ON ELLIPTIC $K3$ SURFACES

ICHIRO SHIMADA

Abstract. We make a complete list of all possible ADE-types of singular fibers of complex elliptic $K3$ surfaces and the torsion parts of their Mordell-Weil groups.

1. Introduction

By virtue of Torelli theorem for the period map on the moduli of complex $K3$ surfaces ([4], [13], [18]), we can study many aspects of $K3$ surfaces from the lattice-theoretic point of view. In this paper, we determine all possible ADE-types of singular fibers of elliptic $K3$ surfaces using Nikulin's theory of discriminant forms of even integral lattices. We also determine, for each ADE-type of singular fibers, all possible torsion parts of the Mordell-Weil groups. Throughout this paper, we use the term "an elliptic $K3$ surface" for "a complex elliptic $K3$ surface with a distinguished zero section" and the term "an elliptic fibration" for "a complex Jacobian elliptic fibration".

A finite formal sum of the symbols A_l $(l \ge 1)$, D_m $(m \ge 4)$ and E_n $(n = 6, 7, 8)$ with non-negative integer coefficients is called an ADE-type. For an ADE-type

$$\Sigma := \sum a_l A_l + \sum d_m D_m + \sum e_n E_n,$$

we denote by $L(\Sigma)^-$ the negative-definite root lattice generated by a root system of type Σ, and by $\operatorname{rank}\Sigma$ the rank of $L(\Sigma)^-$. By definition, we have $\operatorname{rank}\Sigma = \sum a_l l + \sum d_m m + \sum e_n n$.

Let $f : X \to \mathbb{P}^1$ be an elliptic $K3$ surface, and $O : \mathbb{P}^1 \to X$ the zero section of f. Let MW_f be the Mordell-Weil group of f. The torsion part of MW_f is a finite abelian group, which we shall denote by G_f. We put

$$R_f := \{\, p \in \mathbb{P}^1 \ :\ f^{-1}(p) \text{ is reducible } \},$$

and, for each $p \in R_f$, we denote by $f^{-1}(p)^\sharp$ the union of irreducible components of $f^{-1}(p)$ that are disjoint from the zero section. It is known that the cohomology classes of irreducible components of $f^{-1}(p)^\sharp$ span a negative-definite root lattice generated by an indecomposable root system of type A_l, D_m or E_n. Let $\tau_{f,p}$ be the type. The type of singular fiber $f^{-1}(p)$ in the list of Kodaira's classification [7] is related to $\tau_{f,p}$ in an almost one-to-one way (cf. Table 2.2). We define the ADE-type Σ_f of $f : X \to \mathbb{P}^1$ by

$$\Sigma_f := \sum_{p \in R_f} \tau_{f,p}.$$

1991 *Mathematics Subject Classification.* Primary 14J28; Secondary 14Q10.

The Néron-Severi lattice NS_X of X contains the sublattice S_f generated by the cohomology classes of the irreducible components of $\cup_{p\in R_f} f^{-1}(p)^\sharp$, which is isomorphic to $L(\Sigma_f)^-$.

Through computer-aided calculation, we have made the complete list of pairs (Σ, G) of an ADE-type Σ and a finite abelian group G that can be realized as the data (Σ_f, G_f) of an elliptic $K3$ surface $f : X \to \mathbb{P}^1$. This list $\mathcal{P}$ consists of 3693 pairs, and is given at the end of this paper. We deduce some geometric facts from it, and explain the algorithm for obtaining it.

An elliptic $K3$ surface $f : X \to \mathbb{P}^1$ is said to be extremal if the sublattice S_f attains the maximal rank 18. After the work of Miranda and Persson [10], supplemented by Artal-Bartolo, Tokunaga and Zhang [1] and Ye [23], the ADE-types of singular fibers of extremal elliptic $K3$ surfaces and their Mordell-Weil groups were completely determined in [16]. The list consists of 336 pairs.

By Nishiyama [12] and by Besser [2] , the technique of discriminant forms was used to find out all possible elliptic fibrations on special $K3$ surfaces. In [19, 20], Urabe investigated possible configurations of singular points on $K3$ surfaces and suggested an existence of a set of simple rules that generates all possible configurations. In [21, 22], Yang made the complete list of all possible configurations of singularities of ADE-type on plane sextic curves and quartic surfaces using the technique of discriminant forms and a computer.

This paper is organized as follows. In § 2, we state some facts about elliptic $K3$ surfaces that can be derived from the list $\mathcal{P}$. In § 3, we recall the definition and properties of local invariants of lattices over $\mathbb{Z}$ according to Conway and Sloane [6, Chapter 15]. In § 4 and 5, we review Nikulin's theory [11] of discriminant forms of even lattices over $\mathbb{Z}$. A criterion whether there exists an even integral lattice of a given signature and a discriminant form is described in detail in § 5. This criterion is slightly different from [11, Theorem 1.10.1], and is more suited to machine calculation. In § 6, we recall the properties of root lattices. In § 7, we show that it is possible to determine by a purely lattice-theoretic calculation whether a given pair (Σ, G) can be realized as (Σ_f, G_f) of an elliptic $K3$ surface $f : X \to \mathbb{P}^1$. Here we use Kondo-Nishiyama's lemma on the Néron-Severi lattice of an elliptic $K3$ surface. In § 8, we explain our algorithm.

The program for making $\mathcal{P}$ was written by Maple V. The author would like to thank Waterloo Maple Incorporation for developing the nice software. The author also would like to thank the referee for suggesting some improvements on the first version of the paper.

2. Main results

All results in this section are obtained simply by looking at the list $\mathcal{P}$.

2.1. **Torsion parts of Mordell-Weil groups.**

TABLE 2.1. Cardinalities of $\mathcal{P}^G$

G	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[2, 2]	[4, 2]	[6, 2]	[3, 3]	[4, 4]	total
$\|\mathcal{P}^G\|$	2746	732	85	41	6	10	1	1	61	5	1	3	1	3693

Theorem 2.1. *The torsion part of the Mordell-Weil group of an elliptic $K3$ surface is isomorphic to one of the following*:

$$\begin{aligned} &(0),\ \ \mathbb{Z}/(2),\ \ \mathbb{Z}/(3),\ \ \mathbb{Z}/(4),\ \ \mathbb{Z}/(5),\ \ \mathbb{Z}/(6),\ \ \mathbb{Z}/(7),\ \ \mathbb{Z}/(8),\\ &\mathbb{Z}/(2)\times\mathbb{Z}/(2),\ \ \mathbb{Z}/(4)\times\mathbb{Z}/(2),\ \ \mathbb{Z}/(6)\times\mathbb{Z}/(2),\\ &\mathbb{Z}/(3)\times\mathbb{Z}/(3),\ \ \mathbb{Z}/(4)\times\mathbb{Z}/(4). \end{aligned} \tag{2.1}$$

For a group G in (2.1), we denote by $\mathcal{P}^G$ the set of all ADE-types Σ such that there exists an elliptic $K3$ surface $f : X \to \mathbb{P}^1$ with $\Sigma_f = \Sigma$ and $G_f \cong G$. The cardinalities of $\mathcal{P}^G$ are given in Table 2.1. Here, $[a]$ denotes the cyclic group $\mathbb{Z}/(a)$, and $[a, b]$ denotes $\mathbb{Z}/(a) \times \mathbb{Z}/(b)$. In particular, [1] denotes the trivial group.

Theorem 2.2. *Let $f : X \to \mathbb{P}^1$ be an elliptic $K3$ surface. Then the following hold.*

- $G_f \cong \mathbb{Z}/(7) \iff \Sigma_f = 3A_6$.
- $G_f \cong \mathbb{Z}/(8) \iff \Sigma_f = 2A_7 + A_3 + A_1$.
- $G_f \cong \mathbb{Z}/(6) \times \mathbb{Z}/(2) \iff \Sigma_f = 3A_5 + 3A_1$.
- $G_f \cong \mathbb{Z}/(4) \times \mathbb{Z}/(4) \iff \Sigma_f = 6A_3$.
- $G_f \cong \mathbb{Z}/(3) \times \mathbb{Z}/(3) \iff \Sigma_f \in \{2A_5 + 4A_2, A_5 + 6A_2, 8A_2\}$.

Remark 2.3. Elliptic $K3$ surfaces with $G_f = [7], [8], [6, 2], [4, 4]$ are constructed as elliptic modular surfaces (cf. [17, 14]). The corresponding congruence groups $\Gamma \subset \mathrm{SL}_2(\mathbb{Z})$ are as follows.

G_f	[7]	[8]	[6, 2]	[4, 4]
Γ	$\Gamma_1(7)$	$\Gamma_1(8)$	$\Gamma_0(3) \cap \Gamma(2)$	$\Gamma(4)$

2.2. **From ADE-types to configurations of singular fibers.** The correspondence between the type (in the notation of Kodaira) of a singular fiber of an elliptic fibration and an ADE-type is shown in Table 2.2. There are following ambiguities in recovering the configurations of singular fibers from its ADE-type.

- An irreducible singular fiber is of type either I_1 or II.
- A singular fiber of ADE-type A_1 is of type either I_2 or III.
- A singular fiber of ADE-type A_2 is of type either I_3 or IV.

We present some restrictions on the possibilities of configuration of singular fibers of an elliptic $K3$ surface $f : X \to \mathbb{P}^1$ with a given ADE-type.

Let i_b be the number of singular fibers of f of type I_b. We define similarly i_b^*, ii, ii^*, iii, iii^*, iv, iv^*. Miranda and Persson gave a formula for the degree of the modulus function $J_f : \mathbb{P}^1 \to \mathbb{P}^1 := \mathfrak{H}/\mathrm{SL}_2(\mathbb{Z})$ associated with $f : X \to \mathbb{P}^1$:

$$\deg J_f := \sum_{b\geq 1} b(i_b + i_b^*).$$

TABLE 2.2. Singular fibers of elliptic fibration.

Singular fiber	ADE-type	Euler number	Possible Torsion Parts
I_0	regular	0	all
I_1	irreducible	1	$\diamondsuit$
$\mathrm{I}_b \quad (b \geq 2)$	A_{b-1}	b	
$\mathrm{I}_b^* \quad (b \geq 0)$	D_{4+b}	$6+b$	$\begin{cases} [1],[2],[2,2] & \text{if } b \text{ is even} \\ [1],[2],[4] & \text{if } b \text{ is odd} \end{cases}$
II	irreducible	2	$[1]$
II^*	E_8	10	$[1]$
III	A_1	3	$[1],[2]$
III^*	E_7	9	$[1],[2]$
IV	A_2	4	$[1],[3]$
IV^*	E_6	8	$[1],[3]$

$$\diamondsuit \begin{cases} [a] \text{ is possible for } a = 1, \dots, 8, \\ [2a, 2] \text{ is possible for } a = 1, \dots, 3 \text{ if and only if } b = 0 \bmod 2, \\ [3, 3] \text{ is possible if and only if } b = 0 \bmod 3, \\ [4, 4] \text{ is possible if and only if } b = 0 \bmod 4. \end{cases}$$

By the Hurwitz formula, they obtained the following necessary condition for configurations; if $\deg J_f > 0$, then

$$\deg J_f \leq 6 \sum_{b \geq 1} (i_b + i_b^*) + 4(ii + iv^*) + 3(iii + iii^*) + 2(iv + ii^*) - 12.$$

See [9, §3] for the proof.

The euler number 24 of the $K3$ surface X is equal to the sum of euler numbers of singular fibers of f. The third column of Table 2.2 shows the euler number of a singular fiber of each type. We define the euler number euler(Σ) of an ADE-type $\Sigma := \sum a_l A_l + \sum d_m D_m + \sum e_n E_n$ by

$$\text{euler}(\Sigma) := \sum a_l \cdot (l+1) + \sum d_m \cdot (m+2) + \sum e_n \cdot (n+2).$$

Then euler(Σ_f) is less than or equal to the sum of euler numbers of reducible singular fibers. Hence we always have

$$\text{euler}(\Sigma_f) \leq 24.$$

We can deduce from Table 2.2, for example, that, if euler$(\Sigma_f) = 24$, then$f : X \to \mathbb{P}^1$ has no irreducible fibers nor fibers of type III or IV.

Table 2.3. Cardinalities of $\mathcal{R}_r$, $\mathcal{E}_r$ and $\mathcal{P}_r$

r	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	total
$\|\mathcal{R}_r\|$	1	2	3	6	9	16	24	39	57	88	128	193	276	403	570	815	1137	1599	5366
$\|\mathcal{E}_r\|$	1	2	3	6	9	16	24	39	57	88	128	193	274	393	531	688	773	712	3937
$\|\mathcal{P}_r\|$	1	2	3	6	9	16	24	39	57	88	128	193	274	392	518	624	580	325	3279
$\|\mathcal{T}_r\|$	1	2	3	6	9	16	24	38	55	82	115	162	217	289	362	419	372	188	2360

When G_f is non-trivial, certain types of singular fibers cannot appear. Let $g : S \to \Delta$ be an elliptic fibration over an open unit disk Δ such that g is smooth over $\Delta^\times := \Delta \setminus \{0\}$, and let $E := g^{-1}(p)$ be the fiber over a point $p \in \Delta^\times$. Looking at the monodromy action of $\pi_1(\Delta^\times, p)$ on the set of torsion points of E, we can determine whether a finite abelian group can be embedded into the Mordell-Weil group of g. The fourth column of Table 2.2 shows the groups among the list (2.1) that can be isomorphic to the torsion part of the Mordell-Weil group of an elliptic surface having the singular fiber. We see, for example, that, if G_f is non-trivial, then every irreducible singular fiber must be of type I_1.

2.3. **Miscellaneous facts.** For an integer r with $1 \le r \le 18$, we put as follows.

$\mathcal{R}_r := \{\ \Sigma\ ;\ \Sigma \text{ is an } ADE\text{-type with } \mathrm{rank}(\Sigma) = r\ \ \}$,

$\mathcal{E}_r := \{\ \Sigma \in \mathcal{R}_r\ ;\ \mathrm{euler}(\Sigma) \le 24\ \}$, and

$\mathcal{P}_r := \{\ \Sigma \in \mathcal{E}_r\ ;\ \text{there exists an elliptic } K3 \text{ surface } f : X \to \mathbb{P}^1 \text{ with } \Sigma_f = \Sigma\ \} = \cup_G \mathcal{P}_r^G$.

For $\Sigma \in \cup_{r=1}^{18} \mathcal{P}_r$, we denote by $\mathcal{G}(\Sigma)$ the set of isomorphism classes of finite abelian groups G such that $(\Sigma, G) \in \mathcal{P}$. For each r, we denote by $\mathcal{T}_r$ the set of $\Sigma \in \mathcal{P}_r$ such that $\mathcal{G}(\Sigma)$ consists of only the trivial group $[1]$. The cardinalities of these sets are given in Table 2.3. Note that, if $\mathrm{rank}(\Sigma) \le 12$, then $\mathrm{euler}(\Sigma) \le 24$ holds automatically.

Theorem 2.4. *Let Σ be an ADE-type with* $\mathrm{euler}(\Sigma) \le 24$. *Suppose that* $\mathrm{rank}(\Sigma) \le 13$. *Then there exists an elliptic $K3$ surface $f : X \to \mathbb{P}^1$ with $\Sigma_f = \Sigma$.*

Remark 2.5. The complement of $\mathcal{P}_{14}$ in $\mathcal{E}_{14}$ consists of a single element $E_6 + 8A_1$. Hence, when $\mathrm{euler}(\Sigma) \le 24$ and $\mathrm{rank}(\Sigma) = 14$, there exists an elliptic $K3$ surface $f : X \to \mathbb{P}^1$ with $\Sigma_f = \Sigma$ if and only if $\Sigma \ne E_6 + 8\ A_1$.

Theorem 2.6. *Suppose that* $\mathrm{rank}(\Sigma) \le 10$. *Then there exists an elliptic $K3$ surface $f : X \to \mathbb{P}^1$ with $G_f = [1]$ and $\Sigma_f = \Sigma$.*

Remark 2.7. The complement $\mathcal{P}_{11} \setminus \mathcal{P}_{11}^{[1]}$ consists of a single element $11A_1$. We have $\mathcal{G}(11A_1) = \{[2]\}$.

Theorem 2.8. *Let $f : X \to \mathbb{P}^1$ be an elliptic $K3$ surface. If* $\mathrm{rank}(\Sigma_f) \le 7$, *then G_f must be trivial.*

Remark 2.9. The complement $\mathcal{P}_8^{[1]} \setminus \mathcal{T}_8$ consists of a single element $8A_1$, and the complement $\mathcal{P}_9^{[1]} \setminus \mathcal{T}_9$ consists of two elements $9A_1$ and $A_3 + 6A_1$. We have

$$\mathcal{G}(8A_1) = \mathcal{G}(9A_1) = \mathcal{G}(A_3 + 6A_1) = \{[1], [2]\}.$$

Remark 2.10. There are several ADE-types Σ with $|\mathcal{G}(\Sigma)| \geq 3$. For example,

$$\mathcal{G}(2A_5 + 2A_2 + 2A_1) = \mathcal{G}(A_{11} + 2A_2 + 2A_1) = \{[1], [2], [3], [6]\}.$$

3. Local invariants of lattices

First we fix some terminologies about lattices.

Let R be either $\mathbb{Z}$ or $\mathbb{Z}_p$. A *lattice* over R is, by definition, a free R-module L of finite rank equipped with a non-degenerate symmetric bilinear form $(\ ,\ ) : L \times L \to R$. For $\alpha \in R \setminus \{0\}$, let αL denote the lattice obtained from L by multiplying the symmetric bilinear form by α. We will denote L^- for $(-1)\, L$. We often express a lattice by the intersection matrix with respect to a certain basis of L. For example, (a) is the lattice of rank 1 generated by a vector e such that $(e, e) = a$. A sublattice N of L is said to be *primitive* if L/N is torsion free. A lattice L over R is said to be *even* if $(v, v) \in 2\,R$ holds for any $v \in L$. Note that, when R is $\mathbb{Z}_p$ with p an odd prime, every lattice over R is even. The *discriminant* $\operatorname{disc}(L)$ of a lattice L is considered as an element of $(R \setminus \{0\})/(R^\times)^2$. A lattice L is said to be *unimodular* if $\operatorname{disc}(L) \in R^\times/(R^\times)^2$.

Suppose that $R = \mathbb{Z}_p$. Then we have $\operatorname{disc}(L) = p^\nu u$ for some $\nu \geq 0$, where $u \in \mathbb{Z}_p^\times/(\mathbb{Z}_p^\times)^2$. We denote the element u by $\operatorname{reddisc}(L)$ and call it the *reduced discriminant* of L.

Let k be the quotient field of R. The k-vector space $L \otimes_R k$ has a natural symmetric bilinear form with values in k. We denote by $L^\vee$ the R-submodule of $L \otimes_R k$ consisting of all vectors v such that $(v, w) \in R$ holds for every $w \in L$, and call it the *dual lattice* of L. An R-submodule M of $L^\vee$ is said to be an *overlattice* of L if M contains L and the symmetric bilinear form restricted to M takes values in R. Two lattices L and M over R are said to be *k-equivalent* if $L \otimes_R k$ and $M \otimes_R k$ together with their symmetric k-valued bilinear forms are isomorphic.

For a detailed account of the following definitions and theorems, see Conway and Sloane [6, Chapter 15] and Cassels [5, Chapters 8 and 9].

3.1. **Local invariants.** Let Λ be a lattice over $\mathbb{Z}_p$. Then Λ is decomposed into the orthogonal direct sum $\Lambda = \bigoplus_{\nu \geq 0} p^\nu \Lambda_\nu$ with each Λ_ν being unimodular. This decomposition is called a *Jordan decomposition* of Λ, and each $p^\nu \Lambda_\nu$ is called a *Jordan component* of Λ. Note that the reduced discriminant of Λ is the product of the discriminants of Λ_ν.

Suppose that p is odd. Then a lattice Λ over $\mathbb{Z}_p$ is isomorphic to an orthogonal direct sum $\oplus_i p^{\nu_i}(a_i)$, where $a_i \in \mathbb{Z}_p^\times$. The *$p$-excess* of Λ is defined to be

$$-\operatorname{rank}(\Lambda) + 4m + \sum_i p^{\nu_i} \in \mathbb{Z}/(8),$$

where m is the number of orthogonal direct summands $p^{\nu_i}(a_i)$ such that ν_i is odd and that a_i is not square in $\mathbb{Z}_p^\times$. It is known that the p-excess is a well-defined invariant of $\mathbb{Q}_p$-equivalence classes of lattices over $\mathbb{Z}_p$.

Suppose that $p = 2$. We put

$$U := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad V := \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix},$$

both of which are unimodular lattices of rank 2 over $\mathbb{Z}_2$. Then a lattice over $\mathbb{Z}_2$ is decomposed into the orthogonal direct sum of lattices such that each direct summand is isomorphic to $2^\nu(a)$ ($a \in \mathbb{Z}_2^\times$), $2^\nu U$ or $2^\nu V$. We define the 2-excesses of these lattices by

$$\begin{aligned} \text{2-excess}(2^\nu(a)) &= \begin{cases} 1-a \bmod 8 & \text{if } \nu \text{ is even or } a = \pm 1 \bmod 8, \\ 5-a \bmod 8 & \text{if } \nu \text{ is odd and } a = \pm 3 \bmod 8, \end{cases} \\ \text{2-excess}(2^\nu U) &= \ 2 \bmod 8, \\ \text{2-excess}(2^\nu V) &= \begin{cases} 2 \bmod 8 & \text{if } \nu \text{ is even}, \\ 6 \bmod 8 & \text{if } \nu \text{ is odd}. \end{cases} \end{aligned}$$

Then we define the 2-excess of

$$\Lambda \cong \bigoplus_i 2^{\nu_i}(a_i) \oplus \bigoplus_j 2^{\nu_j} U \oplus \bigoplus_k 2^{\nu_k} V \tag{3.1}$$

to be the sum of the 2-excesses of direct summands in the decomposition (3.1). Even though the decomposition (3.1) is not unique in general, it turns out that the 2-excess is a well-defined invariant of $\mathbb{Q}_2$-equivalence classes of lattices over $\mathbb{Z}_2$. (Note that U and V are $\mathbb{Q}_2$-equivalent to $2(1) \oplus 2(7)$ and $2(1) \oplus 2(3)$, respectively.)

3.2. **Existence of lattices over $\mathbb{Z}$ with given local data.** Combining [6, Chapter 15, Theorem 5] and [5, Chapter 9, Theorem 1.2], we obtain the following:

Theorem 3.1. *Let d be a non-zero integer, and (r, s) a pair of non-negative integers such that $n := r + s$ is positive and that $d = (-1)^s|d|$ holds. Suppose that, for each prime divisor p of $2d$, a lattice $\Lambda^{(p)}$ of rank n over $\mathbb{Z}_p$ is given. Then there exists a lattice L over $\mathbb{Z}$ with discriminant d and signature (r, s) such that $L \otimes_{\mathbb{Z}} \mathbb{Z}_p$ is isomorphic to $\Lambda^{(p)}$ for each p if and only if the following two conditions are satisfied:*

(i) $\operatorname{disc}(\Lambda^{(p)})$ *is equal to* $d \cdot (\mathbb{Z}_p^\times)^2$ *for each* p, *and*
(ii) $r - s + \sum_{p|2d} p\text{-excess}(\Lambda^{(p)}) = n \bmod 8$ *holds.* □

4. Theory of discriminant forms

4.1. **Definitions.** Let R and k be as above. Let D be a finite abelian group. A finite symmetric bilinear form on D with values in k/R is, by definition, a homomorphism $b : D \times D \to k/R$ such that $b(x, y) = b(y, x)$ holds for any $x, y \in D$. A finite quadratic form on D with values in $k/2R$ is a map $q : D \to k/2R$ with the following properties:

(i) $q(nx) = n^2 q(x)$ for $n \in \mathbb{Z}$ and $x \in D$, and
(ii) the map $b[q] : D \times D \to k/R$ defined by $(x, y) \mapsto (q(x + y) - q(x) - q(y))/2$ is a finite symmetric bilinear form.

Let H be a subgroup of D. The orthogonal complement $H^\perp$ of H with respect to q is the subgroup of D consisting of elements y such that $b[q](x, y) = 0$ holds for any $x \in H$. We say that q is non-degenerate if $D^\perp = (0)$. Note that, if $D = H \oplus H^\perp$, then q is written as $q|_H \oplus q|_{H^\perp}$, because the homomorphism $a \mapsto a/2$ from $k/2R$ to k/R is injective.

The length of D is, by definition, the minimal number of generators of D. A subset $\{\gamma_1, \dots, \gamma_l\}$ of D is said to be a *reduced set of generators* of D if l is the length of D and $D = \langle\gamma_1\rangle \times \cdots \times \langle\gamma_l\rangle$ holds. Let $\{\gamma_1, \dots, \gamma_l\}$ be a reduced set of

TABLE 4.1. Discriminant forms of lattices over $\mathbb{Z}_p$

Λ	$p^\nu(a)$	$2^\nu U$	$2^\nu V$
D_Λ	$\mathbb{Z}/(p^\nu)$	$(\mathbb{Z}/(2^\nu))^{\oplus 2}$	$(\mathbb{Z}/(2^\nu))^{\oplus 2}$
q_Λ	$\begin{bmatrix}\frac{a}{p^\nu}\end{bmatrix}$	$\frac{1}{2^\nu}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$	$\frac{1}{2^\nu}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$

generators of D. Then a finite quadratic form q on D is expressed by a symmetric $l \times l$ matrix whose diagonal entries are $q(\gamma_i) \in k/2R$ and whose off-diagonal entries are $b[q](\gamma_i, \gamma_j) \in k/R$.

Let L be a lattice over R. The discriminant group D_L of L is, by definition, the quotient group $L^\vee/L$. We denote by $\Psi_L : L^\vee \to D_L$ the natural projection. Suppose that L is even. Then we can define a finite quadratic form q_L on D_L with values in $k/2R$ by $q_L(x) := (x', x') \bmod 2R$, where x' is a vector of $L^\vee$ such that $\Psi_L(x') = x$. We call q_L the *discriminant form* of L. Because L is non-degenerate, q_L is also non-degenerate. By definition, we have $(D_{L\oplus M}, q_{L\oplus M}) = (D_L, q_L) \oplus (D_M, q_M)$.

4.2. **Discriminant forms and overlattices.** The following two propositions, due to Nikulin, play a central role in making the list $\mathcal{P}$.

Proposition 4.1 ([11] Proposition 1.4.1)**.** *Let L be an even lattice over $\mathbb{Z}$.*

(1) *If $H \subset D_L$ is a subgroup isotopic with respect to q_L, then $M := \Psi_L^{-1}(H)$ is an even overlattice of L, and the discriminant form of M is isomorphic to $(H^\perp/H, q_L|_{H^\perp/H})$.*

(2) *The map $H \mapsto \Psi_L^{-1}(H)$ establishes a bijection between the set of isotopic subgroups of (D_L, q_L) and the set of even overlattices of L.* □

Proposition 4.2 ([11] Proposition 1.6.1)**.** *Let L and M be even lattices over $\mathbb{Z}$. Then the following are equivalent.*

(i) *The two finite quadratic forms (D_L, q_L) and $(D_M, -q_M)$ are isomorphic.*
(ii) *There exists an even unimodular overlattice of $L \oplus M$ into which L and M are embedded primitively.*

□

4.3. **Localization and discriminant form.** Let L be an even lattice over $\mathbb{Z}$. We decompose D_L into the direct sum of its p-Sylow subgroups $D_L^{(p)}$, where p runs through the set of prime divisors of $|D_L| = |\operatorname{disc}(L)|$. These p-parts are orthogonal to each other with respect to q_L, and hence q_L is also decomposed into the p-parts; $q_L = \oplus_p q_L^{(p)}$, where $q_L^{(p)}$ is the restriction of q_L to $D_L^{(p)}$. By the definition of the discriminant form, we can easily prove the following:

Lemma 4.3. *The image of $q_L^{(p)}$ is contained in $2\,\mathbb{Z}[1/p]/2\,\mathbb{Z} \subset \mathbb{Q}/2\,\mathbb{Z}$. The natural inclusion $2\,\mathbb{Z}[1/p] \hookrightarrow \mathbb{Q}_p$ induces an isomorphism $2\,\mathbb{Z}[1/p]/2\,\mathbb{Z} \cong \mathbb{Q}_p/2\,\mathbb{Z}_p$ Under this identification, $(D_L^{(p)}, q_L^{(p)})$ is isomorphic to $(D_{L\otimes\mathbb{Z}_p}, q_{L\otimes\mathbb{Z}_p})$.* □

The discriminant form of an even lattice Λ over $\mathbb{Z}_p$ is calculated by Table 4.1. In particular, D_Λ is a p-group of length equal to $\operatorname{rank}(\Lambda) - \operatorname{rank}(\Lambda_0)$, where Λ_0 is the first Jordan component of Λ. We also have $\operatorname{disc}(\Lambda) = |D_\Lambda| \cdot \operatorname{reddisc}(\Lambda)$.

5. Existence of lattices with a given discriminant form

5.1. **Over $\mathbb{Z}_p$.** Suppose that a finite abelian p-group D and a non-degenerate finite quadratic form $q : D \to \mathbb{Q}_p/2\,\mathbb{Z}_p$ are given. It is known that, if $n \ge \operatorname{length}(D)$, then there exists an even lattice Λ of rank n over $\mathbb{Z}_p$ such that (D_Λ, q_Λ) is isomorphic to (D, q). The purpose of this subsection is to describe a method to determine the set $\mathcal{L}^{(p)}(n, D, q)$ of all $[\sigma, u] \in \mathbb{Z}/(8) \times \mathbb{Z}_p^\times/(\mathbb{Z}_p^\times)^2$ such that there exists an even lattice Λ of rank n over $\mathbb{Z}_p$ with $(D_\Lambda, q_\Lambda) \cong (D, q)$, p-excess$(\Lambda) = \sigma$ and $\operatorname{reddisc}(\Lambda) = u$.

Note that

$$p\text{-excess}(\Lambda_1 \oplus \Lambda_2) = p\text{-excess}(\Lambda_1) + p\text{-excess}(\Lambda_2), \quad \text{and}$$
$$\operatorname{reddisc}(\Lambda_1 \oplus \Lambda_2) = \operatorname{reddisc}(\Lambda_1) \cdot \operatorname{reddisc}(\Lambda_2).$$

Taking these into account, for sets $\mathcal{L}$ and $\mathcal{L}'$ of elements of $\mathbb{Z}/(8) \times \mathbb{Z}_p^\times/(\mathbb{Z}_p^\times)^2$, we define $\mathcal{L} * \mathcal{L}'$ to be the set

$$\{\ [\sigma + \sigma', uu']\ ;\ [\sigma, u] \in \mathcal{L},\ [\sigma', u'] \in \mathcal{L}'\ \}.$$

We also put $\mathcal{L}_0^{(p)} := \{[0, 1]\}$. Then $\mathcal{L} * \mathcal{L}_0^{(p)} = \mathcal{L}$ holds for any $\mathcal{L}$.

Lemma 5.1. *Let l be the length of D. Then we have*

$$\mathcal{L}^{(p)}(n, D, q) = \mathcal{L}^{(p)}(n - l, (0), [0]) * \mathcal{L}^{(p)}(l, D, q). \tag{5.1}$$

If p is odd, then

$$\mathcal{L}^{(p)}(n - l, (0), [0]) = \begin{cases} \emptyset & \text{if } n < l, \\ \mathcal{L}_0^{(p)} & \text{if } n = l, \\ \{[0, 1], [0, v_p]\} & \text{if } n > l, \end{cases} \tag{5.2}$$

where v_p is the unique non-trivial element of $\mathbb{Z}_p^\times/(\mathbb{Z}_p^\times)^2$. If $p = 2$, then

$$\mathcal{L}^{(2)}(n - l, (0), [0]) = \begin{cases} \emptyset & \text{if } n < l \text{ or } n - l \bmod 2 = 1, \\ \mathcal{L}_0^{(2)} & \text{if } n = l, \\ \{[n - l, 1], [n - l, 5]\} & \text{if } n > l \text{ and } n - l \bmod 4 = 0, \\ \{[n - l, 3], [n - l, 7]\} & \text{if } n > l \text{ and } n - l \bmod 4 = 2. \end{cases}$$

Proof. Let $\Lambda = \Lambda_0 \oplus \bigoplus_{\nu > 0} p^\nu \Lambda_\nu$ be a Jordan decomposition of a lattice Λ over $\mathbb{Z}_p$ with $(D_\Lambda, q_\Lambda) \cong (D, q)$. We put $\Lambda_{>0} := \Lambda_0^\perp = \bigoplus_{\nu > 0} p^\nu \Lambda_\nu$. Then we have $\operatorname{rank}(\Lambda_{>0}) = l$, $(D_{\Lambda_0}, q_{\Lambda_0}) = ((0), [0])$ and $(D_{\Lambda_{>0}}, q_{\Lambda_{>0}}) = (D_\Lambda, q_\Lambda) \cong (D, q)$. Hence (5.1) holds. The statement (5.2) is obvious. A lattice Λ over $\mathbb{Z}_2$ is even if and only if Λ_0 is of even rank and is isomorphic to an orthogonal direct sum of copies of U and V. Because of

$$[2\text{-excess}(U), \operatorname{reddisc}(U)] = [2, 7] \quad \text{and} \quad [2\text{-excess}(V), \operatorname{reddisc}(V)] = [2, 3],$$

we can easily prove the last statement. □

Lemma 5.2. *Suppose that ν is a positive integer. Let u be an integer prime to p. Then*

$$\mathcal{L}^{(p)}(1, \mathbb{Z}/(p^\nu), \left[\frac{u}{p^\nu}\right]) = \begin{cases} \{[p^\nu - 1, u]\} & \text{if } p \text{ is odd}, \\ \{[1 - u, u]\} & \text{if } p = 2 \text{ and } \nu > 1, \\ \{[1 - u, u], [1 - u, 5u]\} & \text{if } p = 2 \text{ and } \nu = 1. \end{cases}$$

Let u, v and w be integers with v being odd. Then

$$\mathcal{L}^{(2)}(2, (\mathbb{Z}/(2^\nu))^{\oplus 2}, \frac{1}{2^\nu}\begin{bmatrix} 2u & v \\ v & 2w \end{bmatrix}) = \begin{cases} \{[2,7]\} & \text{if } uw \text{ is even,} \\ \{[2,3]\} & \text{if } uw \text{ is odd.} \end{cases}$$

Proof. Two non-degenerate quadratic forms $[u/p^\nu]$ and $[u'/p^\nu]$ on $\mathbb{Z}/(p^\nu)$ with values in $\mathbb{Q}_p/2\,\mathbb{Z}_p$ are isomorphic if and only if

$$uu' \in (\mathbb{Z}_p^\times)^2, \qquad \text{or} \qquad (\quad p = 2, \quad \nu = 1, \quad \text{and} \quad u = u' \bmod 4 \quad)$$

is satisfied. On the other hand, two lattices $p^\nu(u)$ and $p^\nu(u')$ with u, $u' \in \mathbb{Z}_p^\times$ of rank 1 over $\mathbb{Z}_p$ are isomorphic if and only if $uu' \in (\mathbb{Z}_p^\times)^2$ holds. Therefore the first statement follows. The finite quadratic form

$$q = \frac{1}{2^\nu}\begin{bmatrix} 2u & v \\ v & 2w \end{bmatrix} \qquad (v : \text{odd})$$

on $(\mathbb{Z}/(2^\nu))^{\oplus 2}$ with values in $\mathbb{Q}_2/2\,\mathbb{Z}_2$ is isomorphic to $q_{2^\nu U}$ (resp. $q_{2^\nu V}$) if and only if $uw \bmod 2 = 0$ (resp. $uw \bmod 2 = 1$). These two forms can never be isomorphic to $[u'/2^\nu] \oplus [w'/2^\nu]$ with u' and w' being odd. Thus the second statement follows. □

Now we state an algorithm to calculate $\mathcal{L}^{(p)}(n, D, q)$. By Lemma 5.1, it is enough to determine $\mathcal{L}^{(p)}(l, D, q)$. Let $\{\gamma_1, \dots, \gamma_l\}$ be a reduced set of generators of D. We denote the order of γ_i by p^{ν_i}, and arrange the generators in such a way that $\nu_1 \ge \cdots \ge \nu_l$ holds. For an element $\alpha \in \mathbb{Q}_p/\mathbb{Z}_p$, we define $\phi_p(\alpha)$ to be the integer such that the order of α is $p^{\phi_p(\alpha)}$. Note that $\phi_p(b[q](\gamma_i, \gamma_j)) \le \min(\nu_i, \nu_j)$ holds for any γ_i and γ_j.

When $l = 1$, $\mathcal{L}^{(p)}(l, D, q)$ is given by Lemma 5.2. Suppose that $l > 1$.

