CONNECTED COMPONENTS OF THE MODULI OF ELLIPTIC $K3$ SURFACES: COMPUTATIONAL DATA

ICHIRO SHIMADA

This note is an explanation of the computational data calculated in the paper [2] and available from the website [3]. This data `EllipticK3s` is written in `GAP` format [1].

We use the notions and notation introduced in [2]. The data `EllipticK3s` is the list of 3693 records, each of which corresponds to a geometrically realizable combinatorial type (Φ, A) of complex elliptic $K3$ surfaces. The complete list of geometrically realizable combinatorial types has been calculated in [4, 6], and in the list `EllipticK3s`, these types are sorted according to the table [5].

Each item `X` in `EllipticK3s` is a record with the following components.

- `ADE`. The ADE-type Φ of reducible fibers.
- `rankADE`. The rank of $L(\Phi)$.
- `GramADE`. The Gram matrix of the positive definite root lattice of type `ADE`. This means that -1 times `GramADE` is the Gram matrix of $L(\Phi)$ with respect to the standard basis (a fundamental root system) of $L(\Phi)$.
- `MWtor`. The torsion part A of the Mordell-Weil group.
- `algequivclasses`. The list of algebraic equivalence classes of connected components of the moduli. Each item in this list is a record with the following components.
  - `torsionclasses`. The list of classes of torsion sections. Let $[\tau] \in U \oplus M(\Phi)$ be the class of a torsion section τ. Each vector in this list is the image of $[\tau]$ by the projection $U \oplus M(\Phi) \to M(\Phi)$, and is written with respect to the basis of $L(\Phi)^\vee$ dual to the fundamental root system of $L(\Phi)$. This list enables us to see how the torsion sections intersect the irreducible components of reducible fibers. This list also recovers the even overlattice $M(\Phi)$ of $L(\Phi)$, and hence the Néron-Severi lattice $U \oplus M(\Phi)$ of the connected components in this algebraic equivalence class.
  - `connectedcomponents`. The list of connected components in the algebraic equivalence class. When `rankADE` $= 18$, this is the list of the pairs $[T, [r, c]]$, where T is the transcendental lattice $[[a, b], [b, c]]$ of the connected component,

r is the number of real connected components, and c is the number of non-real connected components. (Hence c is always even.) When `rankADE` < 18, this is the quartet $[d_1, d_2, d_3, d_4]$, where

$$
\begin{aligned}
d_1 &:= \dim_{\mathbb{F}_2}(\Gamma_{\mathbb{A}} \times \mathrm{Sign})/(\Gamma_{\mathbb{Q}}^{\sim} \cdot (\Sigma(L, \bar{G}^{\lambda}) \times \{1\})),\\
d_2 &:= \dim_{\mathbb{F}_2}((\Gamma_{\mathbb{A}} \times \mathrm{Sign})/(\Gamma_{\mathbb{Q}}^{\sim} \cdot (\Sigma(L, \bar{G}^{\lambda}) \times \{1\})))/\langle c\rangle,\\
d_3 &:= \dim_{\mathbb{F}_2}(\Gamma_{\mathbb{A}} \times \mathrm{Sign})/(\Gamma_{\mathbb{Q}}^{\sim} \cdot (\Sigma(L, \{\mathrm{id}\}) \times \{1\})),\\
d_4 &:= \dim_{\mathbb{F}_2}((\Gamma_{\mathbb{A}} \times \mathrm{Sign})/(\Gamma_{\mathbb{Q}}^{\sim} \cdot (\Sigma(L, \{\mathrm{id}\}) \times \{1\})))/\langle c\rangle,
\end{aligned}
$$

where c is the complex conjugation. Hence 2^{d_1} is the number of connected components in the algebraic equivalence class, 2^{d_2} is the number of connected components modulo the complex conjugation (see Remark 4.16 of [2]), 2^{d_3} is the number of $\{\mathrm{id}\}$-connected components (see Remark 4.15 of [2]), 2^{d_4} is the number of $\{\mathrm{id}\}$-connected components modulo the complex conjugation.

References

[1] The GAP Group. GAP - Groups, Algorithms, and Programming. Version 4.7.9; 2015 (http://www.gap-system.org).

[2] Ichiro Shimada. Connected components of the moduli of elliptic $K3$ surfaces, 2016.

[3] Ichiro Shimada. Connected components of the moduli of elliptic $K3$ surfaces: computational data, 2016. http://www.math.sci.hiroshima-u.ac.jp/∼shimada/K3.html.

[4] Ichiro Shimada. On elliptic $K3$ surfaces. *Michigan Math. J.*, 47(3):423–446, 2000.

[5] Ichiro Shimada. On elliptic $K3$ surfaces (with a large table). http://www.math.sci.hiroshima-u.ac.jp/∼shimada/K3.html.

[6] Ichiro Shimada and De-Qi Zhang. Classification of extremal elliptic $K3$ surfaces and fundamental groups of open $K3$ surfaces. *Nagoya Math. J.*, 161:23–54, 2001.

Department of Mathematics, Graduate School of Science, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima, 739-8526 JAPAN

E-mail address: `shimada@math.sci.hiroshima-u.ac.jp`