A General Equilibrium Approach to Adverse Selection

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In[1]:= Off[General::spell, General::spell1, Power::infy]
Introduction

Contributions by Arrow [1963] and Akerlof [1970] were synthesized by Rothschild and Stiglitz [1976] as the economics of adverse selection in insurance market. When Arrow [1963] initiated the economics of medical insurance, he assumed that the demand for medical treatment is independent of its price. This assumption was criticized by Pauly [1968], who emphasized the negative side of medical insurance. Arrow's insurance should be regarded as the compensation for the income loss caused by physical damage. Rothschild and Stiglitz [1976] follow Arrow in this respect, making risk averter assumption on households. Arrow [1963] examined if the household purchases the fixed amount of insurance proposed by the insurance company.

Akerlof [1970] examined the asymmetric information in the market for the used cars, asserting that there may not exist equilibrium in such a market with asymmetric information, where sellers of used cars know the quality of them, and purchasers don't. Rothschild and Stiglitz [1976] asserted that there may not exist equilibrium in insurance market with asymmetric information, where sellers of insurance do not know the exact probabilities of diseases of the purchasers and insurers cannot but set the insurance premium by the average of those probabilities.

As explained in Fukiharu [2004], the economics of medical insurance should be formulated in general equilibrium framework, if it is to incorporate the special feature of medical services. The main purpose of this paper, is to examine the adverse selection in the framework of general equilibrium. The minor purpose of this paper is to elucidate the relationship between the risk averter assumption on households and strict concavity of traditional utility function with leisure and commodities as variables.

In Section 1, assuming that sickness is the loss of initial endowment as in Fukiharu [2004], the existence of general equilibrium is computed in the model without medical sector where utility and production functions are assumed as Cobb-Douglas type. In this economy, there is a class of households with the same probability of diseases, who maximize utility with leisure and one consumption good as variables, demanding the good, while supplying labor. In Section 2, the insurance system of income compensation is introduced into this model, where the insurer imposes "fair" insurance fee. It is shown that the utility function must have strict concavity in order for the general equilibrium with insurance to exist, as pointed out in the case of rational expectations equilibrium (see Fukiharu [1988] and Grandmont [1983]). In Section 3, following Rothschild and Stiglitz [1976], it is assumed that there are two classes (types) of households with different probabilities of disease, and sellers of insurance do not know the exact probabilities of diseases of the purchasers and insurers cannot but set the insurance fee by the average of those probabilities. In this economy, it is shown that when the government does not intervene, purchasers of insurance engage themselves in a Nash-type noncooperative game, and the optimum insurance for the households with lower probability is zero; the emergence of adverse selection. It is confirmed that when the government intervenes, she can raise the sum of all the households' utilities.

1 Existence of General Equilibrium for the Economy without Insurance and Medical Sector: One-Class-Household-Case

In[2]:= Clear[u1, u2, H0, a2, a1, ip, z2, le2, z1, le1, x, px, pz, Y2, Y1, sol1, sol2, sol3, sol4, sol5, sol6, sol7, sol8]

In this section, a model without insurance is constructed. This economy consists of a class of identical households and one firm. In each period (year) there are a1 "healthy" households and a2 "sick" households, so that each household has $a=a2/(a1+a2)$ probability of being sick. "Healthy" household has the initial endowment of 365 days, while the "sick" household has the initial endowment of 300 days by definition. When a household is healthy, its behavior is stipulated by traditional utility maximization under income constraint:
max $u(z_1, l_e)$  
\[ \text{s.t. } p_z z_1 = w(365 - l_e) + Y_1 \]  
(1)

where $u(z_1, l_e)$ is the utility function, $z_1$ is the consumption of goods, $l_e$ is the leisure consumption, $p_z$ is the price of goods, $w$ is the wage rate, and $Y_1$ is the transfer of income from others, such as profit and tax. In this paper, since simulation approach is utilized, utility function is stipulated by

$$u(z, l_e) = z \times l_e.$$  
(2)

When a household is "sick", its behavior is stipulated by the following utility maximization under income constraint:

max $u(z_2, l_e)$  
\[ \text{s.t. } p_z z_2 = w(300 - l_e) + Y_2 \]  
(3)

where $u(z_2, l_e)$ is utility function, $z_2$ is consumption of goods, $l_e$ is leisure consumption, and $Y_2$ is transfer of income from others, such as profit and tax. Thus, each household, with $a = a_2/(a_1 + a_2)$ probability of being sick, maximizes the following expected utility:

max $(1-a) u(z_1, l_e) + a u(z_2, l_e)$  
\[ \text{s.t. } p_z z_1 = w(365 - l_e) + Y_1, \text{ and } p_z z_2 = w(300 - l_e) + Y_2 \]  
(4)

For the purpose of computation of equilibrium, suppose that $a_1 = 99$ and $a_2 = 1; a = 0.01$. Under (2) and (4) each household's demand function for the goods when it is "healthy", $zhD$, demand function for the goods when it is "sick", $zsD$, supply function of labor when it is "healthy", $lhS$, and supply function of labor when it is "sick", $lsS$, are given by the following.

