A General Equilibrium Approach to Medical Insurance

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Introduction

After Arrow [1963] initiated the medical economics, a lot of contributions have been made in this field. While Arrow stressed the positive side of medical insurance, Pauly [1968] pointed out its negative side. In the present-day textbooks on microeconomics, Pauly’ argument of Moral Hazard is popular, and it is asserted that the medical insurance causes inefficiency of welfare; emergence of Dead Weight Loss. In this paper his argument is reconstructed in the framework of general equilibrium model for the purpose of revealing the essence of medical insurance in the model which incorporates the special feature of medical care.

When a person falls sick, one aspect of sickness is "bad feeling". The sickness, however, is not restricted to such a "feeling". Another important aspect is the loss of working hours. Thus, a doctor's services to patients are not only the alleviation of bad feeling but also the recovery of working hours. In consideration of this feature, Pauly's argument, it appears, needs reconsideration. His argument was constructed in the framework of partial equilibrium. When the medical service charge is reduced by medical insurance, the increased medical care causes dead weight loss, as explained in a simple diagram. This, however, increases the recovery of working hours, which may well increase the consumption of goods. In other words, consumers' surplus in the consumable goods may well increase as "external economy". The final evaluation must be made by considering the above two aspects together. The argument made so far calls for the analysis of medical insurance in the framework of general equilibrium.

In this paper, sickness is defined by the loss of initial endowment of leisure, and the medical care recovers a part of the lost initial endowment, while it is supplied with medical service charge. After computing general equilibrium, final evaluation is made by examining if the sum of households' utility level at the general equilibrium is increased after the introduction of medical insurance.

In Section 1 of this paper, it is assumed that medical service is supplied by the competitive medical sector. A person has a constant probability of sickness, say \( \alpha \), \( 0 < \alpha < 1 \). When this person falls sick, he or she goes to a doctor to recover a part of the lost working hours, paying medical service charge, while the medical service is produced using labor (nurses) and goods (medicines etc) under constant returns to scale. Goods are produced by another sector using solely labor. In a society with 100 persons, say, there are 100\( \alpha \) sick workers and 100\( (1-\alpha) \) healthy workers. Specifying production functions and utility functions by Cobb Douglas type, it is proved that there exists a competitive general equilibrium.

In Section 2, a fair medical insurance is introduced. Since any worker has a constant probability of sickness, it is reasonable for the government to introduce a medical insurance, with the specification that \( k\% \) of medical cost is paid by this insurance when he or she is sick, while when he or she is healthy he or she must pay insurance fee, which is computed under condition of fair insurance. Since the sick persons are exempt from paying insurance fee by assump-
tion, this medical insurance is a variant of subsidy from healthy persons to sick persons. It is shown that there exists a competitive general equilibrium with medical insurance. Furthermore, it is computed that the sum of utility levels in this equilibrium with medical insurance is smaller than the one without medical insurance. This is the general equilibrium version of Pauly's Moral Hazard. When constructed in this way as a general equilibrium model, a natural question emerges: why the sick persons' utility levels must be simply added to the healthy persons' utilities. In other words, it becomes clear that Pauly's argument lacks the social welfare considerations. It is pointed out, incidentally, that there are anomaly cases, in which there exists no general equilibrium with medical insurance.

In Section 3, it is assumed that the medical service is supplied by monopolistic medical sector, while goods-producing sector is competitive. It is shown that by introducing medical insurance system, the sum of utility levels in this equilibrium with medical insurance is larger than the one without medical insurance. This result could be foreseen from the traditional subsidy argument in the framework of monopoly.

In Section 4, problems are examined, which arises when the government considers the arbitrary provision of medical services. In the sections so far, the medical services are provided by the market, whether competitively or monopolistically. The level of medical service provided in this way may be considered insufficient from the viewpoint of social justice. Indeed, in Japan, the necessary (or minimum) level of medical services appears to be pre-determined by the government. Problems in constructing general equilibrium model incorporating this medical system are examined.

1. Privatized Medical Industry:
   No Medical Insurance, Competitive Medical Care Case

In this section, a general equilibrium model incorporating medical sector is constructed, where medical insurance is not available. It is assumed there are two types of households; the "healthy" household and "sick" household. Everyone knows that the distribution of households is constant in each year; a1 households are healthy and a2 households are sick. No one knows whether each household is sick or healthy before the dawn of the particular year. Only when the particular year starts, a2 households know that they are healthy, while the remainder know that they are sick. In this sense, every household knows that each household has the probability, a=a2/(a1+a2), of being sick in each year. When the household is healthy, it has 365 days of initial leisure days. Its behavior is stipulated by traditional utility maximization under income constraint:

\[
\max u(z_1, le_1)\]

s.t. \( p_z z_1 = w(365-le_1)+Y_1 \) (1)

where \( u(z_1, le_1) \) is the utility function, \( z_1 \) is the consumption of goods, \( le_1 \) is the leisure consumption, \( p_z \) is the price of goods, \( w \) is the wage rate, and \( Y_1 \) is the transfer of income from others, such as profit and tax. In this paper, since simulation approach is utilized utility function is stipulated by

\[
u(z, le)=z\times le.\] (2)

Under (1) and (2) the healthy household's demand function for the goods, \( z_1 \), is given by the following.
In[3]:=  
\[u1 = z1 \cdot le1; le1 = (365 \cdot w + Y1 - px \cdot z1)/w;\]  
\[sol2 = Solve[D[u1, z1] == 0, z1][[1]]; z1 = z1 /. sol2\]  
\[Out[3] = 365 w + Y1\]

On the other hand, the labor supply function of healthy household, l1, is given by the following.

