

How Does My Microwave/EM Simulator Define Complex-Referenced S-Parameters?

Shuhei AMAKAWA[†]

[†] Graduate School of Advanced Sciences of Matter, Hiroshima University, Higashihiroshima, 739-8530 Japan

E-mail: [†]amakawa@ieee.org

Abstract S-parameters are usually referenced to a real reference resistance of $50\ \Omega$. At times, S-parameters normalized to some complex reference impedance are used out of necessity or for convenience' sake. Since there are incompatible definitions of complex-referenced S-parameters, it is important to be sure which definition is adopted in one's microwave or electromagnetic (EM) simulator. However, most simulators are rather silent about which variant they implement. This paper suggests a procedure for identifying it through simulation of transmission lines and postprocessing of resulting S-parameters. Examples of applying the procedure are given and it is shown that two EM simulators from a single vendor adopt incompatible definitions, which could lead to possible mishaps.

Keywords Scattering matrix, traveling waves, power waves, reference impedance, transmission line

1. Introduction

S-parameters are usually referenced to a real reference resistance of $50\ \Omega$ when measurement reference planes are located somewhere along quasi-transverse-electromagnetic (quasi-TEM) transmission lines having a $50\text{-}\Omega$ nominal characteristic resistance. However, S-parameters normalized to some complex reference impedance are used at times out of necessity or for convenience' sake.

Complex-referenced S-parameters can be defined in (at least) two different ways [1]: *traveling-wave S-parameters* and *power-wave S-parameters*. Although traveling waves and power waves have the same dimensions (square-root of power), they have very different properties [2]. Traveling-wave S-parameters and power-wave S-parameters are incompatible with each other [1, 3, 4] and serve different purposes [1, 2]. One must be aware of the differences and use the right one depending on what one wants to do.

Microwave circuit simulators tend to implement power-wave S-parameters as complex-referenced S-parameters. However, many EM simulators do not clearly state which variant of complex-referenced S-parameters they implement (if they do). This article discusses how the user could figure out which complex-referenced S-parameters a given simulator implements when sufficient information is not available from the simulator manual [5].

2. Traveling waves and power waves

The phasors detected by a vector network analyzer (VNA) and used to define S-parameters are the *traveling waves* $a_i(z_{\text{ref}i})$ and $b_j(z_{\text{ref}j})$.

$$a_i(z_{\text{ref}i}) = \frac{\sqrt{\Re(Z_{\text{ref}i})}}{|Z_{\text{ref}i}|} \frac{V_i + Z_{\text{ref}i} I_i}{2} \quad (\text{Port } i, \text{ incident}), \quad (1)$$

$$b_j(z_{\text{ref}j}) = \frac{\sqrt{\Re(Z_{\text{ref}j})}}{|Z_{\text{ref}j}|} \frac{V_j - Z_{\text{ref}j} I_j}{2} \quad (\text{Port } j, \text{ outgoing}), \quad (2)$$

where $Z_{\text{ref}i}$ is the reference impedance of the i th port, V_i and I_i the effective voltage and current of the i th port. Strictly, $a_i(z_{\text{ref}i})$ and $b_j(z_{\text{ref}j})$ can be correlated with electromagnetic traveling waves present in a uniform transmission line (or waveguide) only if the reference impedance equals the characteristic impedance of the line [6]. If not, $a_i(z_{\text{ref}i})$ and $b_j(z_{\text{ref}j})$ should be called *pseudo traveling waves* or *pseudo waves* [6]. The (pseudo-)traveling-wave S-parameter is given by

$$S_{ji}(z_{\text{ref}i}, z_{\text{ref}j}) = \frac{b_j(z_{\text{ref}j})}{a_i(z_{\text{ref}i})}. \quad (3)$$

In (1)–(3), reference impedances are shown explicitly within parentheses in subscripts using the notation shown in Table 1 [7].