In[3]:=  
\[a_2 = 1; a_1 = 99; ip = 2000; u_1 = z_1 \times l_e; l_1 = (365 \times w - p_z \times z_1 + Y_1) / w; l_2 = (300 \times w - p_z \times z_2 + Y_2) / w; u_2 = z_2 \times l_e; u = (a_1 / (a_1 + a_2)) u_1 + (a_2 / (a_1 + a_2)) u_2; \]
\[\text{sol1} = \text{Solve}[[D[u, z_1] = 0, D[u, z_2] = 0], \{z_1, z_2\}] @@ [1]; zhD = z_1 /. \text{sol1}; zsD = z_2 /. \text{sol1}; \]
\[\text{lehD} = l_1 /. \text{sol1}; \text{lesD} = l_2 /. \text{sol1}; \]
\[\text{lhS} = 365 - \text{lehD}; \text{lsS} = 300 - \text{lesD}; \text{Print}[["zhD" \rightarrow \text{zhD}, "zsD" \rightarrow \text{zsD}, "lhS" \rightarrow \text{lhS}, "lsS" \rightarrow \text{lsS}]] \]
\[\{\text{zhD} \rightarrow \frac{365 w + Y_1}{2 p_z}, \text{zsD} \rightarrow \frac{300 w + Y_2}{2 p_z}, \text{lhS} \rightarrow 365 - \frac{365 w}{2 p_z} + \frac{1}{2} \left(\frac{-365 w - Y_1}{w} + Y_1\right), \text{lsS} \rightarrow 300 - \frac{300 w}{2 p_z} + \frac{1}{2} \left(\frac{-300 w - Y_2}{w} + Y_2\right) \}
\]

The behavior of good-producing sector is stipulated by profit maximization where production function is given by

$$z = f(l_g) = l_g^{1/2}$$  
(5)

where $z$ is the output of goods, and $l_g$ is labor input. From the profit maximization under (5) gives rise to the labor demand function of the good-producing sector, $lg1$, supply function of good, $zS$, and profit function, $pt1$, as follows.

In[9]:=  
\[\text{profitg} = p_z \times z - w \times l_g; z = l_g^{1/2}; \]
\[\text{solg} = \text{Solve}[[\text{profitg}, l_g] = 0, l_g] @@ [1]; l_g = l_g /. \text{solg}; zS = z /. l_g \rightarrow l_g; pt1 = \text{profitg} /. \text{solg}; \text{Print}[["zS" \rightarrow \text{zS}, "lgl" \rightarrow l_g, "pt1" \rightarrow \text{pt1}]] \]
\[\{zS \rightarrow \frac{1}{2} \sqrt{\frac{p_z^2}{w^2}}, l_g \rightarrow \frac{p_z^2}{4 w^2}, pt1 \rightarrow \frac{1}{2} p_z \sqrt{\frac{p_z^2}{w^2} - \frac{p_z^2}{4 w}} \}
\]
In this economy, it is assumed that the good-producing firm is owned by the $(a_1 + a_2)$ households with equal share holding, and $Y_1$ and $Y_2$ are given as follows:

\[
Y_1 = pt_1 / (a_1 + a_2); \quad Y_2 = pt_1 / (a_1 + a_2);
\]

In this economy, in each year, there are $a_1$ "healthy" households and $a_2$ "sick" households, so that equilibrium condition for the commodity market is

\[
a_1 x h D + a_2 x s D = z S
\]

Solving (6) with respect to $p_z$ with $w=1$, equilibrium goods price, $p_z$, is given by the following:

\[
\text{In[12]} := \text{sol2 = Solve}[(a_1 x h D + a_2 x s D = z S) / (w -> 1, p_z)][[1]]
\]

Meanwhile, equilibrium condition for the labor market is

\[
a_1 x l h S + a_2 x l s S = l g_1
\]

Solving (7) with respect to $p_z$ with $w=1$, the same equilibrium goods price, $p_z$, is given.

\[
\text{In[13]} := \text{Solve}[(a_1 x l h S + a_2 x l s S = l g_1) / (w -> 1, p_z)][[1]]
\]

Equilibrium expected utility is given by the following:

\[
\text{In[14]} := \text{u / sol1 / sol2 / w -> 1}
\]

Note that this equilibrium utility level is lower than the one with medical sector in Fukiharu [2004].

### 2 Existence of General Equilibrium for the Economy with Insurance and without Medical Sector: One-Class-Household-Case

In this section, it is shown that the strict assumption on the utility function is required in order for the existence of general equilibrium to exist when the insurance is introduced.
a: Non-Existence of General Equilibrium for the Economy with Insurance when the sick household works

Suppose that in this economy, insurance is introduced where the insurer guarantees the "fair" insurance. Following Rothschild and Stiglitz [1976], the insurer pays $Y_1$ for $p_I$ insurance fee when a insurance holder is sick. In other words, if a household plans to obtain $Y_1$ when he or she is "sick", he or she must pay $p_I H$ both when he is sick and when he is healthy. Thus, when the insurance is introduced, each household, with $\alpha=a_2/(a_1+a_2)$ probability of being sick, maximizes the following expected utility:

$$\max (1-\alpha) u(z_1, le_1) + \alpha u(z_2, le_2)$$

s.t. $p_z z_1 + p_I H = w(365-le_1) + Y_1$, and $p_z z_2 + p_I H = w(300-le_2) + H + Y_2$ \hspace{1cm} (8)