In[4]:=  
\[l1 = 365 - le1\]  
\[Out[4] = 365\]

In[5]:=  
\[Clear[lg, z]\]

When the household is sick, it has \(H_0\) days of initial leisure days, say \(H_0 = 300\). It goes to a hospital to see a doctor, in order to recover a part of lost leisure days. It is assumed that a doctor can recover \(x^{1/2}\) days of leisure for the sick family by supplying \(x\) medical treatment, while the doctor receives service charge \(p_x\) per one unit of medical treatment. Sich household behavior is stipulated by the following utility maximization under income constraint:

\[
\text{max } u(z_2, le_2) \\
\text{s.t. } p_z z_2 + p_x x = w(H_0 + x^{1/2} - le_2) + Y_2
\]

where \(u(z_2, le_2)\) is utility function, \(z_2\) is consumption of goods, \(le_2\) is leisure consumption, and \(Y_2\) is transfer of income from others, such as profit and tax. In this paper, since simulation approach is utilized utility function is stipulated by (2). Under (2) and (3) the sick household's demand function for the goods, \(z_2\), is given by the following where \(H_0 = 300\).

In[6]:=  
\[H0 = 300; u2 = z2 \cdot le2; L2[x_] := H0 + x^{1/2}; le2 = (Y2 + w \cdot L2[x] - px \cdot z2 - px \cdot x)/w;\]  
\[sol1 = Solve[[D[u2, z2] == 0, D[u2, x] == 0], {z2, x}][[1]]; z2 = z2 /. sol1\]  
\[Out[6] = 1200 px w + w^2 + 4 px Y2/8 px p_x\]

The sick household's demand function for the medical services, \(x_1\), is given by the following.

In[7]:=  
\[x1 = x /. sol1\]  
\[Out[7] = w^2/4 px^2\]

The sick household's labor supply function, \(l_2\), is given by the following.

In[8]:=  
\[l2 = L2[x] - le2 /. sol1 // Simplify\]  
\[Out[8] = 150 + \frac{3 w}{8 px} - \frac{Y2}{2 w}\]

Alternatively, the representative household's behavior may be formulated by the following maximization of expected utility, with the same demand and supply functions:

\[
\text{max } \alpha u(z_1, le_1) + (1-\alpha) u(z_2, le_2) \\
\text{s.t. } p_z z_1 = w(365 - le_1) + Y_1, \text{ } p_z z_2 + p_x x = w(H_0 + x^{1/2} - le_2) + Y_2
\]

The behavior of good-producing sector is stipulated by profit maximization where production function is given by
where \( z \) is the output of goods, and \( l_g \) is labor input. From the profit maximization under (4) gives rise to the labor demand function of the good-producing sector, \( l_g1 \), as follows.

\[
\text{profit}_g = p_z z - w l_g; \quad z = l_g^{1/2}(1/2);
\]

\[
\text{sol}_g = \text{Solve}[\text{D}[\text{profit}_g, l_g] = 0, l_g][[1]]; \quad l_g1 = l_g / . \text{sol}_g
\]

Its profit function, \( pt_1 \), is given by the following.

\[
\text{pt}_1 = \text{profit}_g / . \text{sol}_g
\]

The behavior of medical sector is stipulated by profit maximization in this section. The production function of medical service is given by

\[
x = g(l_x, z_x) = l_x^{1/2} z_x^{1/2}
\]

where \( l_x \) is the input of labor; e.g. nurses etc., \( z_x \) is the input of goods; e.g. medicines etc. Since (5) is constant returns to scale, the medical sector aims at cost minimization. In order to provide the medical service demanded by the sick households, \( x1 \), the medical sector has the demand function for the goods, \( z_x1 \), as follows.

\[
x2 = (lx)^{(1/2)} * z^x(1/2);
\]

\[
x2 = (lx)^{(1/2)} * z^x(1/2);
\]

\[
sol3 = \text{Solve}[[\text{D}[x2, lx] / \text{D}[x2, z] = w / p_z, x2 = x0], \{lx, z\}][[2]]; \quad z_x1 = z_x / . \text{sol3} / . x0 \rightarrow x1
\]

Under cost minimization, to provide the medical service \( x1 \), the medical sector has the demand function for the labor, \( lx1 \), as follows.

\[
lx1 = lx / . \text{sol3} / . x0 \rightarrow x1
\]

Minimum cost function to provide the medical service \( x1 \), \( c \), is given as follows. In equilibrium, \( c \) must be equal to \( p_x x1 \) with maximum profit for the medical sector being zero.

