A SEIFERT FIBERED MANIFOLD WITH INFINITELY MANY KNOT-SURGERY DESCRIPTIONS

MASAKAZU TERAGAITO

Abstract. Osoinach introduced a way to construct a 3-manifold which can be obtained by the same integral Dehn surgery on an infinite number of knots in the 3-sphere. Using it, he gave such a hyperbolic 3-manifold and a toroidal 3-manifold. In this paper, we give the first example of a Seifert fibered manifold that can be obtained by the same integral surgery on an infinite number of hyperbolic knots. Interestingly, most of those knots have no symmetry. This implies that those knots cannot lie on a genus two Heegaard surface of the 3-sphere.

1. Introduction

By the famous theorem of Lickorish [23], Wallace [42], any closed connected orientable 3-manifold M admits a link-surgery description. That is, M can be obtained by Dehn surgery on some link in the 3-sphere S^3. In this paper, we focus on knot-surgery descriptions. There are some 3-manifolds which have the unique knot-surgery description. The simplest example is $S^2 \times S^1$ [10], and Ghiggini [11] proved that the Poincaré homology 3-sphere does so by using knot Floer homology theory. On the other hand, there are several works on 3-manifolds with multiple knot-surgery descriptions. Lickorish [24] gave the first example of a homology 3-sphere that is obtained by (-1)-surgery on distinct two knots. Next, Brakes [4] showed that for any positive number N, there exists a 3-manifold which admits N knot-surgery descriptions with the same surgery coefficient. Later, various examples with multiple (finite) knot-surgery descriptions were found in [18, 25, 39].

Problem 3.6(D) of [19] asks if there is a homology 3-sphere (or any 3-manifold) which can be obtained by the same surgery on an infinite number of distinct knots. As an answer to this problem, Osoinach [36, 37] introduced a way to construct a 3-manifold that can be obtained by the same integral surgery on an infinitely many knots in S^3. In fact, he gave such a hyperbolic manifold and a toroidal manifold, that is, a manifold containing an incompressible torus. We note that the surgery coefficient is zero in his examples.

As a consequence of Thurston's geometrization conjecture (cf. [7]), the resulting 3-manifold by Dehn surgery on a knot is either hyperbolic, toroidal, a Seifert fibered manifold, or reducible (see [14]). The last possibility is conjectured not to happen for hyperbolic knots [13].

The purpose of this paper is to give the first example of a small Seifert fibered manifold that has infinitely many knot-surgery descriptions. By a *small Seifert fibered manifold*, we mean one with orbit manifold S^2 and just three exceptional fibers, which is denoted by $S^2(p_1, p_2, p_3)$ when three exceptional fibers have indices p_1, p_2, p_3 ($p_i \geq 2$), respectively.

1991 *Mathematics Subject Classification.* Primary 57M25.

Key words and phrases. Dehn surgery, hyperbolic knot, Seifert fibered manifold.

The author was partially supported by Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (C), 16540071.

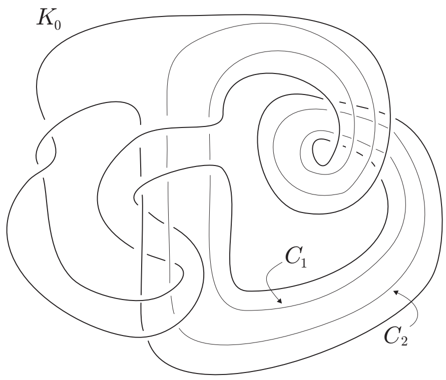

FIGURE 1

Theorem 1.1. *There exists a small Seifert fibered manifold which can be obtained from an infinite number of knots in S^3 by the same integral Dehn surgery.*

In general, for a knot or link K, an orientation-preserving diffeomorphism $f : S^3 \to S^3$ of finite order which leaves K invariant is called a *symmetry* of K.

Theorem 1.1 is an immediate consequence of the following theorem.

Theorem 1.2. *There exist infinitely many hyperbolic knots in S^3 such that*

(1) 4*-surgery on them yield the same Seifert fibered manifold of type $S^2(2,6,7)$;*
(2) *they have no symmetry*; *and*
(3) *they lie on a genus three Heegaard surface of S^3, but cannot lie on a genus two Heegaard surface.*

Any knot which admits a small Seifert fibered surgery was expected to lie on a genus two Heegaard surface of S^3 [15, 26], but Deruelle-Miyazaki-Motegi [8] recently found infinitely many hyperbolic knots which give counterexamples. Their knots are constructed from the $(-3,3,5)$-pretzel knot by twisting along some unknotted simple closed curve disjoint from the knot, called a "seiferter", and have the properties (2) and (3) of Theorem 1.2. However, the surgery coefficients are not constant, and the resulting small Seifert fibered manifolds, which are of type $S^2(3,5,*)$, are mutually distinct. From this point of view, we might say that our examples have different nature from those of [8].

For a knot K in S^3, the resulting 3-manifold by r-Dehn surgery is denoted by $K(r)$ as usual.

The author would like to thank the referee for many valuable comments to make the paper more readable.

2. The construction

Let us consider the link $L = K_0 \cup C_1 \cup C_2$ as illustrated in Figure 1. Note that K_0 is 9_{42}, which is the Montesinos knot of type $(-1/2, 1/3, 2/5)$. Clearly, there is an annulus A between C_1 and C_2, which meets K_0 in two points (see Figure 2).