When there is only one type of households as in this section and the insurer guarantees the "fair" insurance, $p_I=\alpha$. As explained above, in this model, the household selects the optimum insurance holding $H$, as well as optimum commodity consumption, and optimum labor supply. In what follows, first, we derive optimum $zhD$, $zsD$, $lhS$, and $lsS$, given $H$. Next, we derive equilibrium goods price $pz$, given $H$ and $w=1$. This derivation provides the equilibrium expected utility level, $u_1$, given $H$ and $w=1$. Finally, the optimum insurance $H$ is selected by maximizing the equilibrium expected utility level. With $p_I=\alpha$, we have the following optimum $zhD$, $zsD$, $lhS$, and $lsS$, given $H$.

$$\begin{align*}
zhD & \to -H + 36500 w + 100 Y_1 \quad \text{if} \quad z_1 \to \frac{99 H + 30000 w + 100 Y_2}{200 p_z}, \\
lsS & \to 365 - \frac{36500 w + 365 (H - 36500 w - 100 Y_1) + Y_1}{w}, \\
lhS & \to 300 - \frac{99 H + 30000 w - 100 Y_2 + Y_1}{w}.
\end{align*}$$

The same assumptions are made on the production side, so that, the same $zS$, $lg1$, and $pt1$ are given as in Section 1.

$$\begin{align*}
zhD & \to -H + 36500 w + 100 Y_1 \quad \text{if} \quad zhD, \\
zsD & \to 365 - \frac{36500 w + 365 (H - 36500 w - 100 Y_1) + Y_1}{w}, \\
lhS & \to 300 - \frac{99 H + 30000 w - 100 Y_2 + Y_1}{w}.
\end{align*}$$

Thus, the same $Y_1$ and $Y_2$ are derived as in Section 1.
In this economy, in each year, there are \( a_1 \) "healthy" households and \( a_2 \) "sick" households, so that equilibrium condition for the commodity market is (6) in Section 1. Solving (6) with respect to \( p_z \) with \( w=1 \), equilibrium goods price, \( p_z \), is given by the following.

\[
\text{In}[26]:= \text{sol2} = \text{Solve}[a_1 z h D + a_2 z s D = z S / . w \to 1, p_z][[1]]
\]

\[
\text{Out}[26]= \{p_z \to 2 \sqrt{12145}\}
\]

Meanwhile, equilibrium condition for the labor market is (7). Solving (7) with respect to \( p_z \) with \( w=1 \), the same equilibrium commodity price, \( p_z \), is given.

\[
\text{In}[27]:= \text{Solve}[a_1 l h S + a_2 l s S = l g 1 / . w \to 1, p_z][[1]]
\]

\[
\text{Out}[27]= \{p_z \to 2 \sqrt{12145}\}
\]

Since we have derived equilibrium goods price \( p_z \), given \( H \) and \( w=1 \), this derivation provides the equilibrium expected utility level, \( u_1 \), as follows, given \( H \) and \( w=1 \).

\[
\text{In}[28]:= u_1 = \text{Simplify}[u / . \text{sol1} / . \text{sol2} / . w \to 1]
\]

\[
\text{Out}[28]= \frac{2360434675 - 12870 H + 99 H^2}{80000 \sqrt{12145}}
\]

Finally, the optimum insurance \( H \) is selected by maximizing the equilibrium expected utility level, \( u_1 \). This is impossible, however, since \( u_1 \) continuously increases as \( H \) increases, as shown in the following diagram.

\[
\text{In}[29]:= \text{Plot}[u_1, \{H, 0, 200\}];
\]

In the diagram, the value of \( H \), which has the same \( u_1 \) at \( H=0 \), is 130.

\[
\text{In}[30]:= u_{10I} = u_1 / . H \to 0; \text{sol3} = \text{Solve}[u_1 = u_{10I}, H]
\]

\[
\text{Out}[30]= \{\{H \to 0\}, \{H \to 130\}\}
\]

When \( H=130 \), \( le1D \) and \( le2D \) are positive as shown in what follows, and \( H \) can be raised to raise expected utility, \( u_1 \), until \( le1D \) or \( le2D \) becomes zero.

\[
\text{In}[31]:= \{le1D / . \text{sol1} / . \text{sol2} / . w \to 1 / . \text{sol3}[[2]], le2D / . \text{sol1} / . \text{sol2} / . w \to 1 / . \text{sol3}[[2]]\}
\]

\[
\text{Out}[31]= \{\frac{9703}{40}, \frac{11003}{40}\}
\]

In this sense, there is no maximum of \( u_1 \).
b1: The Existence of General Equilibrium without Insurance When the Sick Household Does not Work

In this sub-section, it is assumed that the sick households do not work, consuming 300 initial endowment of leisure days. Thus, each household, with \( a = a_2 / (a_1 + a_2) \) probability of being sick, maximizes the following expected utility:

\[
\max (1-a) u(z_1, le_1) + a u(z_2, 300)
\]

s.t. \( p_z z_1 = w(365-le_1) + Y_1 \), and \( p_z z_2 = Y_2 \) \hspace{1cm} (9)

In the same way above, we have \( zhD \), \( zsD \), and \( lhS \), as in what follows. Note that \( lsS = 0 \) by the assumption.