The other set of waves used to define S-parameters are known as

the *power waves* [2]. These wave variables are defined by

$$a_{pi}(Z_{refi}) = \frac{1}{\sqrt{\Re(Z_{refi})}} \frac{V_i + Z_{refi} I_i}{2} \quad (\text{Port } i, \text{ incident}), \quad (4)$$

$$b_{pj}(Z_{refj}) = \frac{1}{\sqrt{\Re(Z_{refj})}} \frac{V_j - Z_{refj}^* I_j}{2} \quad (\text{Port } j, \text{ outgoing}). \quad (5)$$

The power-wave S-parameter is defined by

$$S_{Pji}(Z_{refi}, Z_{refj}) = \frac{b_{pj}(Z_{refj})}{a_{pi}(Z_{refi})}. \quad (6)$$

Power-wave S-parameters can be useful, for example, for amplifier design [8–11]. However, unlike traveling waves, power waves are not directly measurable [1, 7], nor do they correspond to propagating electromagnetic waves. Power-wave S-parameters are often referred to as the “generalized S-parameters” [8–11], but not always [12, 13]. Therefore, any reference to “generalized S-parameters” in a simulator manual ([14], for example) may or may not mean that the simulator implements power-wave S-parameters.

3. Identifying S-Parameter Definition

We propose to use the known properties of transmission-line S-matrices to identify the definition of complex-referenced S-parameters adopted by a simulator [5].

The pseudo-wave S-matrix of a length, ℓ , of a transmission line is given by (9) on the next page. Z_0 is the characteristic impedance of the line and γ its propagation constant. If $Z_{ref1} = Z_{ref2} = Z_0$, (9) reduces to the following traveling-wave S-matrix.

$$S_{(Z_0)} = \begin{bmatrix} 0 & e^{-\gamma\ell} \\ e^{-\gamma\ell} & 0 \end{bmatrix}. \quad (7)$$

The power-wave S-matrix of a length, ℓ , of a transmission line is given by (10). If $Z_{ref1} = Z_{ref2} = Z_0$, (10) reduces to

$$S_P(Z_0) = \frac{1}{2Z_0} \begin{bmatrix} j\Im(Z_0) & \Re(Z_0) e^{-\gamma\ell} \\ \Re(Z_0) e^{-\gamma\ell} & j\Im(Z_0) \end{bmatrix}. \quad (8)$$

The difference between (7) and (8) gives a clue as to which S-parameter definition a given simulator adopts.

Figure 1 shows a procedure that we followed to find the S-parameter definitions of EMPro and Momentum from Keysight Technologies. The complex- Z_0 -referenced S-parameters from EMPro turned out to be the traveling-wave S-parameters of (7) as shown in Fig. 2. The complex- Z_0 -referenced S-parameters from Momentum turned out to be the power-wave S-parameters of (8) as shown in Fig. 3.

4. Discussion

In this article, we looked at S-parameter definitions by implicitly assuming that (3) and (6) are the only possible candidates. However,

there are further points that should be taken into consideration in general. For example, (4) and (5) are not the unique definition of power waves [1, 2]. Other definitions are also possible because of the arbitrariness of the phase of power waves [1, 2].

In addition, what can be done to check the S-parameter definition depends on the simulator. Not all simulators allow the use of complex reference impedances. If a simulator generates only 50 Ω -referenced S-parameters, we might still want to know, for example, if the results are correctly generated from internally computed Z_0 -referenced S-parameters using the right (not a wrong) conversion formula.

Acknowledgment

This work was supported in part by JSPS.KAKENHI.