For a non-negative integer n, let K_n be the knot obtained from K_0 by twisting along A n times. That is, consider a bicollar $N(A) = A \times [0,1]$ on A. Let $f : N(A) \to N(A)$ be a homeomorphism defined by $f((e^{i\theta}, s), t) = ((e^{i(\theta+2\pi t)}, s), t)$, where A is parametrized as $S^1 \times [0,1] = \{(e^{i\theta}, s) \mid 0 \le \theta \le 2\pi, 0 \le s \le 1\}$. Then K_n is obtained from K_0 by replacing $K_0 \cap N(A)$ with $f^n(K_0 \cap N(A))$. The arrow in Figure 2 indicates the direction of the twist. Equivalently, K_n is the image of K_0 after performing $(n+1)/n$-surgery on C_1 and $(n-1)/n$-surgery on C_2. (Although we can use negative integers n, but it suffices to consider non-negative integers n

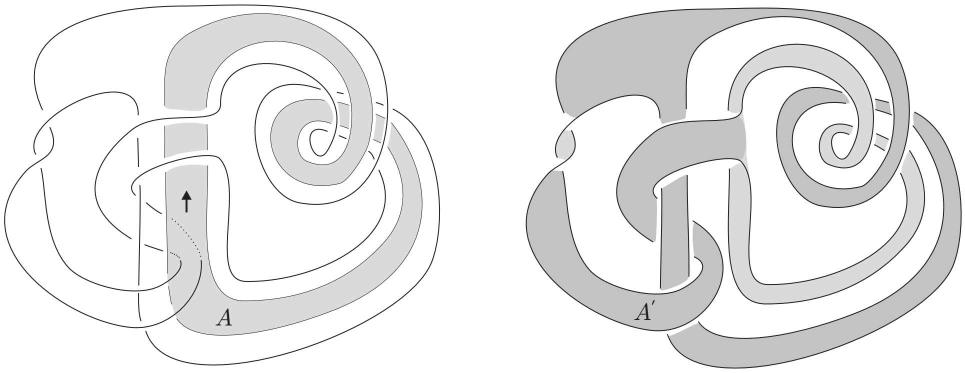

FIGURE 2

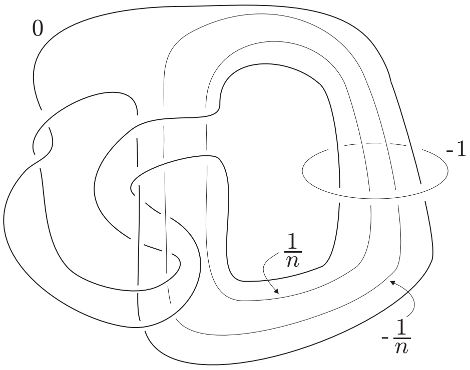

FIGURE 3

for our purpose.) On the other hand, there is a once-punctured annulus A' whose boundary consists of $K_0 \cup C_1 \cup C_2$ as illustrated in Figure 2. Note that the boundary slope of A' on K_0 is 4, and $\partial A'$ defines the same slope on each of $N(C_i)$ as ∂A.

Proposition 2.1. The resulting manifold $K_n(4)$ by 4-surgery on K_n does not depend on n. That is, $K_n(4) \cong K_0(4)$ for any n.

Proof. This follows from [37, Theorem 2.3]. However, we can easily prove this by using Kirby calculus (cf. [12]) here.

Figure 3 shows the surgery description of $K_n(4)$ after inserting an extra unknotted component with coefficient -1. Note that the coefficients 4, $(n+1)/n$ and $(n-1)/n$ on K_0, C_1 and C_2 are changed to 0, $1/n$ and $-1/n$, respectively. More precisely, we should replace the component with coefficient $1/n$ (resp. $-1/n$) by n parallel copies with coefficient 1 (resp. -1). Perform the handle slide as shown in Figure 4 (indeed, n repetitions of this slide). Since the parallel copy of K_0, the band as in Figure 4 and C_2 are contained in the once-punctured annulus A', C_2 becomes isotopic to C_1 after the sliding operation. Hence $K_n(4)$ is the same as $K_0(4)$. □

Proposition 2.2. For $K_0 = 9_{42}$, its 4-surgery yields a Seifert fibered manifold of type $S^2(2,6,7)$.

Proof. This can be seen by the Montesinos trick ([28]). Let us take the quotient by the strong inversion of S^3 with an axis A as shown in Figure 5.

Then it is not hard to see that the resulting link is isotopic to the Montesinos link of type $(1/2, -1/6, -2/7)$. Hence $K_0(4)$ is the double branched cover of S^3