The behavior of good-producing sector is stipulated by profit maximization where production function is given by (5).

From the profit maximization under (5) gives rise to the labor demand function of the good-producing sector, \( lg_1 \), supply function of goods, \( zS \), and profit function, \( pt1 \), as above. In this economy, it is assumed that the good-producing firm is owned by the \((a_1 + a_2) \) households with equal share holding as assumed above, and \( Y_1 \) and \( Y_2 \) are given as follows

\[
Y_1 = pt1 / (a_1 + a_2); \hspace{0.5cm} Y_2 = pt1 / (a_1 + a_2); \hspace{0.5cm} \text{Print}["Y1" \rightarrow Y1, "Y2" \rightarrow Y2]
\]

In this economy, in each year, there are \( a_1 \) "healthy" households and \( a_2 \) "sick" households, so that equilibrium condition for the commodity market is (6). Solving (6) with respect to \( pz \) with \( w = 1 \), equilibrium goods price, \( pz \), is given by the following.

\[
[\text{In}[40]:= \text{profitg} = pz * z - w * lg; \text{solg} = \text{Solve}[\text{profitg}, lg][1]; \text{zS} = z / . \text{solg}; \text{pt1} = \text{profitg} / . \text{solg}; \text{Y1} = pt1 / (a_1 + a_2); \text{Y2} = pt1 / (a_1 + a_2); \text{Print}["Y1" \rightarrow Y1, "Y2" \rightarrow Y2]
\]

In this economy, each year, there are \( a_1 \) "healthy" households and \( a_2 \) "sick" households, so that equilibrium condition for the commodity market is (6). Solving (6) with respect to \( pz \) with \( w = 1 \), equilibrium goods price, \( pz \), is given by the following.

\[
\text{In}[43]:= \text{sol2} = \text{Solve}[(a_1 * zhD + a_2 * zsD = zS) / . w \rightarrow 1, pz][1]
\]

Meanwhile, equilibrium condition for the labor market is (7), where \( lsS = 0 \). Solving (7) with respect to \( pz \) with \( w = 1 \), the same equilibrium commodity price, \( pz \), is given.
In[44]:= \text{sol2} = \text{Solve}[@a1 \ast \text{lhS} m \lg1 \rightarrow 1, \text{pz}][[1]]

Out[44]= \{\text{pz} \rightarrow 60 \sqrt{\frac{4015}{299}}\}

Equilibrium expected utility level is computed as follows.

In[45]:= u \rightarrow . \text{sol1} \rightarrow . \text{sol2} \rightarrow . w \rightarrow 1

Out[45]= \frac{8726439 \sqrt{803}}{23920}

In[46]:= N[4, 30]

Out[46]= 267.370235263324108889668390634

- b2: The Non-Existence of General Equilibrium with Insurance when the Sick Household Does Not Work

In[47]:= \text{Clear}[u, u1, u2, u2k, H0, a2, a1, ip, z2, le2, z1, le1, x, px, pz, Y2, Y1, sol1, sol2, zhD, zsD, lhS, lsS, lg1, le1D, le2D]

In exactly the same way as above, the equilibrium expected utility level, \(u_1\), given \(H\) is computed as in what follows.

In[48]:= \text{a1} = 99; \text{a2} = 1; \text{u1} = z1 \ast \text{le1}; \text{le1} = (365 + w - pz \ast z1 - a2 / (a1 + a2) H + Y1) / w; \text{z2} = (H - a2 / (a1 + a2) H + Y2) / pz; \text{u2} = z2 \ast \text{le2}; \text{le2} = 300; \text{u} = a1 / (a1 + a2) \text{u1} + a2 / (a1 + a2) \text{u2}; \text{sol1} = \text{Solve}[[\text{D}[u, z1] = 0], z1]][[1]]; \text{zhD} = z1 / . \text{sol1}; \text{zsD} = z2; \text{le1D} = le1 / . \text{sol1}; \text{lhS} = 365 - \text{le1D}; \text{le2D} = \text{le2}; \text{profitg} = pz \ast z - w \ast \text{lg}; \text{z} = \text{lg}^1 / 2; \text{solg} = \text{Solve}[[\text{profitg}, \text{lg}] = 0, \text{lg}][[1]]; \text{lg1} = \text{lg} / . \text{solg}; \text{zS} = z / . \text{lg} \rightarrow \text{lg1}; \text{pt1} = \text{profitg} / . \text{solg}; \text{Y1} = \text{pt1} / (a1 + a2); \text{Y2} = \text{pt1} / (a1 + a2); \text{sol2} = \text{Solve}[[\text{a1} \ast \text{zhD} + a2 \ast \text{zsD} = \text{zS} / . \text{w} \rightarrow 1, \text{pz}][[2]]; \text{Solve}[\text{a1} \ast \text{lhS} = 1 \rightarrow . \text{w} \rightarrow 1, \text{pz}][[2]]; \text{u1} = \text{Simplify}[u / . \text{sol1} / . \text{sol2} / . \text{w} \rightarrow 1]

Out[55]= \frac{3 \sqrt{\frac{1}{299}} \left(5308583725 + 123830 H + H^2\right)}{59800 \sqrt{36500 + H}}

Finally, the optimum insurance \(H\) is selected by maximizing the equilibrium expected utility level, \(u_1\). This is impossible, however, since \(u_1\) continuously increases as \(H\) increases, as shown in the following diagram.

