The Economics of Japanese Educational Reform:
21st Century COE Program II

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Introduction

The aim of this paper is to extend Fukiharu [2003]. In Fukiharu [2003], when there are two identical universities, which maximizes each prestige (academic level) given the fixed amount of fund, distributed to universities according to the objective evaluation, the government cannot enhance the national academic level: the sum of academic levels of the universities. In this paper, it is assumed that they are different in its structure of conducting research and teaching, and it is examined if it is possible to enhance the national academic level, by introducing more competition. The introduction of more competition into the higher education is one of the main methods of Japanese educational reform. In section I of this paper, it is examine if the government can enhance the national academic level, by introducing more competition into the universities in terms of simulation when the two universities are different in the above sense and the level of competition is low. In section II of this paper, it is examine if the government can enhance the national academic level, by introducing more competition into the universities in terms of simulation when the two universities are different in the above sense and the level of competition is high.
I. Comparison between $\alpha = 1/10$ and $\alpha = 4/10$: Stable Cases

In Japan, reform of higher education initiated by the Japanese government has been under way. The purpose of this reform is to introduce more competition into the Japanese higher education, and to encourage technological innovations, and finally constructing US-type vigorous economy. As an example of the reform, all the national universities were reconstructed in 2004 as "agencies" of British type. The method of achieving the above goal in the new "agency" system is to rigorously evaluate the academic activity of each university, as well as increasing the financial assistance to universities. Another example is the 21st Century COE (Center of Excellence) program, which selects excellent research projects and concentrates huge amount of money on excellent research institutions. The aim of this paper is to analyse this government policy in economic framework, extending the contribution by Bowen [1980]. Bowen [1980] constructed a simple 1 university behavior model, who maximizes prestige, not profit, under budget constraint. Prestige, $P$, is a function of the quantity and quality of the teaching, $T$, and research, $R$, activity, undertaken by the university; $P=TR$. The unit-cost of teaching is denoted by $c_t$, while that of research is denoted by $c_r$. According to Bowen [1980], this university maximizes prestige under budget constraint, $c_tT+c_rR=B$. An interesting feature in this model is that $B$ is assumed to be a function of $P$; $B=B(P)$, and $B$ is an increasing function of $P$. $B(P)$ is called the donating function. He stipulates that $B(P)=P^a$, $0<\alpha \leq 1/2$, which implies that although the university acts as prestige maximizer, it can raise the budget expenses if the prestige increases, since government, firms, and students provide it with more fund. Thus, the university acts as a prestige maximizer, taking account of this factor. It is easy to solve this maximization problem, and the optimum solution is given by $R^*=((c_r/c_t)^\alpha/2c_t)^{1/(1-2\alpha)}$ and $T^*=c_tR^*/c_r$. If $(c_r/c_t)^\alpha/2c_t>1$, as $\alpha$ approaches $1/2$, $R^*$ and $T^*$ becomes infinite.

In Fukiharu [2003], Bowen model was extended to 2-university behavior model. Most of the assumtions in Fukiharu [2003] are retained in this paper. Suppose that there are 2 universities, university A and university B. Each university maximizes prestige (academic level), under budget constraint. In this model, it is assumed that the total fund for the universities is fixed by K. The government allocates the fund for each university according to the share of evaluation constructed from donating function. Specifically, university A has prestige $P_A$, which is a function of the quantity and quality of the teaching, $T_A$, and research, $R_A$, activity, undertaken by the university: $P_A=T_A^aR_A$. Donating function is the same as in Brown [1980]. From here the modification of the assumptions is made. University A maximizes $P_A=T_A^aR_A$ s.t. $c_{tA}T_A+c_{rA}R_A=B_A$, where $B_A=KB(P_A)/(B(P_A)+B(P_B))$. In the same way, university B maximizes $P_B=T_B^aR_B$ s.t. $c_{tB}T_B+c_{rB}R_B=B_B$, where $B_B=KB(P_B)/(B(P_A)+B(P_B))$. Thus, in this paper, the unit costs of research and education can be different between the two universities. Now, as in Fukiharu [2003], 2-university behavior model is analyzed in the game theoretic framework. University A maximizes her prestige given $P_B$. The optimal prestige is a function of $P_B$: $\phi_A(P_B)$, which is the reaction function of university A. University B maximizes her prestige given $P_A$. The optimal prestige is a function of $P_A$: $\phi_B(P_A)$, which is the reaction function of university B. Equilibrium in 2-university behavior model is Nash non-cooperative solution; i.e. $\{P_A^*,P_B^*\}$ which satisfies $P_A^*=\phi_A(P_B^*)$ and $P_B^*=\phi_B(P_A^*)$. In this approach, the introduction of more competition is defined by the increase of $\alpha$, since the increase of $\alpha$ implies that if $P_A>P_B$ holds, university A receives more fund out of K than before and university B receives less fund out of K than before. In Fukiharu [2003], it was shown that when $c_{tA}=c_{tB}$ and $c_{rA}=c_{rB}$ it is impossible to enhance $P_A+P_B$ by increasing $\alpha$. In this paper, it is assumed that $c_{tA} \neq c_{tB}$ and/or $c_{rA} \neq c_{rB}$ may be possible and it is examined if it is possible to enhance $P_A+P_B$ by increasing $\alpha$.

