EXPLANATION OF THE COMPUTATION DATA FOR THE PAPER "MORDELL-WEIL GROUPS AND AUTOMORPHISM GROUPS OF ELLIPTIC $K3$ SURFACES"

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Abstract. We explain the contents of the computation data written in the file "`CompDataXfg.txt`". These data are about the numerical Néron-Severi lattice S_X, the nef-and-big cone N_X, and the automorphism group $\mathrm{Aut}(X)$ of the $K3$ surface $X = X_{f,g}$ birational to the double plane branched along a 6-cuspidal sextic curve of torus type.

In the text file "`CompDataXfg.txt`", the following data about the $K3$ surface $X = X_{f,g}$ are presented in `GAP` format. (In particular, the `Record` format of `GAP` is heavily used.) These data are obtained and used in the preprint

> [P] Ichiro Shimada: Mordell-Weil groups and automorphism groups of elliptic $K3$ surfaces.

In the following, we freely use the notation in the paper [P]. We fix a basis of S_X and a basis of L_{26}, and use these bases throughout. Vectors are written as row vectors, and matrices act on vector spaces from the right.

- `GramSX` is the Gram matrix of S_X.
- `GramL26` is the Gram matrix of L_{26}.
- `EmbSXL26` is the 13×26 matrix that expresses the primitive embedding $\iota\colon S_X \hookrightarrow L_{26}$.
- `theh` is the class $\boldsymbol{h} \in S_X$.
- `thecusprats` is the list of 6 pairs

$$[\ [\boldsymbol{e}_1^{(+)}, \boldsymbol{e}_1^{(-)}],\ [\boldsymbol{e}_2^{(+)}, \boldsymbol{e}_2^{(-)}],\ \dots,\ [\boldsymbol{e}_6^{(+)}, \boldsymbol{e}_6^{(-)}]\ ].$$

- `thegammas` is the pair $[\boldsymbol{\gamma}^{(+)}, \boldsymbol{\gamma}^{(-)}]$.
- `theample` is the ample class $\boldsymbol{a} \in S_X$.
- `thelines` is the list of classes $\tilde{\ell}_{\alpha\beta} \in S_X$.
- `groupM` is the subgroup M of $\mathrm{O}(S_X, N_X)$.

Other than these small data, we have the following three big lists:

`"V0",   "InvolOverP2s",   "MWs".`

0.1. **The list "V0".** A wall $w = D \cap (v)^\perp$ of an L_{26}/S_X-chamber $D = \mathcal{P}_X \cap \mathbf{C}(\mathbf{w})$ is given by a pair of vectors $[v, r]$, where $v \in S_X^\vee$ is a primitive defining vector of the wall w and r is the Leech root with respect to $\mathbf{w}$ defining the wall $\mathbf{C}(\mathbf{w}) \cap (r)^\perp$ of the Conway chamber $\mathbf{C}(\mathbf{w})$ such that $(v)^\perp = \mathcal{P}_X \cap (r)^\perp$.

An L_{26}/S_X-chamber D is expressed by a record `cham` that has the following items.

- `weyl` is a weyl vector $\mathbf{w} \in L_{26}$ such that $D = \mathcal{P}_X \cap \mathbf{C}(\mathbf{w})$.
- `walls` is the list of pairs $[v, r]$ describing the walls of D in the manner explained above.

We call a record of this type a `cham-record`.

The list `V0` is the list of seven records `Drec` that express L_{26}/S_X-chambers D_0 and $D_1^{(\alpha)}$ for $\alpha = 1, \dots, 6$ in V_0. Each record `Drec` has the following items.

- `name` is the name of the L_{26}/S_X-chamber $D \in V_0$, which is one of the strings `"D0", "D11", ..., "D16"`. Here `"D0"` means D_0, `"D11"` means $D_1^{(1)}$, and so on.
- `cham` is the `cham-record` expressing D.
- `adjrecs` is the list of records `adjrec`. Each record `adjrec` describes the L_{26}/S_X-chamber D' adjacent to D across a wall w of D, and has the following items.
  - `wallvect` is the primitive defining vector $v \in S_X^\vee$ of the wall w of D.
  - `israt` is `true` if the wall w is expressed as $D \cap (r)^\perp$ by some $r \in \mathrm{Rats}(X)$ and hence $D' \notin V$. Otherwise, `israt` is `false` and `adjrec` has further items expressing $D' \in V$ as follows.
  - `cham` is the `cham-record` expressing D'.
  - `isomto` is the `name` of the representative L_{26}/S_X-chamber $D'' \in V_0$ that is G-equivalent to D'.
  - `isomby` is an automorphism $g \in G$ such that $(D'')^g = D'$.