Case 1. Suppose that there exists a generator γ_i such that $\phi_p(b[q](\gamma_i, \gamma_i)) = \nu_1$. Then we have $\nu_i = \nu_1$. Interchanging γ_1 and γ_i, we will assume that $\phi_p(b[q](\gamma_1, \gamma_1)) = \nu_1$. Let u be an integer such that $b[q](\gamma_1, \gamma_1) = u/p^{\nu_1} \bmod \mathbb{Z}_p$. Then u is prime to p, and hence there is an integer v such that $uv = 1 \bmod p^{\nu_1}$ holds. Since $\phi_p(b[q](\gamma_j, \gamma_1)) \le \min(\nu_j, \nu_1) = \nu_j$, we can write $b[q](\gamma_j, \gamma_1)$ in the form w_j/p^{ν_1} mod $\mathbb{Z}_p$ by some integer w_j that is divisible by $p^{\nu_1 - \nu_j}$. For $j \ge 2$, we put $\gamma'_j := \gamma_j - v w_j \gamma_1$. Because γ_1 is of order p^{ν_1} in D, γ'_j is independent of the choice of u, v and w_j. Moreover, γ'_j is of order p^{ν_j}, and $\{\gamma_1, \gamma'_2, \dots, \gamma'_l\}$ is again a reduced set of generators. By definition, we have $b[q](\gamma'_j, \gamma_1) = 0$ for any $j \ge 2$. We put

$$(D_1, q_1) := (\langle \gamma_1 \rangle, q|_{\langle \gamma_1 \rangle}) \cong (\mathbb{Z}/(p^{\nu_1}), [u/p^{\nu_1}])$$

and $(D_2, q_2) := (\langle \gamma'_2, \dots, \gamma'_l \rangle, q|_{\langle \gamma'_2, \dots, \gamma'_l \rangle})$. Then (D, q) is decomposed into the orthogonal direct sum of (D_1, q_1) and (D_2, q_2).

Let Λ be a lattice of rank l over $\mathbb{Z}_p$ such that there exists an isomorphism $h : (D_\Lambda, q_\Lambda) \xrightarrow{\sim} (D, q)$. Let $e^* \in \Lambda^\vee$ be a vector such that $h \circ \Psi_\Lambda(e^*) = \gamma_1$, and $\Lambda'_1 \subset \Lambda^\vee$ the $\mathbb{Z}_p$-submodule generated by e^*. Then $\Lambda_1 := \Lambda'_1 \cap \Lambda$ is a sublattice of rank 1 generated by $e := p^{\nu_1} e^*$. Let x be an arbitrary vector of Λ. Because of $\mathrm{ord}_p((x, e)) \ge \nu_1 = \mathrm{ord}_p((e, e))$, the vector

$$x' := x - \frac{(x, e)}{(e, e)} e$$

is in Λ and orthogonal to Λ_1. Hence we obtain an orthogonal decomposition $\Lambda = \Lambda_1 \oplus \Lambda_1^\perp$. The homomorphism $h \circ \Psi_\Lambda : \Lambda^\vee = \Lambda_1^\vee \oplus (\Lambda_1^\perp)^\vee \to D$ induces isomorphisms $(D_{\Lambda_1}, q_{\Lambda_1}) \cong (D_1, q_1)$ and $(D_{\Lambda_1^\perp}, q_{\Lambda_1^\perp}) \cong (D_2, q_2)$. It follows that

$$\mathcal{L}^{(p)}(l, D, q) = \mathcal{L}^{(p)}(1, D_1, q_1) * \mathcal{L}^{(p)}(l-1, D_2, q_2).$$

Thus $\mathcal{L}^{(p)}(l, D, q)$ is calculated by Lemma 5.2 and the induction hypothesis on l.

Case 2. Suppose that $\phi_p(b[q](\gamma_i, \gamma_i)) < \nu_1$ holds for any generator γ_i. Since q is non-degenerate, there exists at least one generator γ_k that satisfies $\phi_p(b[q](\gamma_1, \gamma_k)) = \nu_1$. Because of $\phi_p(b[q](\gamma_1, \gamma_k)) \le \nu_k$, we have $\nu_k = \nu_1$.

Case 2.1. Suppose that p is odd. We replace γ_1 by $\gamma_1' := \gamma_1 + \gamma_k$, which is an element of order p^{ν_1}. It is obvious that $\{\gamma_1', \gamma_2, \dots, \gamma_l\}$ is again a reduced set of generators of D. Moreover we have $\phi_p(b[q](\gamma_1', \gamma_1')) = \nu_1$. Therefore we are led to Case 1.

Case 2.2. Suppose that $p = 2$. We replace γ_2 by γ_k. There exist integers u, v and w with v being odd such that

$$b[q](\gamma_1, \gamma_1) = \frac{2\,u}{2^{\nu_1}}, \quad b[q](\gamma_1, \gamma_2) = \frac{v}{2^{\nu_1}}, \quad \text{and} \quad b[q](\gamma_2, \gamma_2) = \frac{2\,w}{2^{\nu_1}}$$

hold modulo $\mathbb{Z}_2$. Note that $q(\gamma_1) = 2\tilde{u}/2^{\nu_1}$ and $q(\gamma_2) = 2\tilde{w}/2^{\nu_1}$ hold modulo $2\,\mathbb{Z}_2$ for some integers $\tilde{u}$ and $\tilde{w}$ with $u = \tilde{u} \bmod 2^{\nu_1 - 1}$ and $w = \tilde{w} \bmod 2^{\nu_1 - 1}$.

If $l = 2$, then $\mathcal{L}^{(2)}(l, D, q)$ is determined by Lemma 5.2. Suppose that $l \ge 3$. There exists an integer t such that $(v^2 - uw)t = 1 \bmod 2^{\nu_1}$ holds. For each $j \ge 3$, we choose integers s_{j1} and s_{j2} such that $b[q](\gamma_j, \gamma_1) = s_{j1}/2^{\nu_1} \bmod \mathbb{Z}_2$ and $b[q](\gamma_j, \gamma_2) = s_{j2}/2^{\nu_1} \bmod \mathbb{Z}_2$ hold, and calculate

$$\begin{pmatrix} \beta_{j1} \\ \beta_{j2} \end{pmatrix} := t \cdot \begin{pmatrix} 2w & -v \\ -v & 2u \end{pmatrix} \cdot \begin{pmatrix} s_{j1} \\ s_{j2} \end{pmatrix}.$$

Then β_{j1} and β_{j2} are divisible by $2^{\nu_1 - \nu_j}$. Hence $\gamma_j' := \gamma_j - \beta_{j1}\gamma_1 - \beta_{j2}\gamma_2$ is an element of order 2^{ν_j}, which is independent of the choice of the integers. The set $\{\gamma_1, \gamma_2, \gamma_3', \dots, \gamma_l'\}$ is again a reduced set of generators of D, and the two subgroups $\langle \gamma_1, \gamma_2 \rangle$ and $\langle \gamma_3', \dots, \gamma_l' \rangle$ of D are orthogonal with respect to q. Therefore, putting

$$(D_1, q_1) := (\langle \gamma_1, \gamma_2 \rangle, q|_{\langle \gamma_1, \gamma_2 \rangle}) \cong \left((\mathbb{Z}/(p^{\nu_1}))^{\oplus 2}, \frac{1}{2^{\nu_1}} \begin{bmatrix} 2\tilde{u} & v \\ v & 2\tilde{w} \end{bmatrix} \right)$$

and $(D_2, q_2) := (\langle \gamma_3', \dots, \gamma_l' \rangle, q|_{\langle \gamma_3', \dots, \gamma_l' \rangle})$, we obtain an orthogonal decomposition $(D, q) = (D_1, q_1) \oplus (D_2, q_2)$.

Let Λ be a lattice of rank l over $\mathbb{Z}_2$ such that there exists an isomorphism $h : (D_\Lambda, q_\Lambda) \xrightarrow{\sim} (D, q)$. We pick up two vectors $e_1^*, e_2^* \in \Lambda^\vee$ such that $h \circ \Psi_\Lambda(e_1^*) = \gamma_1$ and $h \circ \Psi_\Lambda(e_2^*) = \gamma_2$. Let $\Lambda_1' \subset \Lambda^\vee$ be the $\mathbb{Z}_2$-submodule of $\Lambda^\vee$ generated by e_1^* and e_2^*. Then $\Lambda_1 := \Lambda_1' \cap \Lambda$ is a sublattice of Λ generated by $e_1 := 2^{\nu_1} e_1^*$ and $e_2 := 2^{\nu_1} e_2^*$. The intersection matrix M_1 of Λ_1 with respect to e_1 and e_2 satisfies $\mathrm{ord}_2(\det M_1^{-1}) = -\nu_1$. Because $\mathrm{ord}_2((x, e_1)) \ge \nu_1$ and $\mathrm{ord}_2((x, e_2)) \ge \nu_1$ hold for any vector $x \in \Lambda$, we have $((x, e_1), (x, e_2)) \cdot M_1^{-1} \in \mathbb{Z}_2^{\oplus 2}$. Therefore Λ is decomposed into the orthogonal direct sum of Λ_1 and $\Lambda_1^\perp$. The homomorphism $h \circ \Psi_\Lambda$ induces isomorphisms $(D_{\Lambda_1}, q_{\Lambda_1}) \cong (D_1, q_1)$ and $(D_{\Lambda_1^\perp}, q_{\Lambda_1^\perp}) \cong (D_2, q_2)$. It follows that

$$\mathcal{L}^{(2)}(l, D, q) = \mathcal{L}^{(2)}(2, D_1, q_1) * \mathcal{L}^{(2)}(l-2, D_2, q_2).$$

Thus $\mathcal{L}^{(2)}(l, D, q)$ is calculated by Lemma 5.2 and the induction hypothesis on l.

5.2. **Over $\mathbb{Z}$.** Let D be a finite abelian group, and $q : D \to \mathbb{Q}/2\,\mathbb{Z}$ a non-degenerate finite quadratic form. Let (r, s) be a pair of non-negative integers such that $n := r + s > 0$. We will describe a criterion to determine whether there exists a lattice L over $\mathbb{Z}$ with signature (r, s) such that (D_L, q_L) is isomorphic to the given finite quadratic form (D, q).

We put $d := (-1)^s|D|$. Let P be the set of prime divisors of $2\,d$, and let $(D, q) = \oplus_{p\in P}(D^{(p)}, q^{(p)})$ be the orthogonal decomposition of (D, q) into the p-parts. If d is odd, then we put $(D^{(2)}, q^{(2)}) = ((0), [0])$. By Lemma 4.3 and Theorem 3.1, a lattice L over $\mathbb{Z}$ with signature (r, s) and $(D_L, q_L) \cong (D, q)$ exists if and only if the following claim is verified:

($\sharp$) For each $p \in P$, there exists a lattice $\Lambda^{(p)}$ of rank n over $\mathbb{Z}_p$ such that
  (i) $\mathrm{disc}(\Lambda^{(p)}) = d \cdot (\mathbb{Z}_p^\times)^2$ and
  (ii) $(D_{\Lambda^{(p)}}, q_{\Lambda^{(p)}}) \cong (D^{(p)}, q^{(p)})$ hold,

and they satisfy

$$r - s + \sum_{p\in P} p\text{-excess}(\Lambda^{(p)}) = n \mod 8.$$

We put $\delta_p := d/p^{\mathrm{ord}_p(d)} \in \mathbb{Z}$. Under the condition (ii), which implies $|D_{\Lambda^{(p)}}| = d/\delta_p$, the condition (i) is equivalent to the condition

$$\mathrm{reddisc}(\Lambda^{(p)}) = \delta_p \cdot (\mathbb{Z}_p^\times)^2.$$

Therefore we can check the claim ($\sharp$) by the following method. First we calculate $\mathcal{L}^{(p)}(n, D^{(p)}, q^{(p)})$ for each $p \in P$. Then we search for an element $(\ [\sigma_p, u_p]\ ;\ p \in P\ )$ of the Cartesian product of the sets $\mathcal{L}^{(p)}(n, D^{(p)}, q^{(p)})$ that satisfies $u_p = \delta_p \cdot (\mathbb{Z}_p^\times)^2$ for each $p \in P$ and $r - s + \sum \sigma_p = n \bmod 8$. The claim ($\sharp$) is true if and only if we find such an element.

6. Roots

For the following, we refer to [3], [6, Chapter 4] or [12].

6.1. **Root system of a positive-definite even lattice over $\mathbb{Z}$.** Let L be a positive-definite even lattice over $\mathbb{Z}$. A vector of L is said to be a *root* if its norm is 2. We denote by L_{root} the sublattice of L generated by roots. A lattice L is said to be a *root lattice* if $L = L_{\mathrm{root}}$ holds. Let $\mathrm{Roots}(L)$ be the set of roots of L. We define $\sim$ to be the finest equivalence relation on $\mathrm{Roots}(L)$ that satisfies $(v, w) \neq 0 \Longrightarrow v \sim w$. Let $I_1, \dots, I_k$ be the equivalence classes of roots under the relation $\sim$, and let L_i be the sublattice of L_{root} generated by I_i. There exists a basis $B_i \subset I_i$ such that the intersection matrix of L_i with respect to B_i is the Cartan matrix corresponding to a Dynkin diagram of type A_l, D_m or E_n. Let τ_i be the type of the Dynkin diagram of the intersection matrix of L_i. We define the root type of L to be $\sum_{i=1}^k \tau_i$. Conversely, for an ADE-type Σ, there exists a root lattice $L(\Sigma)$, unique up to isomorphism, whose root type is Σ.

The root type of a positive-definite even lattice L over $\mathbb{Z}$ is therefore determined by the following procedure.

(1) Create the list $\mathrm{Roots}(L)$, and decompose it into $I_1, \dots, I_k$.
(2) Calculate the rank of L_i for $i = 1, \dots, k$.

Figure 6.1. Dynkin diagram

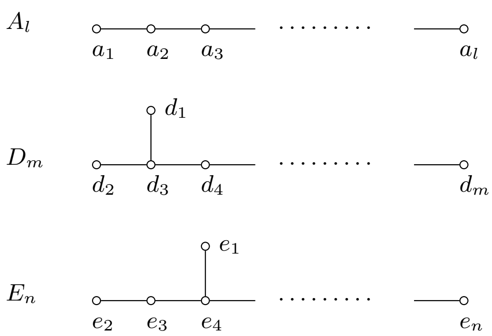

Table 6.1. Number of roots and discriminant forms of root lattices

τ	$\lvert\mathrm{Roots}(L(\tau))\rvert$	$D_{L(\tau)}$	$q_{L(\tau)}$
A_l	$l(l+1)$	$\langle \bar{a}_l^* \rangle \cong \mathbb{Z}/(l+1)$	$\left[\dfrac{l}{l+1}\right]$
D_m $(m:\text{even})$	$2m(m-1)$	$\langle \bar{d}_1^* \rangle \oplus \langle \bar{d}_m^* \rangle \cong (\mathbb{Z}/(2))^{\oplus 2}$	$\begin{bmatrix} m/4 & 1/2 \\ 1/2 & 1 \end{bmatrix}$
D_m $(m:\text{odd})$	$2m(m-1)$	$\langle \bar{d}_1^* \rangle \cong \mathbb{Z}/(4)$	$[m/4]$
E_6	72	$\langle \bar{e}_6^* \rangle \cong \mathbb{Z}/(3)$	$[4/3]$
E_7	126	$\langle \bar{e}_7^* \rangle \cong \mathbb{Z}/(2)$	$[3/2]$
E_8	240	(0)	$[0]$

(3) Determine the type τ_i from $\mathrm{rank}(L_i)$ and $|I_i|$ by using Table 6.1.

6.2. **Discriminant forms of root lattices.** The discriminant form $(D_{L(\tau)}, q_{L(\tau)})$, where τ is A_l, D_m or E_n, is indicated in Table 6.1. In this table, for example, $\{a_1^*, \dots, a_l^*\}$ is the basis of $L(A_l)^\vee$ dual to the basis $\{a_1, \dots, a_l\}$ of $L(A_l)$ given in Figure 6.1, and $\bar{a}_i^* \in D_{L(A_l)}$ is the image of a_i^* by the homomorphism $\Psi_{L(A_l)} : L(A_l)^\vee \to D_{L(A_l)}$.

Let $\Gamma(\tau)$ denote the image of the natural homomorphism from the orthogonal group $O(L(\tau))$ of the lattice $L(\tau)$ to $\mathrm{Aut}(D_{L(\tau)}, q_{L(\tau)})$. The structure of $\Gamma(\tau)$ is given as follows.

- If $\tau = A_1$ or $\tau = E_7$, then $\Gamma(\tau)$ is trivial.
- If $\tau = A_l$ $(l > 1)$ or $\tau = D_m$ $(m : \text{odd})$ or $\tau = E_6$, then $\Gamma(\tau)$ is isomorphic to $\mathbb{Z}/(2)$ generated by the multiplication by -1.
- If $\tau = D_m$ with m being even and > 4, then $\Gamma(\tau)$ is isomorphic to $\mathbb{Z}/(2)$ generated by
$$\bar{d}_1^* \mapsto \bar{d}_1^* + \bar{d}_m^*, \quad \bar{d}_m^* \mapsto \bar{d}_m^*.$$
- If $\tau = D_4$, then $\Gamma(\tau)$ is isomorphic to the full symmetric group acting on the set $\{\bar{d}_1^*, \bar{d}_4^*, \bar{d}_1^* + \bar{d}_4^*\}$ of non-trivial elements of $D_{L(\tau)}$.

7. Existence of an elliptic $K3$ surface with given data

Theorem 7.1. *Let Σ be an ADE-type with* $\mathrm{rank}(\Sigma) \le 18$, *and G a finite abelian group. There exists an elliptic $K3$ surface $f : X \to \mathbb{P}^1$ with $\Sigma_f = \Sigma$ and $G_f \cong G$ if and only if the root lattice $L(\Sigma)$ has an even overlattice M with the following properties.*

(i) $M/L(\Sigma) \cong G$,
(ii) *there exists an even lattice N of signature* $(2, 18 - \mathrm{rank}(\Sigma))$ *such that (D_N, q_N) is isomorphic to (D_M, q_M), and*
(iii) *the sublattice M_{root} of M coincides with $L(\Sigma)$.*

Proof. Suppose that a pair (Σ, G) satisfies the condition of Theorem. By Proposition 4.2, the property (ii) implies that there exists an even unimodular overlattice K' of $M^- \oplus N$ into which M^- and N are primitively embedded. Let H denote the hyperbolic lattice;

$$H := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Then $K := K' \oplus H$ is an even unimodular lattice with signature $(3, 19)$. Hence K is isomorphic to the $K3$ lattice $L(2E_8)^- \oplus H^{\oplus 2}$ by Milnor's structure theorem (cf. [15]). There exists a 2-dimensional linear subspace V of $N \otimes_{\mathbb{Z}} \mathbb{R}$ such that the bilinear form is positive-definite on V and that, if $N' \subset N$ is a sublattice such that $N' \otimes_{\mathbb{Z}} \mathbb{R}$ contains V, then N' coincides with N. By the surjectivity of the period map on the moduli of $K3$ surfaces, there exists a complex $K3$ surface X and an isomorphism $\alpha : H^2(X; \mathbb{Z}) \xrightarrow{\sim} K$ of lattices such that

$$\alpha_{\mathbb{R}}^{-1}(V) = (H^{0,2}(X) \oplus H^{2,0}(X)) \cap H^2(X; \mathbb{R})$$

holds, where $\alpha_{\mathbb{R}} := \alpha \otimes_{\mathbb{Z}} \mathbb{R}$. Then we have

$$\alpha^{-1}(M^- \oplus H) = \mathrm{NS}_X . \tag{7.1}$$

By Kondo's lemma [8, Lemma 2.1], there exists a structure of the elliptic fibration $f : X \to \mathbb{P}^1$ with a section $O : \mathbb{P}^1 \to X$ such that, if F denotes the cohomology class of a general fiber of f, then

$$\mathbb{Z}[F]^{\perp} / \mathbb{Z}[F] \cong M^- \tag{7.2}$$

holds, where $\mathbb{Z}[F]^{\perp}$ is the orthogonal complement of $\mathbb{Z}[F]$ in the Néron-Severi lattice NS_X of X. Let H_f be the sublattice of NS_X spanned by the cohomology classes of the zero section and a general fiber of f, S_f the sublattice of NS_X defined in § 1, and W_f the orthogonal complement of H_f in NS_X. The lattice H_f is isomorphic to the hyperbolic lattice H, and is orthogonal to S_f. By abuse of notation, we denote $(W_f)_{\mathrm{root}}$ for the sublattice of W_f generated by the vectors of norm -2. From (7.2), we have

$$W_f \cong M^-. \tag{7.3}$$

On the other hand, by Nishiyama's lemma [12, Lemma 6.1], we have

$$W_f/(W_f)_{\mathrm{root}} \cong MW_f, \quad \text{and} \tag{7.4}$$

$$S_f = (W_f)_{\mathrm{root}}. \tag{7.5}$$

Combining these with the properties (i) and (iii) of M and the isomorphism (7.3), we have $S_f \cong L(\Sigma)^-$ and $MW_f \cong G$. Hence $\Sigma = \Sigma_f$ and $G \cong G_f$ hold.

Conversely, suppose that there exists an elliptic $K3$ surface $f : X \to \mathbb{P}^1$ with $\Sigma_f = \Sigma$ and $G_f \cong G$. Using Nishiyama's lemma again, we see that the primitive closure $\overline{S}_f$ of S_f in NS_X satisfies $\overline{S}_f/S_f \cong G$ and $(\overline{S}_f)_{\mathrm{root}} = S_f$. We have an isomorphism $S_f \cong L(\Sigma)^-$. Let M^- be the overlattice of $L(\Sigma)^-$ corresponding to $\overline{S}_f$ via this isomorphism. Then $M := (M^-)^-$ is an overlattice of $L(\Sigma)$ that possess the properties (i) and (iii). Moreover, $\overline{S}_f \oplus H_f$ is primitive in the even unimodular lattice $H^2(X;\mathbb{Z})$, and hence Proposition 4.2 implies that the orthogonal complement N_f of $\overline{S}_f \oplus H_f$ in $H^2(X;\mathbb{Z})$ satisfies $(D_{N_f}, q_{N_f}) \cong (D_{\overline{S}_f}, -q_{\overline{S}_f}) \cong (D_M, q_M)$. Because the signature of N_f is $(2, 18-\mathrm{rank}(\Sigma))$, the overlattice M has the property (ii). □

8. Making the list

Recall that, in order for an ADE-type Σ to be an ADE-type of an elliptic $K3$ surface, it is necessary that $\mathrm{rank}(\Sigma) \le 18$ and $\mathrm{euler}(\Sigma) \le 24$. It is obvious that the torsion part of the Mordell-Weil group of an elliptic surface is of length ≤ 2.

First we list up all ADE-types Σ with $\mathrm{rank}(\Sigma) \le 18$ and $\mathrm{euler}(\Sigma) \le 24$. There are 3937 such ADE-types. For each

$$\Sigma := \sum a_l A_l + \sum d_m D_m + \sum e_n E_n$$

in this list, we carry out the following calculation.

Step 1. We calculate the discriminant form $(D_{L(\Sigma)}, q_{L(\Sigma)})$ using Table 6.1. Note that the product of the wreath products

$$\prod_{a_l>0} (\Gamma(A_l) \wr \mathfrak{S}_{a_l}) \times \prod_{d_m>0} (\Gamma(D_m) \wr \mathfrak{S}_{d_m}) \times \prod_{e_n>0} (\Gamma(E_n) \wr \mathfrak{S}_{e_n})$$

acts on $(D_{L(\Sigma)}, q_{L(\Sigma)})$. Here, for example, the full symmetric group $\mathfrak{S}_{a_l}$ acts on $D_{L(\Sigma)}$ as the permutation group on the a_l components of $D_{L(\Sigma)}$ isomorphic to $D_{L(A_l)}$. We denote this group by $\Gamma(\Sigma)$.

Step 2. We make a complete list of representatives of the quotient set $D_{L(\Sigma)}/\Gamma(\Sigma)$ and pick up from this list elements isotopic with respect to $q_{L(\Sigma)}$. Let $\mathcal{V}_\Sigma = \{\bar{v}_1, \dots, \bar{v}_N\}$ be the list of isotopic elements of $D_{L(\Sigma)}$ modulo $\Gamma(\Sigma)$. For each $\bar{v}_i \in \mathcal{V}_\Sigma$, we calculate the stabilizer subgroup $St(\Gamma(\Sigma), \bar{v}_i)$ of $\bar{v}_i$ in $\Gamma(\Sigma)$. Then we make a complete list of representatives of $D_{L(\Sigma)}/St(\Gamma(\Sigma), \bar{v}_i)$, and pick up from this list elements isotopic with respect to $q_{L(\Sigma)}$ and orthogonal to $\bar{v}_i$ with respect to $b[q_{L(\Sigma)}]$. Let $\mathcal{W}_{\Sigma,i}$ be the list of isotopic elements orthogonal to $\bar{v}_i$ modulo $St(\Gamma(\Sigma), \bar{v}_i)$.

Next we make the list $\mathcal{G}'_\Sigma$ of all pairs $[\bar{v}_i, \bar{w}_j]$ of $\bar{v}_i \in \mathcal{V}_\Sigma$ and $\bar{w}_j \in \mathcal{W}_{\Sigma,i}$. Then every isotopic subgroup of $(D_{L(\Sigma)}, q_{L(\Sigma)})$ with length ≤ 2 is conjugate under the action of $\Gamma(\Sigma)$ to a subgroup $\langle \bar{v}_i, \bar{w}_j \rangle$ generated by $\bar{v}_i$ and $\bar{w}_j$ for some $[\bar{v}_i, \bar{w}_j] \in \mathcal{G}'_\Sigma$. Of course, there are several different pairs that generate a same subgroup. We remove this redundancy from $\mathcal{G}'_\Sigma$, and make a list $\mathcal{G}_\Sigma$.

Step 3. For each $[\bar{v}, \bar{w}] \in \mathcal{G}_\Sigma$, we calculate the subgroup $G := \langle \bar{v}, \bar{w} \rangle$ of $D_{L(\Sigma)}$, its orthogonal complement $G^\perp$ in $(D_{L(\Sigma)}, q_{L(\Sigma)})$, and the finite quadratic form $(D_G, q_G) := (G^\perp/G, q_{L(\Sigma)}|_{G^\perp/G})$.

Step 3.1. By the algorithm described in § 5, we determine whether there exists an even lattice N over $\mathbb{Z}$ of signature $(2, 18-\mathrm{rank}(\Sigma))$ such that $(D_N, q_N) \cong (D_G, q_G)$. If the answer is affirmative, we go to the next step.

Step 3.2. We calculate the intersection matrix of the even overlattice M_G of $L(\Sigma)$ generated by $L(\Sigma)$ and v, w in $L(\Sigma)^\vee$, where v and w are vectors of $L(\Sigma)^\vee$ such that $\Psi_{L(\Sigma)}(v) = \bar{v}$ and $\Psi_{L(\Sigma)}(w) = \bar{w}$. Then we calculate the root type of M_G by the algorithm described in § 6. If this root type coincides with the initial ADE-type Σ, then we let the pair (Σ, G) be a member of the list $\mathcal{P}$.

By Theorem 7.1, the list $\mathcal{P}$ thus made is the complete list of the data of elliptic $K3$ surfaces.

The following remarks are useful in checking the program.

Remark 8.1. Note that neither euler$(\Sigma) \leq 24$ nor length$(G) \leq 2$ is contained in the conditions of Theorem 7.1. Therefore, if we input Σ with euler$(\Sigma) > 24$ into the program, then it should return no subgroups G of $D_{L(\Sigma)}$ such that (Σ, G) can be a member of the list $\mathcal{P}$. If we change Step 2 of the program so that it lists up all isotopic subgroups of length ≥ 3, then the result should also be an empty set.

Remark 8.2. Suppose that the root type Σ' of M_G is not equal to Σ in Step 3.2 of the program. Let G' be the finite abelian group $M_G/(M_G)_{\text{root}}$. Then (Σ', G') appears in $\mathcal{P}$.

Remark 8.3. For each $(\Sigma, G) \in \mathcal{P}$, there should be at least one configuration that satisfies the conditions given in § 2.2.

References

[1] E. Artal-Bartolo, H. Tokunaga and D. Q. Zhang. *Miranda-Persson's problem on extremal elliptic K3 surfaces.* preprint. http://xxx.lanl.gov/list/math.AG, 9809065.

[2] A. Besser. *Elliptic fibrations of $K3$ surfaces and QM Kummer surfaces.* Math. Z. **228** (1998), no. 2, 283–308.

[3] N. Bourbaki. Éléments de mathématique. Groupes et algèbres de Lie. Chapitre IV-VI. Hermann, Paris, 1968.

[4] D. Burns and M. Rapoport. *On the Torelli problem for kählerian $K-3$ surfaces.* Ann. Sci. École Norm. Sup. (4) 8 (1975), no. 2, 235–273.

[5] J. W. S. Cassels. Rational quadratic forms. Academic Press, London, 1978.

[6] J. H. Conway and N. J. A. Sloane. *Sphere packings, lattices and groups.* Second edition. Grundlehren der Mathematischen Wissenschaften, **290**, Springer, New York, 1993.

[7] K. Kodaira. *On compact analytic surfaces. II, III.* Ann. of Math. **77** (1963), 563–626; ibid. **78** (1963) 1–40

[8] S. Kondō. *Automorphisms of algebraic $K3$ surfaces which act trivially on Picard groups.* J. Math. Soc. Japan **44** (1992), no. 1, 75–98.

[9] R. Miranda and U. Persson. *On extremal rational elliptic surfaces.* Math. Z. **193** (1986), no. 4, 537–558.

[10] R. Miranda and U. Persson. *Mordell-Weil groups of extremal elliptic $K3$ surfaces.* Problems in the theory of surfaces and their classification (Cortona, 1988), Sympos. Math., XXXII, Academic Press, London, 1991, pp. 167–192.

[11] V. V. Nikulin. *Integer symmetric bilinear forms and some of their applications.* Math. USSR Izvestija **14** (1980), no. 1, 103–167.

[12] K. Nishiyama. *The Jacobian fibrations on some $K3$ surfaces and their Mordell-Weil groups.* Japan. J. Math. (N.S.) **22** (1996), no. 2, 293–347.

[13] I. Piateskii-Shapiro and I. R. Shafarevich. *A Torelli theorem for algebraic surfaces of type $K3$.* Math. USSR Izv. **35** (1971), 530–572.

[14] A. Sebbar. *Congruence groups, Hauptmoduls and elliptic surfaces.* Talk at "Infinite dimensional Lie algebras, the Monster and related topics". July 1999 Hokkaido Univ.

[15] J.-P. Serre. *A course in arithmetic.* Graduate Texts in Mathematics, **7**, Springer, New York, 1973.

[16] I. Shimada and D. Q. Zhang. *Classification of extremal elliptic $K3$ surfaces and fundamental groups of open $K3$ surfaces.* to appear in Nagoya Math. J.

[17] T. Shioda. *On elliptic modular surfaces.* J. Math. Soc. Japan **24** (1972), 20–59.

[18] A. N. Todorov. *Applications of the Kähler-Einstein-Calabi-Yau metric to moduli of $K3$ surfaces.* Invent. Math. **61** (1980), no. 3, 251–265.

[19] T. Urabe. *Elementary transformations of Dynkin graphs and singularities on quartic surfaces.* Invent. Math. **87** (1987), no. 3, 549–572.

[20] T. Urabe. *Tie transformations of Dynkin graphs and singularities on quartic surfaces.* Invent. Math. **100** (1990), no. 1, 207–230.

[21] J.-G. Yang. *Sextic curves with simple singularities.* Tôhoku Math. J. **48** (1996), no. 2, 203-227.

[22] J.-G. Yang. *Enumeration of combinations of rational double points on quartic surfaces.* Singularities and complex geometry (Beijing, 1994), AMS/IP Stud. Adv. Math., **5**, Amer. Math. Soc., Providence, RI, 1997, 275–312.

[23] Q. Ye. *On extremal elliptic $K3$ surfaces.* preprint. http://xxx.lanl.gov/abs/math.AG, 9901081

This is the table of all ADE-types of singular fibers of complex elliptic $K3$ surfaces with a zero section and the torsion parts of their Mordell-Weil groups.

In Table 1, the ADE-types of the singular fibers are listed according to the rank and the lexicographic order. For each ADE-type Σ, the last column shows the list of all abelian groups that can be realized as the torsion part of the Mordell-Weil group of an elliptic $K3$ surface $f : X \to \mathbb{P}^1$ with $\Sigma_f = \Sigma$. Here, [1] is the trivial group, $[a]$ is the cyclic group $\mathbb{Z}/(a)$, and $[a, b]$ is the group $\mathbb{Z}/(a) \times \mathbb{Z}/(b)$.

Table 2 shows, for each abelian group G with order ≥ 3 that appears as G_f of some elliptic $K3$ surface, the list of all ADE-types Σ such that the pair (Σ, G) appears in Table 1.

Table 1.