The following function, constructed in Fukiharu [2003], checkA, computes the optimum $P_A$ when $\alpha$, $c_t$, $c_r$, $K$, and $P_B$ are given arbitrarily, where the initial values in Newton method are fixed at $T_A=20$, $R_A=10$ and $r=10$. Note, first, that this function may not have convergence in the Newton process. Note, furthermore, that this function can be used to compute the optimum $P_B$ when $\alpha$, $c_t$, $c_r$, $K$, and $P_A$ are given arbitrarily.
\textbf{In[1]}:= \texttt{checkA[a0_, ct0_, cr0_, k0_, pb0_] := Module[{f, sol2, d},
    f = RA*TA - r*(cr*RA + ct*TA - K*(RA*TA)^a) / ((RA*TA)^a + PB^a));
    d = {D[f, RA] = 0, D[f, TA] = 0, D[f, r] = 0};
    sol2 = FindRoot[d, {RA, 20}, {TA, 10}, {r, 10}]; (RA/.sol2)*(TA/.sol2)]}

When \(a=4/10\), \(c_{A}=1\), \(c_{A}=2/3\), and \(K=100\,2000\) pairs of \(\{P_{B}, P_{A}\}\), where \(P_{B}=2, ..., 2000\), \(A_{1}\) can be found using \texttt{checkA}, in approximately 22 seconds.

\textbf{In[2]}:= \texttt{Timing[A1 = Table[\{pb0, checkA[4/10, 1, 2/3, 100, pb0]\}, \{pb0, 2, 2000\}];
    A1 is plotted in the following fig11.

\textbf{In[3]}:= \texttt{fig11 = ListPlot[A1, PlotJoined \to True];
    \begin{center}
    \includegraphics[width=\textwidth]{fig11.png}
    \end{center}

When \(a=4/10\), \(c_{B}=1/2\), \(c_{B}=3/2\), and \(K=100\,2000\) pairs of \(\{P_{B}, P_{A}\}\), where \(P_{A}=2, ..., 2000\), \(B_{1}\) can be found using \texttt{checkA}.

\textbf{In[4]}:= \texttt{B1 = Table[checkA[4/10, 1/2, 3/2, 100, pa0], \{pa0, 2, 2000\}];
    B1 is plotted in the following fig12. This dashed line has discontinuity since there are cases in which Newton method cannot have convergence.

\textbf{In[5]}:= \texttt{fig12 = ListPlot[B1, PlotJoined \to True, PlotStyle \to Dashing[{0.01}];
    \begin{center}
    \includegraphics[width=\textwidth]{fig12.png}
    \end{center}

Putting them together, the intersection of the 2 reaction functions is the Nash non-cooperative solution to the prestige game when \(a=4/10\). This solution is stable as shown in the following diagram.

\textbf{In[6]}:= \texttt{figm = Show[Graphics[Text["Em", \{700, 1300\}]], DisplayFunction \to Identity];
    \begin{center}
    \includegraphics[width=\textwidth]{figm.png}
    \end{center}
In what follows, we compute the intersection in the diagram, $E_m$. In the next command, $f_1$ is the Lagrange function of the university $A$, where $\lambda_1$ is the Lagrange multiplier, and $dA_1$ is the set of conditions in deriving university $A$'s reaction function when $a=4/10$. Since this command is used in the computation of the intersection, $PB=PB_1*TB_1$ is substituted.