0.2. **The list `InvolOverP2s`.** The list `InvolOverP2s` is the list of records `involrec` describing involutions $i(h) \in G = \mathrm{Aut}(X)$ of type (a)-(d) obtained from the double coverings $\pi(h)\colon X \to \mathbb{P}^2$ given by the complete linear systems $|h|$ of polarizations $h \in N_X \cap S_X$ of degree 2. Each record `involrec` in this list has the following items:

- `type` is the type of the involution $i(h)$.
  - If $i(h)$ is of type (a), then $h = \boldsymbol{h}$, and `involrec.type` is equal to `["type a"]`.
  - If $i(h)$ is of type (b), then $h = h_{IJ}$, and `involrec.type` is the triple `["type b"`$, I, J]$, where $I = [i_1, i_2]$ and $J = [j_1, j_2]$ with $i_1 < i_2$ and $j_1 < j_2$.

- If $i(h)$ is of type (c), then $h = h^{\sigma}_{\alpha}$, and `involrec.type` is the triple $[$`"type c"`$, \sigma, \alpha]$, where $\sigma \in \{1, -1\}$ indicates the sign $\pm$ and $\alpha \in \{1, \dots, 6\}$.
- If $i(h)$ is of type (d), then $h = h_{\sigma J}$, and `involrec.type` is the triple $[$`"type d"`$, \sigma, J]$, where $\sigma \in \{1, -1\}$ indicates the sign $\pm$ and $J = [[i_1], [i_2, i_3], [i_4, i_5], [i_6]]$ with $i_2 < i_3$ and $i_4 < i_5$.

- `h` is the vector $h \in N_X \cap S_X$.
- `invol` is the matrix representation of $i(h) \in \mathrm{O}(S_X, \mathcal{P}_X)$.
- `singpts` is the list of records `singptrec` describing the singular points $\bar{p} \in \mathrm{Sing}(B(h))$ of the branch curve $B(h) \subset \mathbb{P}^2$ of the double covering $\pi(h)\colon X \to \mathbb{P}^2$. Each `singptrec` has the following items:
  - `ADEtype` is the ADE-type of the singular point $\bar{p}$.
  - `exceps` is the list of classes of smooth rational curves that are contracted to $\bar{p}$ by $\pi(h)\colon X \to \mathbb{P}^2$.

0.3. **The list `MWs`.** The list `MWs` is the list of records `mwrec` describing the 120 Jacobian fibrations $\phi\colon X \to \mathbb{P}^1$ obtained by $f_\phi = f_{\sigma I}$ with the zero section $z_\phi = z_{\sigma I}$, and 6+3 elements of their Mordell-Weil groups MW_ϕ. They give the automorphisms of type (e). Each record `mwrec` in this list has the following items:

- `type` is $[\sigma, I]$, where the sign σ is either 1 or -1, and $I \in \mathcal{I}$ is given by $[[i_1], [i_2, i_3, i_4], [i_5, i_6]]$ with $i_2 < i_3 < i_4$ and $i_5 < i_6$.
- `f` is the class of a fiber of the Jacobian fibration $\phi\colon X \to \mathbb{P}^1$.
- `z` is the class of the zero section of the Jacobian fibration $\phi\colon X \to \mathbb{P}^1$.
- `redfibs` is the list of records `redfib` describing the reducible fibers $\phi^*(p)$ of $\phi\colon X \to \mathbb{P}^1$. Each `redfib` has the following items:
  - `ADEtype` is the ADE-type of the reducible fiber $\phi^*(p)$.
  - `irreds` is the list of classes of irreducible components of $\phi^*(p)$ that are disjoint from the zero section.
  - `connect` is the class of the irreducible component of $\phi^*(p)$ intersecting the zero section.
- `ninesections` is the list of nine records `secrec` describing the 6+3 sections $s = \tilde{\ell}_{j_1 j_2}$ $(j_1 \in \{i_2, i_3, i_4\}, j_2 \in \{i_5, i_6\})$ and $s = \boldsymbol{e}^{(\sigma)}_j$ $(j \in \{i_2, i_3, i_4\})$ of ϕ. Each `secrec` has the following items:
  - `rat` is the class of the section s.
  - `g` is the automorphism $g \in G$ of X obtained from the translation by s.

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