In[56]:= \text{Plot}[[u1, \{H, 0, 10000\}];
When $H$ is large, say, $H=100000$, $\text{le1D}$ is negative.

```
In[57]:= le1D/.sol1/.sol2/.w→1/.H→100000
Out[57]= -2105
```

As explained above, there is no interior maximum of $u_1$.

- **c: the Non-Existence of General Equilibrium with Insurance When the Sick Household Works—Different Utility Function (i)**

```
In[58]:= Clear[u, u1, u2, u2k, H0, a2, a1, ip, z2, le2, z1, le1, x, px, pz, Y2, Y1, sol1, sol2, zhD, zsD, lhS, lsS, lg1, le1D, le2D]
```

In order to examine what strict assumption on utility function is required, suppose that the utility function is of the following form in stead of (2).

$$u(z, le) = z \times \sqrt[2]{le}$$  (10)

With this modification of utility function, the equilibrium expected utility function, given $H$, is computed as in what follows.

```
In[59]:= a1 = 99; a2 = 1;
 u1 = (z1 + le1)^((1/2)); le1 = (365*w - pz*z1 - a2)/(a1 + a2) H + Y1) / w; le2 = (300*w - pz*z2 + H - a2)/(a1 + a2) H + Y2) / w; u2 = (z2 + le2)^((1/2)); u = a1/(a1 + a2) u1 + a2/(a1 + a2) u2;
sol1 = Solve[{D[u, z1] = 0, D[u, z2] = 0}, {z1, z2}][[1]]; zhD = z1/.sol1; zsD = z2/.sol1; le1D = le1/.sol1; lhS = 365 - le1D; le2D = le2/.sol1; lsS = 300 - le2D; profitg = pz*z - w*lg; z = lg^((1/2)); solg = Solve[D[profitg, lg] = 0, lg][[1]]; lg1 = lg/.solg; zS = z/.lg→lg1; pt1 = profitg/.solg; Y1 = pt1/(a1 + a2); Y2 = pt1/(a1 + a2);
sol2 = Solve[a1*zhD + a2*zsD = zS/.w→1, pz][[1]]; Solve[a1*lhS + a2*lsS = lg1/.w→1, px][[1]]; u1 = Simplify[u/.sol1/.sol2/.w→1]; Plot[u1, {H, 0, 100000}];
```

Until $H=48645$ the equilibrium expected utility function, $u_1$, given $H$, is the same as $H=0$. When $H$ is larger than 48645, $u_1$ increases. However, $\text{le1D}$ becomes negative in this case, so that there exists no general equilibrium with insurance when the utility function is specified by (10).
\[\text{In}[70]:=\quad \text{le1D /. sol1 /. sol2 /. w} \to 1 /. H \to 48646\]

\[\text{Out}[70]= -\frac{1}{200}\]

d: the Existence of General Equilibrium with Insurance When the Sick Household Works—Different Utility Function (ii)

\[\text{In}[71]:=\quad \text{Clear}[u, u1, u2, u2k, H0, a2, a1, ip, z2, le2, z1, le1, x,}
\text{ px, pz, Y2, Y1, sol1, sol2, zhD, zsD, lhS, lsS, lg1, le1D, le2D]\]

In order to examine what strict assumption on utility function is required, suppose that the utility function is of the following form in stead of (10).

\[u(z, le) = \sqrt{z} \times \sqrt{le}\]  

With this modification of utility function, the equilibrium expected utility function, given H, is computed as in what follows.

\[\text{In}[72]:=\quad a1 = 99; a2 = 1;\]
\[u1 = (z1 \times le1)^{(1/3)}; \quad le1 = (365 \times w - pz \times z1 - a2) / (a1 + a2) \times H + Y1) / w;\]
\[le2 = (300 \times w - pz \times z2 + H - a2) / (a1 + a2) \times H + Y2) / w;\]
\[u = a1 / (a1 + a2) \times u1 + a2 / (a1 + a2) \times u2;\]
\[\text{sol1 = Solve}[\{D[u, z1] = 0, D[u, z2] = 0, \{z1, z2}\}[[1]]];\]
\[zhD = z1 /. \text{sol1}; \quad zsD = z2 /. \text{sol1}; \quad le1D = le1 /. \text{sol1};\]
\[lhS = 365 - le1D; \quad le2D = le2 /. \text{sol1}; \quad lsS = 300 - le2D;\]
\[\text{profitg} = pz \times z - w \times lg; \quad z = lg^{(1/2)};\]
\[\text{solg} = \text{Solve}[\text{profitg, lg} = 0, lg][[1]]; \quad lg1 = lg /. \text{solg};\]
\[zS = z /. lg \to lg1; \quad pt1 = \text{profitg} /. \text{solg}; \quad Y1 = pt1 / (a1 + a2); \quad Y2 = pt1 / (a1 + a2);\]
\[\text{sol2} = \text{Solve}[a1 \times zhD + a2 \times zsD = zS / . w \to 1, pz][[1]];\]
\[\text{Solve}[a1 + lhS + a2 + lsS = lg1 / . w \to 1, pz][[1]];\]
\[u1 = \text{Simplify}[u /. \text{sol1} /. \text{sol2} /. w \to 1]; \quad \text{Plot}[u1, \{H, 0, 400\}]\]
In the interval $[0,48645]$, the equilibrium expected utility function, $u_1$, given $H$, has interior maximum as shown in the above diagram. When $H$ is larger than 48645, $u_1$ increases. However, $l_{e1D}$ becomes negative in this case, so that there exists general equilibrium with insurance when the utility function is specified by (11).