$$f_1 = RA_1 * TA_1 - \lambda_1 * (cr_1 * RA_1 + ct_1 * TA_1 - K * (RA_1 * TA_1)^a / ((RA_1 * TA_1)^a + PB^a)) / .$$

$$dA_1 = \{D[f_1, RA_1] = 0, D[f_1, TA_1] = 0, D[f_1, \lambda_1] = 0\} / . PB \rightarrow RB_1 * TB_1$$

In what follows, $g_1$ is the Lagrange function of the university $B$, where $\lambda_1$ is the Lagrange multiplier, and $dB_1$ is the set of conditions in deriving university $B$'s reaction function when $a=4/10$. Since this command is used in the computation of the intersection, $PA=PA_1*TA_1$ is substituted.
In[9]:= \[g1 =
RB1 \cdot TB1 - rb1 \cdot (crl1 \cdot RB1 + ct1 \cdot TB1 - K \cdot (RB1 \cdot TB1) ^ (a) / ((RB1 \cdot TB1) ^ (a) + PA ^ (a))) / .
\{a -> 4/10, ct1 -> 1, crl1 -> 2/3, K -> 100\};
dB1 = \{D[g1, RB1] = 0, D[g1, TB1] = 0, D[g1, rb1] = 0\} /. \{PA -> RA1, TA1, \}

Out[9]= \{TB1 - rb1 \cdot 2 + \frac{40\TB1}{(RB1 \TB1)^{1/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} -
\frac{40\TB1}{(RB1 \TB1)^{3/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} = 0,
RB1 - rb1 \cdot 1 + \frac{40\RB1}{(RB1 \TB1)^{1/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} -
\frac{40\RB1}{(RB1 \TB1)^{3/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} = 0,
- \frac{2\RB1}{3} - TB1 + \frac{100\RB1}{(RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5}} \cdot 0} \}

Take the union of these conditions as d1, which computes the optimal values of \(R_A, R_B, T_A,\) and \(T_B\) when \(a=4/10,\)
where \(P_A=R_A, T_A,\) and \(P_B=R_B, T_B\) are university A's prestige (academic level) and university B's prestige (academic level), respectively.

In[10]:= \[d1 = Union[d1, dB1]\]

Out[10]= \{- \frac{3\RA1}{2} - \frac{TA1}{2} + \frac{100\RA1}{(RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5}} \cdot 0,
- \frac{2\RB1}{3} - TB1 + \frac{100\RB1}{(RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5}} \cdot 0,
RA1 - ra1 \cdot 1 + \frac{40\RA1}{(RA1 \TA1)^{1/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} -
\frac{40\RA1}{(RA1 \TA1)^{3/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} = 0,
TA1 - ra1 \cdot 3 + \frac{40\TA1}{(RA1 \TA1)^{1/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} -
\frac{40\TA1}{(RA1 \TA1)^{3/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} = 0,
RB1 - rb1 \cdot 1 + \frac{40\RB1}{(RB1 \TB1)^{1/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} -
\frac{40\RB1}{(RB1 \TB1)^{3/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} = 0,
TB1 - rb1 \cdot 2 + \frac{40\TB1}{(RB1 \TB1)^{1/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} -
\frac{40\TB1}{(RB1 \TB1)^{3/5} \cdot ((RA1 \TA1)^{2/5} + (RB1 \TB1)^{2/5})} = 0\}\}

Using Newton method, we can compute the optimal values of \(R_A, R_B, T_A,\) and \(T_B\) when \(a=4/10\) as in what follows.

In[11]:= \[sol1 = FindRoot[d1, \{RA1, 1\}, \{RB1, 1\}, \{TA1, 1\}, \{TB1, 1\}, \{ra1, 1\}, \{rb1, 1\}\]


Now, university A's prestige (academic level) at the stable Nash solution when \(a=4/10\) is given by the following.

