HOLES OF THE LEECH LATTICE AND THE PROJECTIVE MODELS OF $K3$ SURFACES: COMPUTATIONAL DATA

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We explain the computational data presented in the author's web page

`http://www.math.sci.hiroshima-u.ac.jp/~shimada/K3.html`.

We employ the terminologies and notation defined in [2].

Let `X` denote one of the following symbols:

```
A[1], A[2], ..., A[24], a[1], a[2], ..., a[25],
D[4], D[5], ..., D[24], d[4], d[5], ..., d[25],
E[6], E[7], E[8], e[6], e[7], e[8].
```

The index `i` indicates the `i`th equivalence class $[\mathbf{c}_i]$ of holes in Table 3.1 of [2]. Hence `i` ranges from 1 to $23 + 284 = 307$.

- `GramLeech` is the Gram matrix of Λ with respect to the fixed basis of the Leech lattice Λ; that is, the basis given in Figure 4.12 of [1].
- `CartanMat`[`X`] is the Cartan matrix of the indecomposable Coxeter–Dynkin diagram of type `X`. For example, we have

$$\texttt{CartanMat}[\texttt{A}[3]] := [[2,-1,0,-1], [-1,2,-1,0], [0,-1,2,-1], [-1,0,-1,2]].$$

- `LH_Type`[`i`] is the hole type $\tau(\mathbf{c}_i)$. Each `LH_Type`[`i`] is a list of indecomposable Coxeter–Dynkin types. For example, we have

$$\texttt{LH\_Type}[18] := [\texttt{D}[4], \texttt{A}[5], \texttt{A}[5], \texttt{A}[5], \texttt{A}[5]],$$

  which means that $\tau(\mathbf{c}_{18}) = D_4 A_5^4$.
- `LH_Center`[`i`] is a representative hole $\mathbf{c}_i$ of the equivalence class $[\mathbf{c}_i]$ written as a row vector with respect to the fixed basis of Λ.
- `LH_Vertices`[`i`] is the list of vertices $\boldsymbol{\lambda}_j$ of $P_{\mathbf{c}_i}$, each of which is written as a row vector with respect to the fixed basis of Λ. Suppose that `LH_Type`[`i`] $= [\texttt{X}_1, \dots, \texttt{X}_\texttt{k}]$. Then the vertices of $P_{\mathbf{c}_i}$ are sorted in the list `LH_Vertices`[`i`] $= [\boldsymbol{\lambda}_1, \dots, \boldsymbol{\lambda}_n]$ in such a way that the $n \times n$ matrix

$$[\ \|\boldsymbol{\lambda}_i - \boldsymbol{\lambda}_j\|^2\ ]$$

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is equal to the matrix obtained from

$$\begin{bmatrix} \texttt{CartanMat[X}_1\texttt{]} & & \\ & \cdots & \\ & & \texttt{CartanMat[X}_\texttt{k}\texttt{]} \end{bmatrix}$$

by replacing the entries as follows: $2 \mapsto 0$, $0 \mapsto 4$, $-1 \mapsto 6$, $-2 \mapsto 8$.

- `LH_s[i]` is $s(\mathbf{c}_i)$.
- `LH_m[i]` is $m(\mathbf{c}_i)$.
- `LH_thetasquare[i]` is $\theta(\mathbf{c}_i)^2$.

References

[1] J. H. Conway and N. J. A. Sloane. *Sphere packings, lattices and groups*, volume 290 of *Grundlehren der Mathematischen Wissenschaften*. Springer-Verlag, New York, third edition, 1999. With additional contributions by E. Bannai, R. E. Borcherds, J. Leech, S. P. Norton, A. M. Odlyzko, R. A. Parker, L. Queen and B. B. Venkov.

[2] Ichiro Shimada. Holes of the Leech lattice and the projective models of $K3$ surfaces. Preprint, 2015.

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