No.	rank	ADE-type	G
1	1	A_1	[1]

No.	rank	ADE-type	G
2	2	A_2	[1]
3	2	$2A_1$	[1]

No.	rank	ADE-type	G
4	3	A_3	[1]
5	3	$A_2 + A_1$	[1]
6	3	$3A_1$	[1]

No.	rank	ADE-type	G
7	4	D_4	[1]
8	4	A_4	[1]
9	4	$A_3 + A_1$	[1]
10	4	$2A_2$	[1]
11	4	$A_2 + 2A_1$	[1]
12	4	$4A_1$	[1]

No.	rank	ADE-type	G
13	5	D_5	[1]
14	5	$D_4 + A_1$	[1]
15	5	A_5	[1]

16	5	$A_4 + A_1$	[1]
17	5	$A_3 + A_2$	[1]
18	5	$A_3 + 2A_1$	[1]
19	5	$2A_2 + A_1$	[1]
20	5	$A_2 + 3A_1$	[1]
21	5	$5A_1$	[1]

No.	rank	ADE-type	G
22	6	E_6	[1]
23	6	D_6	[1]
24	6	$D_5 + A_1$	[1]
25	6	$D_4 + A_2$	[1]
26	6	$D_4 + 2A_1$	[1]
27	6	A_6	[1]
28	6	$A_5 + A_1$	[1]
29	6	$A_4 + A_2$	[1]
30	6	$A_4 + 2A_1$	[1]
31	6	$2A_3$	[1]
32	6	$A_3 + A_2 + A_1$	[1]
33	6	$A_3 + 3A_1$	[1]
34	6	$3A_2$	[1]
35	6	$2A_2 + 2A_1$	[1]
36	6	$A_2 + 4A_1$	[1]
37	6	$6A_1$	[1]

No.	rank	ADE-type	G
38	7	E_7	[1]
39	7	$E_6 + A_1$	[1]
40	7	D_7	[1]
41	7	$D_6 + A_1$	[1]
42	7	$D_5 + A_2$	[1]
43	7	$D_5 + 2A_1$	[1]
44	7	$D_4 + A_3$	[1]
45	7	$D_4 + A_2 + A_1$	[1]
46	7	$D_4 + 3A_1$	[1]

47	7	A_7	[1]
48	7	$A_6 + A_1$	[1]
49	7	$A_5 + A_2$	[1]
50	7	$A_5 + 2A_1$	[1]
51	7	$A_4 + A_3$	[1]
52	7	$A_4 + A_2 + A_1$	[1]
53	7	$A_4 + 3A_1$	[1]
54	7	$2A_3 + A_1$	[1]
55	7	$A_3 + 2A_2$	[1]
56	7	$A_3 + A_2 + 2A_1$	[1]
57	7	$A_3 + 4A_1$	[1]
58	7	$3A_2 + A_1$	[1]
59	7	$2A_2 + 3A_1$	[1]
60	7	$A_2 + 5A_1$	[1]
61	7	$7A_1$	[1]

No.	rank	ADE-type	G
62	8	E_8	[1]
63	8	$E_7 + A_1$	[1]
64	8	$E_6 + A_2$	[1]
65	8	$E_6 + 2A_1$	[1]
66	8	D_8	[1]
67	8	$D_7 + A_1$	[1]
68	8	$D_6 + A_2$	[1]
69	8	$D_6 + 2A_1$	[1]
70	8	$D_5 + A_3$	[1]
71	8	$D_5 + A_2 + A_1$	[1]
72	8	$D_5 + 3A_1$	[1]
73	8	$2D_4$	[1]
74	8	$D_4 + A_4$	[1]
75	8	$D_4 + A_3 + A_1$	[1]
76	8	$D_4 + 2A_2$	[1]
77	8	$D_4 + A_2 + 2A_1$	[1]
78	8	$D_4 + 4A_1$	[1]
79	8	A_8	[1]
80	8	$A_7 + A_1$	[1]

81	8	$A_6 + A_2$	[1]
82	8	$A_6 + 2A_1$	[1]
83	8	$A_5 + A_3$	[1]
84	8	$A_5 + A_2 + A_1$	[1]
85	8	$A_5 + 3A_1$	[1]
86	8	$2A_4$	[1]
87	8	$A_4 + A_3 + A_1$	[1]
88	8	$A_4 + 2A_2$	[1]
89	8	$A_4 + A_2 + 2A_1$	[1]
90	8	$A_4 + 4A_1$	[1]
91	8	$2A_3 + A_2$	[1]
92	8	$2A_3 + 2A_1$	[1]
93	8	$A_3 + 2A_2 + A_1$	[1]
94	8	$A_3 + A_2 + 3A_1$	[1]
95	8	$A_3 + 5A_1$	[1]
96	8	$4A_2$	[1]
97	8	$3A_2 + 2A_1$	[1]
98	8	$2A_2 + 4A_1$	[1]
99	8	$A_2 + 6A_1$	[1]
100	8	$8A_1$	[2], [1]

No.	rank	ADE-type	G
101	9	$E_8 + A_1$	[1]
102	9	$E_7 + A_2$	[1]
103	9	$E_7 + 2A_1$	[1]
104	9	$E_6 + A_3$	[1]
105	9	$E_6 + A_2 + A_1$	[1]
106	9	$E_6 + 3A_1$	[1]
107	9	D_9	[1]
108	9	$D_8 + A_1$	[1]
109	9	$D_7 + A_2$	[1]
110	9	$D_7 + 2A_1$	[1]
111	9	$D_6 + A_3$	[1]
112	9	$D_6 + A_2 + A_1$	[1]
113	9	$D_6 + 3A_1$	[1]
114	9	$D_5 + D_4$	[1]

115	9	$D_5 + A_4$	[1]
116	9	$D_5 + A_3 + A_1$	[1]
117	9	$D_5 + 2A_2$	[1]
118	9	$D_5 + A_2 + 2A_1$	[1]
119	9	$D_5 + 4A_1$	[1]
120	9	$2D_4 + A_1$	[1]
121	9	$D_4 + A_5$	[1]
122	9	$D_4 + A_4 + A_1$	[1]
123	9	$D_4 + A_3 + A_2$	[1]
124	9	$D_4 + A_3 + 2A_1$	[1]
125	9	$D_4 + 2A_2 + A_1$	[1]
126	9	$D_4 + A_2 + 3A_1$	[1]
127	9	$D_4 + 5A_1$	[1]
128	9	A_9	[1]
129	9	$A_8 + A_1$	[1]
130	9	$A_7 + A_2$	[1]
131	9	$A_7 + 2A_1$	[1]
132	9	$A_6 + A_3$	[1]
133	9	$A_6 + A_2 + A_1$	[1]
134	9	$A_6 + 3A_1$	[1]
135	9	$A_5 + A_4$	[1]
136	9	$A_5 + A_3 + A_1$	[1]
137	9	$A_5 + 2A_2$	[1]
138	9	$A_5 + A_2 + 2A_1$	[1]
139	9	$A_5 + 4A_1$	[1]
140	9	$2A_4 + A_1$	[1]
141	9	$A_4 + A_3 + A_2$	[1]
142	9	$A_4 + A_3 + 2A_1$	[1]
143	9	$A_4 + 2A_2 + A_1$	[1]
144	9	$A_4 + A_2 + 3A_1$	[1]
145	9	$A_4 + 5A_1$	[1]
146	9	$3A_3$	[1]
147	9	$2A_3 + A_2 + A_1$	[1]
148	9	$2A_3 + 3A_1$	[1]
149	9	$A_3 + 3A_2$	[1]
150	9	$A_3 + 2A_2 + 2A_1$	[1]
151	9	$A_3 + A_2 + 4A_1$	[1]

152	9	$A_3 + 6\,A_1$	[2], [1]
153	9	$4\,A_2 + A_1$	[1]
154	9	$3\,A_2 + 3\,A_1$	[1]
155	9	$2\,A_2 + 5\,A_1$	[1]
156	9	$A_2 + 7\,A_1$	[1]
157	9	$9\,A_1$	[2], [1]

No.	rank	ADE-type	G
158	10	$E_8 + A_2$	[1]
159	10	$E_8 + 2\,A_1$	[1]
160	10	$E_7 + A_3$	[1]
161	10	$E_7 + A_2 + A_1$	[1]
162	10	$E_7 + 3\,A_1$	[1]
163	10	$E_6 + D_4$	[1]
164	10	$E_6 + A_4$	[1]
165	10	$E_6 + A_3 + A_1$	[1]
166	10	$E_6 + 2\,A_2$	[1]
167	10	$E_6 + A_2 + 2\,A_1$	[1]
168	10	$E_6 + 4\,A_1$	[1]
169	10	D_{10}	[1]
170	10	$D_9 + A_1$	[1]
171	10	$D_8 + A_2$	[1]
172	10	$D_8 + 2\,A_1$	[1]
173	10	$D_7 + A_3$	[1]
174	10	$D_7 + A_2 + A_1$	[1]
175	10	$D_7 + 3\,A_1$	[1]
176	10	$D_6 + D_4$	[1]
177	10	$D_6 + A_4$	[1]
178	10	$D_6 + A_3 + A_1$	[1]
179	10	$D_6 + 2\,A_2$	[1]
180	10	$D_6 + A_2 + 2\,A_1$	[1]
181	10	$D_6 + 4\,A_1$	[1]
182	10	$2\,D_5$	[1]
183	10	$D_5 + D_4 + A_1$	[1]
184	10	$D_5 + A_5$	[1]
185	10	$D_5 + A_4 + A_1$	[1]

186	10	$D_5 + A_3 + A_2$	[1]
187	10	$D_5 + A_3 + 2A_1$	[1]
188	10	$D_5 + 2A_2 + A_1$	[1]
189	10	$D_5 + A_2 + 3A_1$	[1]
190	10	$D_5 + 5A_1$	[1]
191	10	$2D_4 + A_2$	[1]
192	10	$2D_4 + 2A_1$	[1]
193	10	$D_4 + A_6$	[1]
194	10	$D_4 + A_5 + A_1$	[1]
195	10	$D_4 + A_4 + A_2$	[1]
196	10	$D_4 + A_4 + 2A_1$	[1]
197	10	$D_4 + 2A_3$	[1]
198	10	$D_4 + A_3 + A_2 + A_1$	[1]
199	10	$D_4 + A_3 + 3A_1$	[1]
200	10	$D_4 + 3A_2$	[1]
201	10	$D_4 + 2A_2 + 2A_1$	[1]
202	10	$D_4 + A_2 + 4A_1$	[1]
203	10	$D_4 + 6A_1$	[2], [1]
204	10	A_{10}	[1]
205	10	$A_9 + A_1$	[1]
206	10	$A_8 + A_2$	[1]
207	10	$A_8 + 2A_1$	[1]
208	10	$A_7 + A_3$	[1]
209	10	$A_7 + A_2 + A_1$	[1]
210	10	$A_7 + 3A_1$	[1]
211	10	$A_6 + A_4$	[1]
212	10	$A_6 + A_3 + A_1$	[1]
213	10	$A_6 + 2A_2$	[1]
214	10	$A_6 + A_2 + 2A_1$	[1]
215	10	$A_6 + 4A_1$	[1]
216	10	$2A_5$	[1]
217	10	$A_5 + A_4 + A_1$	[1]
218	10	$A_5 + A_3 + A_2$	[1]
219	10	$A_5 + A_3 + 2A_1$	[1]
220	10	$A_5 + 2A_2 + A_1$	[1]
221	10	$A_5 + A_2 + 3A_1$	[1]
222	10	$A_5 + 5A_1$	[2], [1]

223	10	$2A_4 + A_2$	[1]
224	10	$2A_4 + 2A_1$	[1]
225	10	$A_4 + 2A_3$	[1]
226	10	$A_4 + A_3 + A_2 + A_1$	[1]
227	10	$A_4 + A_3 + 3A_1$	[1]
228	10	$A_4 + 3A_2$	[1]
229	10	$A_4 + 2A_2 + 2A_1$	[1]
230	10	$A_4 + A_2 + 4A_1$	[1]
231	10	$A_4 + 6A_1$	[1]
232	10	$3A_3 + A_1$	[1]
233	10	$2A_3 + 2A_2$	[1]
234	10	$2A_3 + A_2 + 2A_1$	[1]
235	10	$2A_3 + 4A_1$	[2], [1]
236	10	$A_3 + 3A_2 + A_1$	[1]
237	10	$A_3 + 2A_2 + 3A_1$	[1]
238	10	$A_3 + A_2 + 5A_1$	[1]
239	10	$A_3 + 7A_1$	[2], [1]
240	10	$5A_2$	[1]
241	10	$4A_2 + 2A_1$	[1]
242	10	$3A_2 + 4A_1$	[1]
243	10	$2A_2 + 6A_1$	[1]
244	10	$A_2 + 8A_1$	[2], [1]
245	10	$10A_1$	[2], [1]

No.	rank	ADE-type	G
246	11	$E_8 + A_3$	[1]
247	11	$E_8 + A_2 + A_1$	[1]
248	11	$E_8 + 3A_1$	[1]
249	11	$E_7 + D_4$	[1]
250	11	$E_7 + A_4$	[1]
251	11	$E_7 + A_3 + A_1$	[1]
252	11	$E_7 + 2A_2$	[1]
253	11	$E_7 + A_2 + 2A_1$	[1]
254	11	$E_7 + 4A_1$	[1]
255	11	$E_6 + D_5$	[1]
256	11	$E_6 + D_4 + A_1$	[1]

257	11	$E_6 + A_5$	[1]
258	11	$E_6 + A_4 + A_1$	[1]
259	11	$E_6 + A_3 + A_2$	[1]
260	11	$E_6 + A_3 + 2A_1$	[1]
261	11	$E_6 + 2A_2 + A_1$	[1]
262	11	$E_6 + A_2 + 3A_1$	[1]
263	11	$E_6 + 5A_1$	[1]
264	11	D_{11}	[1]
265	11	$D_{10} + A_1$	[1]
266	11	$D_9 + A_2$	[1]
267	11	$D_9 + 2A_1$	[1]
268	11	$D_8 + A_3$	[1]
269	11	$D_8 + A_2 + A_1$	[1]
270	11	$D_8 + 3A_1$	[1]
271	11	$D_7 + D_4$	[1]
272	11	$D_7 + A_4$	[1]
273	11	$D_7 + A_3 + A_1$	[1]
274	11	$D_7 + 2A_2$	[1]
275	11	$D_7 + A_2 + 2A_1$	[1]
276	11	$D_7 + 4A_1$	[1]
277	11	$D_6 + D_5$	[1]
278	11	$D_6 + D_4 + A_1$	[1]
279	11	$D_6 + A_5$	[1]
280	11	$D_6 + A_4 + A_1$	[1]
281	11	$D_6 + A_3 + A_2$	[1]
282	11	$D_6 + A_3 + 2A_1$	[1]
283	11	$D_6 + 2A_2 + A_1$	[1]
284	11	$D_6 + A_2 + 3A_1$	[1]
285	11	$D_6 + 5A_1$	[2], [1]
286	11	$2D_5 + A_1$	[1]
287	11	$D_5 + D_4 + A_2$	[1]
288	11	$D_5 + D_4 + 2A_1$	[1]
289	11	$D_5 + A_6$	[1]
290	11	$D_5 + A_5 + A_1$	[1]
291	11	$D_5 + A_4 + A_2$	[1]
292	11	$D_5 + A_4 + 2A_1$	[1]
293	11	$D_5 + 2A_3$	[1]

294	11	$D_5 + A_3 + A_2 + A_1$	[1]
295	11	$D_5 + A_3 + 3A_1$	[1]
296	11	$D_5 + 3A_2$	[1]
297	11	$D_5 + 2A_2 + 2A_1$	[1]
298	11	$D_5 + A_2 + 4A_1$	[1]
299	11	$D_5 + 6A_1$	[2], [1]
300	11	$2D_4 + A_3$	[1]
301	11	$2D_4 + A_2 + A_1$	[1]
302	11	$2D_4 + 3A_1$	[1]
303	11	$D_4 + A_7$	[1]
304	11	$D_4 + A_6 + A_1$	[1]
305	11	$D_4 + A_5 + A_2$	[1]
306	11	$D_4 + A_5 + 2A_1$	[1]
307	11	$D_4 + A_4 + A_3$	[1]
308	11	$D_4 + A_4 + A_2 + A_1$	[1]
309	11	$D_4 + A_4 + 3A_1$	[1]
310	11	$D_4 + 2A_3 + A_1$	[1]
311	11	$D_4 + A_3 + 2A_2$	[1]
312	11	$D_4 + A_3 + A_2 + 2A_1$	[1]
313	11	$D_4 + A_3 + 4A_1$	[2], [1]
314	11	$D_4 + 3A_2 + A_1$	[1]
315	11	$D_4 + 2A_2 + 3A_1$	[1]
316	11	$D_4 + A_2 + 5A_1$	[1]
317	11	$D_4 + 7A_1$	[2], [1]
318	11	A_{11}	[1]
319	11	$A_{10} + A_1$	[1]
320	11	$A_9 + A_2$	[1]
321	11	$A_9 + 2A_1$	[1]
322	11	$A_8 + A_3$	[1]
323	11	$A_8 + A_2 + A_1$	[1]
324	11	$A_8 + 3A_1$	[1]
325	11	$A_7 + A_4$	[1]
326	11	$A_7 + A_3 + A_1$	[1]
327	11	$A_7 + 2A_2$	[1]
328	11	$A_7 + A_2 + 2A_1$	[1]
329	11	$A_7 + 4A_1$	[2], [1]
330	11	$A_6 + A_5$	[1]

331	11	$A_6 + A_4 + A_1$	[1]
332	11	$A_6 + A_3 + A_2$	[1]
333	11	$A_6 + A_3 + 2A_1$	[1]
334	11	$A_6 + 2A_2 + A_1$	[1]
335	11	$A_6 + A_2 + 3A_1$	[1]
336	11	$A_6 + 5A_1$	[1]
337	11	$2A_5 + A_1$	[1]
338	11	$A_5 + A_4 + A_2$	[1]
339	11	$A_5 + A_4 + 2A_1$	[1]
340	11	$A_5 + 2A_3$	[1]
341	11	$A_5 + A_3 + A_2 + A_1$	[1]
342	11	$A_5 + A_3 + 3A_1$	[2], [1]
343	11	$A_5 + 3A_2$	[1]
344	11	$A_5 + 2A_2 + 2A_1$	[1]
345	11	$A_5 + A_2 + 4A_1$	[1]
346	11	$A_5 + 6A_1$	[2], [1]
347	11	$2A_4 + A_3$	[1]
348	11	$2A_4 + A_2 + A_1$	[1]
349	11	$2A_4 + 3A_1$	[1]
350	11	$A_4 + 2A_3 + A_1$	[1]
351	11	$A_4 + A_3 + 2A_2$	[1]
352	11	$A_4 + A_3 + A_2 + 2A_1$	[1]
353	11	$A_4 + A_3 + 4A_1$	[1]
354	11	$A_4 + 3A_2 + A_1$	[1]
355	11	$A_4 + 2A_2 + 3A_1$	[1]
356	11	$A_4 + A_2 + 5A_1$	[1]
357	11	$A_4 + 7A_1$	[1]
358	11	$3A_3 + A_2$	[1]
359	11	$3A_3 + 2A_1$	[2], [1]
360	11	$2A_3 + 2A_2 + A_1$	[1]
361	11	$2A_3 + A_2 + 3A_1$	[1]
362	11	$2A_3 + 5A_1$	[2], [1]
363	11	$A_3 + 4A_2$	[1]
364	11	$A_3 + 3A_2 + 2A_1$	[1]
365	11	$A_3 + 2A_2 + 4A_1$	[1]
366	11	$A_3 + A_2 + 6A_1$	[2], [1]
367	11	$A_3 + 8A_1$	[2], [1]

368	11	$5\,A_2 + A_1$	[1]
369	11	$4\,A_2 + 3\,A_1$	[1]
370	11	$3\,A_2 + 5\,A_1$	[1]
371	11	$2\,A_2 + 7\,A_1$	[1]
372	11	$A_2 + 9\,A_1$	[2], [1]
373	11	$11\,A_1$	[2]

No.	rank	ADE-type	G
374	12	$E_8 + D_4$	[1]
375	12	$E_8 + A_4$	[1]
376	12	$E_8 + A_3 + A_1$	[1]
377	12	$E_8 + 2\,A_2$	[1]
378	12	$E_8 + A_2 + 2\,A_1$	[1]
379	12	$E_8 + 4\,A_1$	[1]
380	12	$E_7 + D_5$	[1]
381	12	$E_7 + D_4 + A_1$	[1]
382	12	$E_7 + A_5$	[1]
383	12	$E_7 + A_4 + A_1$	[1]
384	12	$E_7 + A_3 + A_2$	[1]
385	12	$E_7 + A_3 + 2\,A_1$	[1]
386	12	$E_7 + 2\,A_2 + A_1$	[1]
387	12	$E_7 + A_2 + 3\,A_1$	[1]
388	12	$E_7 + 5\,A_1$	[2], [1]
389	12	$2\,E_6$	[1]
390	12	$E_6 + D_6$	[1]
391	12	$E_6 + D_5 + A_1$	[1]
392	12	$E_6 + D_4 + A_2$	[1]
393	12	$E_6 + D_4 + 2\,A_1$	[1]
394	12	$E_6 + A_6$	[1]
395	12	$E_6 + A_5 + A_1$	[1]
396	12	$E_6 + A_4 + A_2$	[1]
397	12	$E_6 + A_4 + 2\,A_1$	[1]
398	12	$E_6 + 2\,A_3$	[1]
399	12	$E_6 + A_3 + A_2 + A_1$	[1]
400	12	$E_6 + A_3 + 3\,A_1$	[1]
401	12	$E_6 + 3\,A_2$	[1]

402	12	$E_6 + 2A_2 + 2A_1$	[1]
403	12	$E_6 + A_2 + 4A_1$	[1]
404	12	$E_6 + 6A_1$	[1]
405	12	D_{12}	[1]
406	12	$D_{11} + A_1$	[1]
407	12	$D_{10} + A_2$	[1]
408	12	$D_{10} + 2A_1$	[1]
409	12	$D_9 + A_3$	[1]
410	12	$D_9 + A_2 + A_1$	[1]
411	12	$D_9 + 3A_1$	[1]
412	12	$D_8 + D_4$	[1]
413	12	$D_8 + A_4$	[1]
414	12	$D_8 + A_3 + A_1$	[1]
415	12	$D_8 + 2A_2$	[1]
416	12	$D_8 + A_2 + 2A_1$	[1]
417	12	$D_8 + 4A_1$	[2], [1]
418	12	$D_7 + D_5$	[1]
419	12	$D_7 + D_4 + A_1$	[1]
420	12	$D_7 + A_5$	[1]
421	12	$D_7 + A_4 + A_1$	[1]
422	12	$D_7 + A_3 + A_2$	[1]
423	12	$D_7 + A_3 + 2A_1$	[1]
424	12	$D_7 + 2A_2 + A_1$	[1]
425	12	$D_7 + A_2 + 3A_1$	[1]
426	12	$D_7 + 5A_1$	[1]
427	12	$2D_6$	[1]
428	12	$D_6 + D_5 + A_1$	[1]
429	12	$D_6 + D_4 + A_2$	[1]
430	12	$D_6 + D_4 + 2A_1$	[1]
431	12	$D_6 + A_6$	[1]
432	12	$D_6 + A_5 + A_1$	[1]
433	12	$D_6 + A_4 + A_2$	[1]
434	12	$D_6 + A_4 + 2A_1$	[1]
435	12	$D_6 + 2A_3$	[1]
436	12	$D_6 + A_3 + A_2 + A_1$	[1]
437	12	$D_6 + A_3 + 3A_1$	[2], [1]
438	12	$D_6 + 3A_2$	[1]

439	12	$D_6 + 2A_2 + 2A_1$	[1]
440	12	$D_6 + A_2 + 4A_1$	[1]
441	12	$D_6 + 6A_1$	[2], [1]
442	12	$2D_5 + A_2$	[1]
443	12	$2D_5 + 2A_1$	[1]
444	12	$D_5 + D_4 + A_3$	[1]
445	12	$D_5 + D_4 + A_2 + A_1$	[1]
446	12	$D_5 + D_4 + 3A_1$	[1]
447	12	$D_5 + A_7$	[1]
448	12	$D_5 + A_6 + A_1$	[1]
449	12	$D_5 + A_5 + A_2$	[1]
450	12	$D_5 + A_5 + 2A_1$	[1]
451	12	$D_5 + A_4 + A_3$	[1]
452	12	$D_5 + A_4 + A_2 + A_1$	[1]
453	12	$D_5 + A_4 + 3A_1$	[1]
454	12	$D_5 + 2A_3 + A_1$	[1]
455	12	$D_5 + A_3 + 2A_2$	[1]
456	12	$D_5 + A_3 + A_2 + 2A_1$	[1]
457	12	$D_5 + A_3 + 4A_1$	[2], [1]
458	12	$D_5 + 3A_2 + A_1$	[1]
459	12	$D_5 + 2A_2 + 3A_1$	[1]
460	12	$D_5 + A_2 + 5A_1$	[1]
461	12	$D_5 + 7A_1$	[2], [1]
462	12	$3D_4$	[1]
463	12	$2D_4 + A_4$	[1]
464	12	$2D_4 + A_3 + A_1$	[1]
465	12	$2D_4 + 2A_2$	[1]
466	12	$2D_4 + A_2 + 2A_1$	[1]
467	12	$2D_4 + 4A_1$	[2], [1]
468	12	$D_4 + A_8$	[1]
469	12	$D_4 + A_7 + A_1$	[1]
470	12	$D_4 + A_6 + A_2$	[1]
471	12	$D_4 + A_6 + 2A_1$	[1]
472	12	$D_4 + A_5 + A_3$	[1]
473	12	$D_4 + A_5 + A_2 + A_1$	[1]
474	12	$D_4 + A_5 + 3A_1$	[2], [1]
475	12	$D_4 + 2A_4$	[1]

476	12	$D_4 + A_4 + A_3 + A_1$	[1]
477	12	$D_4 + A_4 + 2A_2$	[1]
478	12	$D_4 + A_4 + A_2 + 2A_1$	[1]
479	12	$D_4 + A_4 + 4A_1$	[1]
480	12	$D_4 + 2A_3 + A_2$	[1]
481	12	$D_4 + 2A_3 + 2A_1$	[2], [1]
482	12	$D_4 + A_3 + 2A_2 + A_1$	[1]
483	12	$D_4 + A_3 + A_2 + 3A_1$	[1]
484	12	$D_4 + A_3 + 5A_1$	[2], [1]
485	12	$D_4 + 4A_2$	[1]
486	12	$D_4 + 3A_2 + 2A_1$	[1]
487	12	$D_4 + 2A_2 + 4A_1$	[1]
488	12	$D_4 + A_2 + 6A_1$	[2], [1]
489	12	$D_4 + 8A_1$	[2]
490	12	A_{12}	[1]
491	12	$A_{11} + A_1$	[1]
492	12	$A_{10} + A_2$	[1]
493	12	$A_{10} + 2A_1$	[1]
494	12	$A_9 + A_3$	[1]
495	12	$A_9 + A_2 + A_1$	[1]
496	12	$A_9 + 3A_1$	[2], [1]
497	12	$A_8 + A_4$	[1]
498	12	$A_8 + A_3 + A_1$	[1]
499	12	$A_8 + 2A_2$	[1]
500	12	$A_8 + A_2 + 2A_1$	[1]
501	12	$A_8 + 4A_1$	[1]
502	12	$A_7 + A_5$	[1]
503	12	$A_7 + A_4 + A_1$	[1]
504	12	$A_7 + A_3 + A_2$	[1]
505	12	$A_7 + A_3 + 2A_1$	[2], [1]
506	12	$A_7 + 2A_2 + A_1$	[1]
507	12	$A_7 + A_2 + 3A_1$	[1]
508	12	$A_7 + 5A_1$	[2], [1]
509	12	$2A_6$	[1]
510	12	$A_6 + A_5 + A_1$	[1]
511	12	$A_6 + A_4 + A_2$	[1]
512	12	$A_6 + A_4 + 2A_1$	[1]

513	12	$A_6 + 2A_3$	[1]
514	12	$A_6 + A_3 + A_2 + A_1$	[1]
515	12	$A_6 + A_3 + 3A_1$	[1]
516	12	$A_6 + 3A_2$	[1]
517	12	$A_6 + 2A_2 + 2A_1$	[1]
518	12	$A_6 + A_2 + 4A_1$	[1]
519	12	$A_6 + 6A_1$	[1]
520	12	$2A_5 + A_2$	[1]
521	12	$2A_5 + 2A_1$	[2], [1]
522	12	$A_5 + A_4 + A_3$	[1]
523	12	$A_5 + A_4 + A_2 + A_1$	[1]
524	12	$A_5 + A_4 + 3A_1$	[1]
525	12	$A_5 + 2A_3 + A_1$	[2], [1]
526	12	$A_5 + A_3 + 2A_2$	[1]
527	12	$A_5 + A_3 + A_2 + 2A_1$	[1]
528	12	$A_5 + A_3 + 4A_1$	[2], [1]
529	12	$A_5 + 3A_2 + A_1$	[1]
530	12	$A_5 + 2A_2 + 3A_1$	[1]
531	12	$A_5 + A_2 + 5A_1$	[2], [1]
532	12	$A_5 + 7A_1$	[2], [1]
533	12	$3A_4$	[1]
534	12	$2A_4 + A_3 + A_1$	[1]
535	12	$2A_4 + 2A_2$	[1]
536	12	$2A_4 + A_2 + 2A_1$	[1]
537	12	$2A_4 + 4A_1$	[1]
538	12	$A_4 + 2A_3 + A_2$	[1]
539	12	$A_4 + 2A_3 + 2A_1$	[1]
540	12	$A_4 + A_3 + 2A_2 + A_1$	[1]
541	12	$A_4 + A_3 + A_2 + 3A_1$	[1]
542	12	$A_4 + A_3 + 5A_1$	[1]
543	12	$A_4 + 4A_2$	[1]
544	12	$A_4 + 3A_2 + 2A_1$	[1]
545	12	$A_4 + 2A_2 + 4A_1$	[1]
546	12	$A_4 + A_2 + 6A_1$	[1]
547	12	$A_4 + 8A_1$	[2], [1]
548	12	$4A_3$	[2], [1]
549	12	$3A_3 + A_2 + A_1$	[1]

550	12	$3A_3 + 3A_1$	[2], [1]
551	12	$2A_3 + 3A_2$	[1]
552	12	$2A_3 + 2A_2 + 2A_1$	[1]
553	12	$2A_3 + A_2 + 4A_1$	[2], [1]
554	12	$2A_3 + 6A_1$	[2], [1]
555	12	$A_3 + 4A_2 + A_1$	[1]
556	12	$A_3 + 3A_2 + 3A_1$	[1]
557	12	$A_3 + 2A_2 + 5A_1$	[1]
558	12	$A_3 + A_2 + 7A_1$	[2], [1]
559	12	$A_3 + 9A_1$	[2]
560	12	$6A_2$	[3], [1]
561	12	$5A_2 + 2A_1$	[1]
562	12	$4A_2 + 4A_1$	[1]
563	12	$3A_2 + 6A_1$	[1]
564	12	$2A_2 + 8A_1$	[2], [1]
565	12	$A_2 + 10A_1$	[2]
566	12	$12A_1$	$[2,2]$

No.	rank	ADE-type	G
567	13	$E_8 + D_5$	[1]
568	13	$E_8 + D_4 + A_1$	[1]
569	13	$E_8 + A_5$	[1]
570	13	$E_8 + A_4 + A_1$	[1]
571	13	$E_8 + A_3 + A_2$	[1]
572	13	$E_8 + A_3 + 2A_1$	[1]
573	13	$E_8 + 2A_2 + A_1$	[1]
574	13	$E_8 + A_2 + 3A_1$	[1]
575	13	$E_8 + 5A_1$	[1]
576	13	$E_7 + E_6$	[1]
577	13	$E_7 + D_6$	[1]
578	13	$E_7 + D_5 + A_1$	[1]
579	13	$E_7 + D_4 + A_2$	[1]
580	13	$E_7 + D_4 + 2A_1$	[1]
581	13	$E_7 + A_6$	[1]
582	13	$E_7 + A_5 + A_1$	[1]
583	13	$E_7 + A_4 + A_2$	[1]

584	13	$E_7 + A_4 + 2A_1$	[1]
585	13	$E_7 + 2A_3$	[1]
586	13	$E_7 + A_3 + A_2 + A_1$	[1]
587	13	$E_7 + A_3 + 3A_1$	[2], [1]
588	13	$E_7 + 3A_2$	[1]
589	13	$E_7 + 2A_2 + 2A_1$	[1]
590	13	$E_7 + A_2 + 4A_1$	[1]
591	13	$E_7 + 6A_1$	[2], [1]
592	13	$2E_6 + A_1$	[1]
593	13	$E_6 + D_7$	[1]
594	13	$E_6 + D_6 + A_1$	[1]
595	13	$E_6 + D_5 + A_2$	[1]
596	13	$E_6 + D_5 + 2A_1$	[1]
597	13	$E_6 + D_4 + A_3$	[1]
598	13	$E_6 + D_4 + A_2 + A_1$	[1]
599	13	$E_6 + D_4 + 3A_1$	[1]
600	13	$E_6 + A_7$	[1]
601	13	$E_6 + A_6 + A_1$	[1]
602	13	$E_6 + A_5 + A_2$	[1]
603	13	$E_6 + A_5 + 2A_1$	[1]
604	13	$E_6 + A_4 + A_3$	[1]
605	13	$E_6 + A_4 + A_2 + A_1$	[1]
606	13	$E_6 + A_4 + 3A_1$	[1]
607	13	$E_6 + 2A_3 + A_1$	[1]
608	13	$E_6 + A_3 + 2A_2$	[1]
609	13	$E_6 + A_3 + A_2 + 2A_1$	[1]
610	13	$E_6 + A_3 + 4A_1$	[1]
611	13	$E_6 + 3A_2 + A_1$	[1]
612	13	$E_6 + 2A_2 + 3A_1$	[1]
613	13	$E_6 + A_2 + 5A_1$	[1]
614	13	$E_6 + 7A_1$	[1]
615	13	D_{13}	[1]
616	13	$D_{12} + A_1$	[1]
617	13	$D_{11} + A_2$	[1]
618	13	$D_{11} + 2A_1$	[1]
619	13	$D_{10} + A_3$	[1]
620	13	$D_{10} + A_2 + A_1$	[1]

621	13	$D_{10} + 3A_1$	[2], [1]
622	13	$D_9 + D_4$	[1]
623	13	$D_9 + A_4$	[1]
624	13	$D_9 + A_3 + A_1$	[1]
625	13	$D_9 + 2A_2$	[1]
626	13	$D_9 + A_2 + 2A_1$	[1]
627	13	$D_9 + 4A_1$	[1]
628	13	$D_8 + D_5$	[1]
629	13	$D_8 + D_4 + A_1$	[1]
630	13	$D_8 + A_5$	[1]
631	13	$D_8 + A_4 + A_1$	[1]
632	13	$D_8 + A_3 + A_2$	[1]
633	13	$D_8 + A_3 + 2A_1$	[2], [1]
634	13	$D_8 + 2A_2 + A_1$	[1]
635	13	$D_8 + A_2 + 3A_1$	[1]
636	13	$D_8 + 5A_1$	[2], [1]
637	13	$D_7 + D_6$	[1]
638	13	$D_7 + D_5 + A_1$	[1]
639	13	$D_7 + D_4 + A_2$	[1]
640	13	$D_7 + D_4 + 2A_1$	[1]
641	13	$D_7 + A_6$	[1]
642	13	$D_7 + A_5 + A_1$	[1]
643	13	$D_7 + A_4 + A_2$	[1]
644	13	$D_7 + A_4 + 2A_1$	[1]
645	13	$D_7 + 2A_3$	[1]
646	13	$D_7 + A_3 + A_2 + A_1$	[1]
647	13	$D_7 + A_3 + 3A_1$	[1]
648	13	$D_7 + 3A_2$	[1]
649	13	$D_7 + 2A_2 + 2A_1$	[1]
650	13	$D_7 + A_2 + 4A_1$	[1]
651	13	$D_7 + 6A_1$	[2], [1]
652	13	$2D_6 + A_1$	[1]
653	13	$D_6 + D_5 + A_2$	[1]
654	13	$D_6 + D_5 + 2A_1$	[1]
655	13	$D_6 + D_4 + A_3$	[1]
656	13	$D_6 + D_4 + A_2 + A_1$	[1]
657	13	$D_6 + D_4 + 3A_1$	[2], [1]

658	13	$D_6 + A_7$	[1]
659	13	$D_6 + A_6 + A_1$	[1]
660	13	$D_6 + A_5 + A_2$	[1]
661	13	$D_6 + A_5 + 2\,A_1$	[2], [1]
662	13	$D_6 + A_4 + A_3$	[1]
663	13	$D_6 + A_4 + A_2 + A_1$	[1]
664	13	$D_6 + A_4 + 3\,A_1$	[1]
665	13	$D_6 + 2\,A_3 + A_1$	[2], [1]
666	13	$D_6 + A_3 + 2\,A_2$	[1]
667	13	$D_6 + A_3 + A_2 + 2\,A_1$	[1]
668	13	$D_6 + A_3 + 4\,A_1$	[2], [1]
669	13	$D_6 + 3\,A_2 + A_1$	[1]
670	13	$D_6 + 2\,A_2 + 3\,A_1$	[1]
671	13	$D_6 + A_2 + 5\,A_1$	[2], [1]
672	13	$D_6 + 7\,A_1$	[2]
673	13	$2\,D_5 + A_3$	[1]
674	13	$2\,D_5 + A_2 + A_1$	[1]
675	13	$2\,D_5 + 3\,A_1$	[1]
676	13	$D_5 + 2\,D_4$	[1]
677	13	$D_5 + D_4 + A_4$	[1]
678	13	$D_5 + D_4 + A_3 + A_1$	[1]
679	13	$D_5 + D_4 + 2\,A_2$	[1]
680	13	$D_5 + D_4 + A_2 + 2\,A_1$	[1]
681	13	$D_5 + D_4 + 4\,A_1$	[2], [1]
682	13	$D_5 + A_8$	[1]
683	13	$D_5 + A_7 + A_1$	[1]
684	13	$D_5 + A_6 + A_2$	[1]
685	13	$D_5 + A_6 + 2\,A_1$	[1]
686	13	$D_5 + A_5 + A_3$	[1]
687	13	$D_5 + A_5 + A_2 + A_1$	[1]
688	13	$D_5 + A_5 + 3\,A_1$	[2], [1]
689	13	$D_5 + 2\,A_4$	[1]
690	13	$D_5 + A_4 + A_3 + A_1$	[1]
691	13	$D_5 + A_4 + 2\,A_2$	[1]
692	13	$D_5 + A_4 + A_2 + 2\,A_1$	[1]
693	13	$D_5 + A_4 + 4\,A_1$	[1]
694	13	$D_5 + 2\,A_3 + A_2$	[1]

695	13	$D_5 + 2A_3 + 2A_1$	[2], [1]
696	13	$D_5 + A_3 + 2A_2 + A_1$	[1]
697	13	$D_5 + A_3 + A_2 + 3A_1$	[1]
698	13	$D_5 + A_3 + 5A_1$	[2], [1]
699	13	$D_5 + 4A_2$	[1]
700	13	$D_5 + 3A_2 + 2A_1$	[1]
701	13	$D_5 + 2A_2 + 4A_1$	[1]
702	13	$D_5 + A_2 + 6A_1$	[2], [1]
703	13	$D_5 + 8A_1$	[2]
704	13	$3D_4 + A_1$	[1]
705	13	$2D_4 + A_5$	[1]
706	13	$2D_4 + A_4 + A_1$	[1]
707	13	$2D_4 + A_3 + A_2$	[1]
708	13	$2D_4 + A_3 + 2A_1$	[2], [1]
709	13	$2D_4 + 2A_2 + A_1$	[1]
710	13	$2D_4 + A_2 + 3A_1$	[1]
711	13	$2D_4 + 5A_1$	[2]
712	13	$D_4 + A_9$	[1]
713	13	$D_4 + A_8 + A_1$	[1]
714	13	$D_4 + A_7 + A_2$	[1]
715	13	$D_4 + A_7 + 2A_1$	[2], [1]
716	13	$D_4 + A_6 + A_3$	[1]
717	13	$D_4 + A_6 + A_2 + A_1$	[1]
718	13	$D_4 + A_6 + 3A_1$	[1]
719	13	$D_4 + A_5 + A_4$	[1]
720	13	$D_4 + A_5 + A_3 + A_1$	[2], [1]
721	13	$D_4 + A_5 + 2A_2$	[1]
722	13	$D_4 + A_5 + A_2 + 2A_1$	[1]
723	13	$D_4 + A_5 + 4A_1$	[2], [1]
724	13	$D_4 + 2A_4 + A_1$	[1]
725	13	$D_4 + A_4 + A_3 + A_2$	[1]
726	13	$D_4 + A_4 + A_3 + 2A_1$	[1]
727	13	$D_4 + A_4 + 2A_2 + A_1$	[1]
728	13	$D_4 + A_4 + A_2 + 3A_1$	[1]
729	13	$D_4 + A_4 + 5A_1$	[1]
730	13	$D_4 + 3A_3$	[2], [1]
731	13	$D_4 + 2A_3 + A_2 + A_1$	[1]