In[12]:= \[solA1 = T_CI\]
Meanwhile, university B’s prestige (academic level) at the stable Nash solution when \( \alpha = 4/10 \) is given by the following.

\[
\text{In}[13] := \text{PB1} = \text{RB1} \times \text{TB1} /. \text{solA1}
\]

\[
\text{Out}[13] = 1170.21
\]

Thus, the sum of the 2 universities’s prestiges (academic level) at the stable Nash solution when \( \alpha = 4/10 \) is given by the following.

\[
\text{In}[14] := \text{PA1} + \text{PB1}
\]

\[
\text{Out}[14] = 1819.6
\]

Next, we compute the sum of the 2 universities’s prestiges (academic level) at the stable Nash solution when \( \alpha = 1/10 \). When \( \alpha = 1/10, c_B = 1, c_A = 2/3, \) and \( K = 100 \), 2000 pairs of \( \{P_B, P_A\} \), where \( P_B = 2, \ldots, 2000, A3 \), can be found using checkA, in approximately 22 seconds.

\[
\text{In}[15] := \text{Timing[A3 = Table[\{checkA[1\/10, 1, 2/3, 100, pb0], \{pb0, 2, 2000\}\];]}
\]

\[
\text{Out}[15] = \{22.152 \text{ Second}, \text{Null}\}
\]

A3 is plotted in the following fig13.

\[
\text{In}[16] := \text{fig13 = ListPlot[A3, PlotJoined \to \text{True}, PlotStyle \to \text{Thickness}[0.01]]};
\]

![Plot](plot13.png)

When \( \alpha = 1/10, c_B = 1/2, c_A = 3/2, \) and \( K = 100 \), 2000 pairs of \( \{P_B, P_A\} \), where \( P_A = 2, \ldots, 2000, B1 \), can be found using checkA.

\[
\text{In}[17] := A4 = \text{Table[\{checkA[1\/10, 1\/2, 3/2, 100, pb0], \{pb0, 2, 2000\}\];}
\]

A4 is plotted in the following fig14.
Putting them together, the intersection of the 2 reaction functions, El, is the Nash non-cooperative solution to the prestige game when $\alpha=1/10$. This solution is stable as shown in the following diagram.

Intersections, Em and El, are depicted as in what follows.

In what follows, we compute the intersection, El, in the diagram. In the next command, $f_2$ is the Lagrangean function of the university A, where $ra2$ is the Lagrangean multiplier, and $dA2$ is the set of conditions in deriving university A's reaction function when $\alpha=1/10$. Since this command is used in the computation of the intersection, PB=PB2*TB2 is substituted.
In what follows, \( g_2 \) is the Lagrangean function of the university B, where \( r_b2 \) is the Lagrangean multiplier, and \( d_B2 \) is the set of conditions in deriving university B's reaction function when \( \alpha=1/10 \). Since this command is used in the computation of the intersection, \( P_A=P_A1*TA1 \) is substituted.

\[
\text{In[23]}:= g_2 = \\
\begin{align*}
\text{RB2}+\text{TB2} - r_b2^* (\text{cr2} + \text{RB2} + \text{ct2} + \text{TB2} - K * (\text{RB2} + \text{TB2})^\alpha) / ((\text{RB2} + \text{TB2})^\alpha + PA^\alpha) / . \\
(a \rightarrow 1/10, \text{ct2} \rightarrow 1/2, \text{cr2} \rightarrow 2/3, K \rightarrow 100); \\
\text{d_B2} = \{D[g2, \text{RB2}] = 0, D[g2, \text{TB2}] = 0, D[g2, \text{rb2}] = 0\} / . \text{PA} \rightarrow \text{RA2} + \text{TA2}
\end{align*}
\]

\[
\text{Out[23]}= \begin{cases}
\text{TB2} - r_b2 \left(\frac{2}{3} + \frac{10 \text{TB2}}{(\text{RB2} \text{TB2})^{4/5} (\text{RA2} \text{TA2})^{1/10} + (\text{RB2} \text{TB2})^{1/10}}\right) = 0, \\
\text{TB2} - r_b2 \left(1 + \frac{10 \text{RB2}}{(\text{RB2} \text{TB2})^{4/5} (\text{RA2} \text{TA2})^{1/10} + (\text{RB2} \text{TB2})^{1/10}}\right) = 0, \\
\text{TB2} - r_b2 \left(\frac{2}{3} \text{TB2} + \frac{100 \text{RB2}}{(\text{RB2} \text{TB2})^{1/10} + (\text{RB2} \text{TB2})^{1/10}}\right) = 0
\end{cases}
\]
This function computes prestige (academic level) increases. In order to check the robustness, the following function, checkB, is constructed.

In this simulation, as the more competition is introduced into the higher education, the sum of the 2 universities's prestige (academic level) increases. In order to check the robustness, the following function, checkB, is constructed.

Thus, the sum of the 2 universities's prestige (academic level) at the stable Nash solution when $\alpha = 1/10$ is given by the following.