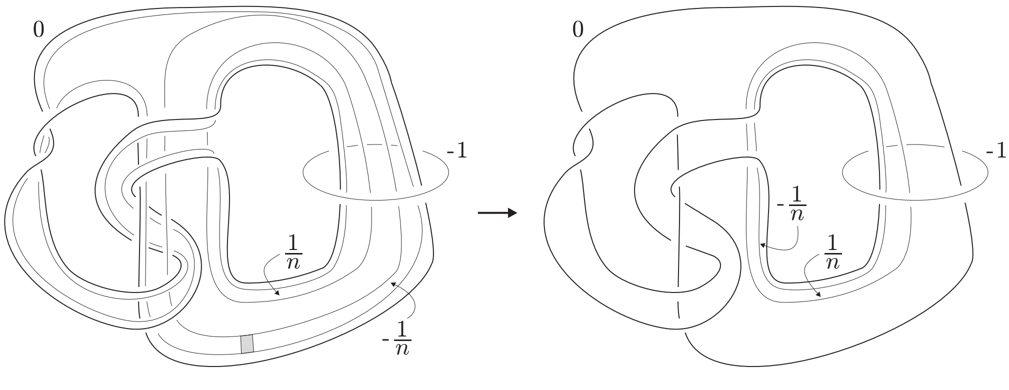

FIGURE 4

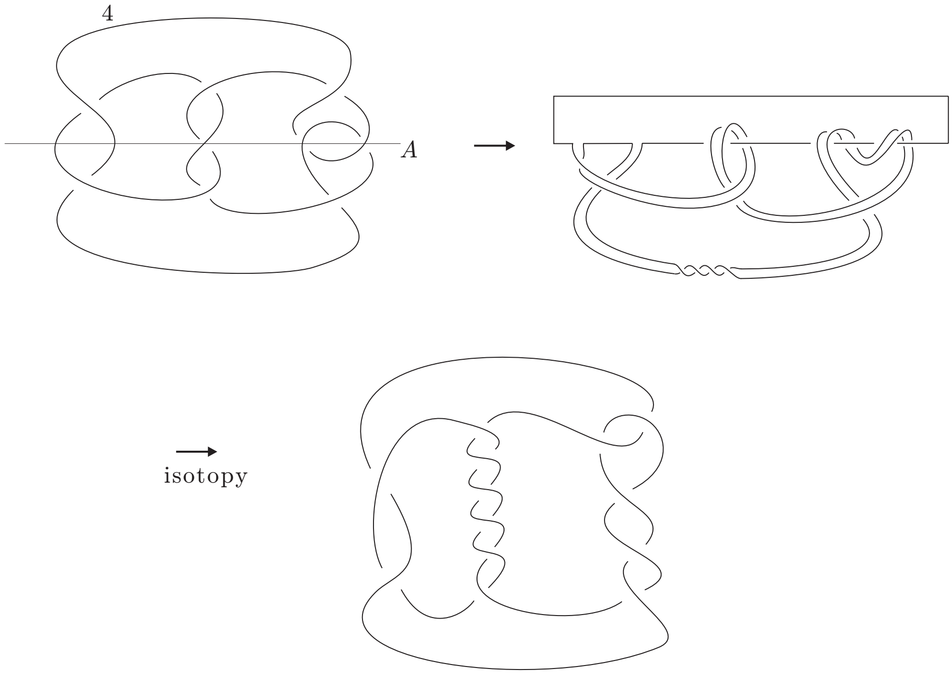

FIGURE 5

branched over this link, which is a Seifert fibered manifold of type $S^2(2,6,7)$ (see [6]). □

3. HYPERBOLICITY OF THE LINK L

In this section, we will prove that the link $L = K_0 \cup C_1 \cup C_2$ of Figure 1 is hyperbolic. We take advantage of the technique of tangle decomposition. See [20, 31, 33, 34, 38].

An *m-string tangle* T, $m \geq 1$, is a pair (B, s) where B is a 3-ball and s is a finite disjoint union of simple closed curves and m properly embedded arcs. We say that T is *essential* if $\partial B - \mathrm{Int}N(s)$ is incompressible and boundary-incompressible in $B - \mathrm{Int}N(s)$. In fact, the boundary-incompressibility follows from the incompressibility when $m > 1$. Also, T is *atoroidal* if $B - \mathrm{Int}N(s)$ does not contain a non-peripheral incompressible torus.

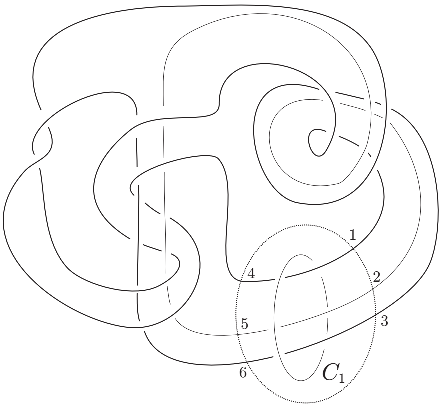

FIGURE 6

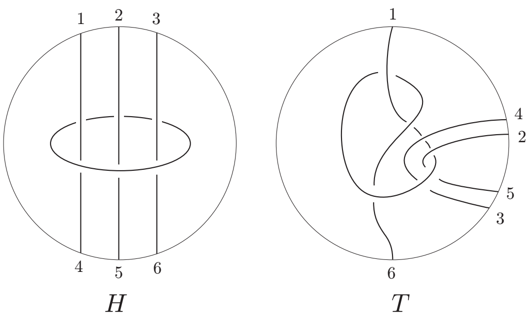

FIGURE 7

The pair (S^3, L) can be expressed as the union of two 3-string tangles H and T, where $H = (B, h)$ is the 3-string Hopf tangle and $T = (B, t)$. See Figures 6 and 7.

Let $\alpha_1, \alpha_2, \alpha_3$ be the arc components of h. Clearly, $(B, \alpha_1, \alpha_2, \alpha_3)$ has a product structure $(D^2, p_1, p_2, p_3) \times I$, where p_1, p_2, p_3 are points in $\text{Int}D^2$. We say that an annulus in $B - h$ is *vertical* if it is of the form $\omega \times I$ for some simple closed curve $\omega \subset \text{Int}D^2 - \{p_1, p_2, p_3\}$.