732	13	$D_4 + 2A_3 + 3A_1$	[2], [1]
733	13	$D_4 + A_3 + 3A_2$	[1]
734	13	$D_4 + A_3 + 2A_2 + 2A_1$	[1]
735	13	$D_4 + A_3 + A_2 + 4A_1$	[2], [1]
736	13	$D_4 + A_3 + 6A_1$	[2]
737	13	$D_4 + 4A_2 + A_1$	[1]
738	13	$D_4 + 3A_2 + 3A_1$	[1]
739	13	$D_4 + 2A_2 + 5A_1$	[1]
740	13	$D_4 + A_2 + 7A_1$	[2]
741	13	$D_4 + 9A_1$	$[2, 2]$
742	13	A_{13}	[1]
743	13	$A_{12} + A_1$	[1]
744	13	$A_{11} + A_2$	[1]
745	13	$A_{11} + 2A_1$	[2], [1]
746	13	$A_{10} + A_3$	[1]
747	13	$A_{10} + A_2 + A_1$	[1]
748	13	$A_{10} + 3A_1$	[1]
749	13	$A_9 + A_4$	[1]
750	13	$A_9 + A_3 + A_1$	[2], [1]
751	13	$A_9 + 2A_2$	[1]
752	13	$A_9 + A_2 + 2A_1$	[1]
753	13	$A_9 + 4A_1$	[2], [1]
754	13	$A_8 + A_5$	[1]
755	13	$A_8 + A_4 + A_1$	[1]
756	13	$A_8 + A_3 + A_2$	[1]
757	13	$A_8 + A_3 + 2A_1$	[1]
758	13	$A_8 + 2A_2 + A_1$	[1]
759	13	$A_8 + A_2 + 3A_1$	[1]
760	13	$A_8 + 5A_1$	[1]
761	13	$A_7 + A_6$	[1]
762	13	$A_7 + A_5 + A_1$	[2], [1]
763	13	$A_7 + A_4 + A_2$	[1]
764	13	$A_7 + A_4 + 2A_1$	[1]
765	13	$A_7 + 2A_3$	[2], [1]
766	13	$A_7 + A_3 + A_2 + A_1$	[1]
767	13	$A_7 + A_3 + 3A_1$	[2], [1]
768	13	$A_7 + 3A_2$	[1]

769	13	$A_7 + 2A_2 + 2A_1$	[1]
770	13	$A_7 + A_2 + 4A_1$	[2], [1]
771	13	$A_7 + 6A_1$	[2], [1]
772	13	$2A_6 + A_1$	[1]
773	13	$A_6 + A_5 + A_2$	[1]
774	13	$A_6 + A_5 + 2A_1$	[1]
775	13	$A_6 + A_4 + A_3$	[1]
776	13	$A_6 + A_4 + A_2 + A_1$	[1]
777	13	$A_6 + A_4 + 3A_1$	[1]
778	13	$A_6 + 2A_3 + A_1$	[1]
779	13	$A_6 + A_3 + 2A_2$	[1]
780	13	$A_6 + A_3 + A_2 + 2A_1$	[1]
781	13	$A_6 + A_3 + 4A_1$	[1]
782	13	$A_6 + 3A_2 + A_1$	[1]
783	13	$A_6 + 2A_2 + 3A_1$	[1]
784	13	$A_6 + A_2 + 5A_1$	[1]
785	13	$A_6 + 7A_1$	[1]
786	13	$2A_5 + A_3$	[2], [1]
787	13	$2A_5 + A_2 + A_1$	[1]
788	13	$2A_5 + 3A_1$	[2], [1]
789	13	$A_5 + 2A_4$	[1]
790	13	$A_5 + A_4 + A_3 + A_1$	[1]
791	13	$A_5 + A_4 + 2A_2$	[1]
792	13	$A_5 + A_4 + A_2 + 2A_1$	[1]
793	13	$A_5 + A_4 + 4A_1$	[1]
794	13	$A_5 + 2A_3 + A_2$	[1]
795	13	$A_5 + 2A_3 + 2A_1$	[2], [1]
796	13	$A_5 + A_3 + 2A_2 + A_1$	[1]
797	13	$A_5 + A_3 + A_2 + 3A_1$	[2], [1]
798	13	$A_5 + A_3 + 5A_1$	[2], [1]
799	13	$A_5 + 4A_2$	[3], [1]
800	13	$A_5 + 3A_2 + 2A_1$	[1]
801	13	$A_5 + 2A_2 + 4A_1$	[1]
802	13	$A_5 + A_2 + 6A_1$	[2], [1]
803	13	$A_5 + 8A_1$	[2]
804	13	$3A_4 + A_1$	[1]
805	13	$2A_4 + A_3 + A_2$	[1]

806	13	$2A_4 + A_3 + 2A_1$	[1]
807	13	$2A_4 + 2A_2 + A_1$	[1]
808	13	$2A_4 + A_2 + 3A_1$	[1]
809	13	$2A_4 + 5A_1$	[1]
810	13	$A_4 + 3A_3$	[1]
811	13	$A_4 + 2A_3 + A_2 + A_1$	[1]
812	13	$A_4 + 2A_3 + 3A_1$	[1]
813	13	$A_4 + A_3 + 3A_2$	[1]
814	13	$A_4 + A_3 + 2A_2 + 2A_1$	[1]
815	13	$A_4 + A_3 + A_2 + 4A_1$	[1]
816	13	$A_4 + A_3 + 6A_1$	[2], [1]
817	13	$A_4 + 4A_2 + A_1$	[1]
818	13	$A_4 + 3A_2 + 3A_1$	[1]
819	13	$A_4 + 2A_2 + 5A_1$	[1]
820	13	$A_4 + A_2 + 7A_1$	[1]
821	13	$A_4 + 9A_1$	[2]
822	13	$4A_3 + A_1$	[2], [1]
823	13	$3A_3 + 2A_2$	[1]
824	13	$3A_3 + A_2 + 2A_1$	[2], [1]
825	13	$3A_3 + 4A_1$	[2], [1]
826	13	$2A_3 + 3A_2 + A_1$	[1]
827	13	$2A_3 + 2A_2 + 3A_1$	[1]
828	13	$2A_3 + A_2 + 5A_1$	[2], [1]
829	13	$2A_3 + 7A_1$	[2]
830	13	$A_3 + 5A_2$	[1]
831	13	$A_3 + 4A_2 + 2A_1$	[1]
832	13	$A_3 + 3A_2 + 4A_1$	[1]
833	13	$A_3 + 2A_2 + 6A_1$	[2], [1]
834	13	$A_3 + A_2 + 8A_1$	[2]
835	13	$A_3 + 10A_1$	$[2,2]$
836	13	$6A_2 + A_1$	[3], [1]
837	13	$5A_2 + 3A_1$	[1]
838	13	$4A_2 + 5A_1$	[1]
839	13	$3A_2 + 7A_1$	[1]
840	13	$2A_2 + 9A_1$	[2]

No.	rank	ADE-type	G
841	14	$E_8 + E_6$	[1]
842	14	$E_8 + D_6$	[1]
843	14	$E_8 + D_5 + A_1$	[1]
844	14	$E_8 + D_4 + A_2$	[1]
845	14	$E_8 + D_4 + 2\,A_1$	[1]
846	14	$E_8 + A_6$	[1]
847	14	$E_8 + A_5 + A_1$	[1]
848	14	$E_8 + A_4 + A_2$	[1]
849	14	$E_8 + A_4 + 2\,A_1$	[1]
850	14	$E_8 + 2\,A_3$	[1]
851	14	$E_8 + A_3 + A_2 + A_1$	[1]
852	14	$E_8 + A_3 + 3\,A_1$	[1]
853	14	$E_8 + 3\,A_2$	[1]
854	14	$E_8 + 2\,A_2 + 2\,A_1$	[1]
855	14	$E_8 + A_2 + 4\,A_1$	[1]
856	14	$E_8 + 6\,A_1$	[1]
857	14	$2\,E_7$	[1]
858	14	$E_7 + E_6 + A_1$	[1]
859	14	$E_7 + D_7$	[1]
860	14	$E_7 + D_6 + A_1$	[1]
861	14	$E_7 + D_5 + A_2$	[1]
862	14	$E_7 + D_5 + 2\,A_1$	[1]
863	14	$E_7 + D_4 + A_3$	[1]
864	14	$E_7 + D_4 + A_2 + A_1$	[1]
865	14	$E_7 + D_4 + 3\,A_1$	[2], [1]
866	14	$E_7 + A_7$	[1]
867	14	$E_7 + A_6 + A_1$	[1]
868	14	$E_7 + A_5 + A_2$	[1]
869	14	$E_7 + A_5 + 2\,A_1$	[2], [1]
870	14	$E_7 + A_4 + A_3$	[1]
871	14	$E_7 + A_4 + A_2 + A_1$	[1]
872	14	$E_7 + A_4 + 3\,A_1$	[1]
873	14	$E_7 + 2\,A_3 + A_1$	[2], [1]
874	14	$E_7 + A_3 + 2\,A_2$	[1]
875	14	$E_7 + A_3 + A_2 + 2\,A_1$	[1]
876	14	$E_7 + A_3 + 4\,A_1$	[2], [1]

877	14	$E_7 + 3A_2 + A_1$	[1]
878	14	$E_7 + 2A_2 + 3A_1$	[1]
879	14	$E_7 + A_2 + 5A_1$	[2], [1]
880	14	$E_7 + 7A_1$	[2]
881	14	$2E_6 + A_2$	[1]
882	14	$2E_6 + 2A_1$	[1]
883	14	$E_6 + D_8$	[1]
884	14	$E_6 + D_7 + A_1$	[1]
885	14	$E_6 + D_6 + A_2$	[1]
886	14	$E_6 + D_6 + 2A_1$	[1]
887	14	$E_6 + D_5 + A_3$	[1]
888	14	$E_6 + D_5 + A_2 + A_1$	[1]
889	14	$E_6 + D_5 + 3A_1$	[1]
890	14	$E_6 + 2D_4$	[1]
891	14	$E_6 + D_4 + A_4$	[1]
892	14	$E_6 + D_4 + A_3 + A_1$	[1]
893	14	$E_6 + D_4 + 2A_2$	[1]
894	14	$E_6 + D_4 + A_2 + 2A_1$	[1]
895	14	$E_6 + D_4 + 4A_1$	[1]
896	14	$E_6 + A_8$	[1]
897	14	$E_6 + A_7 + A_1$	[1]
898	14	$E_6 + A_6 + A_2$	[1]
899	14	$E_6 + A_6 + 2A_1$	[1]
900	14	$E_6 + A_5 + A_3$	[1]
901	14	$E_6 + A_5 + A_2 + A_1$	[1]
902	14	$E_6 + A_5 + 3A_1$	[1]
903	14	$E_6 + 2A_4$	[1]
904	14	$E_6 + A_4 + A_3 + A_1$	[1]
905	14	$E_6 + A_4 + 2A_2$	[1]
906	14	$E_6 + A_4 + A_2 + 2A_1$	[1]
907	14	$E_6 + A_4 + 4A_1$	[1]
908	14	$E_6 + 2A_3 + A_2$	[1]
909	14	$E_6 + 2A_3 + 2A_1$	[1]
910	14	$E_6 + A_3 + 2A_2 + A_1$	[1]
911	14	$E_6 + A_3 + A_2 + 3A_1$	[1]
912	14	$E_6 + A_3 + 5A_1$	[1]
913	14	$E_6 + 4A_2$	[3], [1]

914	14	$E_6 + 3A_2 + 2A_1$	[1]
915	14	$E_6 + 2A_2 + 4A_1$	[1]
916	14	$E_6 + A_2 + 6A_1$	[1]
917	14	D_{14}	[1]
918	14	$D_{13} + A_1$	[1]
919	14	$D_{12} + A_2$	[1]
920	14	$D_{12} + 2A_1$	[2], [1]
921	14	$D_{11} + A_3$	[1]
922	14	$D_{11} + A_2 + A_1$	[1]
923	14	$D_{11} + 3A_1$	[1]
924	14	$D_{10} + D_4$	[1]
925	14	$D_{10} + A_4$	[1]
926	14	$D_{10} + A_3 + A_1$	[2], [1]
927	14	$D_{10} + 2A_2$	[1]
928	14	$D_{10} + A_2 + 2A_1$	[1]
929	14	$D_{10} + 4A_1$	[2], [1]
930	14	$D_9 + D_5$	[1]
931	14	$D_9 + D_4 + A_1$	[1]
932	14	$D_9 + A_5$	[1]
933	14	$D_9 + A_4 + A_1$	[1]
934	14	$D_9 + A_3 + A_2$	[1]
935	14	$D_9 + A_3 + 2A_1$	[1]
936	14	$D_9 + 2A_2 + A_1$	[1]
937	14	$D_9 + A_2 + 3A_1$	[1]
938	14	$D_9 + 5A_1$	[1]
939	14	$D_8 + D_6$	[1]
940	14	$D_8 + D_5 + A_1$	[1]
941	14	$D_8 + D_4 + A_2$	[1]
942	14	$D_8 + D_4 + 2A_1$	[2], [1]
943	14	$D_8 + A_6$	[1]
944	14	$D_8 + A_5 + A_1$	[2], [1]
945	14	$D_8 + A_4 + A_2$	[1]
946	14	$D_8 + A_4 + 2A_1$	[1]
947	14	$D_8 + 2A_3$	[2], [1]
948	14	$D_8 + A_3 + A_2 + A_1$	[1]
949	14	$D_8 + A_3 + 3A_1$	[2], [1]
950	14	$D_8 + 3A_2$	[1]

951	14	$D_8 + 2A_2 + 2A_1$	[1]
952	14	$D_8 + A_2 + 4A_1$	[2], [1]
953	14	$D_8 + 6A_1$	[2]
954	14	$2D_7$	[1]
955	14	$D_7 + D_6 + A_1$	[1]
956	14	$D_7 + D_5 + A_2$	[1]
957	14	$D_7 + D_5 + 2A_1$	[1]
958	14	$D_7 + D_4 + A_3$	[1]
959	14	$D_7 + D_4 + A_2 + A_1$	[1]
960	14	$D_7 + D_4 + 3A_1$	[1]
961	14	$D_7 + A_7$	[1]
962	14	$D_7 + A_6 + A_1$	[1]
963	14	$D_7 + A_5 + A_2$	[1]
964	14	$D_7 + A_5 + 2A_1$	[1]
965	14	$D_7 + A_4 + A_3$	[1]
966	14	$D_7 + A_4 + A_2 + A_1$	[1]
967	14	$D_7 + A_4 + 3A_1$	[1]
968	14	$D_7 + 2A_3 + A_1$	[1]
969	14	$D_7 + A_3 + 2A_2$	[1]
970	14	$D_7 + A_3 + A_2 + 2A_1$	[1]
971	14	$D_7 + A_3 + 4A_1$	[2], [1]
972	14	$D_7 + 3A_2 + A_1$	[1]
973	14	$D_7 + 2A_2 + 3A_1$	[1]
974	14	$D_7 + A_2 + 5A_1$	[1]
975	14	$D_7 + 7A_1$	[2]
976	14	$2D_6 + A_2$	[1]
977	14	$2D_6 + 2A_1$	[2], [1]
978	14	$D_6 + D_5 + A_3$	[1]
979	14	$D_6 + D_5 + A_2 + A_1$	[1]
980	14	$D_6 + D_5 + 3A_1$	[2], [1]
981	14	$D_6 + 2D_4$	[1]
982	14	$D_6 + D_4 + A_4$	[1]
983	14	$D_6 + D_4 + A_3 + A_1$	[2], [1]
984	14	$D_6 + D_4 + 2A_2$	[1]
985	14	$D_6 + D_4 + A_2 + 2A_1$	[1]
986	14	$D_6 + D_4 + 4A_1$	[2]
987	14	$D_6 + A_8$	[1]

988	14	$D_6 + A_7 + A_1$	[2], [1]
989	14	$D_6 + A_6 + A_2$	[1]
990	14	$D_6 + A_6 + 2A_1$	[1]
991	14	$D_6 + A_5 + A_3$	[2], [1]
992	14	$D_6 + A_5 + A_2 + A_1$	[1]
993	14	$D_6 + A_5 + 3A_1$	[2], [1]
994	14	$D_6 + 2A_4$	[1]
995	14	$D_6 + A_4 + A_3 + A_1$	[1]
996	14	$D_6 + A_4 + 2A_2$	[1]
997	14	$D_6 + A_4 + A_2 + 2A_1$	[1]
998	14	$D_6 + A_4 + 4A_1$	[1]
999	14	$D_6 + 2A_3 + A_2$	[1]
1000	14	$D_6 + 2A_3 + 2A_1$	[2], [1]
1001	14	$D_6 + A_3 + 2A_2 + A_1$	[1]
1002	14	$D_6 + A_3 + A_2 + 3A_1$	[2], [1]
1003	14	$D_6 + A_3 + 5A_1$	[2]
1004	14	$D_6 + 4A_2$	[1]
1005	14	$D_6 + 3A_2 + 2A_1$	[1]
1006	14	$D_6 + 2A_2 + 4A_1$	[1]
1007	14	$D_6 + A_2 + 6A_1$	[2]
1008	14	$D_6 + 8A_1$	$[2, 2]$
1009	14	$2D_5 + D_4$	[1]
1010	14	$2D_5 + A_4$	[1]
1011	14	$2D_5 + A_3 + A_1$	[1]
1012	14	$2D_5 + 2A_2$	[1]
1013	14	$2D_5 + A_2 + 2A_1$	[1]
1014	14	$2D_5 + 4A_1$	[2], [1]
1015	14	$D_5 + 2D_4 + A_1$	[1]
1016	14	$D_5 + D_4 + A_5$	[1]
1017	14	$D_5 + D_4 + A_4 + A_1$	[1]
1018	14	$D_5 + D_4 + A_3 + A_2$	[1]
1019	14	$D_5 + D_4 + A_3 + 2A_1$	[2], [1]
1020	14	$D_5 + D_4 + 2A_2 + A_1$	[1]
1021	14	$D_5 + D_4 + A_2 + 3A_1$	[1]
1022	14	$D_5 + D_4 + 5A_1$	[2]
1023	14	$D_5 + A_9$	[1]
1024	14	$D_5 + A_8 + A_1$	[1]

1025	14	$D_5 + A_7 + A_2$	[1]
1026	14	$D_5 + A_7 + 2\,A_1$	[2], [1]
1027	14	$D_5 + A_6 + A_3$	[1]
1028	14	$D_5 + A_6 + A_2 + A_1$	[1]
1029	14	$D_5 + A_6 + 3\,A_1$	[1]
1030	14	$D_5 + A_5 + A_4$	[1]
1031	14	$D_5 + A_5 + A_3 + A_1$	[2], [1]
1032	14	$D_5 + A_5 + 2\,A_2$	[1]
1033	14	$D_5 + A_5 + A_2 + 2\,A_1$	[1]
1034	14	$D_5 + A_5 + 4\,A_1$	[2], [1]
1035	14	$D_5 + 2\,A_4 + A_1$	[1]
1036	14	$D_5 + A_4 + A_3 + A_2$	[1]
1037	14	$D_5 + A_4 + A_3 + 2\,A_1$	[1]
1038	14	$D_5 + A_4 + 2\,A_2 + A_1$	[1]
1039	14	$D_5 + A_4 + A_2 + 3\,A_1$	[1]
1040	14	$D_5 + A_4 + 5\,A_1$	[1]
1041	14	$D_5 + 3\,A_3$	[2], [1]
1042	14	$D_5 + 2\,A_3 + A_2 + A_1$	[1]
1043	14	$D_5 + 2\,A_3 + 3\,A_1$	[2], [1]
1044	14	$D_5 + A_3 + 3\,A_2$	[1]
1045	14	$D_5 + A_3 + 2\,A_2 + 2\,A_1$	[1]
1046	14	$D_5 + A_3 + A_2 + 4\,A_1$	[2], [1]
1047	14	$D_5 + A_3 + 6\,A_1$	[2]
1048	14	$D_5 + 4\,A_2 + A_1$	[1]
1049	14	$D_5 + 3\,A_2 + 3\,A_1$	[1]
1050	14	$D_5 + 2\,A_2 + 5\,A_1$	[1]
1051	14	$D_5 + A_2 + 7\,A_1$	[2]
1052	14	$3\,D_4 + A_2$	[1]
1053	14	$3\,D_4 + 2\,A_1$	[2]
1054	14	$2\,D_4 + A_6$	[1]
1055	14	$2\,D_4 + A_5 + A_1$	[2], [1]
1056	14	$2\,D_4 + A_4 + A_2$	[1]
1057	14	$2\,D_4 + A_4 + 2\,A_1$	[1]
1058	14	$2\,D_4 + 2\,A_3$	[2], [1]
1059	14	$2\,D_4 + A_3 + A_2 + A_1$	[1]
1060	14	$2\,D_4 + A_3 + 3\,A_1$	[2]
1061	14	$2\,D_4 + 3\,A_2$	[1]

1062	14	$2\,D_4 + 2\,A_2 + 2\,A_1$	[1]
1063	14	$2\,D_4 + A_2 + 4\,A_1$	[2]
1064	14	$2\,D_4 + 6\,A_1$	$[2,2]$
1065	14	$D_4 + A_{10}$	[1]
1066	14	$D_4 + A_9 + A_1$	[2], [1]
1067	14	$D_4 + A_8 + A_2$	[1]
1068	14	$D_4 + A_8 + 2\,A_1$	[1]
1069	14	$D_4 + A_7 + A_3$	[2], [1]
1070	14	$D_4 + A_7 + A_2 + A_1$	[1]
1071	14	$D_4 + A_7 + 3\,A_1$	[2], [1]
1072	14	$D_4 + A_6 + A_4$	[1]
1073	14	$D_4 + A_6 + A_3 + A_1$	[1]
1074	14	$D_4 + A_6 + 2\,A_2$	[1]
1075	14	$D_4 + A_6 + A_2 + 2\,A_1$	[1]
1076	14	$D_4 + A_6 + 4\,A_1$	[1]
1077	14	$D_4 + 2\,A_5$	[2], [1]
1078	14	$D_4 + A_5 + A_4 + A_1$	[1]
1079	14	$D_4 + A_5 + A_3 + A_2$	[1]
1080	14	$D_4 + A_5 + A_3 + 2\,A_1$	[2], [1]
1081	14	$D_4 + A_5 + 2\,A_2 + A_1$	[1]
1082	14	$D_4 + A_5 + A_2 + 3\,A_1$	[2], [1]
1083	14	$D_4 + A_5 + 5\,A_1$	[2]
1084	14	$D_4 + 2\,A_4 + A_2$	[1]
1085	14	$D_4 + 2\,A_4 + 2\,A_1$	[1]
1086	14	$D_4 + A_4 + 2\,A_3$	[1]
1087	14	$D_4 + A_4 + A_3 + A_2 + A_1$	[1]
1088	14	$D_4 + A_4 + A_3 + 3\,A_1$	[1]
1089	14	$D_4 + A_4 + 3\,A_2$	[1]
1090	14	$D_4 + A_4 + 2\,A_2 + 2\,A_1$	[1]
1091	14	$D_4 + A_4 + A_2 + 4\,A_1$	[1]
1092	14	$D_4 + A_4 + 6\,A_1$	[2]
1093	14	$D_4 + 3\,A_3 + A_1$	[2], [1]
1094	14	$D_4 + 2\,A_3 + 2\,A_2$	[1]
1095	14	$D_4 + 2\,A_3 + A_2 + 2\,A_1$	[2], [1]
1096	14	$D_4 + 2\,A_3 + 4\,A_1$	[2]
1097	14	$D_4 + A_3 + 3\,A_2 + A_1$	[1]
1098	14	$D_4 + A_3 + 2\,A_2 + 3\,A_1$	[1]

1099	14	$D_4 + A_3 + A_2 + 5\,A_1$	[2]
1100	14	$D_4 + A_3 + 7\,A_1$	[2, 2]
1101	14	$D_4 + 5\,A_2$	[1]
1102	14	$D_4 + 4\,A_2 + 2\,A_1$	[1]
1103	14	$D_4 + 3\,A_2 + 4\,A_1$	[1]
1104	14	$D_4 + 2\,A_2 + 6\,A_1$	[2]
1105	14	A_{14}	[1]
1106	14	$A_{13} + A_1$	[2], [1]
1107	14	$A_{12} + A_2$	[1]
1108	14	$A_{12} + 2\,A_1$	[1]
1109	14	$A_{11} + A_3$	[2], [1]
1110	14	$A_{11} + A_2 + A_1$	[1]
1111	14	$A_{11} + 3\,A_1$	[2], [1]
1112	14	$A_{10} + A_4$	[1]
1113	14	$A_{10} + A_3 + A_1$	[1]
1114	14	$A_{10} + 2\,A_2$	[1]
1115	14	$A_{10} + A_2 + 2\,A_1$	[1]
1116	14	$A_{10} + 4\,A_1$	[1]
1117	14	$A_9 + A_5$	[2], [1]
1118	14	$A_9 + A_4 + A_1$	[1]
1119	14	$A_9 + A_3 + A_2$	[1]
1120	14	$A_9 + A_3 + 2\,A_1$	[2], [1]
1121	14	$A_9 + 2\,A_2 + A_1$	[1]
1122	14	$A_9 + A_2 + 3\,A_1$	[2], [1]
1123	14	$A_9 + 5\,A_1$	[2], [1]
1124	14	$A_8 + A_6$	[1]
1125	14	$A_8 + A_5 + A_1$	[1]
1126	14	$A_8 + A_4 + A_2$	[1]
1127	14	$A_8 + A_4 + 2\,A_1$	[1]
1128	14	$A_8 + 2\,A_3$	[1]
1129	14	$A_8 + A_3 + A_2 + A_1$	[1]
1130	14	$A_8 + A_3 + 3\,A_1$	[1]
1131	14	$A_8 + 3\,A_2$	[3], [1]
1132	14	$A_8 + 2\,A_2 + 2\,A_1$	[1]
1133	14	$A_8 + A_2 + 4\,A_1$	[1]
1134	14	$A_8 + 6\,A_1$	[1]
1135	14	$2\,A_7$	[2], [1]

1136	14	$A_7 + A_6 + A_1$	[1]
1137	14	$A_7 + A_5 + A_2$	[1]
1138	14	$A_7 + A_5 + 2A_1$	[2], [1]
1139	14	$A_7 + A_4 + A_3$	[1]
1140	14	$A_7 + A_4 + A_2 + A_1$	[1]
1141	14	$A_7 + A_4 + 3A_1$	[1]
1142	14	$A_7 + 2A_3 + A_1$	[2], [1]
1143	14	$A_7 + A_3 + 2A_2$	[1]
1144	14	$A_7 + A_3 + A_2 + 2A_1$	[2], [1]
1145	14	$A_7 + A_3 + 4A_1$	[2], [1]
1146	14	$A_7 + 3A_2 + A_1$	[1]
1147	14	$A_7 + 2A_2 + 3A_1$	[1]
1148	14	$A_7 + A_2 + 5A_1$	[2], [1]
1149	14	$A_7 + 7A_1$	[2]
1150	14	$2A_6 + A_2$	[1]
1151	14	$2A_6 + 2A_1$	[1]
1152	14	$A_6 + A_5 + A_3$	[1]
1153	14	$A_6 + A_5 + A_2 + A_1$	[1]
1154	14	$A_6 + A_5 + 3A_1$	[1]
1155	14	$A_6 + 2A_4$	[1]
1156	14	$A_6 + A_4 + A_3 + A_1$	[1]
1157	14	$A_6 + A_4 + 2A_2$	[1]
1158	14	$A_6 + A_4 + A_2 + 2A_1$	[1]
1159	14	$A_6 + A_4 + 4A_1$	[1]
1160	14	$A_6 + 2A_3 + A_2$	[1]
1161	14	$A_6 + 2A_3 + 2A_1$	[1]
1162	14	$A_6 + A_3 + 2A_2 + A_1$	[1]
1163	14	$A_6 + A_3 + A_2 + 3A_1$	[1]
1164	14	$A_6 + A_3 + 5A_1$	[1]
1165	14	$A_6 + 4A_2$	[1]
1166	14	$A_6 + 3A_2 + 2A_1$	[1]
1167	14	$A_6 + 2A_2 + 4A_1$	[1]
1168	14	$A_6 + A_2 + 6A_1$	[1]
1169	14	$A_6 + 8A_1$	[2]
1170	14	$2A_5 + A_4$	[1]
1171	14	$2A_5 + A_3 + A_1$	[2], [1]
1172	14	$2A_5 + 2A_2$	[3], [1]

1173	14	$2\,A_5 + A_2 + 2\,A_1$	[2], [1]
1174	14	$2\,A_5 + 4\,A_1$	[2], [1]
1175	14	$A_5 + 2\,A_4 + A_1$	[1]
1176	14	$A_5 + A_4 + A_3 + A_2$	[1]
1177	14	$A_5 + A_4 + A_3 + 2\,A_1$	[1]
1178	14	$A_5 + A_4 + 2\,A_2 + A_1$	[1]
1179	14	$A_5 + A_4 + A_2 + 3\,A_1$	[1]
1180	14	$A_5 + A_4 + 5\,A_1$	[2], [1]
1181	14	$A_5 + 3\,A_3$	[1]
1182	14	$A_5 + 2\,A_3 + A_2 + A_1$	[2], [1]
1183	14	$A_5 + 2\,A_3 + 3\,A_1$	[2], [1]
1184	14	$A_5 + A_3 + 3\,A_2$	[1]
1185	14	$A_5 + A_3 + 2\,A_2 + 2\,A_1$	[1]
1186	14	$A_5 + A_3 + A_2 + 4\,A_1$	[2], [1]
1187	14	$A_5 + A_3 + 6\,A_1$	[2]
1188	14	$A_5 + 4\,A_2 + A_1$	[3], [1]
1189	14	$A_5 + 3\,A_2 + 3\,A_1$	[1]
1190	14	$A_5 + 2\,A_2 + 5\,A_1$	[2], [1]
1191	14	$A_5 + A_2 + 7\,A_1$	[2]
1192	14	$A_5 + 9\,A_1$	$[2,2]$
1193	14	$3\,A_4 + A_2$	[1]
1194	14	$3\,A_4 + 2\,A_1$	[1]
1195	14	$2\,A_4 + 2\,A_3$	[1]
1196	14	$2\,A_4 + A_3 + A_2 + A_1$	[1]
1197	14	$2\,A_4 + A_3 + 3\,A_1$	[1]
1198	14	$2\,A_4 + 3\,A_2$	[1]
1199	14	$2\,A_4 + 2\,A_2 + 2\,A_1$	[1]
1200	14	$2\,A_4 + A_2 + 4\,A_1$	[1]
1201	14	$2\,A_4 + 6\,A_1$	[1]
1202	14	$A_4 + 3\,A_3 + A_1$	[1]
1203	14	$A_4 + 2\,A_3 + 2\,A_2$	[1]
1204	14	$A_4 + 2\,A_3 + A_2 + 2\,A_1$	[1]
1205	14	$A_4 + 2\,A_3 + 4\,A_1$	[2], [1]
1206	14	$A_4 + A_3 + 3\,A_2 + A_1$	[1]
1207	14	$A_4 + A_3 + 2\,A_2 + 3\,A_1$	[1]
1208	14	$A_4 + A_3 + A_2 + 5\,A_1$	[1]
1209	14	$A_4 + A_3 + 7\,A_1$	[2]

1210	14	$A_4 + 5\,A_2$	[1]
1211	14	$A_4 + 4\,A_2 + 2\,A_1$	[1]
1212	14	$A_4 + 3\,A_2 + 4\,A_1$	[1]
1213	14	$A_4 + 2\,A_2 + 6\,A_1$	[1]
1214	14	$A_4 + A_2 + 8\,A_1$	[2]
1215	14	$4\,A_3 + A_2$	[2], [1]
1216	14	$4\,A_3 + 2\,A_1$	[4], [2], [1]
1217	14	$3\,A_3 + 2\,A_2 + A_1$	[1]
1218	14	$3\,A_3 + A_2 + 3\,A_1$	[2], [1]
1219	14	$3\,A_3 + 5\,A_1$	[2]
1220	14	$2\,A_3 + 4\,A_2$	[1]
1221	14	$2\,A_3 + 3\,A_2 + 2\,A_1$	[1]
1222	14	$2\,A_3 + 2\,A_2 + 4\,A_1$	[2], [1]
1223	14	$2\,A_3 + A_2 + 6\,A_1$	[2]
1224	14	$2\,A_3 + 8\,A_1$	$[2, 2]$
1225	14	$A_3 + 5\,A_2 + A_1$	[1]
1226	14	$A_3 + 4\,A_2 + 3\,A_1$	[1]
1227	14	$A_3 + 3\,A_2 + 5\,A_1$	[1]
1228	14	$A_3 + 2\,A_2 + 7\,A_1$	[2]
1229	14	$7\,A_2$	[3]
1230	14	$6\,A_2 + 2\,A_1$	[3], [1]
1231	14	$5\,A_2 + 4\,A_1$	[1]
1232	14	$4\,A_2 + 6\,A_1$	[1]

No.	rank	ADE-type	G
1233	15	$E_8 + E_7$	[1]
1234	15	$E_8 + E_6 + A_1$	[1]
1235	15	$E_8 + D_7$	[1]
1236	15	$E_8 + D_6 + A_1$	[1]
1237	15	$E_8 + D_5 + A_2$	[1]
1238	15	$E_8 + D_5 + 2\,A_1$	[1]
1239	15	$E_8 + D_4 + A_3$	[1]
1240	15	$E_8 + D_4 + A_2 + A_1$	[1]
1241	15	$E_8 + D_4 + 3\,A_1$	[1]
1242	15	$E_8 + A_7$	[1]
1243	15	$E_8 + A_6 + A_1$	[1]

1244	15	$E_8 + A_5 + A_2$	[1]
1245	15	$E_8 + A_5 + 2A_1$	[1]
1246	15	$E_8 + A_4 + A_3$	[1]
1247	15	$E_8 + A_4 + A_2 + A_1$	[1]
1248	15	$E_8 + A_4 + 3A_1$	[1]
1249	15	$E_8 + 2A_3 + A_1$	[1]
1250	15	$E_8 + A_3 + 2A_2$	[1]
1251	15	$E_8 + A_3 + A_2 + 2A_1$	[1]
1252	15	$E_8 + A_3 + 4A_1$	[1]
1253	15	$E_8 + 3A_2 + A_1$	[1]
1254	15	$E_8 + 2A_2 + 3A_1$	[1]
1255	15	$E_8 + A_2 + 5A_1$	[1]
1256	15	$2E_7 + A_1$	[1]
1257	15	$E_7 + E_6 + A_2$	[1]
1258	15	$E_7 + E_6 + 2A_1$	[1]
1259	15	$E_7 + D_8$	[1]
1260	15	$E_7 + D_7 + A_1$	[1]
1261	15	$E_7 + D_6 + A_2$	[1]
1262	15	$E_7 + D_6 + 2A_1$	[2], [1]
1263	15	$E_7 + D_5 + A_3$	[1]
1264	15	$E_7 + D_5 + A_2 + A_1$	[1]
1265	15	$E_7 + D_5 + 3A_1$	[2], [1]
1266	15	$E_7 + 2D_4$	[1]
1267	15	$E_7 + D_4 + A_4$	[1]
1268	15	$E_7 + D_4 + A_3 + A_1$	[2], [1]
1269	15	$E_7 + D_4 + 2A_2$	[1]
1270	15	$E_7 + D_4 + A_2 + 2A_1$	[1]
1271	15	$E_7 + D_4 + 4A_1$	[2]
1272	15	$E_7 + A_8$	[1]
1273	15	$E_7 + A_7 + A_1$	[2], [1]
1274	15	$E_7 + A_6 + A_2$	[1]
1275	15	$E_7 + A_6 + 2A_1$	[1]
1276	15	$E_7 + A_5 + A_3$	[2], [1]
1277	15	$E_7 + A_5 + A_2 + A_1$	[1]
1278	15	$E_7 + A_5 + 3A_1$	[2], [1]
1279	15	$E_7 + 2A_4$	[1]
1280	15	$E_7 + A_4 + A_3 + A_1$	[1]