References

- [1] S. Amakawa, “Scattered reflections on scattering parameters—Demystifying complex-referenced S parameters—,” *IEICE Trans. Electron.*, vol. E99-C, no. 10, pp. 1100–1112, Oct. 2016.
- [2] K. Kurokawa, “Power waves and the scattering matrix,” *IEEE Trans. Microw. Theory Tech.*, vol. 13, no. 2, pp. 194–202, Mar. 1965.
- [3] D. Williams, “Traveling waves and power waves,” *IEEE Microw. Mag.*, vol. 14, no. 7, pp. 38–45, 2013.
- [4] S. Llorente-Romano, A. Garca-Lampérez, T. K. Sarkar, and M. Salazar-Palma, “An exposition on the choice of the proper S parameters in characterizing devices including transmission lines with complex reference impedances and a general methodology for computing them,” *IEEE Antennas Propag. Mag.*, vol. 55, no. 4, pp. 94–112, Aug. 2013.
- [5] Y. Kobayashi and S. Amakawa, “Prescriptions for identifying the definition of complex-referenced S-parameters in commercial EM simulators,” *38th Progress In Electromagnetics Research Symposium (PIERS)*, May 2017.
- [6] R. B. Marks and D. F. Williams, “A general waveguide circuit theory,” *J. Res. Natl. Inst. Stand. Technol.*, vol. 97, no. 5, pp. 533–562, 1992.
- [7] D. Woods, “Multiport-network analysis by matrix renormalisation employing voltage-wave S-parameters with complex normalisation,” *Proc. IEE*, vol. 124, no. 3, pp. 198–204, Mar. 1977.
- [8] G. Gonzalez, *Microwave Transistor Amplifiers: Analysis and Design*, 2nd edition, Prentice Hall, 1996.
- [9] R. E. Collin, *Foundations for Microwave Engineering*, 2nd edition, Wiley-Interscience, 2001.
- [10] R. Mavaddat, *Network Scattering Parameters*, World Scientific, 1996.
- [11] D. M. Pozar, *Microwave Engineering*, 4th edition, Wiley, 2011.
- [12] D. M. Pozar, *Microwave Engineering*, 3rd edition, Wiley, 2005.
- [13] J. A. Dobrowolski, *Microwave Network Design Using the Scattering Matrix*, Artech House, 2010.
- [14] EMPro 2015.01 Documentation, Keysight Technologies.

Table 1: Notation for showing reference impedances [7].

Example	Description
$S_{ij}(50\Omega)$	S parameter with ports i and j referenced to 50Ω
$S_{(R_{\text{ref}})}$	S matrix with all ports referenced to R_{ref}
$S_{21}(R_{\text{ref}1}, R_{\text{ref}2})$	Port 1 referenced to $R_{\text{ref}1}$, port 2 referenced to $R_{\text{ref}2}$
$S_{(Z_{\text{ref}1}, Z_{\text{ref}2})}$	2-port S matrix referenced to $Z_{\text{ref}1}$ and $Z_{\text{ref}2}$
$S_{(Z_{\text{ref}})}$	S matrix referenced to reference impedance matrix $Z_{\text{ref}} = \text{diag}(Z_{\text{ref}1}, Z_{\text{ref}2}, \dots)$