\[
c = w * lx1 + p_z * z_x1
\]

\[
c = w * lx1 + p_z * z_x1
\]

\[
\text{sol4} = \text{Solve}[c = p_x, p_z][[1]]
\]

\[
\{p_z \rightarrow \frac{4 p_x^6}{w^6}\}
\]
In what follows, equilibrium price of $p_z$ is computed with $w=1$. Suppose that the good-producing sector is possessed by all the households with the equal share holding, where $a_1=99$ and $a_2=1$. One sick household and 99 healthy households have the same individual profit income, $Y_1$ and $Y_2$ as in what follows.

\begin{align*}
\text{In}[16]:= & \quad Y_1 = pt1 / (a1 + a2); \quad Y2 = pt1 / (a1 + a2); \{Y1, Y2\} \\
\text{Out}[16]= & \quad \left\{ \frac{1}{100}, \left( \frac{1}{2} p_z \sqrt{\frac{p_z^2}{w^2} - \frac{p_z^2}{4 w}} \right), \frac{1}{100} \left( \frac{1}{2} p_z \sqrt{\frac{p_z^2}{w^2} - \frac{p_z^2}{4 w}} \right) \right\}
\end{align*}

Equilibrium condition for the good market, $eqz$, is given by the following equation.

\begin{align*}
\text{In}[17]:= & \quad eqz = (\text{Simplify}[\{lg1^\left(1/2\right) \left(a1 \times z1 + a2 \times z2 + z x1\right)\} /. w \rightarrow 1])[[1]] \\
\text{Out}[17]= & \quad \frac{-p x - 2 \sqrt{p z} + p x^2 \left( -145740 + p z^2 + 2 \ p z \sqrt{p z^2} \right)}{p x \sqrt{p z}} = 0
\end{align*}

Meanwhile, equilibrium condition for the labor market, $eql$, is given by the following equation.

\begin{align*}
\text{In}[18]:= & \quad eql = \text{Simplify}[lg1 + lx1 = a1 \times l1 + a2 \times l2] /. w \rightarrow 1 \\
\text{Out}[18]= & \quad 3 \ p x - 2 \sqrt{p z} - p x^2 \left( -145740 + p z^2 + 2 \ p z \sqrt{p z^2} \right) = 0
\end{align*}

Solving $eqz$ with respect to $p_z$, substitute it into $eql$. Then, we have the following, $eql2$.

\begin{align*}
\text{In}[19]:= & \quad \text{sol4} = \text{Solve}[eqz, p x]; \quad eql2 = \text{Simplify}[eql /. \{\text{sol4}[2]\}] \\
\text{Out}[19]= & \quad \frac{1 + 582960 \sqrt{p z} - 4 \ p z^{5/2} - 8 \ p z^{3/2} \sqrt{p z^2} + \sqrt{1 + 8 \sqrt{p z} \left( -145740 + p z^2 + 2 \ p z \sqrt{p z^2} \right)}}{1 + \sqrt{1 + 8 \sqrt{p z} \left( -145740 + p z^2 + 2 \ p z \sqrt{p z^2} \right)}} = 0
\end{align*}

\begin{align*}
\text{In}[20]:= & \quad \text{sol6} = \text{FindRoot}[eql2, \{p x, 1\}, \text{AccuracyGoal} \rightarrow 24, \text{WorkingPrecision} \rightarrow 34, \text{MaxIterations} \rightarrow 100]; \quad \text{sol7} = \text{sol4}[2] /. \text{sol6}; \{\text{sol6}, \text{sol7}\} \\
\text{Out}[20]= & \quad \{\{p x \rightarrow 220.4087621953355163026712637703577 - 3.4626638324372484977183298343309 \times 10^{-35} i\}, \{p x \rightarrow 29.6923399007444690340846263 + 0. \times 10^{-27} i\}\}
\end{align*}

Using the Newton method, $eql2$ can be solved. Solutions for $p_z$ and $p_x$ are given as follows. Here, the initial value in applying the Newton method is ip=2000.

\begin{align*}
\{\{p z \rightarrow 220.4087621953355163026712637703577 - 3.4626638324372484977183298343309 \times 10^{-35} i\}, \{p x \rightarrow 29.6923399007444690340846263 + 0. \times 10^{-27} i\}\}
\end{align*}

It is easy to check that in equilibrium, $c$ is indeed equal to $p_z \times x1$.

\begin{align*}
\text{In}[21]:= & \quad (p x \times x1 - c) /. \text{sol6} /. \text{sol7} /. \text{w} \rightarrow 1 \\
\text{Out}[21]= & \quad -0. \times 10^{-29} + 0. \times 10^{-30} i
\end{align*}

Now, the utility level for the healthy household in this general equilibrium is given by $u_{10}$.

\begin{align*}
\text{In}[22]:= & \quad u_{10} = u_{1} /. x \rightarrow x1 /. \text{w} \rightarrow 1 /. \text{sol6} /. \text{sol7} \\
\text{Out}[22]= & \quad 268.403187279293414945697659141707 + 0. \times 10^{-32} i
\end{align*}
Meanwhile the utility level for the sick household in this general equilibrium is given by $u_{20}$.

```
In[23]:= u20 = u2 . x1 /. w -> 1 /. sol6 /. sol7
Out[23]= 201.474801936035605130549444 + 0. \times 10^{-25} i
```