1281	15	$E_7 + A_4 + 2A_2$	[1]
1282	15	$E_7 + A_4 + A_2 + 2A_1$	[1]
1283	15	$E_7 + A_4 + 4A_1$	[1]
1284	15	$E_7 + 2A_3 + A_2$	[1]
1285	15	$E_7 + 2A_3 + 2A_1$	[2], [1]
1286	15	$E_7 + A_3 + 2A_2 + A_1$	[1]
1287	15	$E_7 + A_3 + A_2 + 3A_1$	[2], [1]
1288	15	$E_7 + A_3 + 5A_1$	[2]
1289	15	$E_7 + 4A_2$	[1]
1290	15	$E_7 + 3A_2 + 2A_1$	[1]
1291	15	$E_7 + 2A_2 + 4A_1$	[1]
1292	15	$E_7 + A_2 + 6A_1$	[2]
1293	15	$2E_6 + A_3$	[1]
1294	15	$2E_6 + A_2 + A_1$	[1]
1295	15	$2E_6 + 3A_1$	[1]
1296	15	$E_6 + D_9$	[1]
1297	15	$E_6 + D_8 + A_1$	[1]
1298	15	$E_6 + D_7 + A_2$	[1]
1299	15	$E_6 + D_7 + 2A_1$	[1]
1300	15	$E_6 + D_6 + A_3$	[1]
1301	15	$E_6 + D_6 + A_2 + A_1$	[1]
1302	15	$E_6 + D_6 + 3A_1$	[1]
1303	15	$E_6 + D_5 + D_4$	[1]
1304	15	$E_6 + D_5 + A_4$	[1]
1305	15	$E_6 + D_5 + A_3 + A_1$	[1]
1306	15	$E_6 + D_5 + 2A_2$	[1]
1307	15	$E_6 + D_5 + A_2 + 2A_1$	[1]
1308	15	$E_6 + D_5 + 4A_1$	[1]
1309	15	$E_6 + 2D_4 + A_1$	[1]
1310	15	$E_6 + D_4 + A_5$	[1]
1311	15	$E_6 + D_4 + A_4 + A_1$	[1]
1312	15	$E_6 + D_4 + A_3 + A_2$	[1]
1313	15	$E_6 + D_4 + A_3 + 2A_1$	[1]
1314	15	$E_6 + D_4 + 2A_2 + A_1$	[1]
1315	15	$E_6 + D_4 + A_2 + 3A_1$	[1]
1316	15	$E_6 + A_9$	[1]
1317	15	$E_6 + A_8 + A_1$	[1]

1318	15	$E_6 + A_7 + A_2$	[1]
1319	15	$E_6 + A_7 + 2A_1$	[1]
1320	15	$E_6 + A_6 + A_3$	[1]
1321	15	$E_6 + A_6 + A_2 + A_1$	[1]
1322	15	$E_6 + A_6 + 3A_1$	[1]
1323	15	$E_6 + A_5 + A_4$	[1]
1324	15	$E_6 + A_5 + A_3 + A_1$	[1]
1325	15	$E_6 + A_5 + 2A_2$	[3], [1]
1326	15	$E_6 + A_5 + A_2 + 2A_1$	[1]
1327	15	$E_6 + A_5 + 4A_1$	[1]
1328	15	$E_6 + 2A_4 + A_1$	[1]
1329	15	$E_6 + A_4 + A_3 + A_2$	[1]
1330	15	$E_6 + A_4 + A_3 + 2A_1$	[1]
1331	15	$E_6 + A_4 + 2A_2 + A_1$	[1]
1332	15	$E_6 + A_4 + A_2 + 3A_1$	[1]
1333	15	$E_6 + A_4 + 5A_1$	[1]
1334	15	$E_6 + 3A_3$	[1]
1335	15	$E_6 + 2A_3 + A_2 + A_1$	[1]
1336	15	$E_6 + 2A_3 + 3A_1$	[1]
1337	15	$E_6 + A_3 + 3A_2$	[1]
1338	15	$E_6 + A_3 + 2A_2 + 2A_1$	[1]
1339	15	$E_6 + A_3 + A_2 + 4A_1$	[1]
1340	15	$E_6 + 4A_2 + A_1$	[3], [1]
1341	15	$E_6 + 3A_2 + 3A_1$	[1]
1342	15	$E_6 + 2A_2 + 5A_1$	[1]
1343	15	D_{15}	[1]
1344	15	$D_{14} + A_1$	[2], [1]
1345	15	$D_{13} + A_2$	[1]
1346	15	$D_{13} + 2A_1$	[1]
1347	15	$D_{12} + A_3$	[2], [1]
1348	15	$D_{12} + A_2 + A_1$	[1]
1349	15	$D_{12} + 3A_1$	[2], [1]
1350	15	$D_{11} + D_4$	[1]
1351	15	$D_{11} + A_4$	[1]
1352	15	$D_{11} + A_3 + A_1$	[1]
1353	15	$D_{11} + 2A_2$	[1]
1354	15	$D_{11} + A_2 + 2A_1$	[1]

1355	15	$D_{11} + 4A_1$	[1]
1356	15	$D_{10} + D_5$	[1]
1357	15	$D_{10} + D_4 + A_1$	[2], [1]
1358	15	$D_{10} + A_5$	[2], [1]
1359	15	$D_{10} + A_4 + A_1$	[1]
1360	15	$D_{10} + A_3 + A_2$	[1]
1361	15	$D_{10} + A_3 + 2A_1$	[2], [1]
1362	15	$D_{10} + 2A_2 + A_1$	[1]
1363	15	$D_{10} + A_2 + 3A_1$	[2], [1]
1364	15	$D_{10} + 5A_1$	[2]
1365	15	$D_9 + D_6$	[1]
1366	15	$D_9 + D_5 + A_1$	[1]
1367	15	$D_9 + D_4 + A_2$	[1]
1368	15	$D_9 + D_4 + 2A_1$	[1]
1369	15	$D_9 + A_6$	[1]
1370	15	$D_9 + A_5 + A_1$	[1]
1371	15	$D_9 + A_4 + A_2$	[1]
1372	15	$D_9 + A_4 + 2A_1$	[1]
1373	15	$D_9 + 2A_3$	[1]
1374	15	$D_9 + A_3 + A_2 + A_1$	[1]
1375	15	$D_9 + A_3 + 3A_1$	[1]
1376	15	$D_9 + 3A_2$	[1]
1377	15	$D_9 + 2A_2 + 2A_1$	[1]
1378	15	$D_9 + A_2 + 4A_1$	[1]
1379	15	$D_9 + 6A_1$	[2]
1380	15	$D_8 + D_7$	[1]
1381	15	$D_8 + D_6 + A_1$	[2], [1]
1382	15	$D_8 + D_5 + A_2$	[1]
1383	15	$D_8 + D_5 + 2A_1$	[2], [1]
1384	15	$D_8 + D_4 + A_3$	[2], [1]
1385	15	$D_8 + D_4 + A_2 + A_1$	[1]
1386	15	$D_8 + D_4 + 3A_1$	[2]
1387	15	$D_8 + A_7$	[2], [1]
1388	15	$D_8 + A_6 + A_1$	[1]
1389	15	$D_8 + A_5 + A_2$	[1]
1390	15	$D_8 + A_5 + 2A_1$	[2], [1]
1391	15	$D_8 + A_4 + A_3$	[1]

1392	15	$D_8 + A_4 + A_2 + A_1$	[1]
1393	15	$D_8 + A_4 + 3A_1$	[1]
1394	15	$D_8 + 2A_3 + A_1$	[2], [1]
1395	15	$D_8 + A_3 + 2A_2$	[1]
1396	15	$D_8 + A_3 + A_2 + 2A_1$	[2], [1]
1397	15	$D_8 + A_3 + 4A_1$	[2]
1398	15	$D_8 + 3A_2 + A_1$	[1]
1399	15	$D_8 + 2A_2 + 3A_1$	[1]
1400	15	$D_8 + A_2 + 5A_1$	[2]
1401	15	$D_8 + 7A_1$	$[2,2]$
1402	15	$2D_7 + A_1$	[1]
1403	15	$D_7 + D_6 + A_2$	[1]
1404	15	$D_7 + D_6 + 2A_1$	[1]
1405	15	$D_7 + D_5 + A_3$	[1]
1406	15	$D_7 + D_5 + A_2 + A_1$	[1]
1407	15	$D_7 + D_5 + 3A_1$	[1]
1408	15	$D_7 + 2D_4$	[1]
1409	15	$D_7 + D_4 + A_4$	[1]
1410	15	$D_7 + D_4 + A_3 + A_1$	[1]
1411	15	$D_7 + D_4 + 2A_2$	[1]
1412	15	$D_7 + D_4 + A_2 + 2A_1$	[1]
1413	15	$D_7 + D_4 + 4A_1$	[2]
1414	15	$D_7 + A_8$	[1]
1415	15	$D_7 + A_7 + A_1$	[1]
1416	15	$D_7 + A_6 + A_2$	[1]
1417	15	$D_7 + A_6 + 2A_1$	[1]
1418	15	$D_7 + A_5 + A_3$	[1]
1419	15	$D_7 + A_5 + A_2 + A_1$	[1]
1420	15	$D_7 + A_5 + 3A_1$	[2], [1]
1421	15	$D_7 + 2A_4$	[1]
1422	15	$D_7 + A_4 + A_3 + A_1$	[1]
1423	15	$D_7 + A_4 + 2A_2$	[1]
1424	15	$D_7 + A_4 + A_2 + 2A_1$	[1]
1425	15	$D_7 + A_4 + 4A_1$	[1]
1426	15	$D_7 + 2A_3 + A_2$	[1]
1427	15	$D_7 + 2A_3 + 2A_1$	[2], [1]
1428	15	$D_7 + A_3 + 2A_2 + A_1$	[1]

1429	15	$D_7 + A_3 + A_2 + 3A_1$	[1]
1430	15	$D_7 + A_3 + 5A_1$	[2]
1431	15	$D_7 + 4A_2$	[1]
1432	15	$D_7 + 3A_2 + 2A_1$	[1]
1433	15	$D_7 + 2A_2 + 4A_1$	[1]
1434	15	$D_7 + A_2 + 6A_1$	[2]
1435	15	$2D_6 + A_3$	[2], [1]
1436	15	$2D_6 + A_2 + A_1$	[1]
1437	15	$2D_6 + 3A_1$	[2]
1438	15	$D_6 + D_5 + D_4$	[1]
1439	15	$D_6 + D_5 + A_4$	[1]
1440	15	$D_6 + D_5 + A_3 + A_1$	[2], [1]
1441	15	$D_6 + D_5 + 2A_2$	[1]
1442	15	$D_6 + D_5 + A_2 + 2A_1$	[1]
1443	15	$D_6 + D_5 + 4A_1$	[2]
1444	15	$D_6 + 2D_4 + A_1$	[2]
1445	15	$D_6 + D_4 + A_5$	[2], [1]
1446	15	$D_6 + D_4 + A_4 + A_1$	[1]
1447	15	$D_6 + D_4 + A_3 + A_2$	[1]
1448	15	$D_6 + D_4 + A_3 + 2A_1$	[2]
1449	15	$D_6 + D_4 + 2A_2 + A_1$	[1]
1450	15	$D_6 + D_4 + A_2 + 3A_1$	[2]
1451	15	$D_6 + D_4 + 5A_1$	$[2, 2]$
1452	15	$D_6 + A_9$	[2], [1]
1453	15	$D_6 + A_8 + A_1$	[1]
1454	15	$D_6 + A_7 + A_2$	[1]
1455	15	$D_6 + A_7 + 2A_1$	[2], [1]
1456	15	$D_6 + A_6 + A_3$	[1]
1457	15	$D_6 + A_6 + A_2 + A_1$	[1]
1458	15	$D_6 + A_6 + 3A_1$	[1]
1459	15	$D_6 + A_5 + A_4$	[1]
1460	15	$D_6 + A_5 + A_3 + A_1$	[2], [1]
1461	15	$D_6 + A_5 + 2A_2$	[1]
1462	15	$D_6 + A_5 + A_2 + 2A_1$	[2], [1]
1463	15	$D_6 + A_5 + 4A_1$	[2]
1464	15	$D_6 + 2A_4 + A_1$	[1]
1465	15	$D_6 + A_4 + A_3 + A_2$	[1]

1466	15	$D_6 + A_4 + A_3 + 2A_1$	[1]
1467	15	$D_6 + A_4 + 2A_2 + A_1$	[1]
1468	15	$D_6 + A_4 + A_2 + 3A_1$	[1]
1469	15	$D_6 + A_4 + 5A_1$	[2]
1470	15	$D_6 + 3A_3$	[2], [1]
1471	15	$D_6 + 2A_3 + A_2 + A_1$	[2], [1]
1472	15	$D_6 + 2A_3 + 3A_1$	[2]
1473	15	$D_6 + A_3 + 3A_2$	[1]
1474	15	$D_6 + A_3 + 2A_2 + 2A_1$	[1]
1475	15	$D_6 + A_3 + A_2 + 4A_1$	[2]
1476	15	$D_6 + A_3 + 6A_1$	$[2,2]$
1477	15	$D_6 + 4A_2 + A_1$	[1]
1478	15	$D_6 + 3A_2 + 3A_1$	[1]
1479	15	$D_6 + 2A_2 + 5A_1$	[2]
1480	15	$3D_5$	[1]
1481	15	$2D_5 + D_4 + A_1$	[1]
1482	15	$2D_5 + A_5$	[1]
1483	15	$2D_5 + A_4 + A_1$	[1]
1484	15	$2D_5 + A_3 + A_2$	[1]
1485	15	$2D_5 + A_3 + 2A_1$	[2], [1]
1486	15	$2D_5 + 2A_2 + A_1$	[1]
1487	15	$2D_5 + A_2 + 3A_1$	[1]
1488	15	$2D_5 + 5A_1$	[2]
1489	15	$D_5 + 2D_4 + A_2$	[1]
1490	15	$D_5 + 2D_4 + 2A_1$	[2]
1491	15	$D_5 + D_4 + A_6$	[1]
1492	15	$D_5 + D_4 + A_5 + A_1$	[2], [1]
1493	15	$D_5 + D_4 + A_4 + A_2$	[1]
1494	15	$D_5 + D_4 + A_4 + 2A_1$	[1]
1495	15	$D_5 + D_4 + 2A_3$	[2], [1]
1496	15	$D_5 + D_4 + A_3 + A_2 + A_1$	[1]
1497	15	$D_5 + D_4 + A_3 + 3A_1$	[2]
1498	15	$D_5 + D_4 + 3A_2$	[1]
1499	15	$D_5 + D_4 + 2A_2 + 2A_1$	[1]
1500	15	$D_5 + D_4 + A_2 + 4A_1$	[2]
1501	15	$D_5 + A_{10}$	[1]
1502	15	$D_5 + A_9 + A_1$	[2], [1]

1503	15	$D_5 + A_8 + A_2$	[1]
1504	15	$D_5 + A_8 + 2A_1$	[1]
1505	15	$D_5 + A_7 + A_3$	[2], [1]
1506	15	$D_5 + A_7 + A_2 + A_1$	[1]
1507	15	$D_5 + A_7 + 3A_1$	[2], [1]
1508	15	$D_5 + A_6 + A_4$	[1]
1509	15	$D_5 + A_6 + A_3 + A_1$	[1]
1510	15	$D_5 + A_6 + 2A_2$	[1]
1511	15	$D_5 + A_6 + A_2 + 2A_1$	[1]
1512	15	$D_5 + A_6 + 4A_1$	[1]
1513	15	$D_5 + 2A_5$	[2], [1]
1514	15	$D_5 + A_5 + A_4 + A_1$	[1]
1515	15	$D_5 + A_5 + A_3 + A_2$	[1]
1516	15	$D_5 + A_5 + A_3 + 2A_1$	[2], [1]
1517	15	$D_5 + A_5 + 2A_2 + A_1$	[1]
1518	15	$D_5 + A_5 + A_2 + 3A_1$	[2], [1]
1519	15	$D_5 + A_5 + 5A_1$	[2]
1520	15	$D_5 + 2A_4 + A_2$	[1]
1521	15	$D_5 + 2A_4 + 2A_1$	[1]
1522	15	$D_5 + A_4 + 2A_3$	[1]
1523	15	$D_5 + A_4 + A_3 + A_2 + A_1$	[1]
1524	15	$D_5 + A_4 + A_3 + 3A_1$	[1]
1525	15	$D_5 + A_4 + 3A_2$	[1]
1526	15	$D_5 + A_4 + 2A_2 + 2A_1$	[1]
1527	15	$D_5 + A_4 + A_2 + 4A_1$	[1]
1528	15	$D_5 + A_4 + 6A_1$	[2]
1529	15	$D_5 + 3A_3 + A_1$	[4], [2], [1]
1530	15	$D_5 + 2A_3 + 2A_2$	[1]
1531	15	$D_5 + 2A_3 + A_2 + 2A_1$	[2], [1]
1532	15	$D_5 + 2A_3 + 4A_1$	[2]
1533	15	$D_5 + A_3 + 3A_2 + A_1$	[1]
1534	15	$D_5 + A_3 + 2A_2 + 3A_1$	[1]
1535	15	$D_5 + A_3 + A_2 + 5A_1$	[2]
1536	15	$D_5 + 5A_2$	[1]
1537	15	$D_5 + 4A_2 + 2A_1$	[1]
1538	15	$D_5 + 3A_2 + 4A_1$	[1]
1539	15	$3D_4 + A_3$	[2]

1540	15	$3\,D_4 + 3\,A_1$	$[2,2]$
1541	15	$2\,D_4 + A_7$	[2], [1]
1542	15	$2\,D_4 + A_6 + A_1$	[1]
1543	15	$2\,D_4 + A_5 + A_2$	[1]
1544	15	$2\,D_4 + A_5 + 2\,A_1$	[2]
1545	15	$2\,D_4 + A_4 + A_3$	[1]
1546	15	$2\,D_4 + A_4 + A_2 + A_1$	[1]
1547	15	$2\,D_4 + 2\,A_3 + A_1$	[2]
1548	15	$2\,D_4 + A_3 + A_2 + 2\,A_1$	[2]
1549	15	$2\,D_4 + A_3 + 4\,A_1$	$[2,2]$
1550	15	$2\,D_4 + 3\,A_2 + A_1$	[1]
1551	15	$D_4 + A_{11}$	[2], [1]
1552	15	$D_4 + A_{10} + A_1$	[1]
1553	15	$D_4 + A_9 + A_2$	[1]
1554	15	$D_4 + A_9 + 2\,A_1$	[2], [1]
1555	15	$D_4 + A_8 + A_3$	[1]
1556	15	$D_4 + A_8 + A_2 + A_1$	[1]
1557	15	$D_4 + A_8 + 3\,A_1$	[1]
1558	15	$D_4 + A_7 + A_4$	[1]
1559	15	$D_4 + A_7 + A_3 + A_1$	[2], [1]
1560	15	$D_4 + A_7 + 2\,A_2$	[1]
1561	15	$D_4 + A_7 + A_2 + 2\,A_1$	[2], [1]
1562	15	$D_4 + A_7 + 4\,A_1$	[2]
1563	15	$D_4 + A_6 + A_5$	[1]
1564	15	$D_4 + A_6 + A_4 + A_1$	[1]
1565	15	$D_4 + A_6 + A_3 + A_2$	[1]
1566	15	$D_4 + A_6 + A_3 + 2\,A_1$	[1]
1567	15	$D_4 + A_6 + 2\,A_2 + A_1$	[1]
1568	15	$D_4 + A_6 + A_2 + 3\,A_1$	[1]
1569	15	$D_4 + 2\,A_5 + A_1$	[2], [1]
1570	15	$D_4 + A_5 + A_4 + A_2$	[1]
1571	15	$D_4 + A_5 + A_4 + 2\,A_1$	[1]
1572	15	$D_4 + A_5 + 2\,A_3$	[1]
1573	15	$D_4 + A_5 + A_3 + A_2 + A_1$	[2], [1]
1574	15	$D_4 + A_5 + A_3 + 3\,A_1$	[2]
1575	15	$D_4 + A_5 + 3\,A_2$	[1]
1576	15	$D_4 + A_5 + 2\,A_2 + 2\,A_1$	[1]

1577	15	$D_4 + A_5 + A_2 + 4A_1$	[2]
1578	15	$D_4 + A_5 + 6A_1$	$[2, 2]$
1579	15	$D_4 + 2A_4 + A_3$	[1]
1580	15	$D_4 + 2A_4 + A_2 + A_1$	[1]
1581	15	$D_4 + 2A_4 + 3A_1$	[1]
1582	15	$D_4 + A_4 + 2A_3 + A_1$	[1]
1583	15	$D_4 + A_4 + A_3 + 2A_2$	[1]
1584	15	$D_4 + A_4 + A_3 + A_2 + 2A_1$	[1]
1585	15	$D_4 + A_4 + A_3 + 4A_1$	[2]
1586	15	$D_4 + A_4 + 3A_2 + A_1$	[1]
1587	15	$D_4 + A_4 + 2A_2 + 3A_1$	[1]
1588	15	$D_4 + 3A_3 + A_2$	[2]
1589	15	$D_4 + 3A_3 + 2A_1$	[2]
1590	15	$D_4 + 2A_3 + 2A_2 + A_1$	[1]
1591	15	$D_4 + 2A_3 + A_2 + 3A_1$	[2]
1592	15	$D_4 + 2A_3 + 5A_1$	$[2, 2]$
1593	15	$D_4 + A_3 + 4A_2$	[1]
1594	15	$D_4 + A_3 + 3A_2 + 2A_1$	[1]
1595	15	$D_4 + A_3 + 2A_2 + 4A_1$	[2]
1596	15	$D_4 + 4A_2 + 3A_1$	[1]
1597	15	A_{15}	[2], [1]
1598	15	$A_{14} + A_1$	[1]
1599	15	$A_{13} + A_2$	[1]
1600	15	$A_{13} + 2A_1$	[2], [1]
1601	15	$A_{12} + A_3$	[1]
1602	15	$A_{12} + A_2 + A_1$	[1]
1603	15	$A_{12} + 3A_1$	[1]
1604	15	$A_{11} + A_4$	[1]
1605	15	$A_{11} + A_3 + A_1$	[2], [1]
1606	15	$A_{11} + 2A_2$	[3], [1]
1607	15	$A_{11} + A_2 + 2A_1$	[2], [1]
1608	15	$A_{11} + 4A_1$	[2], [1]
1609	15	$A_{10} + A_5$	[1]
1610	15	$A_{10} + A_4 + A_1$	[1]
1611	15	$A_{10} + A_3 + A_2$	[1]
1612	15	$A_{10} + A_3 + 2A_1$	[1]
1613	15	$A_{10} + 2A_2 + A_1$	[1]

1614	15	$A_{10} + A_2 + 3\,A_1$	[1]
1615	15	$A_{10} + 5\,A_1$	[1]
1616	15	$A_9 + A_6$	[1]
1617	15	$A_9 + A_5 + A_1$	[2], [1]
1618	15	$A_9 + A_4 + A_2$	[1]
1619	15	$A_9 + A_4 + 2\,A_1$	[1]
1620	15	$A_9 + 2\,A_3$	[1]
1621	15	$A_9 + A_3 + A_2 + A_1$	[2], [1]
1622	15	$A_9 + A_3 + 3\,A_1$	[2], [1]
1623	15	$A_9 + 3\,A_2$	[1]
1624	15	$A_9 + 2\,A_2 + 2\,A_1$	[1]
1625	15	$A_9 + A_2 + 4\,A_1$	[2], [1]
1626	15	$A_9 + 6\,A_1$	[2]
1627	15	$A_8 + A_7$	[1]
1628	15	$A_8 + A_6 + A_1$	[1]
1629	15	$A_8 + A_5 + A_2$	[3], [1]
1630	15	$A_8 + A_5 + 2\,A_1$	[1]
1631	15	$A_8 + A_4 + A_3$	[1]
1632	15	$A_8 + A_4 + A_2 + A_1$	[1]
1633	15	$A_8 + A_4 + 3\,A_1$	[1]
1634	15	$A_8 + 2\,A_3 + A_1$	[1]
1635	15	$A_8 + A_3 + 2\,A_2$	[1]
1636	15	$A_8 + A_3 + A_2 + 2\,A_1$	[1]
1637	15	$A_8 + A_3 + 4\,A_1$	[1]
1638	15	$A_8 + 3\,A_2 + A_1$	[3], [1]
1639	15	$A_8 + 2\,A_2 + 3\,A_1$	[1]
1640	15	$A_8 + A_2 + 5\,A_1$	[1]
1641	15	$2\,A_7 + A_1$	[2], [1]
1642	15	$A_7 + A_6 + A_2$	[1]
1643	15	$A_7 + A_6 + 2\,A_1$	[1]
1644	15	$A_7 + A_5 + A_3$	[1]
1645	15	$A_7 + A_5 + A_2 + A_1$	[2], [1]
1646	15	$A_7 + A_5 + 3\,A_1$	[2], [1]
1647	15	$A_7 + 2\,A_4$	[1]
1648	15	$A_7 + A_4 + A_3 + A_1$	[1]
1649	15	$A_7 + A_4 + 2\,A_2$	[1]
1650	15	$A_7 + A_4 + A_2 + 2\,A_1$	[1]

1651	15	$A_7 + A_4 + 4A_1$	[2], [1]
1652	15	$A_7 + 2A_3 + A_2$	[2], [1]
1653	15	$A_7 + 2A_3 + 2A_1$	[4], [2], [1]
1654	15	$A_7 + A_3 + 2A_2 + A_1$	[1]
1655	15	$A_7 + A_3 + A_2 + 3A_1$	[2], [1]
1656	15	$A_7 + A_3 + 5A_1$	[2]
1657	15	$A_7 + 4A_2$	[1]
1658	15	$A_7 + 3A_2 + 2A_1$	[1]
1659	15	$A_7 + 2A_2 + 4A_1$	[2], [1]
1660	15	$A_7 + A_2 + 6A_1$	[2]
1661	15	$A_7 + 8A_1$	$[2, 2]$
1662	15	$2A_6 + A_3$	[1]
1663	15	$2A_6 + A_2 + A_1$	[1]
1664	15	$2A_6 + 3A_1$	[1]
1665	15	$A_6 + A_5 + A_4$	[1]
1666	15	$A_6 + A_5 + A_3 + A_1$	[1]
1667	15	$A_6 + A_5 + 2A_2$	[1]
1668	15	$A_6 + A_5 + A_2 + 2A_1$	[1]
1669	15	$A_6 + A_5 + 4A_1$	[1]
1670	15	$A_6 + 2A_4 + A_1$	[1]
1671	15	$A_6 + A_4 + A_3 + A_2$	[1]
1672	15	$A_6 + A_4 + A_3 + 2A_1$	[1]
1673	15	$A_6 + A_4 + 2A_2 + A_1$	[1]
1674	15	$A_6 + A_4 + A_2 + 3A_1$	[1]
1675	15	$A_6 + A_4 + 5A_1$	[1]
1676	15	$A_6 + 3A_3$	[1]
1677	15	$A_6 + 2A_3 + A_2 + A_1$	[1]
1678	15	$A_6 + 2A_3 + 3A_1$	[1]
1679	15	$A_6 + A_3 + 3A_2$	[1]
1680	15	$A_6 + A_3 + 2A_2 + 2A_1$	[1]
1681	15	$A_6 + A_3 + A_2 + 4A_1$	[1]
1682	15	$A_6 + A_3 + 6A_1$	[2]
1683	15	$A_6 + 4A_2 + A_1$	[1]
1684	15	$A_6 + 3A_2 + 3A_1$	[1]
1685	15	$A_6 + 2A_2 + 5A_1$	[1]
1686	15	$3A_5$	[3], [1]
1687	15	$2A_5 + A_4 + A_1$	[1]

1688	15	$2A_5 + A_3 + A_2$	[2], [1]
1689	15	$2A_5 + A_3 + 2A_1$	[2], [1]
1690	15	$2A_5 + 2A_2 + A_1$	[3], [1]
1691	15	$2A_5 + A_2 + 3A_1$	[2], [1]
1692	15	$2A_5 + 5A_1$	[2]
1693	15	$A_5 + 2A_4 + A_2$	[1]
1694	15	$A_5 + 2A_4 + 2A_1$	[1]
1695	15	$A_5 + A_4 + 2A_3$	[1]
1696	15	$A_5 + A_4 + A_3 + A_2 + A_1$	[1]
1697	15	$A_5 + A_4 + A_3 + 3A_1$	[2], [1]
1698	15	$A_5 + A_4 + 3A_2$	[1]
1699	15	$A_5 + A_4 + 2A_2 + 2A_1$	[1]
1700	15	$A_5 + A_4 + A_2 + 4A_1$	[1]
1701	15	$A_5 + A_4 + 6A_1$	[2]
1702	15	$A_5 + 3A_3 + A_1$	[2], [1]
1703	15	$A_5 + 2A_3 + 2A_2$	[1]
1704	15	$A_5 + 2A_3 + A_2 + 2A_1$	[2], [1]
1705	15	$A_5 + 2A_3 + 4A_1$	[2]
1706	15	$A_5 + A_3 + 3A_2 + A_1$	[1]
1707	15	$A_5 + A_3 + 2A_2 + 3A_1$	[2], [1]
1708	15	$A_5 + A_3 + A_2 + 5A_1$	[2]
1709	15	$A_5 + A_3 + 7A_1$	$[2, 2]$
1710	15	$A_5 + 5A_2$	[3]
1711	15	$A_5 + 4A_2 + 2A_1$	[3], [1]
1712	15	$A_5 + 3A_2 + 4A_1$	[1]
1713	15	$A_5 + 2A_2 + 6A_1$	[2]
1714	15	$3A_4 + A_3$	[1]
1715	15	$3A_4 + A_2 + A_1$	[1]
1716	15	$3A_4 + 3A_1$	[1]
1717	15	$2A_4 + 2A_3 + A_1$	[1]
1718	15	$2A_4 + A_3 + 2A_2$	[1]
1719	15	$2A_4 + A_3 + A_2 + 2A_1$	[1]
1720	15	$2A_4 + A_3 + 4A_1$	[1]
1721	15	$2A_4 + 3A_2 + A_1$	[1]
1722	15	$2A_4 + 2A_2 + 3A_1$	[1]
1723	15	$2A_4 + A_2 + 5A_1$	[1]
1724	15	$A_4 + 3A_3 + A_2$	[1]

1725	15	$A_4 + 3A_3 + 2A_1$	[2], [1]
1726	15	$A_4 + 2A_3 + 2A_2 + A_1$	[1]
1727	15	$A_4 + 2A_3 + A_2 + 3A_1$	[1]
1728	15	$A_4 + 2A_3 + 5A_1$	[2]
1729	15	$A_4 + A_3 + 4A_2$	[1]
1730	15	$A_4 + A_3 + 3A_2 + 2A_1$	[1]
1731	15	$A_4 + A_3 + 2A_2 + 4A_1$	[1]
1732	15	$A_4 + A_3 + A_2 + 6A_1$	[2]
1733	15	$A_4 + 5A_2 + A_1$	[1]
1734	15	$A_4 + 4A_2 + 3A_1$	[1]
1735	15	$A_4 + 3A_2 + 5A_1$	[1]
1736	15	$5A_3$	[4]
1737	15	$4A_3 + A_2 + A_1$	[2], [1]
1738	15	$4A_3 + 3A_1$	[4], [2]
1739	15	$3A_3 + 3A_2$	[1]
1740	15	$3A_3 + 2A_2 + 2A_1$	[2], [1]
1741	15	$3A_3 + A_2 + 4A_1$	[2]
1742	15	$3A_3 + 6A_1$	$[2, 2]$
1743	15	$2A_3 + 4A_2 + A_1$	[1]
1744	15	$2A_3 + 3A_2 + 3A_1$	[1]
1745	15	$2A_3 + 2A_2 + 5A_1$	[2]
1746	15	$A_3 + 6A_2$	[3]
1747	15	$A_3 + 5A_2 + 2A_1$	[1]
1748	15	$A_3 + 4A_2 + 4A_1$	[1]
1749	15	$7A_2 + A_1$	[3]
1750	15	$6A_2 + 3A_1$	[3]

No.	rank	ADE-type	G
1751	16	$2E_8$	[1]
1752	16	$E_8 + E_7 + A_1$	[1]
1753	16	$E_8 + E_6 + A_2$	[1]
1754	16	$E_8 + E_6 + 2A_1$	[1]
1755	16	$E_8 + D_8$	[1]
1756	16	$E_8 + D_7 + A_1$	[1]
1757	16	$E_8 + D_6 + A_2$	[1]
1758	16	$E_8 + D_6 + 2A_1$	[1]

1759	16	$E_8 + D_5 + A_3$	[1]
1760	16	$E_8 + D_5 + A_2 + A_1$	[1]
1761	16	$E_8 + D_5 + 3\,A_1$	[1]
1762	16	$E_8 + 2\,D_4$	[1]
1763	16	$E_8 + D_4 + A_4$	[1]
1764	16	$E_8 + D_4 + A_3 + A_1$	[1]
1765	16	$E_8 + D_4 + 2\,A_2$	[1]
1766	16	$E_8 + D_4 + A_2 + 2\,A_1$	[1]
1767	16	$E_8 + A_8$	[1]
1768	16	$E_8 + A_7 + A_1$	[1]
1769	16	$E_8 + A_6 + A_2$	[1]
1770	16	$E_8 + A_6 + 2\,A_1$	[1]
1771	16	$E_8 + A_5 + A_3$	[1]
1772	16	$E_8 + A_5 + A_2 + A_1$	[1]
1773	16	$E_8 + A_5 + 3\,A_1$	[1]
1774	16	$E_8 + 2\,A_4$	[1]
1775	16	$E_8 + A_4 + A_3 + A_1$	[1]
1776	16	$E_8 + A_4 + 2\,A_2$	[1]
1777	16	$E_8 + A_4 + A_2 + 2\,A_1$	[1]
1778	16	$E_8 + A_4 + 4\,A_1$	[1]
1779	16	$E_8 + 2\,A_3 + A_2$	[1]
1780	16	$E_8 + 2\,A_3 + 2\,A_1$	[1]
1781	16	$E_8 + A_3 + 2\,A_2 + A_1$	[1]
1782	16	$E_8 + A_3 + A_2 + 3\,A_1$	[1]
1783	16	$E_8 + 4\,A_2$	[1]
1784	16	$E_8 + 3\,A_2 + 2\,A_1$	[1]
1785	16	$E_8 + 2\,A_2 + 4\,A_1$	[1]
1786	16	$2\,E_7 + A_2$	[1]
1787	16	$2\,E_7 + 2\,A_1$	[2], [1]
1788	16	$E_7 + E_6 + A_3$	[1]
1789	16	$E_7 + E_6 + A_2 + A_1$	[1]
1790	16	$E_7 + E_6 + 3\,A_1$	[1]
1791	16	$E_7 + D_9$	[1]
1792	16	$E_7 + D_8 + A_1$	[2], [1]
1793	16	$E_7 + D_7 + A_2$	[1]
1794	16	$E_7 + D_7 + 2\,A_1$	[1]
1795	16	$E_7 + D_6 + A_3$	[2], [1]

1796	16	$E_7 + D_6 + A_2 + A_1$	[1]
1797	16	$E_7 + D_6 + 3\,A_1$	[2]
1798	16	$E_7 + D_5 + D_4$	[1]
1799	16	$E_7 + D_5 + A_4$	[1]
1800	16	$E_7 + D_5 + A_3 + A_1$	[2], [1]
1801	16	$E_7 + D_5 + 2\,A_2$	[1]
1802	16	$E_7 + D_5 + A_2 + 2\,A_1$	[1]
1803	16	$E_7 + D_5 + 4\,A_1$	[2]
1804	16	$E_7 + 2\,D_4 + A_1$	[2]
1805	16	$E_7 + D_4 + A_5$	[2], [1]
1806	16	$E_7 + D_4 + A_4 + A_1$	[1]
1807	16	$E_7 + D_4 + A_3 + A_2$	[1]
1808	16	$E_7 + D_4 + A_3 + 2\,A_1$	[2]
1809	16	$E_7 + D_4 + 2\,A_2 + A_1$	[1]
1810	16	$E_7 + D_4 + A_2 + 3\,A_1$	[2]
1811	16	$E_7 + A_9$	[2], [1]
1812	16	$E_7 + A_8 + A_1$	[1]
1813	16	$E_7 + A_7 + A_2$	[1]
1814	16	$E_7 + A_7 + 2\,A_1$	[2], [1]
1815	16	$E_7 + A_6 + A_3$	[1]
1816	16	$E_7 + A_6 + A_2 + A_1$	[1]
1817	16	$E_7 + A_6 + 3\,A_1$	[1]
1818	16	$E_7 + A_5 + A_4$	[1]
1819	16	$E_7 + A_5 + A_3 + A_1$	[2], [1]
1820	16	$E_7 + A_5 + 2\,A_2$	[1]
1821	16	$E_7 + A_5 + A_2 + 2\,A_1$	[2], [1]
1822	16	$E_7 + A_5 + 4\,A_1$	[2]
1823	16	$E_7 + 2\,A_4 + A_1$	[1]
1824	16	$E_7 + A_4 + A_3 + A_2$	[1]
1825	16	$E_7 + A_4 + A_3 + 2\,A_1$	[1]
1826	16	$E_7 + A_4 + 2\,A_2 + A_1$	[1]
1827	16	$E_7 + A_4 + A_2 + 3\,A_1$	[1]
1828	16	$E_7 + A_4 + 5\,A_1$	[2]
1829	16	$E_7 + 3\,A_3$	[1]
1830	16	$E_7 + 2\,A_3 + A_2 + A_1$	[2], [1]
1831	16	$E_7 + 2\,A_3 + 3\,A_1$	[2]
1832	16	$E_7 + A_3 + 3\,A_2$	[1]