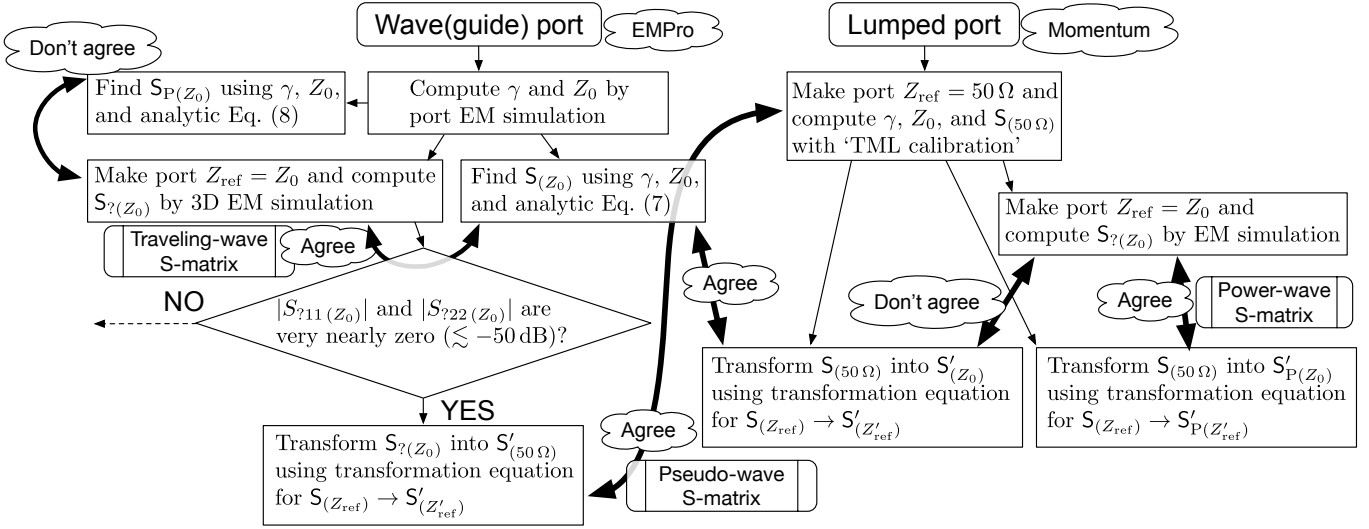


Figure 1: Procedure applied to EMPro and Momentum from Keysight Technologies [5]. Complex-referenced S-matrices in EMPro and Momentum turned out to be traveling-wave and power-wave S-matrices, respectively. Z_{ref} : Reference impedance. γ : Propagation constant. Z_0 : Characteristic impedance.

$$S_{(Z_{\text{ref}1}, Z_{\text{ref}2})} = \frac{1}{(Z_0^2 + Z_{\text{ref}1} Z_{\text{ref}2}) \tanh \gamma \ell + Z_0 (Z_{\text{ref}1} + Z_{\text{ref}2})} \times \begin{bmatrix} (Z_0^2 - Z_{\text{ref}1} Z_{\text{ref}2}) \tanh \gamma \ell + Z_0 (Z_{\text{ref}2} - Z_{\text{ref}1}) & \frac{\sqrt{\Re(Z_{\text{ref}1})} |Z_{\text{ref}2}| 2Z_0 Z_{\text{ref}1}}{\sqrt{\Re(Z_{\text{ref}2})} |Z_{\text{ref}1}| \cosh \gamma \ell} \\ \frac{\sqrt{\Re(Z_{\text{ref}2})} |Z_{\text{ref}1}| 2Z_0 Z_{\text{ref}2}}{\sqrt{\Re(Z_{\text{ref}1})} |Z_{\text{ref}2}| \cosh \gamma \ell} & (Z_0^2 - Z_{\text{ref}1} Z_{\text{ref}2}) \tanh \gamma \ell + Z_0 (Z_{\text{ref}1} - Z_{\text{ref}2}) \end{bmatrix} \quad (9)$$

$$S_{P(Z_{\text{ref}1}, Z_{\text{ref}2})} = \frac{1}{(Z_0^2 + Z_{\text{ref}1} Z_{\text{ref}2}) \tanh \gamma \ell + Z_0 (Z_{\text{ref}1} + Z_{\text{ref}2})} \times \begin{bmatrix} (Z_0^2 - Z_{\text{ref}1}^* Z_{\text{ref}2}) \tanh \gamma \ell + Z_0 (Z_{\text{ref}2} - Z_{\text{ref}1}^*) & \frac{2Z_0 \sqrt{\Re(Z_{\text{ref}1})} \Re(Z_{\text{ref}2})}{\cosh \gamma \ell} \\ \frac{2Z_0 \sqrt{\Re(Z_{\text{ref}1})} \Re(Z_{\text{ref}2})}{\cosh \gamma \ell} & (Z_0^2 - Z_{\text{ref}1} Z_{\text{ref}2}^*) \tanh \gamma \ell + Z_0 (Z_{\text{ref}1} - Z_{\text{ref}2}^*) \end{bmatrix} \quad (10)$$

