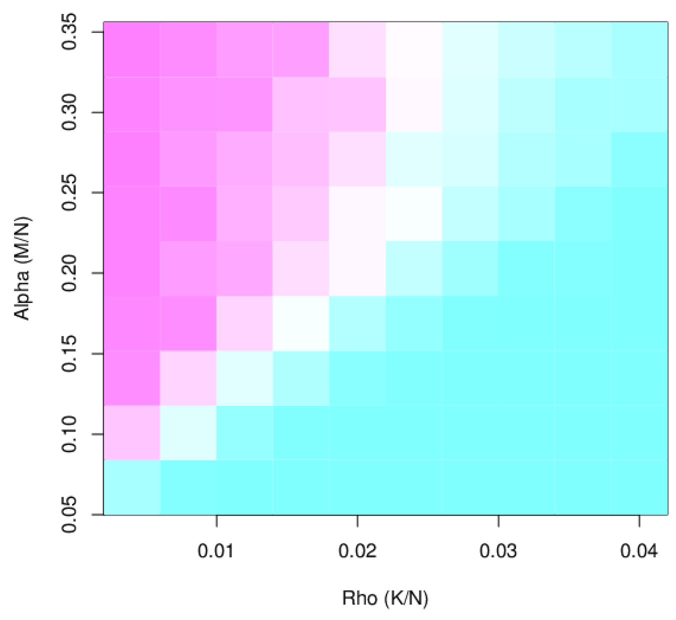
- Condition for reconstruction is much worse in the case that the assumed periods are shorter than 1 day (middle panel). This is mostly due to strong 1-day aliases (left panel).
- Multi-longitude observations (right panel for two observatories) may overcome this situation even if the total number of the data is the same.



Example of the failed reconstruction for short period variables and one observatory. The upper and lower panels show the power spectra and light curve.

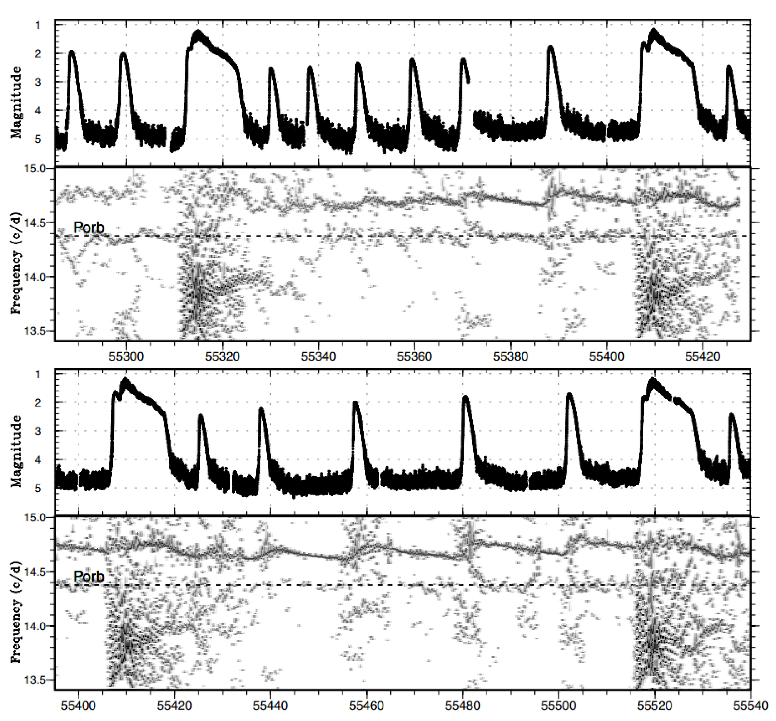


The data size (alpha) v.s. sparsity of spectra (rho) for short period variables, astronomical sampling, and one observatory.



The data size (alpha) v.s. sparsity of spectra (rho) for short period variables, astronomical sampling, and two observatories.

4. Applications



Light curve (upper) and LASSO 2D periodogram (lower) of the dwarf nova V1504 Cyg. Small changes in frequency can be clearly seen.[4]

Osaki & Kato (2013)
- Search for period changes in short-term (~1.7 hours) humps associated with long variation pattern, the so-called "outbursts".
- Clear pattern of the period change can be seen in the 2D Lasso periodogram.
- The normal Fourier spectra form much broader features around the signal, making the detection of period change difficult.

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References: [1] Bourguignon, Carfantan, & Bohm, 2007, A&A, 462, 379, [2] Kato & Uemura, 2012, PASJ, 64, 122, [3] Yamamoto, 2015, Bachelor thesis (Hiroshima University), [4] Osaki & Kato, 2013, PASJ, 65, 95