Similarly, let the unknotted arc components of t be α'_1, α'_2, and the knotted component β. Similar to H, $(B, \alpha'_1, \alpha'_2)$ has a product structure $(D^2, p'_1, p'_2) \times I$, where p'_1, p'_2 are points in $\text{Int}D^2$. An annulus in $B - t$ is *vertical* if it is either of the form $\omega \times I$ for some simple closed curve $\omega \subset \text{Int}D^2 - \{p'_1, p'_2\}$, or parallel to $\partial N(\beta) - \text{Int}(N(\beta) \cap \partial B)$, where $N(\beta)$ is a regular neighborhood of β in $B - \{\alpha'_1, \alpha'_2\}$.

Lemma 3.1. *Both tangles H and T are essential and atoroidal. Furthermore, any essential (incompressible and non boundary-parallel) annulus in $B - h$ or $B - t$ is vertical.*

Proof. Our tangles H and T coincide with H_3 and T_3 of [16, p.402]. Hence this lemma is shown in Lemmas 5.1, 5.2 and 5.3 of [16]. □

Proposition 3.2. The link $L = K_0 \cup C_1 \cup C_2$ is hyperbolic.

Proof. This is equivalent to the conditions that $E(L) = S^3 - \text{Int}N(L)$ is irreducible, atoroidal, and not a Seifert fibered manifold.

Since C_1 has non-zero linking number with each of K_0 and C_2 (after suitably oriented), L is not a split link, and the exterior $E(L)$ is irreducible. The atoroidality of $E(L)$ is shown by Lemma 3.1 as follows. Let F be an essential torus embedded

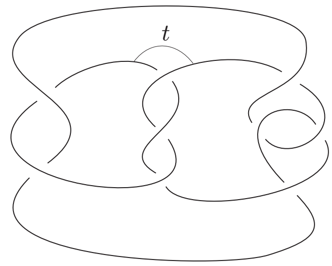

FIGURE 8

in $E(L)$. Let S be the decomposing sphere of the tangle decomposition $(S^3, L) = H \cup T$. By an isotopy of F, we may assume that F intersects S transversely in a finite number of simple closed curves, and the number of components in $F \cap S$ is minimal. Since F is incompressible and both tangles are essential, each component of $F \cap S$ does not bound a disk on F. Hence the components are mutually parallel on F. By Lemma 3.1, $F \cap S \neq \emptyset$. Thus F intersects H and T in properly embedded essential annuli, each of which is vertical in $B - h$ or $B - t$ by Lemma 3.1 again. However, as seen from the labeling in Figure 7, the only possibility is the case where F is boundary-parallel, a contradiction.

Finally, if $E(L)$ is a Seifert fibered manifold, then each component of the link must be a torus knot [5]. However, this is not the case, because K_0 is not a torus knot. (For, K_0 has unknotting number one. If K_0 is a torus knot, then it must be a trefoil [22], which is absurd.) □

4. HEEGAARD SURFACE

In this section, we prove that our knots can lie on a genus three Heegaard surface of S^3.

Recall that an *unknotting tunnel system* for a knot K is a collection of disjoint arcs $\{t_1, \dots, t_s\}$ such that $t_i \cap K = \partial t_i$ $(1 \leq i \leq s)$ and $S^3 - \mathrm{Int}N(K \cup t_1 \cup \cdots \cup t_s)$ is a handlebody. Then the *tunnel number* of K, denoted by $t(K)$, is the minimal number of arcs required in an unknotting tunnel system for K. It is easy to see that K can lie on a genus $t(K) + 1$ Heegaard surface of S^3.

First, Figure 8 shows the unknotting tunnel t for $K_0 = 9_{42}$, hence $t(K_0) = 1$. Thus K_0 lies on a genus two Heegaard surface of S^3.

Proposition 4.1. For any $n \geq 1$, K_n can lie on a genus three Heegaard surface of S^3.

Proof. It suffices to show that the tunnel number of K_n is at most two. Figure 9 shows the unknotting tunnel system $\{t_1, t_2\}$ for K_n, and the deformation of $K_n \cup t_1 \cup t_2$ to a standard bouquet. (For simplicity, K_2 is drawn there.) □

5. SYMMETRY

The computer experiment using the software SnapPea [43] suggests that our knot K_n is always hyperbolic, and more interestingly, K_n has no symmetry whenever $n \geq 1$. (We remark that K_0 is strongly-invertible.) In this section, we prove a weaker result that the family $\{K_n\}$ contains infinitely many knots without symmetry by suitably modifying the arguments of [8].

First, we will show that the link $L = K_0 \cup C_1 \cup C_2$ (Figure 1) has no symmetry.

Lemma 5.1. *There is no orientation-preserving diffeomorphism $f : S^3 \to S^3$ such that $f(K_0) = K_0$, $f(C_1) = C_2$ and $f(C_2) = C_1$.*