Now, university A's prestige (academic level) at the stable Nash solution when $\alpha = 1/10$ is given by the following.

Meanwhile, university B's prestige (academic level) at the stable Nash solution when $\alpha = 1/10$ is given by the following.

Thus, the sum of the 2 universities's prestige (academic level) at the stable Nash solution when $\alpha = 1/10$ is given by the following.

In this simulation, as the more competition is introduced into the higher education, the sum of the 2 universities's prestige (academic level) increases. In order to check the robustness, the following function, checkB, is constructed.

This function computes $P_A + P_B$, the sum of the 2 universities's prestige (academic level) at the stable Nash solution, when $\alpha, c_A, c_B, c_A, c_B$, and $K$ are given arbitrarily.
In[29]:= checkB[a0, ctA0, crA0, ctB0, crB0, K0] :=
Module[{f1, dA1, gl, dB1, d1, solA1, pA1, pB1},
  f1 = RA1*TA1 - ra1*H*ra1*H*RA1*TA1 -
    (ct1*TA1 - K*H*RA1*TA1)^H +
    (PB*H)^H.
  dA1 = {D[f1, RA1] = 0, D[f1, TA1] = 0, D[f1, ra1] = 0} /. PB -> RB1*TB1;
  gl = RB1*TB1 - rb1*(ct1*TB1 - K*(RB1*TB1))
    ((RB1*TB1)^H (a) + PA^H (a)) /. (a -> a0, ct1 -> ctB0, cr1 -> crB0, K -> K0);
  dB1 = {D[gl, RB1] = 0, D[gl, TB1] = 0, D[gl, rb1] = 0} /. PA -> RA1*TA1;
  d1 = Union[dA1, dB1];
  solA1 = FindRoot[d1, {RA1, 1}, {RB1, 1}, {TA1, 1}, {TB1, 1}, {ra1, 1}, {rb1, 1}];
  PA1 = RA1*TA1 /. solA1;
  PB1 = RB1*TB1 /. solA1;
  PA1 + PB1]

First of all, it is checked that the same solution is computed when \(\alpha=1/10, \ c_{\alpha A}=1/2, \ c_{\alpha B}=1, \ c_{tA}=3/2, \ c_{tB}=2/3, \) and \(K=100.\)

In[30]:= checkB[1/10, 1/2, 3/2, 1, 2/3, 100]
Out[30]= 1772.46

Next, it is checked that the same solution is computed when \(\alpha=4/10, \ c_{\alpha A}=1/2, \ c_{\alpha B}=1, \ c_{tA}=3/2, \ c_{tB}=2/3, \) and \(K=100.\)

In[31]:= checkB[4/10, 1/2, 3/2, 1, 2/3, 100]
Out[31]= 1819.6

Incidentally, it is checked that the same solution is computed when \(\alpha=1/10, \ c_{\alpha A}=1, \ c_{\alpha B}=1, \ c_{tA}=1, \ c_{tB}=1, \) and \(K=100\) (See Fukiharu [2003]).

In[32]:= checkB[1/10, 1, 1, 1, 1, 100]

It is also checked that the same solution is computed when \(\alpha=4/10, \ c_{\alpha A}=1, \ c_{\alpha B}=1, \ c_{tA}=1, \ c_{tB}=1, \) and \(K=100\) (See Fukiharu [2003]).

In[33]:= checkB[4/10, 1, 1, 1, 1, 100]
Out[33]= 1250.

For an other example, when \(\alpha=1/10, \ c_{\alpha A}=1/2, \ c_{\alpha B}=1, \ c_{tA}=1/2, \ c_{tB}=1, \) and \(K=100, \) checkB computes \(P_A+P_B\) as follows.

In[34]:= checkB[1/10, 1/2, 1/2, 1, 1, 100]
Out[34]= 3472.44

As \(\alpha\) increases from 1/10 to 4/10, \(P_A+P_B\) increases as follows.