Sum of these households in this general equilibrium is given by $W_0$.

```
In[24]:= W0 = a1 u10 + a2 u20
Out[24]= 26773.390342586083684754617699 + 0. \times 10^{-25} i
```

The following function, check1[H0,a1,a2,ip], the collection of the above series of programmings, computes $u_{10}$, $u_{20}$, $W_0$, etc, when the initial endowment of leisure days for the sick households, $H_0$, the number of healthy households, $a_1$, the number of sick households, $a_2$, and the initial value used in applying the Newton method, $ip$, are stipulated.
It is easy to confirm that check1[300,99,1,2000] gives rise to the desired equilibrium values.
For later use, suppose that there are 10 healthy households and 100 sick households with $H_0=1$. Using check1, general equilibrium without medical insurance can be computed as follows. In this specification $W_0=4920$.

\[ \text{In}[31]:= \text{check1}[1, 10, 100, 20000] \]

\[ \text{Out}[31]= 
\begin{align*}
8H_{12.2478314545453407591927208678708691+0.\times10^{-33}}&:Y_10, \\
(188.62391572726703795963604339354345+0.\times10^{-33})&:z_0, \\
(2.569453824616063385864183812037727+0.\times10^{-35})&:z_10, \\
(484.660441679484319932440592084780+0.\times10^{-33})&:u_10, \\
(0.7365762097607832259939918256674+0.\times10^{-33})&:u_20, \\
(4920.26203777092152192380510341454+0.\times10^{-31})&:W_0
\end{align*} \]

Incidentally, the level of medical care, determined in the market, is given by the following.

\[ \text{In}[28]:= x_1/.w\rightarrow1/.sol6/.sol7 \]

\[ \text{Out}[28]= 0.0002835640442670326703538081+0.\times10^{-31}\]

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2. Privatized Medical Industry: Medical Insurance, Competitive Medical Care Case

In this section, "fair" medical insurance is introduced into the general equilibrium model constructed in section 1. In this medical insurance system, the sick household receives $k\%$ medical charge reduction: i.e. it pays $(1-k)p_x x$ for the medical service, $x$. Since there are $a_2$ sick households, $a_2 \times k \times p_x x$ must be supplied by the healthy households. Thus, $a_2 \times k \times p_x x/a_1$ is the insurance fee for each healthy household, if the "fair" insurance is to be constructed. Except for the specification of $Y_1$, no modification in (1) and (2) is required for the healthy household. The modification in (3) is required for the sick household as follows.

\[
\begin{align*}
&\max_{z_2, \ell_2} u(z_2, \ell_2) \\
&\text{s.t. } p_x z_1 + (1-k)p_x x = w(H_0 + x^{1/2} - \ell_2) + Y_2
\end{align*}
\]

(3')

If $k=10/100$, the sick household must pay 90% of the medical charge. This implies that $a_2 \times k \times p_x x/a_1$ must be deducted from the healthy household's transfer function as the insurance fee. Thus, the specification of $Y_1$ must be modified as follows.

\[ Y_1 = pt_1/(a_1+a_2)-a_2 \times k \times p_x x/a_1. \]

With these modifications, the series of programmings in section 1 can compute the general equilibrium values for the utility level of healthy household, $u_1$, the utility level of sick household, $u_2$, the sum of all the household utilities, $W_1$.

The following function, check2[H0,a1,a2,ip,k], the collection of the modified series of programmings, computes $u_1$, $u_2$, $W_1$, etc, when the initial endowment of leisure days for the sick households, $H_0$, the number of healthy households, $a_1$, the number of sick households, $a_2$, the initial value used in applying the Newton method, $ip$, and the reduction rate for the medical charge, $k$, are stipulated.
In[32]:=
check2[0_, a1_, a2_, ip_, k_] :=
  Module[{u2, L2, le2, sol1, x1, z2, l2, le1, u1, sol2, z1, l1,
    profitm, solg, pt1, lg1, x2, sol3, zx1, lx1, c, Y1, eqz, sol4,
    eql, eql1, eql2, sol6, sol7, uA, u11, u21, W1, Y11, le11, z11},
    u2 = z2 + le2;
    L2[x_] := H0 + x^(1/2);
    le2 = (Y2 + w*L2[x] - px*z2 - (1 - k)*px*x) / w;
    sol1 = Solve[{D[u2, z2] == 0, D[u2, x] == 0}, {z2, x}][[1]];
    x1 = x /. sol1;
    z2 = z2 /. sol1;
    l2 = L2[x] - le2 /. sol1;
    le1 = (365*w + Y1 - px*z1) / w;
    u1 = z1 + le1;
    sol2 = Solve[D[u1, z1] == 0, z1][[1]];
    z1 = z1 /. sol2;
    l1 = 365 - le1 /. sol2;
    profitm = px*z - w*lg; z = lg^((1/2));
    solg = Solve[D[proftim, lg] == 0, lg][[1]];
    pt1 = profitm /. solg;
    lg1 = lg /. solg;
    x2 = (lx)^(1/2)*zx^(1/2);
    sol3 = Solve[{D[x2, lx]/D[x2, zx] == w/px, x2 == x0}, {lx, zx}][[2]];
    zx1 = zx /. sol3 /. x0 -> x1;
    lx1 = lx /. sol3 /. x0 -> x1;
    c = w*lxl + px*zx1;
    Y1 = pt1 / (a1 + a2) - a2*k*px*x / a1 /. x -> x1;
    Y2 = pt1 / (a1 + a2);
    eqz = (Simplify[(lg1^((1/2)) == a1*z1 + a2*z2 + zx1)] /. w -> 1)[[1]];
    sol4 = Solve[eqz, px];
    eql = Simplify[lg1 + lx1 = a1 + 1 + a2 + 12] /. w -> 1;
    eql1 = eql1 /. (sol4[[2]]);
    eql2 = eql2 /. (sol4[[2]]);
    sol6 = FindRoot[eql2, {pz, ip}, AccuracyGoal -> 24, WorkingPrecision -> 34,
      MaxIterations -> 100];
    eqm3 = (lg1 + lx1 - (a1 + 1 + a2 + 12)) /. w -> 1 /. (sol4[[2]]);
    sol7 = sol4[[2]] /. sol6;
    u11 = u1 /. x -> x1 /. w -> 1 /. sol6 /. sol7;
    u21 = u2 /. x -> x1 /. w -> 1 /. sol6 /. sol7;
    W1 = a1 + u11 + a2*u21;
    Y11 = Y1 /. w -> 1 /. sol6 /. sol7;
    le11 = le1 /. w -> 1 /. sol6 /. sol7;
    z11 = z1 /. w -> 1 /. sol6 /. sol7;
    {"Y11" Y11, "le11" le11, "z11" z11, "u11" u11, ":u21" u21, ":W1" W1}]

