Embedding Nonsmooth Systems in Digital Controllers

Ryo Kikuuwe
Nonsmooth Systems?

- Systems subject to discontinuous differential equations. A good example is ...
  - Coulomb friction

\[
M \ddot{v} = \begin{cases} 
-\frac{Fv}{|v|} & \text{if } v \neq 0 \\
\in [-F, F] & \text{if } v = 0
\end{cases}
\]

- Mathematically very cumbersome.
- but very familiar in our daily lives.
- and useful for some control applications
Today's Talk

1. about natural nonsmooth systems

2. about artificial nonsmooth systems
Simulation of Natural Nonsmooth Systems: Systems Subject to Coulomb Friction
Motivation: Request from Automobile Industry

- Robotic assistance for manual fine positioning in assembling.
- Problem: What kind of resistive forces are appropriate for enhancing the performance?
Haptic Feedback

- To realize a prescribed position-force relation at the interface between a device and an operator contacting therewith.

- Applications:
  - Robotic assistance of manual tasks
  - Virtual reality (involving haptics)
    - surgery simulators, computer games, CAD interfaces, ...
Idea: “Coulomb Friction”

- It cancels external forces below static friction level. (Thus, effective for fine positioning.)
- Most familiar dynamics in our daily lives.

\[ f = F \text{sgn}(v) \]

- But producing such force-velocity relation by digital control is very challenging!!
Why Is Coulomb Friction Problematic?

- In discrete time, the force is determined according to the relative velocity.
- Due to the latency, chattering (repeated zero-velocity crossings) occurs.

Idea: put a delayless feedback in the loop to remove chattering.
Delayless Feedback Including Signum?

\[ y = \text{sgn}(x - y) \]

where \( \text{sgn}(x) = \begin{cases} 
1 & \text{if } x > 0 \\
[-1, 1] & \text{if } x = 0 \\
-1 & \text{if } x < 0 
\end{cases} \)

\[ \iff (y = 1 \land x - y > 0) \]
\[ \lor (-1 < 0 < 1 \land x - y = 0) \]
\[ \lor (y = -1 \land x - y < 0) \]

\[ \iff y = \text{sat}(x) \]

where \( \text{sat}(x) = \begin{cases} 
1 & \text{if } x > 1 \\
x & \text{if } x \in [-1, 1] \\
-1 & \text{if } x < -1 
\end{cases} \)

(unit saturation function)
Solution

- Rigid, discontinuous friction between objects: difficult to simulate.

Put a delayless feedback as a virtual viscoelastic element.
Derivation of Simulation Algorithm

- **step 1**: continuous-time representation
  - differential algebraic equations
    \[ f = Ke + Be \dot{e} \]
    \[ f = Fs \text{sgn}(u - \dot{e}) \]

- **step 2**: discrete-time representation
  - backward Euler; algebraic equations
    \[ f_k = Ke_k + B(e_k - e_{k-1})/T \]
    \[ f_k = Fs \text{sgn}(u_k - (e_k - e_{k-1})/T) \]

- **step 3**: Algorithm (discrete-time)
  - solution to the above
    \[ f_k := Fs \text{sat}(((KT + B)u_k + Ke_{k-1})/F) \]
    \[ e_k := (Be_{k-1} + Tf_k)/(KT + B) \]
Applications

friction force on the needle shaft
Implementation of Artificial Nonsmooth Systems: “Proxy-based Sliding Mode Control”
Motivation

- **Coulomb Friction**

  \[ f = \text{sgn}(v) \]

- **What happens?**

  \[ f = \text{sgn}(p + Hv) \]

- Physical characteristic that does not exist in nature but can be produced by digital control

  Sliding Mode Control?
Sliding Mode Control

- A “nonsmooth” control scheme.
  - The actuator force is “discontinuously” switched on a “sliding surface” in the state space.

Simplest Example:

\[ f = F \text{sgn}(p_d - p - H\dot{p}) \]

- \( F \): upper bound of force
- \( H \): time constant
- exponentially converges to the target position with time constant \( H \)

- In discrete time, however, there is chattering due to the discontinuity!!
**Proxy-based Sliding Mode Control**

- **step 1:** continuous-time representation
  - differential algebraic equations
    
    $$ f = Ke + B\dot{e} + L \int e dt $$
    
    $$ f = F \text{sgn}(p_d - q + H(\dot{p}_d - \dot{q})) $$

- **step 2:** discrete-time representation
  - algebraic equations
    
    $$ f_k = La_k + K \nabla a_k / T + B \nabla^2 a_k / T^2 $$
    
    $$ f_k = F \text{sgn}(p_{d,k} - q_k + H \nabla (p_{d,k} - q_k) / T) $$

- **step 3:** Discrete-time representation
  - solution to the above
    
    $$ \sigma_k := p_{d,k} - p_k + H(\nabla p_{d,k} / T - \nabla p_k / T) $$
    
    $$ f_k^* := \frac{B + KT + LT^2}{H + T} \sigma_k + \frac{KH - B + LT(2H + T)}{(H + T)T} a_{k-1} - \frac{KH - B + LTH}{(H + T)T} a_k $$
    
    $$ f_k := F \text{sat}(f_k^* / F) $$
    
    $$ a_k := \frac{(2B + KT)a_{k-1} - Ba_{k-2} + T^2 f_k}{B + KT + LT^2}. $$
Benefit of PSMC

- **PSMC is as accurate as but much safer than PID!!**
  - Before saturation, equivalent to PID control.
    - fast response to small positional error
  - After saturation, SMC-like response is dominant
    - moderate recovery from large positional error without overshoots.

![Diagram of control system concepts](image)
PID-C vs PSMC

- Response to Step Input
- Response to Large Disturbance

- PSMC is as accurate as but safer than PID
Proxy-Based Sliding Mode Control of a Manipulator Actuated by Pleated Pneumatic Artificial Muscles

M. Van Damme, B. Vanderborght, R. Van Ham, B. Verrelst, F. Daeleen, D. Lefeber
Robotics and Multibody Mechanics Research Group
Department of Mechanical Engineering
Vrije Universiteit Brussel
Pleinlaan 2, 1050 Brussel
michael.vandamme@vub.ac.be

Abstract—Recently, Kikuuwe and Fujimoto have introduced Proxy-Based Sliding Mode Control. It combines responsive and accurate tracking during normal operation with smooth, slow recovery from large position errors that can sometimes occur after abnormal events. The method can be seen as an extension to both conventional PID control and sliding mode control.

In this paper, Proxy-Based Sliding Mode Control is used to control a 2-DOF planar manipulator actuated by Pleated Pneumatic Artificial Muscles (PPAMs). The principal advantage of this control method is increased safety for people interacting with the manipulator.

Two different forms of Proxy-Based Sliding Mode Control were implemented on the system, and their performance was experimentally evaluated. Both forms performed very well with respect to safety. Good tracking was also obtained, especially for the manipulator.

Fig. 1. The manipulator. The series arrangement of Pleated Pneumatic Artificial Muscles is clearly visible.
Conclusions

- Introduced an algorithm for producing Coulomb friction characteristics in discrete-time controller.
- Introduced a new position control scheme, "Proxy-based sliding mode control."

Future (Current Ongoing) Projects

- Utilization of friction algorithms
  - Driving simulator with realistic steering resistance.
  - Low-cost robot equipped with standardized gear transmissions.