In[35]:= checkB[4/10, 1/2, 1/2, 1, 1, 100]
Out[35]= 10000.
Furthermore, another example is examined, when $\alpha=1/10$, $c_{tA}=1/4$, $c_{tB}=3$, $c_{rA}=2$, $c_{rB}=1/5$, and $K=100$. The function `checkB` computes $P_A + P_B$ as follows.

```
In[36]:= checkB[1/10, 1/4, 2, 3, 1/5, 100]
Out[36]= 2296.71
```

As $\alpha$ increases from $1/10$ to $4/10$, $P_A + P_B$ increases as follows.

```
In[37]:= checkB[4/10, 1/4, 2, 3, 1/5, 100]
Out[37]= 2441.32
```

It appears that we have a conclusion that as more competition is introduced into the higher education the sum of academic levels increases. How robust is this tentative conclusion? Suppose that $c_{tA}=2.5$, $c_{tB}=1.2$, $c_{rA}=1.8$, $c_{rB}=0.6$, and $K=100$. If $\alpha$ increases from $0.2$ to $0.4$, the sum of academic levels declines, as shown by the following command.

```
In[38]:= checkB[4/10, 2.5, 1.8, 1.2, 0.6, 100] - checkB[2/10, 2.5, 1.8, 1.2, 0.6, 100]
Out[38]= -971.843
```

Is this a counter example to our tentative conclusion? It is not the case. When $\alpha=4/10$, $P_A + P_B$ is computed as the following $A_{11}=555.556$.

```
In[39]:= f11 = RA1*TA1 - ra1*(ct1*RA1 + cr1*TA1 - K*(RA1*TA1)^a)/(RA1*TA1)^a + PB^a)
   {a \to 4/10, ct1 \to 2.5, cr1 \to 1.8, K \to 100};
   dA11 = {D[f11, RA1] = 0, D[f11, TA1] = 0, D[f11, ra1] = 0}. PB \to RB1*TB1;
   g11 = RB1*TB1 - rb1*(ct1*RB1 + cr1*TB1 - K*(RB1*TB1)^a)/(RB1*TB1)^a + PA^a)
   {a \to 4/10, ct1 \to 1.2, cr1 \to 0.6, K \to 100};
   dB11 = {D[g11, RB1] = 0, D[g11, TB1] = 0, D[g11, rb1] = 0}. PA \to RA1*TA1;
   d11 = Union[dA11, dB11];
   solA11 = FindRoot[d11, {RA1, 1}, {RB1, 1}, {TA1, 1}, {TB1, 1}, {ra1, 1}, {rb1, 1}];
   PA11 = RA1*TA1 /. solA11;
   PB11 = RB1*TB1 /. solA11;
   A11 = PA11 + PB11
Out[45]= 555.556
```

On the other hand, when $\alpha=2/10$, $P_A + P_B$ is computed as the following $A_{12}=1527.4$. Thus, the difference is $-971.843$. 

Bowen3A.nb
Finally, selecting 1000 parameters \( c_A, c_B, c_A, \) and \( c_B, \) randomly, from the closed interval, [0,3], the function checkB computes the difference of \( P_A + P_B \) between \( \alpha = 11/100 \) and \( \alpha = 1/10 \) for each pair of parameter as follows.
In some cases, the difference of $P_A + P_B$ exhibits minus number. It may be checked that the minus value shows up since in these cases the Newton method may not converge or the boundary values may well appear, as is clear from the following example and comments by Mathematica.

In[64]:= \[ \text{Length}[\text{Select}[\text{data3}, \#[[2]] > 0 \&]] \]

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In[65]:= \[ \text{Length}[\text{Select}[\text{data3}, \#[[2]] < 0 \&]] \]

Out[65]= 40

Thus, we may safely conclude that as more competition is introduced into the higher education the sum of academic levels increases.

II. Comparison between $\alpha=8/10$ and $\alpha=85/100$ : UnStable Cases

In Fukiharu [2003] there emerged the non-existence of Nash non-cooperative game solution when $\alpha$ is large. What would happen in this modified model when $\alpha$ is large? First of all, it is examined if the government can enhance the national academic level by introducing more competition into the academic world when the competition itself is already in high level. With the same stipulation on the unit costs for the two universities, when $\alpha=8/10$, the national academic level is computable using checkB.

In[66]:= \[ \text{checkB}[8/10, 1/2, 3/2, 1, 2/3, 100] \]

Out[66]= 1765.38

Note that the convergence of Newton method is guaranteed. Next, with the same stipulation on the unit costs for the two universities, suppose that $\alpha$ is increased to 85/100. In this case, too, the national academic level is computable using checkB. Note that the convergence of Newton method is guaranteed in this case, too.