\[ l_{e1D} = \frac{1}{200} \]

The maximand of $u_1$ is $H=65$ as shown as follows.

\[ \text{Solve}[D[u_1, H] == 0, H] \]

\[ \{\{H -> 65\}\} \]

Note that (11): i.e. strict concavity; corresponds with the risk aversion assumption: i.e. $u''(Y)<0$ where $Y$ is income; for Arrow [1963] and Rothschild and Stiglitz [1976]. The strict concavity of utility function was also required in proving the existence of general equilibrium with money, as pointed out by Fukiharu [1988] and Grandmont [1983].

3 Assymmetric Information and the Adverse Selection in Insurance Market: Two-Class-Household-Case

\[ \text{Clear}[u, u_1, u_2, u_{2k}, H_0, a_2, a_1, ip, z_2, z_1, le_1, x, px, pz, Y_2, Y_1, sol_1, sol_2, zhD, zsD, lhS, lsS, lg1, le_1D, le_2D] \]
Akerlof [1970] examined the asymmetric information in the market for the used cars, asserting that there may not exist equilibrium in such a market with asymmetric information, where the sellers of used cars know the quality of them, while the purchasers don’t. Rothschild and Stiglitz [1976] asserted that there may not exist equilibrium in insurance market with asymmetric information, where the sellers of insurance do not know the exact probabilities of diseases of the purchasers and the insurers cannot but set the insurance fee by the average of those probabilities.

As explained in Fukiharu [2004], the economics of medical insurance should be formulated in general equilibrium framework, if it is to incorporate the special feature of medical services. In this paper, following Fukiharu [2004], the model in Rothschild and Stiglitz [1976] is reformulated in the framework of traditional general equilibrium. In this section, following Rothschild and Stiglitz [1976], it is assumed that there are two classes of households with different probabilities of disease, and the sellers of insurance do not know the exact probabilities of diseases of the purchasers and insurers cannot but set the insurance fee by the average of those probabilities. In this economy, it is shown that the optimum insurance for these two classes is zero; the emergence of adverse selection.

\[ a_1 = 99; \ a_2 = 1; \ b_1 = 90; \ b_2 = 10; \]

Utility functions are assumed to be specified by (11). There are two types of households. Type A-households have the same characters as specified in Sections 1 and 2. Specifically, they have the probability \( a = 1/100 \) of being sick. For simplicity, it is assumed that there are 100 type A-households. As in the preceding sections, suppose that \( a1 = 99 \) "healthy" households and \( a2 = 1 \) "sick" household in this class. In this section, another different type of households is added. Suppose that there are 100 type B-households, with the probability \( b = 1/10 \) of being sick. For simplicity, it is assumed that there are 100 type B-households. Suppose furthermore that \( b1 = 90 \) "healthy" households and \( b2 = 10 \) "sick" households in this class.

- **Type A-Household in Two-Class-Household-Case**

In this subsection, the behavior of type A-households is examined. Their behavior is stipulated by (8), where \( p_f = a \) does not hold when there are two types of households as in this section even if the insurer guarantees the fair insurance. Their purchase of insurance is \( Ha \), the demand function for the consumption good when they are "healthy" is \( zhaD \), the one when they are "sick" is \( zsaD \), the supply function of labor when they are "healthy" is \( lhaS \), and the one when they are "sick" is \( lsaS \). Assuming that the consumption-goods producing firm is owned by 200 households with the equal share holding, these functions, given \( Ha \), as well as other prices, are computed as follows.
Type B-Household in Two-Class-Household-Case

Meanwhile the behavior of type B-households is examined. Their behavior is stipulated by (8), where $p_I = \alpha$ does not hold when there are two types of households as in this section even if the insurer guarantees the fair insurance. It is assumed that the insurer sets the same insurance fee, $p_I$, for type B-households since the insurer cannot distinguish type-A and type-B. Type B-households’ purchase of insurance is Hb, the demand function for the consumption good when they are "healthy" is zhbD, the one when they are "sick" is zsbD, the supply function of labor when they are "healthy" is lhbS, and the one when they are "sick" is lsbS. Assuming that the consumption-good producing firm is owned by 200 households with the equal share holding, these functions, given Hb, as well as other prices, are computed as follows.