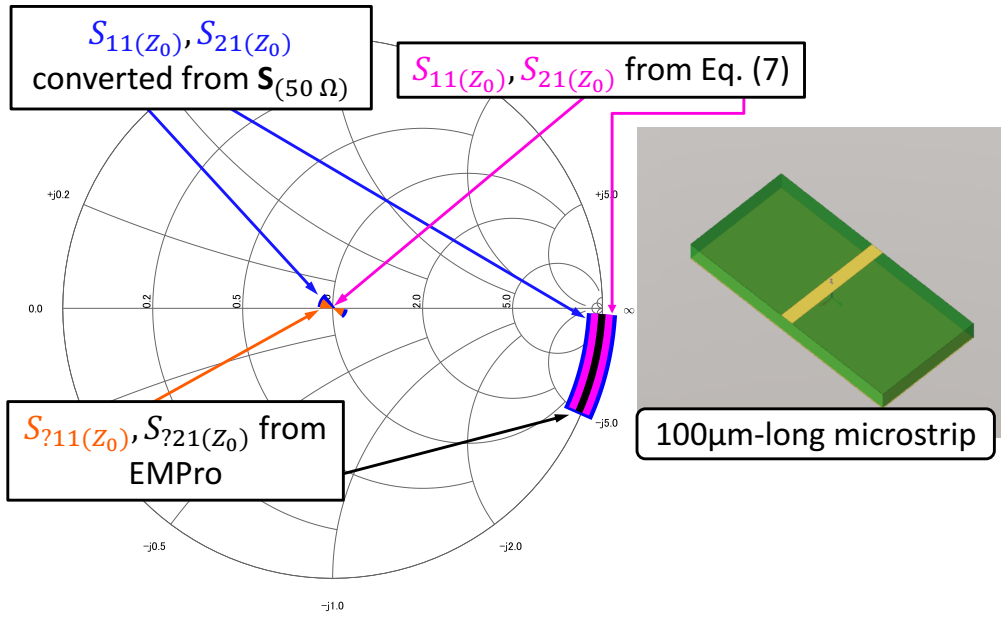


Figure 2: Transmission-line S-parameters from EMPro up to 100 GHz. $S_{?11}(z_0)$ is at the center and $S_{?21}(z_0)$ shows phase delay. $S_{?11}(z_0)$ and $S_{?21}(z_0)$ are traveling-wave S-parameters given by (7).

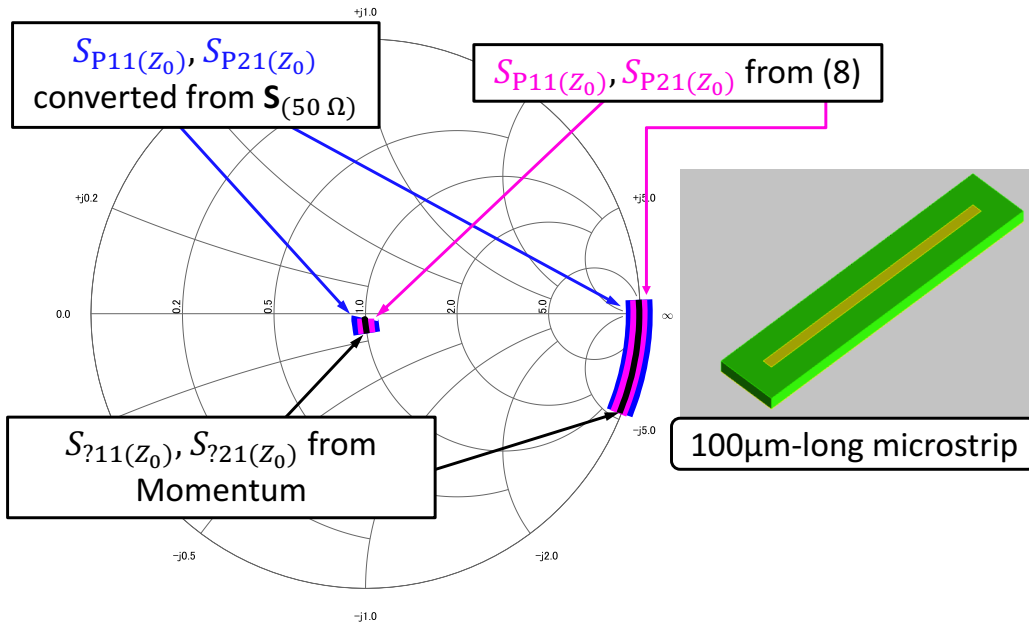


Figure 3: Transmission-line S-parameters from Momentum up to 100 GHz. $S_{?11}(z_0)$ is not quite at the center and $S_{?21}(z_0)$ shows phase advance at low frequencies. $S_{?11}(z_0)$ and $S_{?21}(z_0)$ are power-wave S-parameters given by (8).