In[67]:= \[ \text{checkB}[85/100, 1/2, 3/2, 1, 2/3, 100] \]

Out[67]= 1764.99

In this simulation, as more competition is introduced into the academic world, the national academic level falls. This result appears to negate our conclusion. In this case, the stability must be checked. If the stability of Nash non-cooperative game solution is guaranteed in both case, our conclusion is negated. To examine the stability, first of all, the reaction function of University A is computed using checkA1, which has somewhat different initial values in applying Newton method. The reaction function is named C31. It requires 33 seconds to compute C31.

II. Comparison between $\alpha=8/10$ and $\alpha=85/100$ : UnStable Cases

In Fukiharu [2003] there emerged the non-existence of Nash non-cooperative game solution when $\alpha$ is large. What would happen in this modified model when $\alpha$ is large? First of all, it is examined if the government can enhance the national academic level by introducing more competition into the academic world when the competition itself is already in high level. With the same stipulation on the unit costs for the two universities, when $\alpha=8/10$, the national academic level is computable using checkB.

In[66]:= \[ \text{checkB}[8/10, 1/2, 3/2, 1, 2/3, 100] \]

Out[66]= 1765.38

Note that the convergence of Newton method is guaranteed. Next, with the same stipulation on the unit costs for the two universities, suppose that $\alpha$ is increased to 85/100. In this case, too, the national academic level is computable using checkB. Note that the convergence of Newton method is guaranteed in this case, too.

In[67]:= \[ \text{checkB}[85/100, 1/2, 3/2, 1, 2/3, 100] \]

Out[67]= 1764.99

In this simulation, as more competition is introduced into the academic world, the national academic level falls. This result appears to negate our conclusion. In this case, the stability must be checked. If the stability of Nash non-cooperative game solution is guaranteed in both case, our conclusion is negated. To examine the stability, first of all, the reaction function of University A is computed using checkA1, which has somewhat different initial values in applying Newton method. The reaction function is named C31. It requires 33 seconds to compute C31.

In[68]:= \[ \text{checkA1}[a0\_, ct0\_, cr0\_, k0\_, pb0\_] := \text{Module}[\{f, sol2, d\},
    f = RA \cdot TA - r \cdot (cr \cdot RA + ct \cdot TA - K \cdot (RA \cdot TA) \wedge (a)) / ((RA \cdot TA) \wedge (a) + PB \wedge (a)) / .
    \{a \rightarrow a0, ct \rightarrow ct0, cr \rightarrow cr0, K \rightarrow k0, PB \rightarrow pb0\};
    d = \{D[f, RA] = 0, D[f, TA] = 0, D[f, r] = 0\};
    sol2 = \text{FindRoot}[d, \{RA, 200\}, \{TA, 100\}, \{r, 100\}]; (RA /. sol2) \ast (TA /. sol2) \]

In[68]:= \[ \text{checkA1}[a0\_, ct0\_, cr0\_, k0\_, pb0\_] := \text{Module}[\{f, sol2, d\},
    f = RA \cdot TA - r \cdot (cr \cdot RA + ct \cdot TA - K \cdot (RA \cdot TA) \wedge (a)) / ((RA \cdot TA) \wedge (a) + PB \wedge (a)) / .
    \{a \rightarrow a0, ct \rightarrow ct0, cr \rightarrow cr0, K \rightarrow k0, PB \rightarrow pb0\};
    d = \{D[f, RA] = 0, D[f, TA] = 0, D[f, r] = 0\};
    sol2 = \text{FindRoot}[d, \{RA, 200\}, \{TA, 100\}, \{r, 100\}]; (RA /. sol2) \ast (TA /. sol2) \]
In[69]:= Timing[C31 = Table[{pb0, checkA[0.8, 1, 2/3, 100, pb0]}, {pb0, 2, 1400}];]

C31 is plotted in the following fig31.

In[70]:= fig31 = ListPlot[C31, PlotJoined -> True];

Next, in the same way, the reaction function of University B is computed using checkA, which is named C32.

In[71]:= C32 = Table[{checkA[0.8, 1/2, 3/2, 100, pb0], pb0}, {pb0, 2, 1200}];

C32 is plotted in the following fig32.

In[72]:= fig32 = ListPlot[C32, PlotJoined -> True, PlotStyle -> Dashing[{0.02}]]; 

Putting them together, the intersection of the two reaction functions is the solution to the Nash non-cooperative game.