If 10% reduction is introduced as the medical insurance, we have the following result, with small increase in the sick household's utility level, small decrease in the healthy household's utility level, and the small decrease in the total utilities.
In what follows, using check2, u11, u21, and W1 are computed when k=0.2, 0.3, ..., 0.9, and 0.99. As is easily seen, this result corresponds with Pauly’s argument of Moral Hazard. By the introduction of medical insurance, the sick household becomes better off. However, since it affects the market price, it produces the decrease in efficiency. Division of W0 and W1 by the number of total households imply the expected utility for each household in the society without medical insurance and the one with medical insurance. W0=W1 implies that each household’s expected utility in a society without medical insurance is larger than the one with medical insurance. As is pointed out always, however, how to determine the weights for this modification is quite difficult. division of W0 and W1 should not be used. Rather, they may propose the social welfare function as the weighted sum of household’s utilities, where the weight for the sick household’s utility should be greater than the one for the healthy household. In this modification, made from the viewpoint of social fairness, the situation with medical insurance may be better than the one without medical insurance. In other words, market mechanism produces Pareto-optimality. From the viewpoint of social fairness, however, some economists may assert that as a social welfare function W0 and W1 should not be used. Rather, they may propose the social welfare function as the weighted sum of household’s utilities, where the weight for the sick household’s utility should be greater than the one for the healthy household. In this modification, made from the viewpoint of social fairness, the situation with medical insurance may be better than the one without medical insurance. As is pointed out always, however, how to determine the weights for this modification is quite difficult.

In what follows, using check2, u11, u21, and W1 are computed when k=0.2, 0.3, ..., 0.9, and 0.99. As is easily seen, the variations are small. W1 continuously decreases, which implies that as k increases the expected utility for each household continuously decreases.
Produced. In Section 1, supposing that there are 10 healthy households and 100 sick households with 

\[
\text{check1} \quad \text{and check2, an example of non-existence of general equilibrium with medical insurance can be con-}
\]

structed. In Section 1, supposing that there are 10 healthy households and 100 sick households with \( H_0=1 \), general equilibrium without medical insurance was computed, with \( W_0=4920 \). Now, suppose that medical insurance is introduced.

Using check1 and check2, an example of non-existence of general equilibrium with medical insurance can be constructed. In Section 1, supposing that there are 10 healthy households and 100 sick households with \( H_0=1 \), general equilibrium without medical insurance was computed, with \( W_0=4920 \). Now, suppose that medical insurance is introduced.
If the reduction rate is 30%, there exists a general equilibrium with medical insurance as follows. The introduction of medical insurance reduces the welfare level to $W_1=4644$, as asserted by Pauly [1967].

```
In[46]:= check2[1, 10, 100, 20000, 30/100]
Out[46]= (3.751385030850615440377724335768 + 0. \times 10^{-31} i) : Y11, 
     (184.375692515425307720188862167884 + 0. \times 10^{-32} i) : le11, 
     (2.47452624713874443378636815665724 + 3. \times 10^{-34} i) : z11, 
     (456.24249046380247738368127276074 + 0. \times 10^{-32} i) : u11, 
     (0.8255848217313910435697571561964 + 0. \times 10^{-32} i) : u21, 
     (4644.983386811163878193788432270 + 0. \times 10^{-30} i) : W1
```

When the reduction rate is 90%, check2 provides the following result with $W_1$ increasing to 17176. Note, however that $Y_1$, $z_1$, and $le_1$ are all negative. Thus, there exists no general equilibrium with medical insurance in this specification.

```
In[44]:= check2[1, 10, 100, 20000, 90/100]
Out[44]= (-1153.28113494716107449080909814474 : Y11, 
     -394.14056747358053724540454907237 : le11, 
     -4.28155889000193257802652412803055 : z11, 
     1687.53605057691529677409915092845 : u11, 
     3.009262050922336960032054332192 : u21, 
     17176.2867107783763373413120526064 : W1)
```