1833	16	$E_7 + A_3 + 2A_2 + 2A_1$	[1]
1834	16	$E_7 + A_3 + A_2 + 4A_1$	[2]
1835	16	$E_7 + 4A_2 + A_1$	[1]
1836	16	$E_7 + 3A_2 + 3A_1$	[1]
1837	16	$2E_6 + D_4$	[1]
1838	16	$2E_6 + A_4$	[1]
1839	16	$2E_6 + A_3 + A_1$	[1]
1840	16	$2E_6 + 2A_2$	[3], [1]
1841	16	$2E_6 + A_2 + 2A_1$	[1]
1842	16	$2E_6 + 4A_1$	[1]
1843	16	$E_6 + D_{10}$	[1]
1844	16	$E_6 + D_9 + A_1$	[1]
1845	16	$E_6 + D_8 + A_2$	[1]
1846	16	$E_6 + D_8 + 2A_1$	[1]
1847	16	$E_6 + D_7 + A_3$	[1]
1848	16	$E_6 + D_7 + A_2 + A_1$	[1]
1849	16	$E_6 + D_7 + 3A_1$	[1]
1850	16	$E_6 + D_6 + D_4$	[1]
1851	16	$E_6 + D_6 + A_4$	[1]
1852	16	$E_6 + D_6 + A_3 + A_1$	[1]
1853	16	$E_6 + D_6 + 2A_2$	[1]
1854	16	$E_6 + D_6 + A_2 + 2A_1$	[1]
1855	16	$E_6 + 2D_5$	[1]
1856	16	$E_6 + D_5 + D_4 + A_1$	[1]
1857	16	$E_6 + D_5 + A_5$	[1]
1858	16	$E_6 + D_5 + A_4 + A_1$	[1]
1859	16	$E_6 + D_5 + A_3 + A_2$	[1]
1860	16	$E_6 + D_5 + A_3 + 2A_1$	[1]
1861	16	$E_6 + D_5 + 2A_2 + A_1$	[1]
1862	16	$E_6 + D_5 + A_2 + 3A_1$	[1]
1863	16	$E_6 + 2D_4 + A_2$	[1]
1864	16	$E_6 + D_4 + A_6$	[1]
1865	16	$E_6 + D_4 + A_5 + A_1$	[1]
1866	16	$E_6 + D_4 + A_4 + A_2$	[1]
1867	16	$E_6 + D_4 + A_4 + 2A_1$	[1]
1868	16	$E_6 + D_4 + 2A_3$	[1]
1869	16	$E_6 + D_4 + A_3 + A_2 + A_1$	[1]

1870	16	$E_6 + D_4 + 2A_2 + 2A_1$	[1]
1871	16	$E_6 + A_{10}$	[1]
1872	16	$E_6 + A_9 + A_1$	[1]
1873	16	$E_6 + A_8 + A_2$	[3], [1]
1874	16	$E_6 + A_8 + 2A_1$	[1]
1875	16	$E_6 + A_7 + A_3$	[1]
1876	16	$E_6 + A_7 + A_2 + A_1$	[1]
1877	16	$E_6 + A_7 + 3A_1$	[1]
1878	16	$E_6 + A_6 + A_4$	[1]
1879	16	$E_6 + A_6 + A_3 + A_1$	[1]
1880	16	$E_6 + A_6 + 2A_2$	[1]
1881	16	$E_6 + A_6 + A_2 + 2A_1$	[1]
1882	16	$E_6 + A_6 + 4A_1$	[1]
1883	16	$E_6 + 2A_5$	[3], [1]
1884	16	$E_6 + A_5 + A_4 + A_1$	[1]
1885	16	$E_6 + A_5 + A_3 + A_2$	[1]
1886	16	$E_6 + A_5 + A_3 + 2A_1$	[1]
1887	16	$E_6 + A_5 + 2A_2 + A_1$	[3], [1]
1888	16	$E_6 + A_5 + A_2 + 3A_1$	[1]
1889	16	$E_6 + 2A_4 + A_2$	[1]
1890	16	$E_6 + 2A_4 + 2A_1$	[1]
1891	16	$E_6 + A_4 + 2A_3$	[1]
1892	16	$E_6 + A_4 + A_3 + A_2 + A_1$	[1]
1893	16	$E_6 + A_4 + A_3 + 3A_1$	[1]
1894	16	$E_6 + A_4 + 3A_2$	[1]
1895	16	$E_6 + A_4 + 2A_2 + 2A_1$	[1]
1896	16	$E_6 + A_4 + A_2 + 4A_1$	[1]
1897	16	$E_6 + 3A_3 + A_1$	[1]
1898	16	$E_6 + 2A_3 + 2A_2$	[1]
1899	16	$E_6 + 2A_3 + A_2 + 2A_1$	[1]
1900	16	$E_6 + A_3 + 3A_2 + A_1$	[1]
1901	16	$E_6 + A_3 + 2A_2 + 3A_1$	[1]
1902	16	$E_6 + 5A_2$	[3]
1903	16	$E_6 + 4A_2 + 2A_1$	[3]
1904	16	D_{16}	[2], [1]
1905	16	$D_{15} + A_1$	[1]
1906	16	$D_{14} + A_2$	[1]

1907	16	$D_{14} + 2\,A_1$	[2], [1]
1908	16	$D_{13} + A_3$	[1]
1909	16	$D_{13} + A_2 + A_1$	[1]
1910	16	$D_{13} + 3\,A_1$	[1]
1911	16	$D_{12} + D_4$	[2], [1]
1912	16	$D_{12} + A_4$	[1]
1913	16	$D_{12} + A_3 + A_1$	[2], [1]
1914	16	$D_{12} + 2\,A_2$	[1]
1915	16	$D_{12} + A_2 + 2\,A_1$	[2], [1]
1916	16	$D_{12} + 4\,A_1$	[2]
1917	16	$D_{11} + D_5$	[1]
1918	16	$D_{11} + D_4 + A_1$	[1]
1919	16	$D_{11} + A_5$	[1]
1920	16	$D_{11} + A_4 + A_1$	[1]
1921	16	$D_{11} + A_3 + A_2$	[1]
1922	16	$D_{11} + A_3 + 2\,A_1$	[1]
1923	16	$D_{11} + 2\,A_2 + A_1$	[1]
1924	16	$D_{11} + A_2 + 3\,A_1$	[1]
1925	16	$D_{10} + D_6$	[2], [1]
1926	16	$D_{10} + D_5 + A_1$	[2], [1]
1927	16	$D_{10} + D_4 + A_2$	[1]
1928	16	$D_{10} + D_4 + 2\,A_1$	[2]
1929	16	$D_{10} + A_6$	[1]
1930	16	$D_{10} + A_5 + A_1$	[2], [1]
1931	16	$D_{10} + A_4 + A_2$	[1]
1932	16	$D_{10} + A_4 + 2\,A_1$	[1]
1933	16	$D_{10} + 2\,A_3$	[1]
1934	16	$D_{10} + A_3 + A_2 + A_1$	[2], [1]
1935	16	$D_{10} + A_3 + 3\,A_1$	[2]
1936	16	$D_{10} + 3\,A_2$	[1]
1937	16	$D_{10} + 2\,A_2 + 2\,A_1$	[1]
1938	16	$D_{10} + A_2 + 4\,A_1$	[2]
1939	16	$D_{10} + 6\,A_1$	$[2, 2]$
1940	16	$D_9 + D_7$	[1]
1941	16	$D_9 + D_6 + A_1$	[1]
1942	16	$D_9 + D_5 + A_2$	[1]
1943	16	$D_9 + D_5 + 2\,A_1$	[1]

1944	16	$D_9 + D_4 + A_3$	[1]
1945	16	$D_9 + D_4 + A_2 + A_1$	[1]
1946	16	$D_9 + A_7$	[1]
1947	16	$D_9 + A_6 + A_1$	[1]
1948	16	$D_9 + A_5 + A_2$	[1]
1949	16	$D_9 + A_5 + 2\,A_1$	[1]
1950	16	$D_9 + A_4 + A_3$	[1]
1951	16	$D_9 + A_4 + A_2 + A_1$	[1]
1952	16	$D_9 + A_4 + 3\,A_1$	[1]
1953	16	$D_9 + 2\,A_3 + A_1$	[1]
1954	16	$D_9 + A_3 + 2\,A_2$	[1]
1955	16	$D_9 + A_3 + A_2 + 2\,A_1$	[1]
1956	16	$D_9 + A_3 + 4\,A_1$	[2]
1957	16	$D_9 + 3\,A_2 + A_1$	[1]
1958	16	$D_9 + 2\,A_2 + 3\,A_1$	[1]
1959	16	$2\,D_8$	[2], [1]
1960	16	$D_8 + D_7 + A_1$	[1]
1961	16	$D_8 + D_6 + A_2$	[1]
1962	16	$D_8 + D_6 + 2\,A_1$	[2]
1963	16	$D_8 + D_5 + A_3$	[2], [1]
1964	16	$D_8 + D_5 + A_2 + A_1$	[1]
1965	16	$D_8 + D_5 + 3\,A_1$	[2]
1966	16	$D_8 + 2\,D_4$	[2]
1967	16	$D_8 + D_4 + A_4$	[1]
1968	16	$D_8 + D_4 + A_3 + A_1$	[2]
1969	16	$D_8 + D_4 + A_2 + 2\,A_1$	[2]
1970	16	$D_8 + D_4 + 4\,A_1$	$[2, 2]$
1971	16	$D_8 + A_8$	[1]
1972	16	$D_8 + A_7 + A_1$	[2], [1]
1973	16	$D_8 + A_6 + A_2$	[1]
1974	16	$D_8 + A_6 + 2\,A_1$	[1]
1975	16	$D_8 + A_5 + A_3$	[1]
1976	16	$D_8 + A_5 + A_2 + A_1$	[2], [1]
1977	16	$D_8 + A_5 + 3\,A_1$	[2]
1978	16	$D_8 + 2\,A_4$	[1]
1979	16	$D_8 + A_4 + A_3 + A_1$	[1]
1980	16	$D_8 + A_4 + 2\,A_2$	[1]

1981	16	$D_8 + A_4 + A_2 + 2A_1$	[1]
1982	16	$D_8 + A_4 + 4A_1$	[2]
1983	16	$D_8 + 2A_3 + A_2$	[2]
1984	16	$D_8 + 2A_3 + 2A_1$	[2]
1985	16	$D_8 + A_3 + 2A_2 + A_1$	[1]
1986	16	$D_8 + A_3 + A_2 + 3A_1$	[2]
1987	16	$D_8 + A_3 + 5A_1$	[2, 2]
1988	16	$D_8 + 4A_2$	[1]
1989	16	$D_8 + 3A_2 + 2A_1$	[1]
1990	16	$D_8 + 2A_2 + 4A_1$	[2]
1991	16	$2D_7 + A_2$	[1]
1992	16	$2D_7 + 2A_1$	[1]
1993	16	$D_7 + D_6 + A_3$	[1]
1994	16	$D_7 + D_6 + A_2 + A_1$	[1]
1995	16	$D_7 + D_6 + 3A_1$	[2]
1996	16	$D_7 + D_5 + D_4$	[1]
1997	16	$D_7 + D_5 + A_4$	[1]
1998	16	$D_7 + D_5 + A_3 + A_1$	[1]
1999	16	$D_7 + D_5 + 2A_2$	[1]
2000	16	$D_7 + D_5 + A_2 + 2A_1$	[1]
2001	16	$D_7 + D_5 + 4A_1$	[2]
2002	16	$D_7 + D_4 + A_5$	[1]
2003	16	$D_7 + D_4 + A_4 + A_1$	[1]
2004	16	$D_7 + D_4 + A_3 + 2A_1$	[2]
2005	16	$D_7 + D_4 + 2A_2 + A_1$	[1]
2006	16	$D_7 + A_9$	[1]
2007	16	$D_7 + A_8 + A_1$	[1]
2008	16	$D_7 + A_7 + A_2$	[1]
2009	16	$D_7 + A_7 + 2A_1$	[2], [1]
2010	16	$D_7 + A_6 + A_3$	[1]
2011	16	$D_7 + A_6 + A_2 + A_1$	[1]
2012	16	$D_7 + A_6 + 3A_1$	[1]
2013	16	$D_7 + A_5 + A_4$	[1]
2014	16	$D_7 + A_5 + A_3 + A_1$	[2], [1]
2015	16	$D_7 + A_5 + 2A_2$	[1]
2016	16	$D_7 + A_5 + A_2 + 2A_1$	[1]
2017	16	$D_7 + A_5 + 4A_1$	[2]

2018	16	$D_7 + 2A_4 + A_1$	[1]
2019	16	$D_7 + A_4 + A_3 + A_2$	[1]
2020	16	$D_7 + A_4 + A_3 + 2A_1$	[1]
2021	16	$D_7 + A_4 + 2A_2 + A_1$	[1]
2022	16	$D_7 + A_4 + A_2 + 3A_1$	[1]
2023	16	$D_7 + 3A_3$	[4]
2024	16	$D_7 + 2A_3 + A_2 + A_1$	[1]
2025	16	$D_7 + 2A_3 + 3A_1$	[2]
2026	16	$D_7 + A_3 + 3A_2$	[1]
2027	16	$D_7 + A_3 + 2A_2 + 2A_1$	[1]
2028	16	$D_7 + A_3 + A_2 + 4A_1$	[2]
2029	16	$D_7 + 3A_2 + 3A_1$	[1]
2030	16	$2D_6 + D_4$	[2]
2031	16	$2D_6 + A_4$	[1]
2032	16	$2D_6 + A_3 + A_1$	[2]
2033	16	$2D_6 + 2A_2$	[1]
2034	16	$2D_6 + A_2 + 2A_1$	[2]
2035	16	$2D_6 + 4A_1$	$[2, 2]$
2036	16	$D_6 + 2D_5$	[1]
2037	16	$D_6 + D_5 + D_4 + A_1$	[2]
2038	16	$D_6 + D_5 + A_5$	[2], [1]
2039	16	$D_6 + D_5 + A_4 + A_1$	[1]
2040	16	$D_6 + D_5 + A_3 + A_2$	[1]
2041	16	$D_6 + D_5 + A_3 + 2A_1$	[2]
2042	16	$D_6 + D_5 + 2A_2 + A_1$	[1]
2043	16	$D_6 + D_5 + A_2 + 3A_1$	[2]
2044	16	$D_6 + 2D_4 + 2A_1$	$[2, 2]$
2045	16	$D_6 + D_4 + A_6$	[1]
2046	16	$D_6 + D_4 + A_5 + A_1$	[2]
2047	16	$D_6 + D_4 + A_4 + A_2$	[1]
2048	16	$D_6 + D_4 + 2A_3$	[2]
2049	16	$D_6 + D_4 + A_3 + A_2 + A_1$	[2]
2050	16	$D_6 + D_4 + A_3 + 3A_1$	$[2, 2]$
2051	16	$D_6 + D_4 + 3A_2$	[1]
2052	16	$D_6 + A_{10}$	[1]
2053	16	$D_6 + A_9 + A_1$	[2], [1]
2054	16	$D_6 + A_8 + A_2$	[1]

2055	16	$D_6 + A_8 + 2A_1$	[1]
2056	16	$D_6 + A_7 + A_3$	[2], [1]
2057	16	$D_6 + A_7 + A_2 + A_1$	[2], [1]
2058	16	$D_6 + A_7 + 3A_1$	[2]
2059	16	$D_6 + A_6 + A_4$	[1]
2060	16	$D_6 + A_6 + A_3 + A_1$	[1]
2061	16	$D_6 + A_6 + 2A_2$	[1]
2062	16	$D_6 + A_6 + A_2 + 2A_1$	[1]
2063	16	$D_6 + 2A_5$	[2], [1]
2064	16	$D_6 + A_5 + A_4 + A_1$	[1]
2065	16	$D_6 + A_5 + A_3 + A_2$	[2], [1]
2066	16	$D_6 + A_5 + A_3 + 2A_1$	[2]
2067	16	$D_6 + A_5 + 2A_2 + A_1$	[1]
2068	16	$D_6 + A_5 + A_2 + 3A_1$	[2]
2069	16	$D_6 + A_5 + 5A_1$	$[2, 2]$
2070	16	$D_6 + 2A_4 + A_2$	[1]
2071	16	$D_6 + 2A_4 + 2A_1$	[1]
2072	16	$D_6 + A_4 + 2A_3$	[1]
2073	16	$D_6 + A_4 + A_3 + A_2 + A_1$	[1]
2074	16	$D_6 + A_4 + A_3 + 3A_1$	[2]
2075	16	$D_6 + A_4 + 3A_2$	[1]
2076	16	$D_6 + A_4 + 2A_2 + 2A_1$	[1]
2077	16	$D_6 + 3A_3 + A_1$	[2]
2078	16	$D_6 + 2A_3 + 2A_2$	[1]
2079	16	$D_6 + 2A_3 + A_2 + 2A_1$	[2]
2080	16	$D_6 + 2A_3 + 4A_1$	$[2, 2]$
2081	16	$D_6 + A_3 + 3A_2 + A_1$	[1]
2082	16	$D_6 + A_3 + 2A_2 + 3A_1$	[2]
2083	16	$D_6 + 4A_2 + 2A_1$	[1]
2084	16	$3D_5 + A_1$	[1]
2085	16	$2D_5 + D_4 + A_2$	[1]
2086	16	$2D_5 + D_4 + 2A_1$	[2]
2087	16	$2D_5 + A_6$	[1]
2088	16	$2D_5 + A_5 + A_1$	[2], [1]
2089	16	$2D_5 + A_4 + A_2$	[1]
2090	16	$2D_5 + A_4 + 2A_1$	[1]
2091	16	$2D_5 + 2A_3$	[4], [2], [1]

2092	16	$2\,D_5 + A_3 + A_2 + A_1$	[1]
2093	16	$2\,D_5 + A_3 + 3\,A_1$	[2]
2094	16	$2\,D_5 + 3\,A_2$	[1]
2095	16	$2\,D_5 + 2\,A_2 + 2\,A_1$	[1]
2096	16	$D_5 + 2\,D_4 + A_3$	[2]
2097	16	$D_5 + D_4 + A_7$	[2], [1]
2098	16	$D_5 + D_4 + A_6 + A_1$	[1]
2099	16	$D_5 + D_4 + A_5 + A_2$	[1]
2100	16	$D_5 + D_4 + A_5 + 2\,A_1$	[2]
2101	16	$D_5 + D_4 + A_4 + A_3$	[1]
2102	16	$D_5 + D_4 + A_4 + A_2 + A_1$	[1]
2103	16	$D_5 + D_4 + 2\,A_3 + A_1$	[2]
2104	16	$D_5 + D_4 + A_3 + A_2 + 2\,A_1$	[2]
2105	16	$D_5 + D_4 + 3\,A_2 + A_1$	[1]
2106	16	$D_5 + A_{11}$	[2], [1]
2107	16	$D_5 + A_{10} + A_1$	[1]
2108	16	$D_5 + A_9 + A_2$	[1]
2109	16	$D_5 + A_9 + 2\,A_1$	[2], [1]
2110	16	$D_5 + A_8 + A_3$	[1]
2111	16	$D_5 + A_8 + A_2 + A_1$	[1]
2112	16	$D_5 + A_8 + 3\,A_1$	[1]
2113	16	$D_5 + A_7 + A_4$	[1]
2114	16	$D_5 + A_7 + A_3 + A_1$	[4], [2], [1]
2115	16	$D_5 + A_7 + 2\,A_2$	[1]
2116	16	$D_5 + A_7 + A_2 + 2\,A_1$	[2], [1]
2117	16	$D_5 + A_7 + 4\,A_1$	[2]
2118	16	$D_5 + A_6 + A_5$	[1]
2119	16	$D_5 + A_6 + A_4 + A_1$	[1]
2120	16	$D_5 + A_6 + A_3 + A_2$	[1]
2121	16	$D_5 + A_6 + A_3 + 2\,A_1$	[1]
2122	16	$D_5 + A_6 + 2\,A_2 + A_1$	[1]
2123	16	$D_5 + A_6 + A_2 + 3\,A_1$	[1]
2124	16	$D_5 + 2\,A_5 + A_1$	[2], [1]
2125	16	$D_5 + A_5 + A_4 + A_2$	[1]
2126	16	$D_5 + A_5 + A_4 + 2\,A_1$	[1]
2127	16	$D_5 + A_5 + 2\,A_3$	[1]
2128	16	$D_5 + A_5 + A_3 + A_2 + A_1$	[2], [1]

2129	16	$D_5 + A_5 + A_3 + 3A_1$	[2]
2130	16	$D_5 + A_5 + 3A_2$	[1]
2131	16	$D_5 + A_5 + 2A_2 + 2A_1$	[1]
2132	16	$D_5 + A_5 + A_2 + 4A_1$	[2]
2133	16	$D_5 + 2A_4 + A_3$	[1]
2134	16	$D_5 + 2A_4 + A_2 + A_1$	[1]
2135	16	$D_5 + 2A_4 + 3A_1$	[1]
2136	16	$D_5 + A_4 + 2A_3 + A_1$	[1]
2137	16	$D_5 + A_4 + A_3 + 2A_2$	[1]
2138	16	$D_5 + A_4 + A_3 + A_2 + 2A_1$	[1]
2139	16	$D_5 + A_4 + A_3 + 4A_1$	[2]
2140	16	$D_5 + A_4 + 3A_2 + A_1$	[1]
2141	16	$D_5 + A_4 + 2A_2 + 3A_1$	[1]
2142	16	$D_5 + 3A_3 + A_2$	[2]
2143	16	$D_5 + 3A_3 + 2A_1$	[4], [2]
2144	16	$D_5 + 2A_3 + 2A_2 + A_1$	[1]
2145	16	$D_5 + 2A_3 + A_2 + 3A_1$	[2]
2146	16	$D_5 + A_3 + 4A_2$	[1]
2147	16	$D_5 + A_3 + 3A_2 + 2A_1$	[1]
2148	16	$4D_4$	$[2,2]$
2149	16	$2D_4 + A_8$	[1]
2150	16	$2D_4 + A_7 + A_1$	[2]
2151	16	$2D_4 + A_5 + A_2 + A_1$	[2]
2152	16	$2D_4 + 2A_4$	[1]
2153	16	$2D_4 + 2A_3 + 2A_1$	$[2,2]$
2154	16	$2D_4 + 4A_2$	[1]
2155	16	$D_4 + A_{12}$	[1]
2156	16	$D_4 + A_{11} + A_1$	[2], [1]
2157	16	$D_4 + A_{10} + A_2$	[1]
2158	16	$D_4 + A_{10} + 2A_1$	[1]
2159	16	$D_4 + A_9 + A_3$	[1]
2160	16	$D_4 + A_9 + A_2 + A_1$	[2], [1]
2161	16	$D_4 + A_9 + 3A_1$	[2]
2162	16	$D_4 + A_8 + A_4$	[1]
2163	16	$D_4 + A_8 + A_3 + A_1$	[1]
2164	16	$D_4 + A_8 + 2A_2$	[1]
2165	16	$D_4 + A_8 + A_2 + 2A_1$	[1]

2166	16	$D_4 + A_7 + A_5$	[1]
2167	16	$D_4 + A_7 + A_4 + A_1$	[1]
2168	16	$D_4 + A_7 + A_3 + A_2$	[2]
2169	16	$D_4 + A_7 + A_3 + 2A_1$	[2]
2170	16	$D_4 + A_7 + 2A_2 + A_1$	[1]
2171	16	$D_4 + A_7 + A_2 + 3A_1$	[2]
2172	16	$D_4 + 2A_6$	[1]
2173	16	$D_4 + A_6 + A_5 + A_1$	[1]
2174	16	$D_4 + A_6 + A_4 + A_2$	[1]
2175	16	$D_4 + A_6 + A_4 + 2A_1$	[1]
2176	16	$D_4 + A_6 + A_3 + A_2 + A_1$	[1]
2177	16	$D_4 + A_6 + 3A_2$	[1]
2178	16	$D_4 + A_6 + 2A_2 + 2A_1$	[1]
2179	16	$D_4 + 2A_5 + A_2$	[2], [1]
2180	16	$D_4 + 2A_5 + 2A_1$	[2]
2181	16	$D_4 + A_5 + A_4 + A_3$	[1]
2182	16	$D_4 + A_5 + A_4 + A_2 + A_1$	[1]
2183	16	$D_4 + A_5 + A_4 + 3A_1$	[2]
2184	16	$D_4 + A_5 + 2A_3 + A_1$	[2]
2185	16	$D_4 + A_5 + A_3 + 2A_2$	[1]
2186	16	$D_4 + A_5 + A_3 + A_2 + 2A_1$	[2]
2187	16	$D_4 + A_5 + A_3 + 4A_1$	$[2, 2]$
2188	16	$D_4 + A_5 + 2A_2 + 3A_1$	[2]
2189	16	$D_4 + 3A_4$	[1]
2190	16	$D_4 + 2A_4 + A_3 + A_1$	[1]
2191	16	$D_4 + 2A_4 + 2A_2$	[1]
2192	16	$D_4 + 2A_4 + A_2 + 2A_1$	[1]
2193	16	$D_4 + A_4 + 2A_3 + 2A_1$	[2]
2194	16	$D_4 + A_4 + A_3 + 2A_2 + A_1$	[1]
2195	16	$D_4 + A_4 + 3A_2 + 2A_1$	[1]
2196	16	$D_4 + 3A_3 + A_2 + A_1$	[2]
2197	16	$D_4 + 3A_3 + 3A_1$	$[2, 2]$
2198	16	$D_4 + 2A_3 + 3A_2$	[1]
2199	16	$D_4 + 2A_3 + 2A_2 + 2A_1$	[2]
2200	16	A_{16}	[1]
2201	16	$A_{15} + A_1$	[2], [1]
2202	16	$A_{14} + A_2$	[3], [1]

2203	16	$A_{14} + 2A_1$	[1]
2204	16	$A_{13} + A_3$	[1]
2205	16	$A_{13} + A_2 + A_1$	[2], [1]
2206	16	$A_{13} + 3A_1$	[2], [1]
2207	16	$A_{12} + A_4$	[1]
2208	16	$A_{12} + A_3 + A_1$	[1]
2209	16	$A_{12} + 2A_2$	[1]
2210	16	$A_{12} + A_2 + 2A_1$	[1]
2211	16	$A_{12} + 4A_1$	[1]
2212	16	$A_{11} + A_5$	[3], [1]
2213	16	$A_{11} + A_4 + A_1$	[1]
2214	16	$A_{11} + A_3 + A_2$	[2], [1]
2215	16	$A_{11} + A_3 + 2A_1$	[4], [2], [1]
2216	16	$A_{11} + 2A_2 + A_1$	[3], [1]
2217	16	$A_{11} + A_2 + 3A_1$	[2], [1]
2218	16	$A_{11} + 5A_1$	[2]
2219	16	$A_{10} + A_6$	[1]
2220	16	$A_{10} + A_5 + A_1$	[1]
2221	16	$A_{10} + A_4 + A_2$	[1]
2222	16	$A_{10} + A_4 + 2A_1$	[1]
2223	16	$A_{10} + 2A_3$	[1]
2224	16	$A_{10} + A_3 + A_2 + A_1$	[1]
2225	16	$A_{10} + A_3 + 3A_1$	[1]
2226	16	$A_{10} + 3A_2$	[1]
2227	16	$A_{10} + 2A_2 + 2A_1$	[1]
2228	16	$A_{10} + A_2 + 4A_1$	[1]
2229	16	$A_9 + A_7$	[1]
2230	16	$A_9 + A_6 + A_1$	[1]
2231	16	$A_9 + A_5 + A_2$	[2], [1]
2232	16	$A_9 + A_5 + 2A_1$	[2], [1]
2233	16	$A_9 + A_4 + A_3$	[1]
2234	16	$A_9 + A_4 + A_2 + A_1$	[1]
2235	16	$A_9 + A_4 + 3A_1$	[2], [1]
2236	16	$A_9 + 2A_3 + A_1$	[2], [1]
2237	16	$A_9 + A_3 + 2A_2$	[1]
2238	16	$A_9 + A_3 + A_2 + 2A_1$	[2], [1]
2239	16	$A_9 + A_3 + 4A_1$	[2]

2240	16	$A_9 + 3A_2 + A_1$	[1]
2241	16	$A_9 + 2A_2 + 3A_1$	[2], [1]
2242	16	$A_9 + A_2 + 5A_1$	[2]
2243	16	$2A_8$	[3], [1]
2244	16	$A_8 + A_7 + A_1$	[1]
2245	16	$A_8 + A_6 + A_2$	[1]
2246	16	$A_8 + A_6 + 2A_1$	[1]
2247	16	$A_8 + A_5 + A_3$	[1]
2248	16	$A_8 + A_5 + A_2 + A_1$	[3], [1]
2249	16	$A_8 + A_5 + 3A_1$	[1]
2250	16	$A_8 + 2A_4$	[1]
2251	16	$A_8 + A_4 + A_3 + A_1$	[1]
2252	16	$A_8 + A_4 + 2A_2$	[1]
2253	16	$A_8 + A_4 + A_2 + 2A_1$	[1]
2254	16	$A_8 + A_4 + 4A_1$	[1]
2255	16	$A_8 + 2A_3 + A_2$	[1]
2256	16	$A_8 + 2A_3 + 2A_1$	[1]
2257	16	$A_8 + A_3 + 2A_2 + A_1$	[1]
2258	16	$A_8 + A_3 + A_2 + 3A_1$	[1]
2259	16	$A_8 + 4A_2$	[3]
2260	16	$A_8 + 3A_2 + 2A_1$	[3], [1]
2261	16	$A_8 + 2A_2 + 4A_1$	[1]
2262	16	$2A_7 + A_2$	[2], [1]
2263	16	$2A_7 + 2A_1$	[4], [2], [1]
2264	16	$A_7 + A_6 + A_3$	[1]
2265	16	$A_7 + A_6 + A_2 + A_1$	[1]
2266	16	$A_7 + A_6 + 3A_1$	[1]
2267	16	$A_7 + A_5 + A_4$	[1]
2268	16	$A_7 + A_5 + A_3 + A_1$	[2], [1]
2269	16	$A_7 + A_5 + 2A_2$	[1]
2270	16	$A_7 + A_5 + A_2 + 2A_1$	[2], [1]
2271	16	$A_7 + A_5 + 4A_1$	[2]
2272	16	$A_7 + 2A_4 + A_1$	[1]
2273	16	$A_7 + A_4 + A_3 + A_2$	[1]
2274	16	$A_7 + A_4 + A_3 + 2A_1$	[2], [1]
2275	16	$A_7 + A_4 + 2A_2 + A_1$	[1]
2276	16	$A_7 + A_4 + A_2 + 3A_1$	[1]

2277	16	$A_7 + A_4 + 5A_1$	[2]
2278	16	$A_7 + 3A_3$	[4]
2279	16	$A_7 + 2A_3 + A_2 + A_1$	[2], [1]
2280	16	$A_7 + 2A_3 + 3A_1$	[4], [2]
2281	16	$A_7 + A_3 + 3A_2$	[1]
2282	16	$A_7 + A_3 + 2A_2 + 2A_1$	[2], [1]
2283	16	$A_7 + A_3 + A_2 + 4A_1$	[2]
2284	16	$A_7 + A_3 + 6A_1$	$[2, 2]$
2285	16	$A_7 + 4A_2 + A_1$	[1]
2286	16	$A_7 + 3A_2 + 3A_1$	[1]
2287	16	$A_7 + 2A_2 + 5A_1$	[2]
2288	16	$2A_6 + A_4$	[1]
2289	16	$2A_6 + A_3 + A_1$	[1]
2290	16	$2A_6 + 2A_2$	[1]
2291	16	$2A_6 + A_2 + 2A_1$	[1]
2292	16	$2A_6 + 4A_1$	[1]
2293	16	$A_6 + 2A_5$	[1]
2294	16	$A_6 + A_5 + A_4 + A_1$	[1]
2295	16	$A_6 + A_5 + A_3 + A_2$	[1]
2296	16	$A_6 + A_5 + A_3 + 2A_1$	[1]
2297	16	$A_6 + A_5 + 2A_2 + A_1$	[1]
2298	16	$A_6 + A_5 + A_2 + 3A_1$	[1]
2299	16	$A_6 + A_5 + 5A_1$	[2]
2300	16	$A_6 + 2A_4 + A_2$	[1]
2301	16	$A_6 + 2A_4 + 2A_1$	[1]
2302	16	$A_6 + A_4 + 2A_3$	[1]
2303	16	$A_6 + A_4 + A_3 + A_2 + A_1$	[1]
2304	16	$A_6 + A_4 + A_3 + 3A_1$	[1]
2305	16	$A_6 + A_4 + 3A_2$	[1]
2306	16	$A_6 + A_4 + 2A_2 + 2A_1$	[1]
2307	16	$A_6 + A_4 + A_2 + 4A_1$	[1]
2308	16	$A_6 + 3A_3 + A_1$	[1]
2309	16	$A_6 + 2A_3 + 2A_2$	[1]
2310	16	$A_6 + 2A_3 + A_2 + 2A_1$	[1]
2311	16	$A_6 + 2A_3 + 4A_1$	[2]
2312	16	$A_6 + A_3 + 3A_2 + A_1$	[1]
2313	16	$A_6 + A_3 + 2A_2 + 3A_1$	[1]

2314	16	$A_6 + 4A_2 + 2A_1$	[1]
2315	16	$A_6 + 3A_2 + 4A_1$	[1]
2316	16	$3A_5 + A_1$	[3], [1]
2317	16	$2A_5 + A_4 + A_2$	[1]
2318	16	$2A_5 + A_4 + 2A_1$	[2], [1]
2319	16	$2A_5 + 2A_3$	[2], [1]
2320	16	$2A_5 + A_3 + A_2 + A_1$	[2], [1]
2321	16	$2A_5 + A_3 + 3A_1$	[2]
2322	16	$2A_5 + 3A_2$	[3]
2323	16	$2A_5 + 2A_2 + 2A_1$	[6], [3], [2], [1]
2324	16	$2A_5 + A_2 + 4A_1$	[2]
2325	16	$2A_5 + 6A_1$	$[2, 2]$
2326	16	$A_5 + 2A_4 + A_3$	[1]
2327	16	$A_5 + 2A_4 + A_2 + A_1$	[1]
2328	16	$A_5 + 2A_4 + 3A_1$	[1]
2329	16	$A_5 + A_4 + 2A_3 + A_1$	[2], [1]
2330	16	$A_5 + A_4 + A_3 + 2A_2$	[1]
2331	16	$A_5 + A_4 + A_3 + A_2 + 2A_1$	[1]
2332	16	$A_5 + A_4 + A_3 + 4A_1$	[2]
2333	16	$A_5 + A_4 + 3A_2 + A_1$	[1]
2334	16	$A_5 + A_4 + 2A_2 + 3A_1$	[1]
2335	16	$A_5 + A_4 + A_2 + 5A_1$	[2]
2336	16	$A_5 + 3A_3 + A_2$	[1]
2337	16	$A_5 + 3A_3 + 2A_1$	[2]
2338	16	$A_5 + 2A_3 + 2A_2 + A_1$	[2], [1]
2339	16	$A_5 + 2A_3 + A_2 + 3A_1$	[2]
2340	16	$A_5 + 2A_3 + 5A_1$	$[2, 2]$
2341	16	$A_5 + A_3 + 4A_2$	[3]
2342	16	$A_5 + A_3 + 3A_2 + 2A_1$	[1]
2343	16	$A_5 + A_3 + 2A_2 + 4A_1$	[2]
2344	16	$A_5 + 5A_2 + A_1$	[3]
2345	16	$A_5 + 4A_2 + 3A_1$	[3]
2346	16	$4A_4$	[5], [1]
2347	16	$3A_4 + A_3 + A_1$	[1]
2348	16	$3A_4 + 2A_2$	[1]
2349	16	$3A_4 + A_2 + 2A_1$	[1]
2350	16	$3A_4 + 4A_1$	[1]