In[95]:=
ub = (zlb + lelb) ^ (1/3); lelb = (365 * w - px * zlb - pi + Hb + Y1b) / w;
le2b = (300 * w - px - zlb + pi + Hb + Y2b) / w;

u2b = (zlb + le2b) ^ (1/3); ub = ub1 / (ub1 + ub2) u1b + b2 / (b1 + b2) u2b;

solb = Solve[{D[ub, zlb] = 0, D[ub, z2b] = 0}, {zlb, z2b}][[1]]; zhbD = zlb /. solb; zsbD = z2b /. solb; le1bD = lelb1 /. solb1; lhbS = 365 - le1bD; le2bD = le2b /. solb1; lsbS = 300 - le2bD;

Y1b = pt1 / (a1 + a2 + b1 + b2); Y2b = pt1 / (a1 + a2 + b1 + b2);
Print["zhbD" -> zhbD, "zsbD" -> zsbD, "lhbS" -> lhbS, "lsbS" -> lsbS]

In[88]:=
ula = (zla + lela) ^ (1/3); lela = (365 * w - px * zla - pi + Ha + Y1a) / w;
le2a = (300 * w - px - z2a + pi + Ha + Y2a) / w;
u2a = (z2a + le2a) ^ (1/3); ua = a1 / (a1 + a2) u1a + a2 / (a1 + a2) u2a;
sola = Solve[{D[ua, zla] = 0, D[ua, z2a] = 0}, {zla, z2a}][[1]]; zhaD = zla /. sola; zsaD = z2a /. sola; le1aD = lela1 /. sola1; lhaS = 365 - le1aD; le2ad = le2a /. sola1; lsaS = 300 - le2ad;

profitg = px * z - w * lg; zg = lg ^ (1/2);

solg = Solve[profitg, lg] = 0, lg][[1]]; lg1 = 1g /. solg;

zS = z / . lg1; pt1 = profityg /. solg;

Y1a = pt1 / (a1 + a2 + b1 + b2); Y2a = pt1 / (a1 + a2 + b1 + b2);
Print["zhaD" -> zhaD, "zsaD" -> zsaD, "lhaS" -> lhaS, "lsaS" -> lsaS]
Equilibrium in Two-Class-Household-Case

Equilibrium condition for the commodity market with two-class-household general equilibrium model is the following.

\[ a_1 \times z_{haD} + a_2 \times z_{saD} + b_1 \times z_{hbD} + b_2 \times z_{sbD} = z_1 \] \hspace{1cm} (12)

From (12), the equilibrium commodity price \( p_z \), when \( H_a \) and \( H_b \) are given and \( w = 1 \), is computed as follows.

```
In[101]:= solA1 = Solve[{a_1 \times z_{haD} + a_2 \times z_{saD} + b_1 \times z_{hbD} + b_2 \times z_{sbD} = z_1 / w \to 1}, p_z]
```

```
Out[101]= \{
{p_z \to \frac{2 \sqrt{72285 + Ha + 10 Hb - 100 Ha p_I - 100 Hb p_I}}{\sqrt{3}}},
{p_z \to -\frac{2 \sqrt{72285 - Ha - 10 Hb + 100 Ha p_I + 100 Hb p_I}}{\sqrt{3}}}
\}
```

Equilibrium condition for the labor market with two-type general equilibrium model is the following.

\[ a_1 \times l_{haS} + a_2 \times l_{saS} + b_1 \times l_{hbS} + b_2 \times l_{sbS} = l_g^1 \] \hspace{1cm} (13)

From (13), the equilibrium goods price \( p_z \), when \( H_a \) and \( H_b \) are given and \( w = 1 \), is computed as follows.

```
In[102]:= Solve[{a_1 \times l_{haS} + a_2 \times l_{saS} + b_1 \times l_{hbS} + b_2 \times l_{sbS} = l_g^1 / w \to 1}, p_z]
```

```
Out[102]= \{
{p_z \to 2 \sqrt{-72285 + Ha + 10 Hb + 100 Ha p_I - 100 Hb p_I}},
{p_z \to 2 \sqrt{-72285 - Ha - 10 Hb + 100 Ha p_I + 100 Hb p_I}}\}
```

The same commodity price must prevail at equilibrium, so that the following condition, \( \text{solA2} \), must hold at equilibrium.

```
In[103]:= solA2 = Solve[{72285 - Ha - 10 Hb + 100 Ha p_I + 100 Hb p_I ==
72285 + Ha + 10 Hb - 100 Ha p_I - 100 Hb p_I, p_I}[[1]]]
```

```
Out[103]= \{p_I \to \frac{Ha + 10 Hb}{100 (Ha + Hb)}\}
```

Equilibrium commodity price when \( w = 1 \) is computed as the following.

```
In[104]:= Simplify[solA1[[1]] / solA2]
```

```
Out[104]= \{p_z \to 2 \sqrt{24095}\}
```

The equilibrium expected utility for household of type A, given \( H_a \) and \( H_b \), is computed as \( u_{1A} \), and its graph is depicted as in what follows.
The indirect expected utility function, $u_{A1}[H_a, H_b]$, is depicted as in what follows.

When $H_b$ is small, $u_{A1}$ has maximum at the positive $H_a$.