3. Privatized Medical Industry:

No Medical Insurance, Monopolistic Case

In this section the medical sector is assumed to be a monopolist. The Japan Medical Association or The American Medical Association may become a monopolist in each country, if they can set the medical charges at will, although in Japan, they are set by the Japanese government. For simplicity, suppose that this sector is owned by the households with equal share holding. Thus monopolistic profit is distributed equally to each household. Other assumptions are the same as in Section 1; e.g. the good-producing sector is assumed to be a competitive firm. In this modified general equilibrium model, what would happen to the above assertion: the introduction of medical insurance causes the welfare inefficiency?

- i: No Medical Insurance Case

First, the existence of monopolistic general equilibrium without medical insurance is examined and comparison is made with the result in Section 1. As in Section 1, suppose that $H_0=300$, there 99 healthy households, and 1 sick household, with the same utility functions and the same production functions. From the same computational programming as in Section 1, the following demand and supply functions for households and competitive firm are derived.

```
In[47]:= Clear[k, H0, a2, a1, ip, z2, le2, z1, le1, x, px, 
     pz, Y2, Y1, sol1, sol2, sol3, sol4, sol5, sol6, sol7, sol8]
```
When the medical sector provides the medical service demanded by the sick household, \( x_1 \), the minimum cost function for the medical sector, \( c \), is given by the following, with \( p_x \), \( p_z \), and \( w \), parameters.

\[
\text{In}[55]:= \quad x_2 = (l x)^{(1/2)} * z x^{(1/2)};
\]
\[
\text{In}[60] := \quad px * (x /. sol1) - c;
\]

The profit function of the medical sector is given by the following function; \( f[p_x,p_z,w] \).

\[
\text{Out}[59] = \quad \frac{\sqrt{pz} w^{5/2}}{2 p x^2}
\]

When \( p_z=1/2 \), \( w=1 \), for example, the profit function with \( p_x \) variable, is depicted in the following diagram.

\[
\text{In}[62] := \quad \text{Plot}[f[px, 1/2, 1], \{px, 1, 20\}];
\]

The maximization of monopolistic profit for the medical sector gives rise to the following pricing rule.

\[
\text{In}[63] := \quad \text{sol4} = \text{Solve}[D[f[px, p_z, w]], px] = 0, px][1];
\]

The maximized monopolistic profit, \( \text{proM} \), is equally distributed to each household.
With this modification, from the equilibrium condition for good market with \( w=1 \), equilibrium goods price, \( p_z \), is computed as follows.

It can be checked that the same equilibrium goods price, \( p_z \), is computed from the equilibrium condition for labor market.

The utility level for the healthy households in this monopolistic case, \( u_{10M} \), is higher than the one in competitive case, \( u_{10} \).

Meanwhile, the utility level for the sick household in this monopolistic case, \( u_{20M} \), is lower than the one in competitive case, \( u_{20} \). This result is a natural one.

In total, the social welfare in this monopolistic case, \( W_{0M} \), is lower than the one in competitive case, \( W_0 \).

ii: The Introduction of Medical Insurance

When the medical insurance is introduced into this monopolistic model, what would happen? Following the definition of the medical insurance, medical service charges for the sick households is reduced \( 100 \times k\% \). The modifications in the computational programs are required for 2 formulations; \( le_2 \) and \( Y_1 \), as in what follows.
With these modifications, from the equilibrium condition for good market with \( w = 1 \), and \( k = 10/100 \) equilibrium goods price, \( p_z \), is computed as follows.

\[
\text{In[93]} := \text{Clear}[x2, le2, zl1, le1, x, xl, ex1, px, pz, Y2, Y1, sol1, sol2, sol3, sol4, sol5, sol6, sol7, sol8]
\]

\[
\text{In[94]} := k = 10/100; u2 = z2 + le2; L2[x_] := H0 + x^ 1(1/2);
le2 = (Y2 + w*L2[x] - px*z2 - (1 - k) * px*x) / w;
sol1 = Solve[{D[u2, z2] = 0, D[u2, x] = 0}, \{z2, x\}][[1]]; x1 = x /. sol1;
z2 = z2 /. sol1; l2 = L2[x] - le2 /. sol1 // Simplify; Clear[z1];
le1 = (365*w + Y1 - px*z1) / w; u1 = z1 + le1; sol2 = Solve[D[u1, z1] = 0, z1][[1]]; z1 = z1 /. sol2; l1 = 365 - le1 /. sol2 // Simplify; Clear[lg, z];
profitg = pz * z - w * lg; lg = Solve[profitg, lg] = 0, lg [[1]];
pt1 = profitg /. solg; lg1 = lg / . solg; Clear[lx]; x2 = (1x)^ 1(1/2) * x2^ 1(1/2);
sol3 = Solve[{D[x2, lx] / D[x2, x2] = w / px, x2 = x0}, \{lx, x2\}][[2]]; zx1 = z2 / . sol3 / . x0 = x1; lx1 = lx / . sol3 / . x0 = x1; c = w * lx1 + px * xz1;
sol4 = Solve[D[px * x1 - c, px] = 0, px] [[1]]; proM = px * x1 - c / . sol4; xz1M = xz1 / . sol4; lxM = lx1 / . sol4;
Y1 = (pt1 + proM) / (a1 + a2) - a2 * k * px * x / a1 /. \{x = x1\} / . sol4;
Y2 = (pt1 + proM) / (a1 + a2) /. sol4;