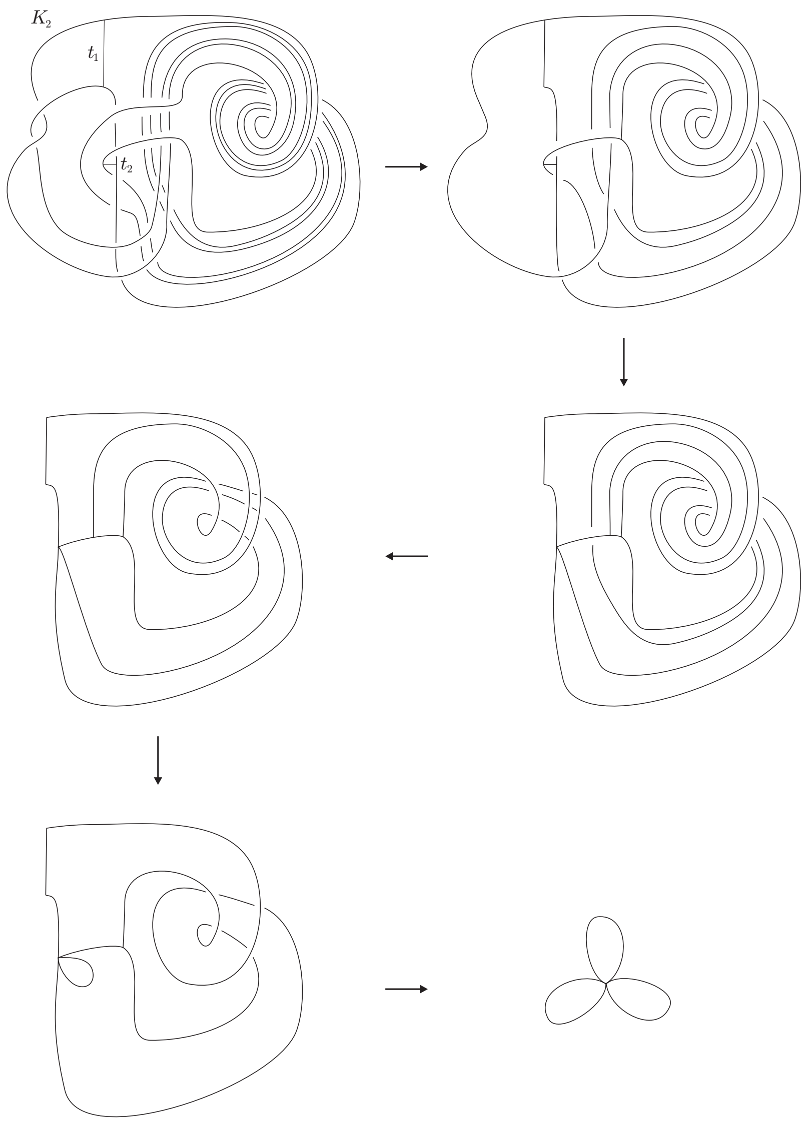

FIGURE 9

Proof. The sublink $K_0 \cup C_1$ of L is isotopic to the link in Figure 10. By performing (-1)-twist along C_1, we have an algebraic link, which is the sum of two Montesinos tangles (see Figure 10). In particular, its double branched cover contains a separating incompressible torus which derives from the sphere giving the tangle decomposition.

On the other hand, it can be seen that the sublink $K_0 \cup C_2$ changes into the Montesinos link of type $(1/3, -1/5, -2/5)$ by performing (-1)-twist along C_2. Its double branched cover is a Seifert fibered manifold of type $S^2(3,5,5)$, which does not contain a separating incompressible surface [9, 40] (see also [17]).

If there is an orientation-preserving diffeomorphism f on S^3 which sends K_0 to itself and exchanges C_1 and C_2, then the two links $K_0 \cup C_1$ and $K_0 \cup C_2$ are equivalent with K_0 invariant. However, this is impossible, because the resulting links after (-1)-twist along C_1, C_2, respectively, are not equivalent as above. $\square$

Lemma 5.2. *The link $K_0 \cup C_2$ is not strongly-invertible.*

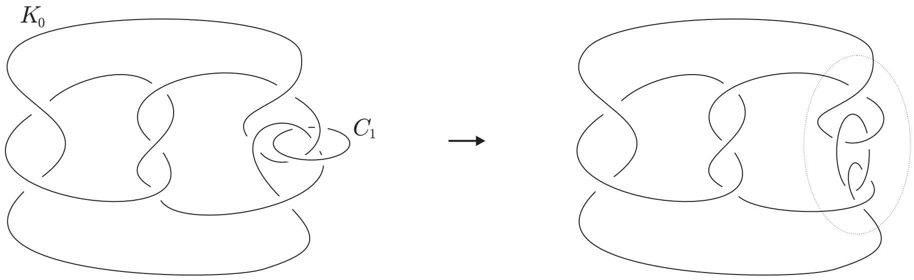

FIGURE 10

Proof. Assume that the link $K_0 \cup C_2$ admits a strong inversion f. Take an f-invariant tubular neighborhood $N(C_2)$ of C_2. Then the involution f restricted to $S^3 - \mathrm{Int}N(C_2)$ extends to an involution after 1-surgery along C_2 (cf. [30]). Since 1-surgery is equivalent to (-1)-twist, this means that the resulting link L' after (-1)-twist along C_2 remains strongly-invertible. This contradicts the fact that the Montesinos link L' of type $(1/3, -1/5, -2/5)$ is not invertible [3]. (It has the unique symmetry that is a cyclic period of order two.) □

Proposition 5.3. The link $L = K_0 \cup C_1 \cup C_2$ has no symmetry.

Proof. Assume that L has a symmetry f. Since K_0 is knotted and both of C_1 and C_2 are unknotted, $f(K_0) = K_0$. According to [21], $K_0 = 9_{42}$ has the unique symmetry that is a strong inversion. Note that f is non-trivial on K_0, because of the positive solution to the Smith conjecture [1]. Hence f gives the unique strong inversion of K_0. Furthermore, $f(C_1) = C_1$ and $f(C_2) = C_2$ by Lemma 5.1.