In[73]:= Show[fig31, fig32];
In what follows, using the program, constructed in section I, the solution to the Nash non-cooperative game is computed as follows.

\[ f_{11} = RA1 * TA1 - ra1 * \left( cr1 * RA1 + ct1 * TA1 - K * (RA1 * TA1)^{(a)} \right) / \left( (RA1 * TA1)^{(a)} + PB^{(a)} \right) \]

\[ a \rightarrow 0.8, \quad ct1 \rightarrow 1, \quad cr1 \rightarrow 2 / 3, \quad K \rightarrow 100; \]

\[ d_{A1} = D[f_{11}, RA1] = 0, \quad D[f_{11}, TA1] = 0, \quad D[f_{11}, ra1] = 0; \quad PB \rightarrow RB1 * TB1; \]

\[ g_{11} = RB1 * TB1 - rb1 * \left( cr1 * RB1 + ct1 * TB1 - K * (RB1 * TB1)^{(a)} \right) / \left( (RB1 * TB1)^{(a)} + PA^{(a)} \right) \]

\[ a \rightarrow 0.8, \quad ct1 \rightarrow 1 / 2, \quad cr1 \rightarrow 3 / 2, \quad K \rightarrow 100; \]

\[ d_{B1} = D[g_{11}, RB1] = 0, \quad D[g_{11}, TB1] = 0, \quad D[g_{11}, rb1] = 0; \quad PA \rightarrow RA1 * TA1; \]

\[ d_{11} = Union[d_{A1}, d_{B1}]; \]

\[ sol_{A1} = FindRoot[d_{11}, {RA1, 1 < PB, 1, 1, 1}, \{TA1, 1 < PB, 1, 1, 1}, \{ra1, 1}, \{rb1, 1}] ; \]

\[ PA1 = RA1 * TA1 / . sol_{A1}; \]

\[ PB1 = RB1 * TB1 / . sol_{A1}; \]

\[ "{\"PA"\":PA1}, "{\"PB"\":PB1} \]

As clear from the diagram, the solution \{969.052, 796.33\} is the unstable one, so that it is not guaranteed that it can be achieved. When \( a = 85/100 \), the reaction functions of the two universities can be depicted in the following diagram.

\[ C51 = Table[{pb0, checkA1[0.85, 1, 2 / 3, 100, pb0]}, {pb0, 2, 1300}]; \]

\[ fig51 = ListPlot[C51, PlotJoined \rightarrow True, DisplayFunction \rightarrow Identity]; \]

\[ C52 = Table[{checkA1[0.85, 1 / 2, 3 / 2, 100, pa0], pa0}, {pa0, 2, 1200}]; \]

\[ fig52 = ListPlot[C52, PlotJoined \rightarrow True, \]

\[ PlotStyle \rightarrow Dashing[(0.02)], DisplayFunction \rightarrow Identity]; \]

\[ Show[{fig51, fig52}, DisplayFunction \rightarrow $DisplayFunction]; \]

In this case, too, the instability of the Nash non-cooperative solution emerges, so that it is not guaranteed that this solution can be achieved when \( a = 85/100 \). Thus, the conclusion in section I is still valid; i.e. we may safely conclude that as more competition is introduced into the higher education the national academic levels increases.
Conclusion

The aim of this paper was to extend Fukiharu [2003]. Fukiharu [2003] extended Bowen [1980], which examined a university's optimal behavior of research and teaching, given the budget constraint. Product of research level and teaching level is defined as the prestige of the university, which may be called the academic level of the university. The prestige (academic level) of the university may be regarded as the utility function of the university. Fukiharu [2003] considered the case in which there are two identical universities, which maximizes each prestige (academic level) given the fixed amount of fund, distributed to universities by the government according to the objective evaluation. Here, it was assumed that the two universities have the same constant unit costs in conducting research and teaching. Applying this two-identical-university model to the Japanese educational reform, in which the government attempts to introduce more competition into the higher education, Fukiharu [2003] examined if the government can enhance the country's national academic level (the sum of the universities' academic levels) by introducing more competition. The conclusion was that the government cannot enhance the national academic level, since in the two-identical-university model, the Nash-noncooperative solution have the same country's academic level, independent of the government's policy. In this paper, it is assumed that two universities are different in its structure of conducting research and teaching: they have different constant unit costs in conducting research and teaching. In section I of this paper, it was shown in terms of simulation that when the two universities are different in the above sense, the government can enhance the national academic level, by introducing more competition into the universities. Of course it must be noted that introduction of too much competition may result in the unstable solution as shown in section II.

References