eqM = \{Simplify[[lg1^ 1(1/2) = a1 * z1 + a2 * z2 * xz1M]] / . sol4 / . w = 1\} [[1]]; sol15 = N[Solve[eqM, pz], 30][[1]]
\]

\[
\text{Out[107]} = \{pz \rightarrow 220.408739558123284103642833549\}
\]

It can be checked that the same equilibrium goods price, \( p_z \), is computed from the equilibrium condition for labor market.

\[
\text{In[108]} :=
\text{eqM} = \text{Simplify}[[lg1 + lx1M = a1 * l1 + a2 * l2]] / . sol4 / . w = 1;
\text{N}[\text{Solve}[eqM, pz], 30][[1]]
\]

\[
\text{Out[109]} = \{pz \rightarrow 220.408739558123284103642833549\}
\]

Using these equilibrium prices, the utility level for the healthy households, \( u1M \), the one for the sick households, \( u2M \), and social welfare, \( W1M \), are computed as follows.

\[
\text{In[110]} :=
\text{u1M} = u1 / . sol5 / . w = 1; \text{u2M} = u2 / . \{x \rightarrow x1\} / . sol4 / . sol5 / . w = 1;
\text{W1M} = a1 * u1M + a2 * u2M; \{" :u1M" u1M, " :u2M" u2M, " :W1M" W1M, " :WOM" WOM\}
\]

\[
\text{Out[111]} = \{268.40321019959338948900389361 :u1M, 201.4712458962756101125221105 :u2M, 26773.389055656021169523907678 :W1M, 26773.38863790811029935986256 :WOM\}
\]

Note that \( W1M \) is higher than \( \text{WOM} \). In other words, the introduction of medical insurance into the monopolistic medical sector model makes this society better off, even if the social welfare is defined by the simple sum of individual utility levels. Since, however, the medical insurance in this paper is nothing but a subsidy to the sick household, this result could be foreseen from the traditional argument on the subsidy in monopoly.
4. Planned Medical Care

In the preceding sections, the level of medical care, \( x^* \), was determined in the market. As computed as a simulation in Section 1, \( x^* \) was 0.000283. Suppose that the government judges this level too low, and it wants to attain \( x^* = 1 \). In Japan, this type of policy appears to have been adopted by the government. Specifically, setting the medical service charges, \( p_x \), and \( k \), the government appears to have the intention of guiding the level of medical care to the desired one. The result is huge amount of budgetary deficit in medical insurance account. Is it possible for the government to arbitrarily set the level of medical care? From the viewpoint of general equilibrium, it could not be done. To show this impossibility, supposing that \( x^* = 1 \), the government is assumed to change \( k \) and find the equilibrium value of \( k \), while \( p_x \) is determined in the market, as well as \( p_z \) and \( w \).

As in Section 1, suppose that \( H_0 = 300 \), there are 99 (=\( a_1 \)) healthy households, and 1 (=\( a_2 \)) sick household, with the same utility functions and the same production functions. As assumed already, the government's target level of medical care is 1 (=\( x_0 \)). Under (3') in Section 2, the sick households's demand for medical care, \( x_1 \), is given by the following.

\[
\text{In[112]:=} \quad \text{Clear}[k, z_2, l_2, z_1, l_1, x, p_x, p_z, Y_2, Y_1, \text{sol1}, \text{sol2}, \text{sol3}, \text{sol4}, \text{sol5}, \text{sol6}, \text{sol7}, \text{sol8}]
\]

\[
\text{In[113]:=} \quad H_0 = 300; a_2 = 1; a_1 = 99; \text{ip} = 2000; x_0 = 1; u_2 = z_2 \times a_2; L_2[x_\_] := H_0 + x^{(1/2)}; \text{le2} = (Y_2 + w \times \text{z2} - p_z \times \text{z2} - p_x \times (1 - k) \times x) / w; \text{sol1} = \text{Solve}[\{D[u_2, z_2] = 0, D[u_2, x] = 0, \{z_2, x\}\}[[1]]; x_1 = x / \text{. sol1}
\]

\[
\text{Out[115]=} \quad \frac{w^2}{4 (-1 + k)^2 p_x^2}
\]

Meanwhile, in order for the medical sector to supply the medical care on the amount of 1, the minimum cost function, \( c \), is given by the following.