Let A denote the axis of f. Then either $A \cap C_2 \neq \emptyset$ or $A \cap C_2 = \emptyset$.

First assume $A = C_2$. Then $\mathrm{lk}(K_0, C_2) = \pm 1$ after giving orientation (see Figure 5). But this is impossible, since $\mathrm{lk}(K_0, C_2)$ is even as seen from Figure 1.

Thus if $A \cap C_2 \neq \emptyset$, then A meets C_2 in two points. Then f reverses an orientation of C_2. Hence f gives a strong inversion of $K_0 \cup C_2$, which contradicts Lemma 5.2. Thus we have $A \cap C_2 = \emptyset$.

Let us take an f-invariant tubular neighborhood $N(C_2)$ of C_2. Then the involution f restricted to $S^3 - \mathrm{Int}N(C_2)$ extends to an involution after 1-surgery along C_2. This means that the resulting link L', which is the Montesinos link of type $(1/3, -1/5, -2/5)$, after 1-surgery along C_2 admits an involution preserving the components. However, L' has the unique symmetry that is a cyclic period of order two and exchanges the components [3]. This is a contradiction. □

Proposition 5.4. There is a constant N such that if $n > N$ then K_n is hyperbolic and has no symmetry.

Proof. Let $M = S^3 - L = S^3 - K_0 \cup C_1 \cup C_2$. By Proposition 3.2, M is a hyperbolic 3-manifold with three cusps. The knot complement $S^3 - K_n$ is obtained from M by $(n+1)/n$-filling along C_1 and $(n-1)/n$-filling along C_2. By Thurston's hyperbolic Dehn surgery theory, there is a constant N such that if $n > N$ then $S^3 - K_n$ is hyperbolic and for some $\varepsilon > 0$ the only simple closed geodesics of length less than ε in $S^3 - K_n$ are the cores C_1^* and C_2^* of the attached solid tori (see [2]). Furthermore, we may assume that the volume of $S^3 - K_n$ is bigger than that of $S^3 - K_0$ for $n > N$ ([35]).

Assume that K_n has a symmetry f for some $n > N$. Let $N(K_n)$ be an f-invariant tubular neighborhood of K_n. Then we have a periodic diffeomorphism $f : S^3 - \mathrm{Int}N(K_n) \to S^3 - \mathrm{Int}N(K_n)$. By Mostow's rigidity theorem, f is homotopic, and isotopic by [41], to an isometry φ. Thus $\varphi(C_1^* \cup C_2^*) = C_1^* \cup C_2^*$, the union of the shortest closed geodesics. Since φ preserves a meridian of K_n, φ restricted to $S^3 - \mathrm{Int}(N(K_n) \cup N(C_1^*) \cup N(C_2^*))$ extends to a periodic diffeomorphism $\bar{\varphi}$: $S^3 - \mathrm{Int}(N(C_1^*) \cup N(C_2^*)) \to S^3 - \mathrm{Int}(N(C_1^*) \cup N(C_2^*))$ such that $\bar{\varphi}(K_n) = K_n$.

On the other hand, there is a diffeomorphism $h : S^3 - \mathrm{Int}(N(C_1) \cup N(C_2)) \to S^3 - \mathrm{Int}(N(C_1^*) \cup N(C_2^*))$ induced by twisting along the annulus A n times (see Figure 2). Note that $h(K_0) = K_n$. Let $g = h^{-1} \circ \bar{\varphi} \circ h : S^3 - \mathrm{Int}(N(C_1) \cup N(C_2)) \to S^3 - \mathrm{Int}(N(C_1) \cup N(C_2))$. Then g is periodic and $g(K_0) = K_0$. Choose (preferred) meridian-longitude systems (μ_i, λ_i) for C_i and (μ_i^*, λ_i^*) for C_i^* for $i = 1, 2$. Here, we can choose their orientations such that $\mathrm{lk}(\mu_i, \lambda_i) = 1$ and $\mathrm{lk}(\mu_i^*, \lambda_i^*) = 1$ for $i = 1, 2$. Then $\partial A \cap \partial N(C_i) = \mu_i + \lambda_i$ in $H_1(\partial N(C_i))$. Hence we may suppose that

$$h(\mu_1) = (1-n)\mu_1^* - n\lambda_1^*, \quad h(\lambda_1) = n\mu_1^* + (n+1)\lambda_1^*,$$
$$h(\mu_2) = (1+n)\mu_2^* + n\lambda_2^*, \quad h(\lambda_2) = (-n)\mu_2^* + (1-n)\lambda_2^*.$$

The argument is divided into two cases.

Case 1. $\varphi(C_i^*) = C_i^*$ for $i = 1, 2$.

In this case, $\bar{\varphi}(\mu_i^*) = \varepsilon_i \mu_i^*$ and $\bar{\varphi}(\lambda_i^*) = \varepsilon_i \lambda_i^*$ for some $\varepsilon_i \in \{1, -1\}$, since $\bar{\varphi}$ is orientation-preserving. Then a simple calculation shows that $g(\mu_i) = \varepsilon_i \mu_i$ for $i = 1, 2$. Hence g can be extended to a periodic diffeomorphism over S^3 leaving K_0, C_1, C_2 invariant, respectively. This means that the link $L = K_0 \cup C_1 \cup C_2$ admits a symmetry, contradicting Proposition 5.3.

Case 2. $\varphi(C_1^*) = C_2^*$ and $\varphi(C_2^*) = C_1^*$.