2351	16	$2\,A_4 + 2\,A_3 + A_2$	[1]
2352	16	$2\,A_4 + 2\,A_3 + 2\,A_1$	[1]
2353	16	$2\,A_4 + A_3 + 2\,A_2 + A_1$	[1]
2354	16	$2\,A_4 + A_3 + A_2 + 3\,A_1$	[1]
2355	16	$2\,A_4 + 4\,A_2$	[1]
2356	16	$2\,A_4 + 3\,A_2 + 2\,A_1$	[1]
2357	16	$2\,A_4 + 2\,A_2 + 4\,A_1$	[1]
2358	16	$A_4 + 3\,A_3 + A_2 + A_1$	[1]
2359	16	$A_4 + 3\,A_3 + 3\,A_1$	[2]
2360	16	$A_4 + 2\,A_3 + 3\,A_2$	[1]
2361	16	$A_4 + 2\,A_3 + 2\,A_2 + 2\,A_1$	[1]
2362	16	$A_4 + 2\,A_3 + A_2 + 4\,A_1$	[2]
2363	16	$A_4 + A_3 + 4\,A_2 + A_1$	[1]
2364	16	$A_4 + A_3 + 3\,A_2 + 3\,A_1$	[1]
2365	16	$A_4 + 6\,A_2$	[3]
2366	16	$5\,A_3 + A_1$	[4]
2367	16	$4\,A_3 + 2\,A_2$	[2], [1]
2368	16	$4\,A_3 + A_2 + 2\,A_1$	[4], [2]
2369	16	$4\,A_3 + 4\,A_1$	$[4,2]$, $[2,2]$
2370	16	$3\,A_3 + 3\,A_2 + A_1$	[1]
2371	16	$3\,A_3 + 2\,A_2 + 3\,A_1$	[2]
2372	16	$2\,A_3 + 4\,A_2 + 2\,A_1$	[1]
2373	16	$A_3 + 6\,A_2 + A_1$	[3]
2374	16	$8\,A_2$	$[3,3]$

No.	rank	ADE-type	G
2375	17	$2\,E_8 + A_1$	[1]
2376	17	$E_8 + E_7 + A_2$	[1]
2377	17	$E_8 + E_7 + 2\,A_1$	[1]
2378	17	$E_8 + E_6 + A_3$	[1]
2379	17	$E_8 + E_6 + A_2 + A_1$	[1]
2380	17	$E_8 + E_6 + 3\,A_1$	[1]
2381	17	$E_8 + D_9$	[1]
2382	17	$E_8 + D_8 + A_1$	[1]
2383	17	$E_8 + D_7 + A_2$	[1]
2384	17	$E_8 + D_7 + 2\,A_1$	[1]

2385	17	$E_8 + D_6 + A_3$	[1]
2386	17	$E_8 + D_6 + A_2 + A_1$	[1]
2387	17	$E_8 + D_5 + D_4$	[1]
2388	17	$E_8 + D_5 + A_4$	[1]
2389	17	$E_8 + D_5 + A_3 + A_1$	[1]
2390	17	$E_8 + D_5 + 2\,A_2$	[1]
2391	17	$E_8 + D_5 + A_2 + 2\,A_1$	[1]
2392	17	$E_8 + D_4 + A_5$	[1]
2393	17	$E_8 + D_4 + A_4 + A_1$	[1]
2394	17	$E_8 + D_4 + 2\,A_2 + A_1$	[1]
2395	17	$E_8 + A_9$	[1]
2396	17	$E_8 + A_8 + A_1$	[1]
2397	17	$E_8 + A_7 + A_2$	[1]
2398	17	$E_8 + A_7 + 2\,A_1$	[1]
2399	17	$E_8 + A_6 + A_3$	[1]
2400	17	$E_8 + A_6 + A_2 + A_1$	[1]
2401	17	$E_8 + A_6 + 3\,A_1$	[1]
2402	17	$E_8 + A_5 + A_4$	[1]
2403	17	$E_8 + A_5 + A_3 + A_1$	[1]
2404	17	$E_8 + A_5 + 2\,A_2$	[1]
2405	17	$E_8 + A_5 + A_2 + 2\,A_1$	[1]
2406	17	$E_8 + 2\,A_4 + A_1$	[1]
2407	17	$E_8 + A_4 + A_3 + A_2$	[1]
2408	17	$E_8 + A_4 + A_3 + 2\,A_1$	[1]
2409	17	$E_8 + A_4 + 2\,A_2 + A_1$	[1]
2410	17	$E_8 + A_4 + A_2 + 3\,A_1$	[1]
2411	17	$E_8 + 2\,A_3 + A_2 + A_1$	[1]
2412	17	$E_8 + A_3 + 3\,A_2$	[1]
2413	17	$E_8 + A_3 + 2\,A_2 + 2\,A_1$	[1]
2414	17	$2\,E_7 + A_3$	[2], [1]
2415	17	$2\,E_7 + A_2 + A_1$	[1]
2416	17	$2\,E_7 + 3\,A_1$	[2]
2417	17	$E_7 + E_6 + D_4$	[1]
2418	17	$E_7 + E_6 + A_4$	[1]
2419	17	$E_7 + E_6 + A_3 + A_1$	[1]
2420	17	$E_7 + E_6 + 2\,A_2$	[1]
2421	17	$E_7 + E_6 + A_2 + 2\,A_1$	[1]

2422	17	$E_7 + D_{10}$	[2], [1]
2423	17	$E_7 + D_9 + A_1$	[1]
2424	17	$E_7 + D_8 + A_2$	[1]
2425	17	$E_7 + D_8 + 2A_1$	[2]
2426	17	$E_7 + D_7 + A_3$	[1]
2427	17	$E_7 + D_7 + A_2 + A_1$	[1]
2428	17	$E_7 + D_7 + 3A_1$	[2]
2429	17	$E_7 + D_6 + D_4$	[2]
2430	17	$E_7 + D_6 + A_4$	[1]
2431	17	$E_7 + D_6 + A_3 + A_1$	[2]
2432	17	$E_7 + D_6 + 2A_2$	[1]
2433	17	$E_7 + D_6 + A_2 + 2A_1$	[2]
2434	17	$E_7 + 2D_5$	[1]
2435	17	$E_7 + D_5 + D_4 + A_1$	[2]
2436	17	$E_7 + D_5 + A_5$	[2], [1]
2437	17	$E_7 + D_5 + A_4 + A_1$	[1]
2438	17	$E_7 + D_5 + A_3 + A_2$	[1]
2439	17	$E_7 + D_5 + A_3 + 2A_1$	[2]
2440	17	$E_7 + D_5 + 2A_2 + A_1$	[1]
2441	17	$E_7 + D_4 + A_6$	[1]
2442	17	$E_7 + D_4 + A_5 + A_1$	[2]
2443	17	$E_7 + D_4 + A_4 + A_2$	[1]
2444	17	$E_7 + D_4 + A_3 + A_2 + A_1$	[2]
2445	17	$E_7 + A_{10}$	[1]
2446	17	$E_7 + A_9 + A_1$	[2], [1]
2447	17	$E_7 + A_8 + A_2$	[1]
2448	17	$E_7 + A_8 + 2A_1$	[1]
2449	17	$E_7 + A_7 + A_3$	[1]
2450	17	$E_7 + A_7 + A_2 + A_1$	[2], [1]
2451	17	$E_7 + A_7 + 3A_1$	[2]
2452	17	$E_7 + A_6 + A_4$	[1]
2453	17	$E_7 + A_6 + A_3 + A_1$	[1]
2454	17	$E_7 + A_6 + 2A_2$	[1]
2455	17	$E_7 + A_6 + A_2 + 2A_1$	[1]
2456	17	$E_7 + 2A_5$	[1]
2457	17	$E_7 + A_5 + A_4 + A_1$	[1]
2458	17	$E_7 + A_5 + A_3 + A_2$	[2], [1]

2459	17	$E_7 + A_5 + A_3 + 2A_1$	[2]
2460	17	$E_7 + A_5 + 2A_2 + A_1$	[1]
2461	17	$E_7 + A_5 + A_2 + 3A_1$	[2]
2462	17	$E_7 + 2A_4 + A_2$	[1]
2463	17	$E_7 + 2A_4 + 2A_1$	[1]
2464	17	$E_7 + A_4 + 2A_3$	[1]
2465	17	$E_7 + A_4 + A_3 + A_2 + A_1$	[1]
2466	17	$E_7 + A_4 + A_3 + 3A_1$	[2]
2467	17	$E_7 + A_4 + 3A_2$	[1]
2468	17	$E_7 + A_4 + 2A_2 + 2A_1$	[1]
2469	17	$E_7 + 3A_3 + A_1$	[2]
2470	17	$E_7 + 2A_3 + 2A_2$	[1]
2471	17	$E_7 + 2A_3 + A_2 + 2A_1$	[2]
2472	17	$E_7 + A_3 + 3A_2 + A_1$	[1]
2473	17	$2E_6 + D_5$	[1]
2474	17	$2E_6 + D_4 + A_1$	[1]
2475	17	$2E_6 + A_5$	[3], [1]
2476	17	$2E_6 + A_4 + A_1$	[1]
2477	17	$2E_6 + A_3 + A_2$	[1]
2478	17	$2E_6 + A_3 + 2A_1$	[1]
2479	17	$2E_6 + 2A_2 + A_1$	[3]
2480	17	$E_6 + D_{11}$	[1]
2481	17	$E_6 + D_{10} + A_1$	[1]
2482	17	$E_6 + D_9 + A_2$	[1]
2483	17	$E_6 + D_9 + 2A_1$	[1]
2484	17	$E_6 + D_8 + A_3$	[1]
2485	17	$E_6 + D_8 + A_2 + A_1$	[1]
2486	17	$E_6 + D_7 + D_4$	[1]
2487	17	$E_6 + D_7 + A_4$	[1]
2488	17	$E_6 + D_7 + A_3 + A_1$	[1]
2489	17	$E_6 + D_7 + A_2 + 2A_1$	[1]
2490	17	$E_6 + D_6 + D_5$	[1]
2491	17	$E_6 + D_6 + A_5$	[1]
2492	17	$E_6 + D_6 + A_4 + A_1$	[1]
2493	17	$E_6 + D_6 + A_3 + A_2$	[1]
2494	17	$E_6 + D_6 + 2A_2 + A_1$	[1]
2495	17	$E_6 + 2D_5 + A_1$	[1]

2496	17	$E_6 + D_5 + D_4 + A_2$	[1]
2497	17	$E_6 + D_5 + A_6$	[1]
2498	17	$E_6 + D_5 + A_5 + A_1$	[1]
2499	17	$E_6 + D_5 + A_4 + A_2$	[1]
2500	17	$E_6 + D_5 + A_4 + 2\,A_1$	[1]
2501	17	$E_6 + D_5 + 2\,A_3$	[1]
2502	17	$E_6 + D_5 + A_3 + A_2 + A_1$	[1]
2503	17	$E_6 + D_4 + A_7$	[1]
2504	17	$E_6 + D_4 + A_6 + A_1$	[1]
2505	17	$E_6 + D_4 + A_4 + A_3$	[1]
2506	17	$E_6 + D_4 + A_4 + A_2 + A_1$	[1]
2507	17	$E_6 + A_{11}$	[3], [1]
2508	17	$E_6 + A_{10} + A_1$	[1]
2509	17	$E_6 + A_9 + A_2$	[1]
2510	17	$E_6 + A_9 + 2\,A_1$	[1]
2511	17	$E_6 + A_8 + A_3$	[1]
2512	17	$E_6 + A_8 + A_2 + A_1$	[3], [1]
2513	17	$E_6 + A_8 + 3\,A_1$	[1]
2514	17	$E_6 + A_7 + A_4$	[1]
2515	17	$E_6 + A_7 + A_3 + A_1$	[1]
2516	17	$E_6 + A_7 + 2\,A_2$	[1]
2517	17	$E_6 + A_7 + A_2 + 2\,A_1$	[1]
2518	17	$E_6 + A_6 + A_5$	[1]
2519	17	$E_6 + A_6 + A_4 + A_1$	[1]
2520	17	$E_6 + A_6 + A_3 + A_2$	[1]
2521	17	$E_6 + A_6 + A_3 + 2\,A_1$	[1]
2522	17	$E_6 + A_6 + 2\,A_2 + A_1$	[1]
2523	17	$E_6 + A_6 + A_2 + 3\,A_1$	[1]
2524	17	$E_6 + 2\,A_5 + A_1$	[3], [1]
2525	17	$E_6 + A_5 + A_4 + A_2$	[1]
2526	17	$E_6 + A_5 + A_4 + 2\,A_1$	[1]
2527	17	$E_6 + A_5 + 2\,A_3$	[1]
2528	17	$E_6 + A_5 + A_3 + A_2 + A_1$	[1]
2529	17	$E_6 + A_5 + 3\,A_2$	[3]
2530	17	$E_6 + A_5 + 2\,A_2 + 2\,A_1$	[3]
2531	17	$E_6 + 2\,A_4 + A_3$	[1]
2532	17	$E_6 + 2\,A_4 + A_2 + A_1$	[1]

2533	17	$E_6 + 2A_4 + 3A_1$	[1]
2534	17	$E_6 + A_4 + 2A_3 + A_1$	[1]
2535	17	$E_6 + A_4 + A_3 + 2A_2$	[1]
2536	17	$E_6 + A_4 + A_3 + A_2 + 2A_1$	[1]
2537	17	$E_6 + 2A_3 + 2A_2 + A_1$	[1]
2538	17	$E_6 + A_3 + 4A_2$	[3]
2539	17	D_{17}	[1]
2540	17	$D_{16} + A_1$	[2], [1]
2541	17	$D_{15} + A_2$	[1]
2542	17	$D_{15} + 2A_1$	[1]
2543	17	$D_{14} + A_3$	[1]
2544	17	$D_{14} + A_2 + A_1$	[2], [1]
2545	17	$D_{14} + 3A_1$	[2]
2546	17	$D_{13} + D_4$	[1]
2547	17	$D_{13} + A_4$	[1]
2548	17	$D_{13} + A_3 + A_1$	[1]
2549	17	$D_{13} + 2A_2$	[1]
2550	17	$D_{13} + A_2 + 2A_1$	[1]
2551	17	$D_{12} + D_5$	[2], [1]
2552	17	$D_{12} + D_4 + A_1$	[2]
2553	17	$D_{12} + A_5$	[1]
2554	17	$D_{12} + A_4 + A_1$	[1]
2555	17	$D_{12} + A_3 + A_2$	[2]
2556	17	$D_{12} + A_3 + 2A_1$	[2]
2557	17	$D_{12} + 2A_2 + A_1$	[1]
2558	17	$D_{12} + A_2 + 3A_1$	[2]
2559	17	$D_{11} + D_6$	[1]
2560	17	$D_{11} + D_5 + A_1$	[1]
2561	17	$D_{11} + A_6$	[1]
2562	17	$D_{11} + A_5 + A_1$	[1]
2563	17	$D_{11} + A_4 + A_2$	[1]
2564	17	$D_{11} + A_4 + 2A_1$	[1]
2565	17	$D_{11} + A_3 + A_2 + A_1$	[1]
2566	17	$D_{11} + 3A_2$	[1]
2567	17	$D_{11} + 2A_2 + 2A_1$	[1]
2568	17	$D_{10} + D_7$	[1]
2569	17	$D_{10} + D_6 + A_1$	[2]

2570	17	$D_{10} + D_5 + A_2$	[1]
2571	17	$D_{10} + D_5 + 2\,A_1$	[2]
2572	17	$D_{10} + D_4 + A_2 + A_1$	[2]
2573	17	$D_{10} + A_7$	[1]
2574	17	$D_{10} + A_6 + A_1$	[1]
2575	17	$D_{10} + A_5 + A_2$	[2], [1]
2576	17	$D_{10} + A_5 + 2\,A_1$	[2]
2577	17	$D_{10} + A_4 + A_3$	[1]
2578	17	$D_{10} + A_4 + A_2 + A_1$	[1]
2579	17	$D_{10} + A_4 + 3\,A_1$	[2]
2580	17	$D_{10} + 2\,A_3 + A_1$	[2]
2581	17	$D_{10} + A_3 + 2\,A_2$	[1]
2582	17	$D_{10} + A_3 + A_2 + 2\,A_1$	[2]
2583	17	$D_{10} + A_3 + 4\,A_1$	$[2, 2]$
2584	17	$D_{10} + 2\,A_2 + 3\,A_1$	[2]
2585	17	$D_9 + D_8$	[1]
2586	17	$D_9 + D_7 + A_1$	[1]
2587	17	$D_9 + D_6 + A_2$	[1]
2588	17	$D_9 + D_5 + A_3$	[1]
2589	17	$D_9 + D_5 + A_2 + A_1$	[1]
2590	17	$D_9 + D_4 + A_4$	[1]
2591	17	$D_9 + A_8$	[1]
2592	17	$D_9 + A_7 + A_1$	[1]
2593	17	$D_9 + A_6 + A_2$	[1]
2594	17	$D_9 + A_6 + 2\,A_1$	[1]
2595	17	$D_9 + A_5 + A_3$	[1]
2596	17	$D_9 + A_5 + A_2 + A_1$	[1]
2597	17	$D_9 + A_5 + 3\,A_1$	[2]
2598	17	$D_9 + 2\,A_4$	[1]
2599	17	$D_9 + A_4 + A_3 + A_1$	[1]
2600	17	$D_9 + A_4 + 2\,A_2$	[1]
2601	17	$D_9 + A_4 + A_2 + 2\,A_1$	[1]
2602	17	$D_9 + 2\,A_3 + 2\,A_1$	[2]
2603	17	$D_9 + A_3 + 2\,A_2 + A_1$	[1]
2604	17	$D_9 + 3\,A_2 + 2\,A_1$	[1]
2605	17	$2\,D_8 + A_1$	[2]
2606	17	$D_8 + D_7 + 2\,A_1$	[2]

2607	17	$D_8 + D_6 + A_3$	[2]
2608	17	$D_8 + D_6 + A_2 + A_1$	[2]
2609	17	$D_8 + D_6 + 3\,A_1$	[2, 2]
2610	17	$D_8 + D_5 + D_4$	[2]
2611	17	$D_8 + D_5 + A_4$	[1]
2612	17	$D_8 + D_5 + A_3 + A_1$	[2]
2613	17	$D_8 + D_5 + A_2 + 2\,A_1$	[2]
2614	17	$D_8 + D_4 + A_3 + 2\,A_1$	[2, 2]
2615	17	$D_8 + A_9$	[1]
2616	17	$D_8 + A_8 + A_1$	[1]
2617	17	$D_8 + A_7 + A_2$	[2]
2618	17	$D_8 + A_7 + 2\,A_1$	[2]
2619	17	$D_8 + A_6 + A_2 + A_1$	[1]
2620	17	$D_8 + A_5 + A_4$	[1]
2621	17	$D_8 + A_5 + A_3 + A_1$	[2]
2622	17	$D_8 + A_5 + 2\,A_2$	[1]
2623	17	$D_8 + A_5 + A_2 + 2\,A_1$	[2]
2624	17	$D_8 + A_5 + 4\,A_1$	[2, 2]
2625	17	$D_8 + 2\,A_4 + A_1$	[1]
2626	17	$D_8 + A_4 + A_3 + 2\,A_1$	[2]
2627	17	$D_8 + A_4 + 2\,A_2 + A_1$	[1]
2628	17	$D_8 + 2\,A_3 + A_2 + A_1$	[2]
2629	17	$D_8 + 2\,A_3 + 3\,A_1$	[2, 2]
2630	17	$D_8 + A_3 + 3\,A_2$	[1]
2631	17	$D_8 + A_3 + 2\,A_2 + 2\,A_1$	[2]
2632	17	$2\,D_7 + A_2 + A_1$	[1]
2633	17	$D_7 + D_6 + A_4$	[1]
2634	17	$D_7 + D_6 + A_3 + A_1$	[2]
2635	17	$D_7 + D_6 + 2\,A_2$	[1]
2636	17	$D_7 + 2\,D_5$	[1]
2637	17	$D_7 + D_5 + A_5$	[1]
2638	17	$D_7 + D_5 + A_4 + A_1$	[1]
2639	17	$D_7 + D_5 + A_3 + 2\,A_1$	[2]
2640	17	$D_7 + D_5 + 2\,A_2 + A_1$	[1]
2641	17	$D_7 + D_4 + A_5 + A_1$	[2]
2642	17	$D_7 + D_4 + 3\,A_2$	[1]
2643	17	$D_7 + A_{10}$	[1]

2644	17	$D_7 + A_9 + A_1$	[2], [1]
2645	17	$D_7 + A_8 + A_2$	[1]
2646	17	$D_7 + A_8 + 2\,A_1$	[1]
2647	17	$D_7 + A_7 + A_3$	[4]
2648	17	$D_7 + A_7 + A_2 + A_1$	[1]
2649	17	$D_7 + A_7 + 3\,A_1$	[2]
2650	17	$D_7 + A_6 + A_4$	[1]
2651	17	$D_7 + A_6 + A_3 + A_1$	[1]
2652	17	$D_7 + A_6 + 2\,A_2$	[1]
2653	17	$D_7 + A_6 + A_2 + 2\,A_1$	[1]
2654	17	$D_7 + 2\,A_5$	[2], [1]
2655	17	$D_7 + A_5 + A_4 + A_1$	[1]
2656	17	$D_7 + A_5 + A_3 + A_2$	[1]
2657	17	$D_7 + A_5 + A_3 + 2\,A_1$	[2]
2658	17	$D_7 + A_5 + A_2 + 3\,A_1$	[2]
2659	17	$D_7 + 2\,A_4 + A_2$	[1]
2660	17	$D_7 + 2\,A_4 + 2\,A_1$	[1]
2661	17	$D_7 + A_4 + A_3 + A_2 + A_1$	[1]
2662	17	$D_7 + A_4 + 2\,A_2 + 2\,A_1$	[1]
2663	17	$D_7 + 3\,A_3 + A_1$	[4]
2664	17	$D_7 + 2\,A_3 + 2\,A_2$	[1]
2665	17	$D_7 + 2\,A_3 + A_2 + 2\,A_1$	[2]
2666	17	$2\,D_6 + D_5$	[2]
2667	17	$2\,D_6 + D_4 + A_1$	$[2,2]$
2668	17	$2\,D_6 + A_5$	[2]
2669	17	$2\,D_6 + A_3 + A_2$	[2]
2670	17	$2\,D_6 + A_3 + 2\,A_1$	$[2,2]$
2671	17	$D_6 + 2\,D_5 + A_1$	[2]
2672	17	$D_6 + D_5 + A_6$	[1]
2673	17	$D_6 + D_5 + A_5 + A_1$	[2]
2674	17	$D_6 + D_5 + A_4 + A_2$	[1]
2675	17	$D_6 + D_5 + 2\,A_3$	[2]
2676	17	$D_6 + D_5 + A_3 + A_2 + A_1$	[2]
2677	17	$D_6 + D_5 + 3\,A_2$	[1]
2678	17	$D_6 + D_4 + A_7$	[2]
2679	17	$D_6 + D_4 + A_5 + A_2$	[2]
2680	17	$D_6 + D_4 + 2\,A_3 + A_1$	$[2,2]$

2681	17	$D_6 + A_{11}$	[2], [1]
2682	17	$D_6 + A_{10} + A_1$	[1]
2683	17	$D_6 + A_9 + A_2$	[2], [1]
2684	17	$D_6 + A_9 + 2A_1$	[2]
2685	17	$D_6 + A_8 + A_3$	[1]
2686	17	$D_6 + A_8 + A_2 + A_1$	[1]
2687	17	$D_6 + A_7 + A_4$	[1]
2688	17	$D_6 + A_7 + A_3 + A_1$	[2]
2689	17	$D_6 + A_7 + 2A_2$	[1]
2690	17	$D_6 + A_7 + A_2 + 2A_1$	[2]
2691	17	$D_6 + A_6 + A_5$	[1]
2692	17	$D_6 + A_6 + A_4 + A_1$	[1]
2693	17	$D_6 + A_6 + A_3 + A_2$	[1]
2694	17	$D_6 + A_6 + 2A_2 + A_1$	[1]
2695	17	$D_6 + 2A_5 + A_1$	[2]
2696	17	$D_6 + A_5 + A_4 + A_2$	[1]
2697	17	$D_6 + A_5 + A_4 + 2A_1$	[2]
2698	17	$D_6 + A_5 + 2A_3$	[2]
2699	17	$D_6 + A_5 + A_3 + A_2 + A_1$	[2]
2700	17	$D_6 + A_5 + A_3 + 3A_1$	$[2, 2]$
2701	17	$D_6 + A_5 + 2A_2 + 2A_1$	[2]
2702	17	$D_6 + 2A_4 + A_3$	[1]
2703	17	$D_6 + 2A_4 + A_2 + A_1$	[1]
2704	17	$D_6 + A_4 + 2A_3 + A_1$	[2]
2705	17	$D_6 + A_4 + A_3 + 2A_2$	[1]
2706	17	$D_6 + A_4 + 3A_2 + A_1$	[1]
2707	17	$D_6 + 3A_3 + A_2$	[2]
2708	17	$D_6 + 3A_3 + 2A_1$	$[2, 2]$
2709	17	$D_6 + 2A_3 + 2A_2 + A_1$	[2]
2710	17	$3D_5 + A_2$	[1]
2711	17	$2D_5 + D_4 + A_3$	[2]
2712	17	$2D_5 + A_7$	[4], [2], [1]
2713	17	$2D_5 + A_6 + A_1$	[1]
2714	17	$2D_5 + A_5 + A_2$	[1]
2715	17	$2D_5 + A_5 + 2A_1$	[2]
2716	17	$2D_5 + A_4 + A_3$	[1]
2717	17	$2D_5 + A_4 + A_2 + A_1$	[1]

2718	17	$2\,D_5 + 2\,A_3 + A_1$	[4]
2719	17	$D_5 + D_4 + A_8$	[1]
2720	17	$D_5 + D_4 + A_7 + A_1$	[2]
2721	17	$D_5 + D_4 + A_5 + A_2 + A_1$	[2]
2722	17	$D_5 + D_4 + 2\,A_4$	[1]
2723	17	$D_5 + A_{12}$	[1]
2724	17	$D_5 + A_{11} + A_1$	[4], [2], [1]
2725	17	$D_5 + A_{10} + A_2$	[1]
2726	17	$D_5 + A_{10} + 2\,A_1$	[1]
2727	17	$D_5 + A_9 + A_3$	[1]
2728	17	$D_5 + A_9 + A_2 + A_1$	[2], [1]
2729	17	$D_5 + A_9 + 3\,A_1$	[2]
2730	17	$D_5 + A_8 + A_4$	[1]
2731	17	$D_5 + A_8 + A_3 + A_1$	[1]
2732	17	$D_5 + A_8 + 2\,A_2$	[1]
2733	17	$D_5 + A_8 + A_2 + 2\,A_1$	[1]
2734	17	$D_5 + A_7 + A_5$	[1]
2735	17	$D_5 + A_7 + A_4 + A_1$	[1]
2736	17	$D_5 + A_7 + A_3 + A_2$	[2]
2737	17	$D_5 + A_7 + A_3 + 2\,A_1$	[4], [2]
2738	17	$D_5 + A_7 + 2\,A_2 + A_1$	[1]
2739	17	$D_5 + A_7 + A_2 + 3\,A_1$	[2]
2740	17	$D_5 + 2\,A_6$	[1]
2741	17	$D_5 + A_6 + A_5 + A_1$	[1]
2742	17	$D_5 + A_6 + A_4 + A_2$	[1]
2743	17	$D_5 + A_6 + A_4 + 2\,A_1$	[1]
2744	17	$D_5 + A_6 + A_3 + A_2 + A_1$	[1]
2745	17	$D_5 + A_6 + 3\,A_2$	[1]
2746	17	$D_5 + A_6 + 2\,A_2 + 2\,A_1$	[1]
2747	17	$D_5 + 2\,A_5 + A_2$	[2], [1]
2748	17	$D_5 + 2\,A_5 + 2\,A_1$	[2]
2749	17	$D_5 + A_5 + A_4 + A_3$	[1]
2750	17	$D_5 + A_5 + A_4 + A_2 + A_1$	[1]
2751	17	$D_5 + A_5 + A_4 + 3\,A_1$	[2]
2752	17	$D_5 + A_5 + 2\,A_3 + A_1$	[2]
2753	17	$D_5 + A_5 + A_3 + 2\,A_2$	[1]
2754	17	$D_5 + A_5 + A_3 + A_2 + 2\,A_1$	[2]

2755	17	$D_5 + 3\,A_4$	[1]
2756	17	$D_5 + 2\,A_4 + A_3 + A_1$	[1]
2757	17	$D_5 + 2\,A_4 + 2\,A_2$	[1]
2758	17	$D_5 + 2\,A_4 + A_2 + 2\,A_1$	[1]
2759	17	$D_5 + A_4 + 2\,A_3 + 2\,A_1$	[2]
2760	17	$D_5 + A_4 + A_3 + 2\,A_2 + A_1$	[1]
2761	17	$D_5 + 3\,A_3 + A_2 + A_1$	[4]
2762	17	$D_5 + 2\,A_3 + 3\,A_2$	[1]
2763	17	$D_4 + A_{13}$	[1]
2764	17	$D_4 + A_{12} + A_1$	[1]
2765	17	$D_4 + A_{11} + A_2$	[2]
2766	17	$D_4 + A_{11} + 2\,A_1$	[2]
2767	17	$D_4 + A_{10} + A_2 + A_1$	[1]
2768	17	$D_4 + A_9 + A_4$	[1]
2769	17	$D_4 + A_9 + A_3 + A_1$	[2]
2770	17	$D_4 + A_9 + 2\,A_2$	[1]
2771	17	$D_4 + A_9 + A_2 + 2\,A_1$	[2]
2772	17	$D_4 + A_8 + A_5$	[1]
2773	17	$D_4 + A_8 + A_4 + A_1$	[1]
2774	17	$D_4 + A_7 + A_4 + 2\,A_1$	[2]
2775	17	$D_4 + A_7 + A_3 + A_2 + A_1$	[2]
2776	17	$D_4 + A_7 + 2\,A_2 + 2\,A_1$	[2]
2777	17	$D_4 + 2\,A_6 + A_1$	[1]
2778	17	$D_4 + A_6 + A_5 + A_2$	[1]
2779	17	$D_4 + A_6 + A_4 + A_2 + A_1$	[1]
2780	17	$D_4 + A_6 + A_3 + 2\,A_2$	[1]
2781	17	$D_4 + 2\,A_5 + A_3$	[2]
2782	17	$D_4 + 2\,A_5 + 3\,A_1$	[2, 2]
2783	17	$D_4 + A_5 + 2\,A_4$	[1]
2784	17	$D_4 + A_5 + A_4 + A_3 + A_1$	[2]
2785	17	$D_4 + A_5 + 2\,A_3 + 2\,A_1$	[2, 2]
2786	17	$D_4 + 2\,A_4 + 2\,A_2 + A_1$	[1]
2787	17	$D_4 + 3\,A_3 + 2\,A_2$	[2]
2788	17	A_{17}	[3], [1]
2789	17	$A_{16} + A_1$	[1]
2790	17	$A_{15} + A_2$	[2], [1]
2791	17	$A_{15} + 2\,A_1$	[4], [2], [1]

2792	17	$A_{14} + A_3$	[1]
2793	17	$A_{14} + A_2 + A_1$	[3], [1]
2794	17	$A_{14} + 3\,A_1$	[1]
2795	17	$A_{13} + A_4$	[1]
2796	17	$A_{13} + A_3 + A_1$	[2], [1]
2797	17	$A_{13} + 2\,A_2$	[1]
2798	17	$A_{13} + A_2 + 2\,A_1$	[2], [1]
2799	17	$A_{13} + 4\,A_1$	[2]
2800	17	$A_{12} + A_5$	[1]
2801	17	$A_{12} + A_4 + A_1$	[1]
2802	17	$A_{12} + A_3 + A_2$	[1]
2803	17	$A_{12} + A_3 + 2\,A_1$	[1]
2804	17	$A_{12} + 2\,A_2 + A_1$	[1]
2805	17	$A_{12} + A_2 + 3\,A_1$	[1]
2806	17	$A_{11} + A_6$	[1]
2807	17	$A_{11} + A_5 + A_1$	[3], [1]
2808	17	$A_{11} + A_4 + A_2$	[1]
2809	17	$A_{11} + A_4 + 2\,A_1$	[2], [1]
2810	17	$A_{11} + 2\,A_3$	[4]
2811	17	$A_{11} + A_3 + A_2 + A_1$	[2], [1]
2812	17	$A_{11} + A_3 + 3\,A_1$	[4], [2]
2813	17	$A_{11} + 3\,A_2$	[3]
2814	17	$A_{11} + 2\,A_2 + 2\,A_1$	[6], [3], [2], [1]
2815	17	$A_{11} + A_2 + 4\,A_1$	[2]
2816	17	$A_{10} + A_7$	[1]
2817	17	$A_{10} + A_6 + A_1$	[1]
2818	17	$A_{10} + A_5 + A_2$	[1]
2819	17	$A_{10} + A_5 + 2\,A_1$	[1]
2820	17	$A_{10} + A_4 + A_3$	[1]
2821	17	$A_{10} + A_4 + A_2 + A_1$	[1]
2822	17	$A_{10} + A_4 + 3\,A_1$	[1]
2823	17	$A_{10} + 2\,A_3 + A_1$	[1]
2824	17	$A_{10} + A_3 + 2\,A_2$	[1]
2825	17	$A_{10} + A_3 + A_2 + 2\,A_1$	[1]
2826	17	$A_{10} + 3\,A_2 + A_1$	[1]
2827	17	$A_{10} + 2\,A_2 + 3\,A_1$	[1]
2828	17	$A_9 + A_8$	[1]

2829	17	$A_9 + A_7 + A_1$	[1]
2830	17	$A_9 + A_6 + A_2$	[1]
2831	17	$A_9 + A_6 + 2\,A_1$	[1]
2832	17	$A_9 + A_5 + A_3$	[2], [1]
2833	17	$A_9 + A_5 + A_2 + A_1$	[2], [1]
2834	17	$A_9 + A_5 + 3\,A_1$	[2]
2835	17	$A_9 + 2\,A_4$	[5], [1]
2836	17	$A_9 + A_4 + A_3 + A_1$	[2], [1]
2837	17	$A_9 + A_4 + 2\,A_2$	[1]
2838	17	$A_9 + A_4 + A_2 + 2\,A_1$	[1]
2839	17	$A_9 + A_4 + 4\,A_1$	[2]
2840	17	$A_9 + 2\,A_3 + A_2$	[1]
2841	17	$A_9 + 2\,A_3 + 2\,A_1$	[2]
2842	17	$A_9 + A_3 + 2\,A_2 + A_1$	[2], [1]
2843	17	$A_9 + A_3 + A_2 + 3\,A_1$	[2]
2844	17	$A_9 + 3\,A_2 + 2\,A_1$	[1]
2845	17	$A_9 + 2\,A_2 + 4\,A_1$	[2]
2846	17	$2\,A_8 + A_1$	[3], [1]
2847	17	$A_8 + A_7 + A_2$	[1]
2848	17	$A_8 + A_7 + 2\,A_1$	[1]
2849	17	$A_8 + A_6 + A_3$	[1]
2850	17	$A_8 + A_6 + A_2 + A_1$	[1]
2851	17	$A_8 + A_6 + 3\,A_1$	[1]
2852	17	$A_8 + A_5 + A_4$	[1]
2853	17	$A_8 + A_5 + A_3 + A_1$	[1]
2854	17	$A_8 + A_5 + 2\,A_2$	[3]
2855	17	$A_8 + A_5 + A_2 + 2\,A_1$	[3], [1]
2856	17	$A_8 + 2\,A_4 + A_1$	[1]
2857	17	$A_8 + A_4 + A_3 + A_2$	[1]
2858	17	$A_8 + A_4 + A_3 + 2\,A_1$	[1]
2859	17	$A_8 + A_4 + 2\,A_2 + A_1$	[1]
2860	17	$A_8 + A_4 + A_2 + 3\,A_1$	[1]
2861	17	$A_8 + 2\,A_3 + A_2 + A_1$	[1]
2862	17	$A_8 + A_3 + 3\,A_2$	[3]
2863	17	$A_8 + A_3 + 2\,A_2 + 2\,A_1$	[1]
2864	17	$A_8 + 4\,A_2 + A_1$	[3]
2865	17	$A_8 + 3\,A_2 + 3\,A_1$	[3]