\[
\text{In[116]:=} \quad z_2 = z_2 / \text{. sol1}; l_2 = L_2[x] - \text{le2} / \text{. sol1} // \text{Simplify}; \text{Clear}[z_1]; \text{le1} = (365 \times w + Y_1 - p_z \times z_1) / w; u_1 = z_1 \times \text{le1}; \text{sol2} = \text{Solve}[\{D[u_1, z_1] = 0, z_1\}[[1]]; z_1 = z_1 / \text{. sol2}; l_1 = 365 - \text{le1} / \text{. sol2} // \text{Simplify}; \text{Clear}[l_1, z]; \text{profitg} = p_z \times z - w \times l_1; g = l_1^{(1/2)}; \text{solg} = \text{Solve}[\{\text{profitg}, l_1\} = 0, l_1\}[[1]]; l_1 = l_1 / \text{. solg}; \text{Clear}[l_1x]; x_2 = (l_1x)^{(1/2)} + \text{z2}^{(1/2)}; \text{sol3} = \text{Solve}[\{D[x_2, l_1x] / D[x_2, x_2] = w / p_z, x_2 = x_0, \{l_1x, x_2\}\}[[2]]; \text{zxl1F} = \text{z2} / \text{. sol3}; \text{lxl1F} = \text{l1x} / \text{. sol3}; c = w \times \text{lxl1F} + p_z \times \text{zxl1F}
\]

\[
\text{Out[122]=} \quad 2 \sqrt{p_z} \sqrt{w}
\]

Since \( p_x \times x_0 = c \) must hold, the following \( \text{sol4} \) must hold.

\[
\text{In[123]:=} \quad p_x = c / x_0; \text{sol4} = \text{Solve}[x_1 = x_0 / a_2, w] [[1]]
\]

\[
\text{Out[123]=} \quad \{w \rightarrow 16 (-1 + k)^2 p_z\}
\]

From the condition of \( w \) in \( \text{sol4} \) being 1 and the equilibrium condition in goods market, \( p_z \) and \( k \) at the equilibrium are computed as follows.
Function f has the unique solution which satisfies f=0, as shown in the following diagram.

\[ \text{Out[128]} = \text{In[128]} = \text{Out[127]} = \text{In[127]} = \text{Out[126]} = \text{In[126]} = \text{Out[125]} = \text{In[125]} = \]

Unfortunately, these values do not satisfy the equilibrium condition in labor market,

\[ \text{In[127]} = \text{FindRoot}[\{w /. \text{sol4} = 1, \text{lg1} + \text{lx1F} = a1 \cdot l1 + a2 \cdot l2\} /. w \to 1, \{pz, 100\}, \{k, 0.9\}, \text{AccuracyGoal} \to 24, \text{WorkingPrecision} \to 50, \text{MaxIterations} \to 100] \]

\[ \text{Out[126]} = \{pz \to 211.7635887304187132231521658018508111575586641938, k \to 0.98282033858764702369939195963874876285569 \} \]

Then, is it possible to find \{w,pz,k\} which satisfies good market equilibrium condition, labor market equilibrium condition, and sol4 ? It is impossible by the following reason. The labor market equilibrium condition, under sol4, implies the following.

\[ \text{In[127]} = \text{Simplify}[\{\text{lg1} + \text{lx1F} = a1 \cdot l1 + a2 \cdot l2\} /. \text{sol4}] \]

\[ \text{Out[127]} = \frac{1}{\left((-1 + k)^2 \right)^{3/2}} (-1 + k) \left[ 1024 (-1 + k)^4 (1 + 48 k) \sqrt{p z} + \right. \]

\[ \left. \begin{array}{c}
37309439 - 2 -\left((1 + k)^4 + 4 \left(-37309440 + \sqrt{(-1 + k)^4} \right) k - \right.
\end{array} \right] = 0 \]

If k=1 or pz=0, \{w,pz,k\} cannot satisfy the above equation. If \{w,pz,k\} with k≠1 and pz≠0 satisfies the above equation, it must satisfy the following equation.

\[ \text{In[128]} = f = \left[ 1024 (-1 + k)^4 (1 + 48 k) + \right. \]

\[ \left. \begin{array}{c}
37309439 - 2 -\left((1 + k)^4 + 4 \left(-37309440 + \sqrt{(-1 + k)^4} \right) k - \right.
\end{array} \right] = 0 \]

\[ \text{Out[128]} = 1024 (-1 + k)^4 (1 + 48 k) + \]

\[ \begin{array}{c}
\sqrt{(-1 + k)^2} \left[ 37309439 - 2 -\left((1 + k)^4 + 4 \left(-37309440 + \sqrt{(-1 + k)^4} \right) k - \right.
\end{array} \right] = 0 \]

Function f has the unique solution which satisfies f=0, as shown in the following diagram.
This unique solution $k$ is computed by the Newton method.

$$g_1 = \text{FindRoot}[f = 0, \{k, 0.8\}, \text{AccuracyGoal} \to 24, \text{WorkingPrecision} \to 50, \text{MaxIterations} \to 100]$$

$$\{k \to 0.98348191743331367078251392593505341098664719906857\}$$

This $k$ cannot satisfy the goods market equilibrium condition as in what follows.

$$\text{NIntegrate}^\#(1/2) - (a1 + a2 + z2 + \text{zx}1F) \text{/. sol4} / . g1$$

$$12.736264643765066512008400318226575072007709472$$

Thus, there is no $\{w, pz, k\}$ which satisfies good market equilibrium condition, labor market equilibrium condition, and sol4.

### References
