COMPUTATIONAL DATA FOR THE PAPER "A NOTE ON CONSTRUCTIONS OF THE LEECH LATTICE"

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Abstract. We explain the contents of the computational data that was used in the paper "A note on constructions of the Leech lattice".

1. Introduction

This paper explains the numerical data for the paper

`[Note]  Ichiro Shimada: A note on constructions of the Leech lattice.`

The data is written in the plain text file

`ConstructionsLeech.txt`

in `GAP` format. The term "record" that appears below means the data structure used by `GAP`. The paper and the data are available from the author's web page.

`http://www.math.sci.hiroshima-u.ac.jp/shimada/lattice.html`

2. The data

In the file `ConstructionsLeech.txt`, the following two data are written.

2.1. **Niemeier lattices.** The data `NiemeierLattices` is the list of 23 records for the isomorphism classes of Niemeier lattices with roots. Each record contains numerical data of a negative-definite Niemeier lattice N^- with roots. These records are sorted in the descending order of the Coxeter number h of N^-.

In this data, each Niemeier lattice N^- is equipped with a certain basis, and vectors of N^- are expressed as row vectors with respect to this basis.

Each record has the following items.

- `tau` is the ADE-type $\tau(N) = \tau(N)_1 + \cdots + \tau(N)_K$. For example, the ADE-type $D_4 + 4A_5$ is written as `[ "D4", "A5", "A5", "A5", "A5" ]`.
- `h` is the Coxeter number h of N^-.
- `Gram` is the Gram matrix of N^- (with respect to the fixed basis of N^-).

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- `roots` is the list R of (-2)-vectors in N^-.
- `Theta` is the list $[\Theta_1, \dots, \Theta_K]$ of lists Θ_i of (-2)-vectors in R. The list Θ_i forms a connected ordinary ADE-configuration of type $\tau(N)_i$, and the disjoint union of $\Theta_1, \dots, \Theta_K$ is a simple root system $\Theta \subset R$ of the negative-definite root lattice $\langle R \rangle$.
- `mus` is the list $[\mu_1, \dots, \mu_K]$ of the highest roots $\mu_i \in \langle \Theta_i \rangle$ with respect to Θ_i.
- `ms` is the list $[m_1, \dots, m_K]$ of functions $m_i \colon \Theta_i \to \mathbb{Z}_{>0}$ defined in Section 2.3 of the paper [Note]. Each function m_i is presented by the list $[m_i(r_1), \dots, m_i(r_{n_i})]$ of integers, where Θ_i is given by the list $[r_1, \dots, r_{n_i}]$ of (-2)-vectors in the item `Theta` above.
- `rhos` is the list $[\rho_1, \dots, \rho_K]$ of the vectors $\rho_i \in \langle \Theta_i \rangle \otimes \mathbb{Q}$ satisfying $\langle r, \rho_i \rangle_N = 1$ for all $r \in \Theta_i$.
- `rho` is the Weyl vector $\rho = \rho_1 + \dots + \rho_K$ of N^- with respect to Θ.
- `code` indicates the isomorphism class of the finite abelian group $N^-/\langle R \rangle$. For example, the empty list $[\,]$ indicates $N^-/\langle R \rangle = 0$, and the list $[2, 3]$ indicates $N^-/\langle R \rangle \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$.
- `codewords` is the list of codewords of the code $N^-/\langle R \rangle$. Each codeword $\gamma \in N^-/\langle R \rangle$ is presented by its canonical representative $\tilde{\gamma} \in N^-$.

2.2. **The Leech lattice.** The data `LeechLattice` is a record that contains numerical data of the negative-definite Leech lattice Λ^-. The Leech lattice Λ^- is equipped with a certain basis, and vectors of $\Lambda^- \otimes \mathbb{Q}$ are expressed as row vectors with respect to this basis. This record has the following items.

- `Gram` is the Gram matrix of Λ^- (with respect to the fixed basis of Λ^-).
- `holes` is the list of Co_∞-equivalence classes of deep holes. These equivalence classes are sorted by the corresponding Niemeier lattices in the same order as in the data `NiemeierLattices`. Each Co_∞-equivalence class is described by a record that contains the following items.
  - `center` is the vector $c \in \Lambda^- \otimes \mathbb{Q}$ of a deep hole in this equivalence class.
  - `tau` is the ADE-type $\tau(c) = \tau(c)_1 + \dots + \tau(c)_K$. For example, the ADE-type $D_4 + 4A_5$ is expressed as `[ "D4", "A5", "A5", "A5", "A5" ]`.
  - `h` is the the Coxeter number h of the deep hole c.
  - `P0` is the list of vectors $\lambda \in \Lambda^-$ such that $d(\lambda, c)^2 = 2$. This list `P0` is given as a list $[\mathtt{P0}_1, \dots, \mathtt{P0}_K]$ in such a way that `P0` is decomposed as the disjoint union of $\mathtt{P0}_1, \dots, \mathtt{P0}_K$, and that, for $i = 1, \dots, K$, the set of Leech roots $\Xi_0(c)_i = \{ r_\lambda \mid \lambda \in \mathtt{P0}_i \}$ of L_Λ with respect to w_Λ forms an extended connected ADE-configuration of type $\tau(c)_i$.

- ms is the list $[m_1, \dots, m_K]$ of functions $m_i \colon \Xi_0(c)_i \to \mathbb{Z}_{>0}$ defined in Section 2.3 of the paper [Note]. Suppose that $\mathtt{P0}_i$ is given by the list $[\lambda_1, \dots, \lambda_n]$ of vectors of Λ in the item P0 above. Then the function m_i is presented by the list $[m_i(r_{\lambda_1}), \dots, m_i(r_{\lambda_n})]$ of integers.
- P1 is the list of vectors $\lambda \in \Lambda^-$ such that $d(\lambda, c)^2 = 2(1 + 1/h)$.

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