For instance, when $H_b=1$, $u_{A1}$ has the local maximum at $H_a=61.6467$, while the global maximum is at $H_a=0$. 
In[109]:=
Plot[uA1[Ha, 1], {Ha, 0, 100}];

In[110]:=
FindRoot[D[uA1[Ha, 1], Ha] == 0, {Ha, 60}]
Out[110]=
{Ha -> 61.6467}

When \( H_b \) is not small, however, \( uA1 \) has maximum at \( Ha=0 \).

In[111]:=
Plot3D[uA1[Ha, Hb], {Ha, 0.1, 1000}, {Hb, 50, 100}];

In[112]:=
figEA1 = Plot[uA1[Ha, 60], {Ha, 0, 1000}];
On the other hand, the indirect expected utility for household of type B, given $H_a$ and $H_b$, is computed as $u_B[H_a,H_b]$, and its graph is depicted as in what follows.

```
In[113]:=
Simplify[ub /. sol1b /. solA1 /. solA2 /. w -> 1];
uB1[H_a_, H_b_] = %
Out[114]=
9
(1/3)

\left( \frac{H_a (-97095.2 H_b) - 5 H_b (-19419.3 H_b)^2}{(H_a H_b)^2} \right) + \frac{\left( 111 H_a (7645.18 H_b) - 5 H_b (16819.36 H_b)^2 \right)}{(H_a H_b)^2})^{1/3}
\frac{400 \sqrt{5} 4819^{1/6}}
```

```
In[115]:=
Plot3D[uB1[H_a, H_b], {H_a, 0.1, 1000}, {H_b, 0.1, 1000}];
```

```
Even when $H_a$ is small, indirected utility has maximum at $H_b \approx 65$. This phenomenon emerges since insurance fee is cheap for type B households, who have higher possibility of illness.

```
In[116]:=
Plot3D[uB1[H_a, H_b], {H_a, 0.1, 5}, {H_b, 0.1, 1000}];
```

For instance, when $H_a=1$, indirected utility has maximum at $H_b=65.3268$. 
In[117]:=
Plot[uB1[1, Hb], {Hb, -500, 10000}];

In[118]:=
FindRoot[D[uB1[1, Hb], Hb] = 0, {Hb, 10}]

Out[118]=
{Hb \rightarrow 65.3268}

Even if \(H_a\) is large, indirect expected utility has maximum at \(H_b>65\).

In[119]:=
Plot3D[uB1[Ha, Hb], {Ha, 500, 550}, {Hb, 0.1, 1000}];

For instance, when \(H_a=500\), indirect utility has maximum at \(H_b=559.333\).
Thus, when the government does not intervene, type-A households and type-B households play a Nash-type noncooperative game, and equilibrium is on the boundary; i.e. $H_a=0$ and $H_b=65$. Indeed, when $H_b=65$, the indirect utility of type-A households is depicted as in figEA1. On the other hand, when $H_a=0$ the indirect expected utility of type-B households has maximum at $H_b=65$.

Thus, when the government does not intervene, the adverse selection emerges in this game-theoretic equilibrium. This is not efficient as is expected. The total social utility (sum of all the households' utilities) is $100*u_{A1}[H_a, H_b]+100*u_{B1}[H_a, H_b]$, as in what follows.

From the following figures, the maximum of the total social utility appears to be achieved at $H_a=65$ and $H_b=65$. 

In[120]:=
```
Plot[uB1[500, Hb], {Hb, 0, 6000}];
```

Out[120]=
```
```

```
In[121]:=
```
FindRoot[D[uB1[500, Hb], Hb] = 0, {Hb, 1000}]
```
Out[121]=
```
{Hb -> 559.333}
```

In[122]:=
```
FindRoot[D[uB1[0, Hb], Hb] = 0, {Hb, 100}]
```
Out[122]=
```
{Hb -> 65.}
```

In this way, when the government does not intervene, the adverse selection emerges in this game-theoretic equilibrium. This is not efficient as is expected. The total social utility (sum of all the households' utilities) is $100*u_{A1}[H_a, H_b]+100*u_{B1}[H_a, H_b]$, as in what follows.

```
In[123]:=
```
Simplify[100*u_{A1}[H_a, H_b] + 100*u_{B1}[H_a, H_b]]
```
Out[123]=
```
```

```
From the following figures, the maximum of the total social utility appears to be achieved at $H_a=65$ and $H_b=65$. 

heathage3.6aaaaa.nb
This anticipation is confirmed by the following.

Solve[{100*D[uA1[Ha, Hb] + 100*uB1[Ha, Hb], Ha] == 0, D[100*uA1[Ha, Hb] + 100*uB1[Ha, Hb], Hb] == 0}, {Ha, Hb}]

$\text{Aborted}$

\begin{align*}
\text{In}[126]:= & \quad \text{FindRoot}\left[\{D[100 \cdot uA1[Ha, Hb] + 100 \cdot uB1[Ha, Hb], Ha] == 0, \right. \\
& \quad D[100 \cdot uA1[Ha, Hb] + 100 \cdot uB1[Ha, Hb], Hb] == 0\}, \{Ha, 60\}, \{Hb, 60\}\right]
\end{align*}

Out[126]=

\{Ha \to 65., Hb \to 65.\}
References


Fukiharu, T. [1988], The Structure of Lucas-Type Neutrality of Money, *Kobe University Economic Review* 34, pp.41-77