Then $\bar{\varphi}(\mu_1^*) = \varepsilon_1 \mu_2^*$, $\bar{\varphi}(\lambda_1^*) = \varepsilon_1 \lambda_2^*$, $\bar{\varphi}(\mu_2^*) = \varepsilon_2 \mu_1^*$ and $\bar{\varphi}(\lambda_2^*) = \varepsilon_2 \lambda_1^*$ for some $\varepsilon_1, \varepsilon_2 \in \{1, -1\}$. A similar calculation shows that $g(\mu_1) = \varepsilon_1((1-2n)\mu_2 - 2n\lambda_2)$ and $g(\mu_2) = \varepsilon_2((1+2n)\mu_1 + 2n\lambda_1)$. By twisting along the annulus A $2n$ times, we see that (S^3, K_0) is equivalent to (S^3, K_{2n}), a contradiction. □

6. Proof of Theorem 1.2 and Questions

Proof of Theorem 1.2. By Proposition 5.4, there is a constant N such that if $n > N$ then K_n is hyperbolic and has no symmetry. By Proposition 4.1, K_n can lie on a genus three Heegaard surface of S^3. If K_n lies on a genus two Heegaard surface, then either it has a cyclic period of order two or it is strongly-invertible (cf. [8]). For any n, 4-surgery on K_n yields the same Seifert fibered manifold of type $S^2(2, 6, 7)$ by Propositions 2.1 and 2.2. Finally, the volume of $S^3 - K_n$ increases monotonically to the volume of $S^3 - L$ for n large enough ([35]). Thus the family $\{K_n\}$ contains infinitely many distinct knots that satisfy the required properties. □

Remark 6.1. Takuji Nakamura [32] informed that K_n can be shown to be fibered and have genus $2n + 2$ by finding a suitable Seifert surface.

As a starting point, we can use the link illustrated in Figure 11. Similar to our previous knot, 4-surgery on K_0, hence on K_n for any $n \geq 0$, yields the same Seifert fibered manifold of type $S^2(3, 4, 8)$ in this case. The computer experiment by SnapPea suggests that K_n has no symmetry if $n \geq 1$. Although we have one more candidate for K_0 with such property, it is not definite that there are infinitely many such knots, and that there are infinitely many small Seifert fibered manifolds with infinitely many knot-surgery descriptions.

We close this paper with the following questions.

Question 6.2. Does there exist a lens space that can be obtained by the same surgery on an infinite number of knots?

Probably, the answer is negative.

Question 6.3. Does there exist a 3-manifold that can be obtained by the same non-integral surgery on an infinite number of knots?

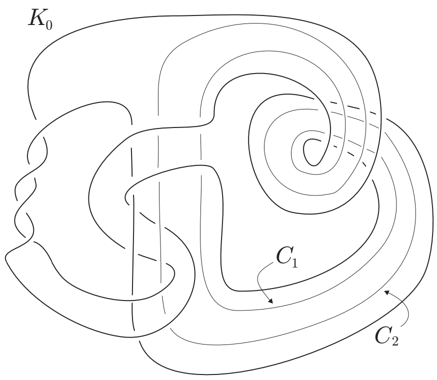

FIGURE 11

It is known that for a given $N > 1$, there exists a 3-manifold that is obtained by the same non-integral surgery on N mutually distinct hyperbolic knots [18].

Assume that r-surgery on a knot K yields a Seifert fibered manifold. If there exists an unknotted simple closed curve C disjoint from K which becomes a fiber in $K(r)$, then C is called a *seiferter* [8]. (The idea of seiferter has already been appeared in [27].) It is expected that a seiferter always exists for any Seifert surgery, but we could not find a seiferter for 4-surgery on K_n when $n > 1$.

Question 6.4. Does there exist a seiferter for 4-surgery on K_n when $n \geq 1$?

REFERENCES

1. H. Bass and J. Morgan (eds.), *The Smith conjecture*, Pure and Applied Mathematics, **112**, Academic Press, Inc., Orlando, FL, 1984.
2. R. Benedetti and C. Petronio, *Lectures on hyperbolic geometry*, Universitext, Springer-Verlag, Berlin, 1992.
3. M. Boileau and B. Zimmermann, *Symmetries of nonelliptic Montesinos links*, Math. Ann. **277** (1987), 563–584.
4. W. R. Brakes, *Manifolds with multiple knot-surgery descriptions*, Math. Proc. Cambridge Philos. Soc. **87** (1980), 443–448.
5. G. Burde and K. Murasugi, *Links and Seifert fiber spaces*, Duke Math. J. **37** (1970), 89–93.
6. G. Burde and H. Zieschang, *Knots*, Second edition, de Gruyter Studies in Mathematics, **5**, Walter de Gruyter and Co., Berlin, 2003.
7. H. D. Cao and X. P. Zhu, *A complete proof of the Poincaré and geometrization conjectures — application of the Hamilton-Perelman theory of the Ricci flow*, Asian J. Math. **10** (2006), 165–492.
8. A. Deruelle, K. Miyazaki and K. Motegi, *Networking Seifert fibered surgeries on knots*, preprint.
9. B. Evans and W. Jaco, *Varieties of groups and three-manifolds*, Topology **12** (1973), 83–97.
10. D. Gabai, *Foliations and the topology of* 3*-manifolds. III*, J. Differential Geom. **26** (1987), 479–536.
11. P. Ghiggini, *Knot Floer homology detects genus-one fibred links*, preprint, `arXiv:math.GT/0603445`.
12. R. Gompf and A. Stipsicz, 4*-manifolds and Kirby calculus*, Graduate Studies in Mathematics, **20**, American Mathematical Society, Providence, RI, 1999.
13. F. González-Acuña and H. Short, *Knot surgery and primeness*, Math. Proc. Cambridge Philos. Soc. **99** (1986), 89–102.
14. C. McA. Gordon, *Small surfaces and Dehn filling*, Proceedings of the Kirbyfest (Berkeley, CA, 1998), 177–199 (electronic), Geom. Topol. Monogr., **2**, Geom. Topol. Publ., Coventry, 1999.
15. C. McA. Gordon and J. Luecke, *Non-integral toroidal Dehn surgeries*, Comm. Anal. Geom. **12** (2004), 417–485.
16. C. McA. Gordon and A. Reid, *Tangle decompositions of tunnel number one knots and links*, J. Knot Theory Ramifications **4** (1995), 389–409.

