ON CHARACTERISTIC POLYNOMIALS OF AUTOMORPHISMS OF ENRIQUES SURFACES: COMPUTATIONAL DATA

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1. Introduction

This note is an explanation of the computational data for the results in the paper

On characteristic polynomials of automorphisms of Enriques surfaces
(joint work with S. Brandhorst and S. Rams).

The data is available at

`http://www.math.sci.hiroshima-u.ac.jp/~shimada/K3andEnriques.html`

in three txt files

`autYrec1.txt, autYrec2.txt, autYrec3.txt.`

Each of these text files contains a data in the `Record`-format of `GAP` [2].

2. `autYrec`

We fix bases of the lattices L_{26}, S_X, S_Y, P, Q. The data below are written with resepct to these bases.

Remark 2.1. An element g_Y of $\mathrm{aut}(Y) \subset \mathrm{O}(S_Y, \mathcal{P}_Y)$ is given by a record with two components `gX` and `gY`. This record indicates the pair $[g_X, g_Y]$ of an isometry $g_X \in \mathrm{O}(S_X, \mathcal{P}_X)$ that preserves the submodule S_Y of S_X, and the isometry $g_Y = g_X | S_Y$ of S_Y.

Each record `autYrec*` has the following components.

- `No`. The number of `autYrec` (1 or 2 or 3).
- `GramL26`. The Gram matrix of L_{26}.
- `GramSX`. The Gram matrix of S_X.
- `GramSY`. The Gram matrix of S_Y.
- `GramP`. The Gram matrix ofthe orthogonal complement P of $S_Y(2)$ in L_{26}.
- `GramQ`. The Gram matrix ofthe orthogonal complement Q of $S_Y(2)$ in S_X.
- `embSXL26`. The embedding ι_X of S_X into L_{26}.
- `embSY2L26`. The embedding ι_Y of $S_Y(2)$ into L_{26}.
- `embSY2SX`. The embedding π^* of $S_Y(2)$ into S_X.
- `embPL26`. The embedding of P into L_{26}.
- `embQSX`. The embedding of Q into S_X.
- `iotaYtype`. The type of the embedding ι_Y in the notation of [1] such as `"40B"`, `"20A"`, `"20D"`.
- `generatorsAutQ`. A generating set of the group $\mathrm{O}(Q)$ of isometries of Q.
- `invol`. The action of the Enriques involution of $X \to Y$ on S_X.
- `ampleY`. An ample class α of Y contained in the interior of the initial chamber D_0.

- `autYgenerators`. The generators of $\mathrm{aut}(Y)$ (see Remark 2.1).
- `weyl0`. The Weyl vector for the Conway chamber inducing the initial chamber D_0 of $\mathcal{P}_Y$ contained in N_Y.
- `OSYD0`. The list of elements of the group $\mathrm{O}(S_Y, D_0)$.
- `Drecs`. The list that gives the list $\mathcal{R}$ of representatives of $\mathrm{aut}(Y)$-equivalence classes of induced chambers contained in N_Y. Each element of `Drecs` is a record `Drec` that has the following components. Let $D \in \mathcal{R}$ be the induced chamber that `Drec` describes.
  - `no`. The number k such that `Drec` appears at the $(k+1)$st position of `Drecs`. The first record in `Drecs` describes the initial induced chamber D_0, and has `Drec.no` $= 0$.
  - `D0toD`. An element $\tau \in \mathrm{O}(S_Y, \mathcal{P}_Y)$ that maps D_0 to D.
  - `DtoD0`. The inverse $\tau^{-1} \in \mathrm{O}(S_Y, \mathcal{P}_Y)$ of `Drec.D0toD`.
  - `autYD`. The list of elements of $\mathrm{aut}(Y, D) := \mathrm{aut}(Y) \cap \mathrm{O}(S_Y, D)$ (see Remark 2.1).
  - `walls`. The list of (-2)-vectors that defines the walls of D.
  - `ampleD`. An interior α_D point of D.
  - `orbitrecs`. The list of records `orbrec` that describe an orbit o_i of the action of $\mathrm{aut}(Y, D)$ on the set of walls of D. Each member of `orbrec` is a record that has the following components. Let o_i be the orbit decribed by `orbrec`.
    * `orbitnumber`. The number i such that `orbrec` appears at the ith position of `orbrecrecs`.
    * `orbit`. The list of (-2)-vectors in o_i.
    * `representative`. The representative vector $r \in o_i$ of this orbit. Let D' be the induced chamber adjacent to D across the wall $D \cap (r)^{\perp}$.
    * `reflection`. The reflection s_r with respect to r. Note that we have $D' = D^{s_r}$.
    * `innout`. If D' is contained in N_Y, then `orbrec.innout` is `"inner"`. Otherwise, this item is a (-2)-vector r' of S_X such that $\langle \pi^*(\alpha_D), r' \rangle > 0$ and $\langle \pi^*({\alpha_D}^{s_r}), r' \rangle < 0$. (Note that ${\alpha_D}^{s_r}$ is an interior point of D'.)
    * `isnew`. If `orbrec.innout` is not `"inner"`, then `orbrec.isnew` is `"null"`. If `orbrec.innout` is `"inner"`, then `orbrec.isnew` is either one of the following:
      · `rec(flag=true, no=k)`, which means that D' is chosen as a representatives of an $\mathrm{aut}(Y)$-equivalence class of induced chambers in N_Y and is put at the $(k+1)$st position of $\mathcal{R}$.
      · `rec(flag=false, isomwith=m, by=g)`, which means that D' is $\mathrm{aut}(Y)$-equivalent with the induced chamber D_m at the $(m+1)$st position of $\mathcal{R}$, and $g \in \mathrm{aut}(Y)$ satisfies $D_m^g = D'$.

References

[1] Brandhorst, S. and Shimada, I. Borcherds' method for Enriques surfaces, 2019. Preprint, arXiv:1903.01087.

[2] The GAP Group. *GAP - Groups, Algorithms, and Programming*. Version 4.8.6; 2016 (`http://www.gap-system.org`).

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