2866	17	$2A_7 + A_3$	[4]
2867	17	$2A_7 + A_2 + A_1$	[2], [1]
2868	17	$2A_7 + 3A_1$	[4]
2869	17	$A_7 + A_6 + A_4$	[1]
2870	17	$A_7 + A_6 + A_3 + A_1$	[1]
2871	17	$A_7 + A_6 + 2A_2$	[1]
2872	17	$A_7 + A_6 + A_2 + 2A_1$	[1]
2873	17	$A_7 + A_6 + 4A_1$	[2]
2874	17	$A_7 + 2A_5$	[1]
2875	17	$A_7 + A_5 + A_4 + A_1$	[2], [1]
2876	17	$A_7 + A_5 + A_3 + A_2$	[1]
2877	17	$A_7 + A_5 + A_3 + 2A_1$	[2]
2878	17	$A_7 + A_5 + 2A_2 + A_1$	[2], [1]
2879	17	$A_7 + A_5 + A_2 + 3A_1$	[2]
2880	17	$A_7 + A_5 + 5A_1$	$[2,2]$
2881	17	$A_7 + 2A_4 + A_2$	[1]
2882	17	$A_7 + 2A_4 + 2A_1$	[1]
2883	17	$A_7 + A_4 + A_3 + A_2 + A_1$	[1]
2884	17	$A_7 + A_4 + A_3 + 3A_1$	[2]
2885	17	$A_7 + A_4 + 3A_2$	[1]
2886	17	$A_7 + A_4 + 2A_2 + 2A_1$	[1]
2887	17	$A_7 + A_4 + A_2 + 4A_1$	[2]
2888	17	$A_7 + 3A_3 + A_1$	[4]
2889	17	$A_7 + 2A_3 + 2A_2$	[2], [1]
2890	17	$A_7 + 2A_3 + A_2 + 2A_1$	[4], [2]
2891	17	$A_7 + 2A_3 + 4A_1$	$[4,2]$, $[2,2]$
2892	17	$A_7 + A_3 + 3A_2 + A_1$	[1]
2893	17	$A_7 + A_3 + 2A_2 + 3A_1$	[2]
2894	17	$2A_6 + A_5$	[1]
2895	17	$2A_6 + A_4 + A_1$	[1]
2896	17	$2A_6 + A_3 + A_2$	[1]
2897	17	$2A_6 + A_3 + 2A_1$	[1]
2898	17	$2A_6 + 2A_2 + A_1$	[1]
2899	17	$2A_6 + A_2 + 3A_1$	[1]
2900	17	$A_6 + 2A_5 + A_1$	[1]
2901	17	$A_6 + A_5 + A_4 + A_2$	[1]
2902	17	$A_6 + A_5 + A_4 + 2A_1$	[1]

2903	17	$A_6 + A_5 + 2A_3$	[1]
2904	17	$A_6 + A_5 + A_3 + A_2 + A_1$	[1]
2905	17	$A_6 + A_5 + A_3 + 3A_1$	[2]
2906	17	$A_6 + A_5 + 2A_2 + 2A_1$	[1]
2907	17	$A_6 + 2A_4 + A_3$	[1]
2908	17	$A_6 + 2A_4 + A_2 + A_1$	[1]
2909	17	$A_6 + 2A_4 + 3A_1$	[1]
2910	17	$A_6 + A_4 + 2A_3 + A_1$	[1]
2911	17	$A_6 + A_4 + A_3 + 2A_2$	[1]
2912	17	$A_6 + A_4 + A_3 + A_2 + 2A_1$	[1]
2913	17	$A_6 + A_4 + 3A_2 + A_1$	[1]
2914	17	$A_6 + A_4 + 2A_2 + 3A_1$	[1]
2915	17	$A_6 + 3A_3 + A_2$	[1]
2916	17	$A_6 + 3A_3 + 2A_1$	[2]
2917	17	$A_6 + 2A_3 + 2A_2 + A_1$	[1]
2918	17	$A_6 + A_3 + 3A_2 + 2A_1$	[1]
2919	17	$3A_5 + A_2$	[3]
2920	17	$3A_5 + 2A_1$	[6], [2]
2921	17	$2A_5 + A_4 + A_3$	[2], [1]
2922	17	$2A_5 + A_4 + A_2 + A_1$	[1]
2923	17	$2A_5 + A_4 + 3A_1$	[2]
2924	17	$2A_5 + 2A_3 + A_1$	[2]
2925	17	$2A_5 + A_3 + 2A_2$	[6], [3]
2926	17	$2A_5 + A_3 + A_2 + 2A_1$	[2]
2927	17	$2A_5 + A_3 + 4A_1$	[2, 2]
2928	17	$2A_5 + 3A_2 + A_1$	[3]
2929	17	$2A_5 + 2A_2 + 3A_1$	[6]
2930	17	$A_5 + 3A_4$	[1]
2931	17	$A_5 + 2A_4 + A_3 + A_1$	[1]
2932	17	$A_5 + 2A_4 + 2A_2$	[1]
2933	17	$A_5 + 2A_4 + A_2 + 2A_1$	[1]
2934	17	$A_5 + A_4 + 2A_3 + A_2$	[1]
2935	17	$A_5 + A_4 + 2A_3 + 2A_1$	[2]
2936	17	$A_5 + A_4 + A_3 + 2A_2 + A_1$	[1]
2937	17	$A_5 + A_4 + A_3 + A_2 + 3A_1$	[2]
2938	17	$A_5 + A_4 + 4A_2$	[3]
2939	17	$A_5 + 3A_3 + A_2 + A_1$	[2]

2940	17	$A_5 + 3A_3 + 3A_1$	$[2,2]$
2941	17	$A_5 + 2A_3 + 2A_2 + 2A_1$	$[2]$
2942	17	$A_5 + A_3 + 4A_2 + A_1$	$[3]$
2943	17	$A_5 + 6A_2$	$[3,3]$
2944	17	$4A_4 + A_1$	$[5]$
2945	17	$3A_4 + A_3 + 2A_1$	$[1]$
2946	17	$3A_4 + A_2 + 3A_1$	$[1]$
2947	17	$2A_4 + 2A_3 + A_2 + A_1$	$[1]$
2948	17	$2A_4 + A_3 + 3A_2$	$[1]$
2949	17	$2A_4 + A_3 + 2A_2 + 2A_1$	$[1]$
2950	17	$A_4 + 3A_3 + 2A_2$	$[1]$
2951	17	$A_4 + 3A_3 + A_2 + 2A_1$	$[2]$
2952	17	$A_4 + 2A_3 + 3A_2 + A_1$	$[1]$
2953	17	$5A_3 + A_2$	$[4]$
2954	17	$5A_3 + 2A_1$	$[4,2]$

No.	rank	ADE-type	G
2955	18	$2E_8 + A_2$	$[1]$
2956	18	$2E_8 + 2A_1$	$[1]$
2957	18	$E_8 + E_7 + A_3$	$[1]$
2958	18	$E_8 + E_7 + A_2 + A_1$	$[1]$
2959	18	$E_8 + E_6 + D_4$	$[1]$
2960	18	$E_8 + E_6 + A_4$	$[1]$
2961	18	$E_8 + E_6 + A_3 + A_1$	$[1]$
2962	18	$E_8 + D_{10}$	$[1]$
2963	18	$E_8 + D_9 + A_1$	$[1]$
2964	18	$E_8 + D_7 + A_2 + A_1$	$[1]$
2965	18	$E_8 + D_6 + A_4$	$[1]$
2966	18	$E_8 + D_6 + 2A_2$	$[1]$
2967	18	$E_8 + 2D_5$	$[1]$
2968	18	$E_8 + D_5 + A_5$	$[1]$
2969	18	$E_8 + D_5 + A_4 + A_1$	$[1]$
2970	18	$E_8 + A_{10}$	$[1]$
2971	18	$E_8 + A_9 + A_1$	$[1]$
2972	18	$E_8 + A_8 + A_2$	$[1]$
2973	18	$E_8 + A_8 + 2A_1$	$[1]$

2974	18	$E_8 + A_7 + A_2 + A_1$	[1]
2975	18	$E_8 + A_6 + A_4$	[1]
2976	18	$E_8 + A_6 + A_3 + A_1$	[1]
2977	18	$E_8 + A_6 + 2\,A_2$	[1]
2978	18	$E_8 + A_6 + A_2 + 2\,A_1$	[1]
2979	18	$E_8 + 2\,A_5$	[1]
2980	18	$E_8 + A_5 + A_4 + A_1$	[1]
2981	18	$E_8 + A_5 + A_3 + A_2$	[1]
2982	18	$E_8 + 2\,A_4 + 2\,A_1$	[1]
2983	18	$E_8 + A_4 + A_3 + A_2 + A_1$	[1]
2984	18	$E_8 + 2\,A_3 + 2\,A_2$	[1]
2985	18	$2\,E_7 + D_4$	[2]
2986	18	$2\,E_7 + A_4$	[1]
2987	18	$2\,E_7 + A_3 + A_1$	[2]
2988	18	$2\,E_7 + 2\,A_2$	[1]
2989	18	$E_7 + E_6 + D_5$	[1]
2990	18	$E_7 + E_6 + A_5$	[1]
2991	18	$E_7 + E_6 + A_4 + A_1$	[1]
2992	18	$E_7 + E_6 + A_3 + A_2$	[1]
2993	18	$E_7 + D_{11}$	[1]
2994	18	$E_7 + D_{10} + A_1$	[2]
2995	18	$E_7 + D_9 + A_2$	[1]
2996	18	$E_7 + D_8 + A_2 + A_1$	[2]
2997	18	$E_7 + D_7 + A_4$	[1]
2998	18	$E_7 + D_7 + A_3 + A_1$	[2]
2999	18	$E_7 + D_6 + D_5$	[2]
3000	18	$E_7 + D_6 + A_5$	[2]
3001	18	$E_7 + D_6 + A_3 + A_2$	[2]
3002	18	$E_7 + D_5 + A_6$	[1]
3003	18	$E_7 + D_5 + A_5 + A_1$	[2]
3004	18	$E_7 + D_5 + A_4 + A_2$	[1]
3005	18	$E_7 + A_{11}$	[1]
3006	18	$E_7 + A_{10} + A_1$	[1]
3007	18	$E_7 + A_9 + A_2$	[2], [1]
3008	18	$E_7 + A_9 + 2\,A_1$	[2]
3009	18	$E_7 + A_8 + A_3$	[1]
3010	18	$E_7 + A_8 + A_2 + A_1$	[1]

3011	18	$E_7 + A_7 + A_4$	[1]
3012	18	$E_7 + A_7 + A_3 + A_1$	[2]
3013	18	$E_7 + A_7 + 2A_2$	[1]
3014	18	$E_7 + A_7 + A_2 + 2A_1$	[2]
3015	18	$E_7 + A_6 + A_5$	[1]
3016	18	$E_7 + A_6 + A_4 + A_1$	[1]
3017	18	$E_7 + A_6 + A_3 + A_2$	[1]
3018	18	$E_7 + A_6 + 2A_2 + A_1$	[1]
3019	18	$E_7 + A_5 + A_4 + A_2$	[1]
3020	18	$E_7 + A_5 + A_4 + 2A_1$	[2]
3021	18	$E_7 + A_5 + 2A_3$	[2]
3022	18	$E_7 + A_5 + A_3 + A_2 + A_1$	[2]
3023	18	$E_7 + A_4 + 2A_3 + A_1$	[2]
3024	18	$E_7 + A_4 + A_3 + 2A_2$	[1]
3025	18	$3E_6$	[3]
3026	18	$2E_6 + D_6$	[1]
3027	18	$2E_6 + A_6$	[1]
3028	18	$2E_6 + A_5 + A_1$	[3]
3029	18	$2E_6 + 2A_3$	[1]
3030	18	$E_6 + D_{12}$	[1]
3031	18	$E_6 + D_{11} + A_1$	[1]
3032	18	$E_6 + D_9 + A_3$	[1]
3033	18	$E_6 + D_9 + A_2 + A_1$	[1]
3034	18	$E_6 + D_8 + A_4$	[1]
3035	18	$E_6 + D_7 + D_5$	[1]
3036	18	$E_6 + D_7 + A_4 + A_1$	[1]
3037	18	$E_6 + D_6 + A_6$	[1]
3038	18	$E_6 + D_6 + A_4 + A_2$	[1]
3039	18	$E_6 + D_5 + A_7$	[1]
3040	18	$E_6 + D_5 + A_6 + A_1$	[1]
3041	18	$E_6 + D_5 + A_4 + A_3$	[1]
3042	18	$E_6 + A_{12}$	[1]
3043	18	$E_6 + A_{11} + A_1$	[3], [1]
3044	18	$E_6 + A_{10} + A_2$	[1]
3045	18	$E_6 + A_{10} + 2A_1$	[1]
3046	18	$E_6 + A_9 + A_3$	[1]
3047	18	$E_6 + A_9 + A_2 + A_1$	[1]

3048	18	$E_6 + A_8 + A_4$	[1]
3049	18	$E_6 + A_8 + A_3 + A_1$	[1]
3050	18	$E_6 + A_8 + 2A_2$	[3]
3051	18	$E_6 + A_8 + A_2 + 2A_1$	[3]
3052	18	$E_6 + A_7 + A_5$	[1]
3053	18	$E_6 + A_7 + A_4 + A_1$	[1]
3054	18	$E_6 + A_6 + A_5 + A_1$	[1]
3055	18	$E_6 + A_6 + A_4 + A_2$	[1]
3056	18	$E_6 + A_6 + A_4 + 2A_1$	[1]
3057	18	$E_6 + A_6 + A_3 + A_2 + A_1$	[1]
3058	18	$E_6 + 2A_5 + A_2$	[3]
3059	18	$E_6 + A_5 + A_4 + A_3$	[1]
3060	18	$E_6 + A_5 + A_3 + 2A_2$	[3]
3061	18	$E_6 + 2A_4 + A_3 + A_1$	[1]
3062	18	D_{18}	[1]
3063	18	$D_{17} + A_1$	[1]
3064	18	$D_{16} + A_2$	[2]
3065	18	$D_{16} + 2A_1$	[2]
3066	18	$D_{15} + A_2 + A_1$	[1]
3067	18	$D_{14} + A_4$	[1]
3068	18	$D_{14} + A_3 + A_1$	[2]
3069	18	$D_{14} + 2A_2$	[1]
3070	18	$D_{14} + A_2 + 2A_1$	[2]
3071	18	$D_{13} + D_5$	[1]
3072	18	$D_{13} + A_5$	[1]
3073	18	$D_{13} + A_4 + A_1$	[1]
3074	18	$D_{12} + D_6$	[2]
3075	18	$D_{12} + D_5 + A_1$	[2]
3076	18	$D_{12} + A_4 + 2A_1$	[2]
3077	18	$D_{12} + A_3 + A_2 + A_1$	[2]
3078	18	$D_{12} + 2A_2 + 2A_1$	[2]
3079	18	$D_{11} + A_6 + A_1$	[1]
3080	18	$D_{11} + A_5 + A_2$	[1]
3081	18	$D_{11} + A_4 + A_2 + A_1$	[1]
3082	18	$D_{11} + A_3 + 2A_2$	[1]
3083	18	$D_{10} + D_7 + A_1$	[2]
3084	18	$D_{10} + D_6 + A_2$	[2]

3085	18	$D_{10} + D_5 + A_2 + A_1$	[2]
3086	18	$D_{10} + A_8$	[1]
3087	18	$D_{10} + A_6 + A_2$	[1]
3088	18	$D_{10} + A_5 + A_3$	[2]
3089	18	$D_{10} + A_5 + 3\,A_1$	[2, 2]
3090	18	$D_{10} + 2\,A_4$	[1]
3091	18	$D_{10} + A_4 + A_3 + A_1$	[2]
3092	18	$D_{10} + 2\,A_3 + 2\,A_1$	[2, 2]
3093	18	$2\,D_9$	[1]
3094	18	$D_9 + D_5 + A_4$	[1]
3095	18	$D_9 + A_9$	[1]
3096	18	$D_9 + A_8 + A_1$	[1]
3097	18	$D_9 + A_7 + 2\,A_1$	[2]
3098	18	$D_9 + A_6 + A_2 + A_1$	[1]
3099	18	$D_9 + A_5 + A_4$	[1]
3100	18	$D_9 + A_5 + A_3 + A_1$	[2]
3101	18	$D_9 + A_4 + 2\,A_2 + A_1$	[1]
3102	18	$2\,D_8 + 2\,A_1$	[2, 2]
3103	18	$D_8 + D_6 + A_3 + A_1$	[2, 2]
3104	18	$D_8 + 2\,D_5$	[2]
3105	18	$D_8 + A_9 + A_1$	[2]
3106	18	$D_8 + A_7 + A_2 + A_1$	[2]
3107	18	$D_8 + A_6 + 2\,A_2$	[1]
3108	18	$D_8 + 2\,A_5$	[2]
3109	18	$D_8 + A_5 + A_4 + A_1$	[2]
3110	18	$D_8 + A_5 + A_3 + 2\,A_1$	[2, 2]
3111	18	$D_8 + 2\,A_3 + 2\,A_2$	[2]
3112	18	$2\,D_7 + 2\,A_2$	[1]
3113	18	$D_7 + D_6 + A_5$	[2]
3114	18	$D_7 + D_5 + A_5 + A_1$	[2]
3115	18	$D_7 + A_{11}$	[4]
3116	18	$D_7 + A_{10} + A_1$	[1]
3117	18	$D_7 + A_9 + A_2$	[1]
3118	18	$D_7 + A_9 + 2\,A_1$	[2]
3119	18	$D_7 + A_7 + A_3 + A_1$	[4]
3120	18	$D_7 + A_7 + A_2 + 2\,A_1$	[2]
3121	18	$D_7 + A_6 + A_5$	[1]

3122	18	$D_7 + A_6 + A_4 + A_1$	[1]
3123	18	$D_7 + A_6 + A_3 + A_2$	[1]
3124	18	$D_7 + 2A_4 + A_2 + A_1$	[1]
3125	18	$D_7 + 3A_3 + A_2$	[4]
3126	18	$3D_6$	[2, 2]
3127	18	$2D_6 + 2A_3$	[2, 2]
3128	18	$D_6 + D_5 + A_7$	[2]
3129	18	$D_6 + D_5 + A_5 + A_2$	[2]
3130	18	$D_6 + A_{12}$	[1]
3131	18	$D_6 + A_{11} + A_1$	[2]
3132	18	$D_6 + A_{10} + A_2$	[1]
3133	18	$D_6 + A_9 + A_3$	[2]
3134	18	$D_6 + A_9 + A_2 + A_1$	[2]
3135	18	$D_6 + A_8 + A_4$	[1]
3136	18	$D_6 + A_7 + A_4 + A_1$	[2]
3137	18	$D_6 + A_7 + A_3 + A_2$	[2]
3138	18	$D_6 + A_7 + 2A_2 + A_1$	[2]
3139	18	$D_6 + 2A_6$	[1]
3140	18	$D_6 + A_6 + A_4 + A_2$	[1]
3141	18	$D_6 + 2A_5 + 2A_1$	[2, 2]
3142	18	$D_6 + A_5 + A_4 + A_3$	[2]
3143	18	$D_6 + A_5 + 2A_3 + A_1$	[2, 2]
3144	18	$D_6 + 2A_4 + 2A_2$	[1]
3145	18	$2D_5 + A_8$	[1]
3146	18	$2D_5 + A_7 + A_1$	[4]
3147	18	$2D_5 + 2A_4$	[1]
3148	18	$D_5 + A_{13}$	[1]
3149	18	$D_5 + A_{12} + A_1$	[1]
3150	18	$D_5 + A_{11} + A_2$	[2]
3151	18	$D_5 + A_{11} + 2A_1$	[4]
3152	18	$D_5 + A_{10} + A_2 + A_1$	[1]
3153	18	$D_5 + A_9 + A_4$	[1]
3154	18	$D_5 + A_9 + A_3 + A_1$	[2]
3155	18	$D_5 + A_9 + 2A_2$	[1]
3156	18	$D_5 + A_9 + A_2 + 2A_1$	[2]
3157	18	$D_5 + A_8 + A_5$	[1]
3158	18	$D_5 + A_8 + A_4 + A_1$	[1]

3159	18	$D_5 + A_7 + A_4 + 2A_1$	[2]
3160	18	$D_5 + A_7 + A_3 + A_2 + A_1$	[4]
3161	18	$D_5 + 2A_6 + A_1$	[1]
3162	18	$D_5 + A_6 + A_5 + A_2$	[1]
3163	18	$D_5 + A_6 + A_4 + A_2 + A_1$	[1]
3164	18	$D_5 + A_6 + A_3 + 2A_2$	[1]
3165	18	$D_5 + 2A_5 + A_3$	[2]
3166	18	$D_5 + A_5 + 2A_4$	[1]
3167	18	$D_5 + A_5 + A_4 + A_3 + A_1$	[2]
3168	18	A_{18}	[1]
3169	18	$A_{17} + A_1$	[3], [1]
3170	18	$A_{16} + A_2$	[1]
3171	18	$A_{16} + 2A_1$	[1]
3172	18	$A_{15} + A_3$	[4]
3173	18	$A_{15} + A_2 + A_1$	[2], [1]
3174	18	$A_{15} + 3A_1$	[4]
3175	18	$A_{14} + A_4$	[1]
3176	18	$A_{14} + A_3 + A_1$	[1]
3177	18	$A_{14} + 2A_2$	[3]
3178	18	$A_{14} + A_2 + 2A_1$	[3], [1]
3179	18	$A_{13} + A_5$	[1]
3180	18	$A_{13} + A_4 + A_1$	[2], [1]
3181	18	$A_{13} + A_3 + A_2$	[1]
3182	18	$A_{13} + A_3 + 2A_1$	[2]
3183	18	$A_{13} + 2A_2 + A_1$	[2], [1]
3184	18	$A_{13} + A_2 + 3A_1$	[2]
3185	18	$A_{12} + A_6$	[1]
3186	18	$A_{12} + A_5 + A_1$	[1]
3187	18	$A_{12} + A_4 + A_2$	[1]
3188	18	$A_{12} + A_4 + 2A_1$	[1]
3189	18	$A_{12} + A_3 + A_2 + A_1$	[1]
3190	18	$A_{12} + 2A_2 + 2A_1$	[1]
3191	18	$A_{11} + A_6 + A_1$	[1]
3192	18	$A_{11} + A_5 + A_2$	[3]
3193	18	$A_{11} + A_5 + 2A_1$	[6], [2]
3194	18	$A_{11} + A_4 + A_2 + A_1$	[1]
3195	18	$A_{11} + A_4 + 3A_1$	[2]

3196	18	$A_{11} + 2A_3 + A_1$	[4]
3197	18	$A_{11} + A_3 + 2A_2$	[6], [3]
3198	18	$A_{11} + A_3 + A_2 + 2A_1$	[4], [2]
3199	18	$A_{11} + 3A_2 + A_1$	[3]
3200	18	$A_{11} + 2A_2 + 3A_1$	[6]
3201	18	$A_{10} + A_8$	[1]
3202	18	$A_{10} + A_7 + A_1$	[1]
3203	18	$A_{10} + A_6 + A_2$	[1]
3204	18	$A_{10} + A_6 + 2A_1$	[1]
3205	18	$A_{10} + A_5 + A_3$	[1]
3206	18	$A_{10} + A_5 + A_2 + A_1$	[1]
3207	18	$A_{10} + 2A_4$	[1]
3208	18	$A_{10} + A_4 + A_3 + A_1$	[1]
3209	18	$A_{10} + A_4 + 2A_2$	[1]
3210	18	$A_{10} + A_4 + A_2 + 2A_1$	[1]
3211	18	$A_{10} + 2A_3 + A_2$	[1]
3212	18	$A_{10} + A_3 + 2A_2 + A_1$	[1]
3213	18	$2A_9$	[5], [1]
3214	18	$A_9 + A_8 + A_1$	[1]
3215	18	$A_9 + A_7 + A_2$	[1]
3216	18	$A_9 + A_6 + A_3$	[1]
3217	18	$A_9 + A_6 + A_2 + A_1$	[1]
3218	18	$A_9 + A_6 + 3A_1$	[2]
3219	18	$A_9 + A_5 + A_4$	[2], [1]
3220	18	$A_9 + A_5 + A_3 + A_1$	[2]
3221	18	$A_9 + A_5 + A_2 + 2A_1$	[2]
3222	18	$A_9 + 2A_4 + A_1$	[5]
3223	18	$A_9 + A_4 + A_3 + 2A_1$	[2]
3224	18	$A_9 + A_4 + A_2 + 3A_1$	[2]
3225	18	$A_9 + 2A_3 + A_2 + A_1$	[2]
3226	18	$A_9 + A_3 + 2A_2 + 2A_1$	[2]
3227	18	$2A_8 + 2A_1$	[3], [1]
3228	18	$A_8 + A_7 + A_2 + A_1$	[1]
3229	18	$A_8 + A_6 + A_4$	[1]
3230	18	$A_8 + A_6 + A_3 + A_1$	[1]
3231	18	$A_8 + A_6 + A_2 + 2A_1$	[1]
3232	18	$A_8 + A_5 + A_4 + A_1$	[1]

3233	18	$A_8 + A_5 + A_3 + A_2$	[3]
3234	18	$A_8 + A_5 + 2A_2 + A_1$	[3]
3235	18	$A_8 + 2A_4 + 2A_1$	[1]
3236	18	$A_8 + A_4 + A_3 + A_2 + A_1$	[1]
3237	18	$A_8 + A_4 + 3A_2$	[3]
3238	18	$A_8 + A_3 + 3A_2 + A_1$	[3]
3239	18	$2A_7 + A_3 + A_1$	[8]
3240	18	$2A_7 + 2A_2$	[2], [1]
3241	18	$2A_7 + 4A_1$	[4, 2]
3242	18	$A_7 + A_6 + A_5$	[1]
3243	18	$A_7 + A_6 + A_4 + A_1$	[1]
3244	18	$A_7 + A_6 + A_3 + A_2$	[1]
3245	18	$A_7 + A_6 + A_3 + 2A_1$	[2]
3246	18	$A_7 + A_6 + 2A_2 + A_1$	[1]
3247	18	$A_7 + 2A_5 + A_1$	[2]
3248	18	$A_7 + A_5 + A_4 + A_2$	[1]
3249	18	$A_7 + A_5 + A_4 + 2A_1$	[2]
3250	18	$A_7 + A_5 + A_3 + A_2 + A_1$	[2]
3251	18	$A_7 + A_5 + A_3 + 3A_1$	[2, 2]
3252	18	$A_7 + A_4 + A_3 + 2A_2$	[1]
3253	18	$A_7 + A_4 + A_3 + A_2 + 2A_1$	[2]
3254	18	$A_7 + 3A_3 + A_2$	[4]
3255	18	$A_7 + 3A_3 + 2A_1$	[4, 2]
3256	18	$3A_6$	[7]
3257	18	$2A_6 + A_4 + A_2$	[1]
3258	18	$2A_6 + 2A_3$	[1]
3259	18	$2A_6 + 2A_2 + 2A_1$	[1]
3260	18	$A_6 + 2A_5 + 2A_1$	[2]
3261	18	$A_6 + A_5 + A_4 + A_3$	[1]
3262	18	$A_6 + A_5 + A_4 + A_2 + A_1$	[1]
3263	18	$A_6 + A_5 + 2A_3 + A_1$	[2]
3264	18	$A_6 + 2A_4 + A_3 + A_1$	[1]
3265	18	$A_6 + 2A_4 + A_2 + 2A_1$	[1]
3266	18	$A_6 + A_4 + 2A_3 + A_2$	[1]
3267	18	$A_6 + A_4 + A_3 + 2A_2 + A_1$	[1]
3268	18	$3A_5 + A_3$	[6]
3269	18	$3A_5 + 3A_1$	[6, 2]

3270	18	$2A_5 + 2A_4$	[1]
3271	18	$2A_5 + A_4 + A_3 + A_1$	[2]
3272	18	$2A_5 + A_4 + 2A_2$	[3]
3273	18	$2A_5 + 2A_3 + 2A_1$	[2, 2]
3274	18	$2A_5 + A_3 + 2A_2 + A_1$	[6]
3275	18	$2A_5 + 4A_2$	[3, 3]
3276	18	$A_5 + A_4 + 2A_3 + A_2 + A_1$	[2]
3277	18	$4A_4 + 2A_1$	[5]
3278	18	$2A_4 + 2A_3 + 2A_2$	[1]
3279	18	$6A_3$	[4, 4]

Table 2.

$G = \mathbb{Z}/(3)$
$3E_6$, $2E_6 + A_5 + A_1$, $2E_6 + A_5$, $2E_6 + 2A_2 + A_1$, $2E_6 + 2A_2$, $E_6 + A_{11} + A_1$, $E_6 + A_{11}$, $E_6 + A_8 + 2A_2$, $E_6 + A_8 + A_2 + 2A_1$, $E_6 + A_8 + A_2 + A_1$, $E_6 + A_8 + A_2$, $E_6 + 2A_5 + A_2$, $E_6 + 2A_5 + A_1$, $E_6 + 2A_5$, $E_6 + A_5 + A_3 + 2A_2$, $E_6 + A_5 + 3A_2$, $E_6 + A_5 + 2A_2 + 2A_1$, $E_6 + A_5 + 2A_2 + A_1$, $E_6 + A_5 + 2A_2$, $E_6 + A_3 + 4A_2$, $E_6 + 5A_2$, $E_6 + 4A_2 + 2A_1$, $E_6 + 4A_2 + A_1$, $E_6 + 4A_2$, $A_{17} + A_1$, A_{17}, $A_{14} + 2A_2$, $A_{14} + A_2 + 2A_1$, $A_{14} + A_2 + A_1$, $A_{14} + A_2$, $A_{11} + A_5 + A_2$, $A_{11} + A_5 + A_1$, $A_{11} + A_5$, $A_{11} + A_3 + 2A_2$, $A_{11} + 3A_2 + A_1$, $A_{11} + 3A_2$, $A_{11} + 2A_2 + 2A_1$, $A_{11} + 2A_2 + A_1$, $A_{11} + 2A_2$, $2A_8 + 2A_1$, $2A_8 + A_1$, $2A_8$, $A_8 + A_5 + A_3 + A_2$, $A_8 + A_5 + 2A_2 + A_1$, $A_8 + A_5 + 2A_2$, $A_8 + A_5 + A_2 + 2A_1$, $A_8 + A_5 + A_2 + A_1$, $A_8 + A_5 + A_2$, $A_8 + A_4 + 3A_2$, $A_8 + A_3 + 3A_2 + A_1$, $A_8 + A_3 + 3A_2$, $A_8 + 4A_2 + A_1$, $A_8 + 4A_2$, $A_8 + 3A_2 + 3A_1$, $A_8 + 3A_2 + 2A_1$, $A_8 + 3A_2 + A_1$, $A_8 + 3A_2$, $3A_5 + A_2$, $3A_5 + A_1$, $3A_5$, $2A_5 + A_4 + 2A_2$, $2A_5 + A_3 + 2A_2$, $2A_5 + 3A_2 + A_1$, $2A_5 + 3A_2$, $2A_5 + 2A_2 + 2A_1$, $2A_5 + 2A_2 + A_1$, $2A_5 + 2A_2$, $A_5 + A_4 + 4A_2$, $A_5 + A_3 + 4A_2 + A_1$, $A_5 + A_3 + 4A_2$, $A_5 + 5A_2 + A_1$, $A_5 + 5A_2$, $A_5 + 4A_2 + 3A_1$, $A_5 + 4A_2 + 2A_1$, $A_5 + 4A_2 + A_1$, $A_5 + 4A_2$, $A_4 + 6A_2$, $A_3 + 6A_2 + A_1$, $A_3 + 6A_2$, $7A_2 + A_1$, $7A_2$, $6A_2 + 3A_1$, $6A_2 + 2A_1$, $6A_2 + A_1$, $6A_2$

$G = \mathbb{Z}/(4)$
$D_7 + A_{11}$, $D_7 + A_7 + A_3 + A_1$, $D_7 + A_7 + A_3$, $D_7 + 3A_3 + A_2$, $D_7 + 3A_3 + A_1$, $D_7 + 3A_3$, $2D_5 + A_7 + A_1$, $2D_5 + A_7$, $2D_5 + 2A_3 + A_1$, $2D_5 + 2A_3$, $D_5 + A_{11} + 2A_1$, $D_5 + A_{11} + A_1$, $D_5 + A_7 + A_3 + A_2 + A_1$, $D_5 + A_7 + A_3 + 2A_1$, $D_5 + A_7 + A_3 + A_1$, $D_5 + 3A_3 + A_2 + A_1$, $D_5 + 3A_3 + 2A_1$, $D_5 + 3A_3 + A_1$, $A_{15} + A_3$, $A_{15} + 3A_1$, $A_{15} + 2A_1$, $A_{11} + 2A_3 + A_1$, $A_{11} + 2A_3$, $A_{11} + A_3 + A_2 + 2A_1$, $A_{11} + A_3 + 3A_1$, $A_{11} + A_3 + 2A_1$, $2A_7 + A_3$, $2A_7 + 3A_1$, $2A_7 + 2A_1$, $A_7 + 3A_3 + A_2$, $A_7 + 3A_3 + A_1$, $A_7 + 3A_3$, $A_7 + 2A_3 + A_2 + 2A_1$, $A_7 + 2A_3 + 3A_1$, $A_7 + 2A_3 + 2A_1$, $5A_3 + A_2$, $5A_3 + A_1$, $5A_3$, $4A_3 + A_2 + 2A_1$, $4A_3 + 3A_1$, $4A_3 + 2A_1$

$G = \mathbb{Z}/(5)$
$2A_9$, $A_9 + 2A_4 + A_1$, $A_9 + 2A_4$, $4A_4 + 2A_1$, $4A_4 + A_1$, $4A_4$

$G = \mathbb{Z}/(6)$
$A_{11} + A_5 + 2A_1$, $A_{11} + A_3 + 2A_2$, $A_{11} + 2A_2 + 3A_1$, $A_{11} + 2A_2 + 2A_1$, $3A_5 + A_3$, $3A_5 + 2A_1$, $2A_5 + A_3 + 2A_2 + A_1$, $2A_5 + A_3 + 2A_2$, $2A_5 + 2A_2 + 3A_1$, $2A_5 + 2A_2 + 2A_1$

$G = \mathbb{Z}/(7)$
$3A_6$

$G = \mathbb{Z}/(8)$
$2A_7 + A_3 + A_1$

$G = \mathbb{Z}/(2) \times \mathbb{Z}/(2)$
$D_{10} + A_5 + 3A_1$, $D_{10} + 2A_3 + 2A_1$, $D_{10} + A_3 + 4A_1$, $D_{10} + 6A_1$, $2D_8 + 2A_1$, $D_8 + D_6 + A_3 + A_1$, $D_8 + D_6 + 3A_1$, $D_8 + D_4 + A_3 + 2A_1$, $D_8 + D_4 + 4A_1$, $D_8 + A_5 + A_3 + 2A_1$, $D_8 + A_5 + 4A_1$, $D_8 + 2A_3 + 3A_1$, $D_8 + A_3 + 5A_1$, $D_8 + 7A_1$, $3D_6$, $2D_6 + D_4 + A_1$, $2D_6 + 2A_3$, $2D_6 + A_3 + 2A_1$, $2D_6 + 4A_1$, $D_6 + 2D_4 + 2A_1$, $D_6 + D_4 + 2A_3 + A_1$, $D_6 + D_4 + A_3 + 3A_1$, $D_6 + D_4 + 5A_1$, $D_6 + 2A_5 + 2A_1$, $D_6 + A_5 + 2A_3 + A_1$, $D_6 + A_5 + A_3 + 3A_1$, $D_6 + A_5 + 5A_1$, $D_6 + 3A_3 + 2A_1$, $D_6 + 2A_3 + 4A_1$, $D_6 + A_3 + 6A_1$, $D_6 + 8A_1$, $4D_4$, $3D_4 + 3A_1$, $2D_4 + 2A_3 + 2A_1$, $2D_4 + A_3 + 4A_1$, $2D_4 + 6A_1$, $D_4 + 2A_5 + 3A_1$, $D_4 + A_5 + 2A_3 + 2A_1$, $D_4 + A_5 + A_3 + 4A_1$, $D_4 + A_5 + 6A_1$, $D_4 + 3A_3 + 3A_1$, $D_4 + 2A_3 + 5A_1$, $D_4 + A_3 + 7A_1$, $D_4 + 9A_1$, $A_7 + A_5 + A_3 + 3A_1$, $A_7 + A_5 + 5A_1$, $A_7 + 2A_3 + 4A_1$, $A_7 + A_3 + 6A_1$, $A_7 + 8A_1$, $2A_5 + 2A_3 + 2A_1$, $2A_5 + A_3 + 4A_1$, $2A_5 + 6A_1$, $A_5 + 3A_3 + 3A_1$, $A_5 + 2A_3 + 5A_1$, $A_5 + A_3 + 7A_1$, $A_5 + 9A_1$, $4A_3 + 4A_1$, $3A_3 + 6A_1$, $2A_3 + 8A_1$, $A_3 + 10A_1$, $12A_1$

$G = \mathbb{Z}/(4) \times \mathbb{Z}/(2)$
$2A_7 + 4A_1$, $A_7 + 3A_3 + 2A_1$, $A_7 + 2A_3 + 4A_1$, $5A_3 + 2A_1$, $4A_3 + 4A_1$

$G = \mathbb{Z}/(6) \times \mathbb{Z}/(2)$
$3A_5 + 3A_1$

$G = \mathbb{Z}/(3) \times \mathbb{Z}/(3)$
$2A_5 + 4A_2$, $A_5 + 6A_2$, $8A_2$

$G = \mathbb{Z}/(4) \times \mathbb{Z}/(4)$
$6A_3$

Department of Mathematics, Faculty of Science, Hokkaido University, Sapporo 060-0810, JAPAN
E-mail address: shimada@math.sci.hokudai.ac.jp