17. W. Jaco, *Lectures on three-manifold topology*, CBMS Regional Conference Series in Mathematics, **43**, American Mathematical Society, Providence, R.I., 1980.
18. A. Kawauchi, *Mutative hyperbolic homology 3-spheres with the same Floer homology*, Geom. Dedicata **61** (1996), 205–217.
19. R. Kirby, *Problems in low-dimensional topology*, AMS/IP Stud. Adv. Math., 2.2, Geometric topology (Athens, GA, 1993), 35–473, Amer. Math. Soc., Providence, RI, 1997.
20. R. Kirby and W. R. Lickorish, *Prime knots and concordance*, Math. Proc. Cambridge Philos. Soc. **86** (1979), 437–441.
21. K. Kodama and M. Sakuma, *Symmetry groups of prime knots up to* 10 *crossings*, Knots 90 (Osaka, 1990), 323–340, de Gruyter, Berlin, 1992.
22. P. Kronheimer and T. Mrowka, *Gauge theory for embedded surfaces. I*, Topology **32** (1993), 773–826.
23. W. B. R. Lickorish, *A representation of orientable combinatorial* 3*-manifolds*, Ann. of Math. **76** (1962), 531–540.
24. W. B. R. Lickorish, *Surgery on knots*, Proc. Amer. Math. Soc. **60** (1976), 296–298.
25. C. Livingston, *More* 3*-manifolds with multiple knot-surgery and branched-cover descriptions*, Math. Proc. Cambridge Philos. Soc. **91** (1982), 473–475.
26. T. Mattman, K. Miyazaki and K. Motegi, *Seifert-fibered surgeries which do not arise from primitive/Seifert-fibered constructions*, Trans. Amer. Math. Soc. **358** (2006), 4045–4055.
27. K. Miyazaki and K. Motegi, *Seifert fibered manifolds and Dehn surgery. III*, Comm. Anal. Geom. **7** (1999), 551–582.
28. J. Montesinos, *Surgery on links and double branched covers of* S^3, Knots, groups, and 3-manifolds (Papers dedicated to the memory of R. H. Fox), 227–259. Ann. of Math. Studies **84**, Princeton Univ. Press, Princeton, N.J., 1975.
29. F. Morgan, *The Smith conjecture*, 3–6, Pure Appl. Math., 112, Academic Press, Orlando, FL, 1984
30. K. Motegi, *Dehn surgeries, group actions and Seifert fiber spaces*, Comm. Anal. Geom. **11** (2003), 343–389.
31. R. Myers, *Homology cobordisms, link concordances, and hyperbolic* 3*-manifolds*, Trans. Amer. Math. Soc. **278** (1983), 271–288.
32. T. Nakamura, a private communication.
33. Y. Nakanishi, *Primeness of links*, Math. Sem. Notes Kobe Univ. **9** (1981), 415–440.
34. Y. Nakanishi, *Prime and simple links*, Math. Sem. Notes Kobe Univ. **11** (1983), 249–256.
35. W. Neumann and D. Zagier, *Volumes of hyperbolic three-manifolds*, Topology **24** (1985), 307–332.
36. J. Osoinach Jr., *Manifolds obtained by Dehn surgery on infinitely many distinct knots in* S^3, dissertation, the University of Texas at Austion, 1998.
37. J. Osoinach Jr., *Manifolds obtained by surgery on an infinite number of knots in* S^3, Topology **45** (2006), 725–733.
38. T. Soma, *Simple links and tangles*, Tokyo J. Math. **6** (1983), 65–73.
39. M. Teragaito, *Homology handles with multiple knot-surgery descriptions*, Topology Appl. **56** (1994),249–257.
40. F. Waldhausen, *Gruppen mit Zentrum und* 3*-dimensionale Mannigfaltigkeiten*, Topology **6** (1967), 505–517.
41. F. Waldhausen, *On irreducible* 3*-manifolds which are sufficiently large*, Ann. of Math. **87** (1968), 56–88.
42. A. H. Wallace, *Modifications and cobounding manifolds*, Canad. J. Math. **12** (1960), 503–528.
43. J. Weeks, SnapPea, a computer program for creating and studying hyperbolic 3-manifolds, available from `http://www.geometrygames.org/SnapPea/`

Department of Mathematics and Mathematics Education, Hiroshima University, Kagamiyama 1-1-1, Higashi-hiroshima, Japan 739-8524.

E-mail address: `teragai@hiroshima-u.ac.jp`