KYUSHU UNIVERSITY 2011
100th Anniversary

Embedding Nonsmooth Systems in Digital Controllers

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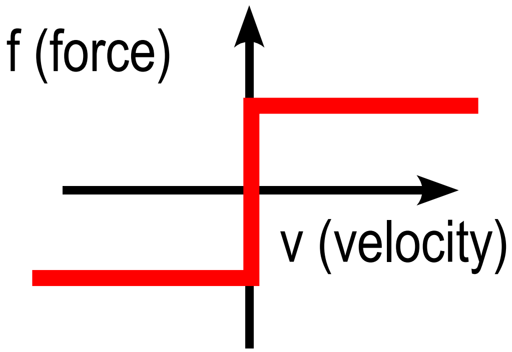
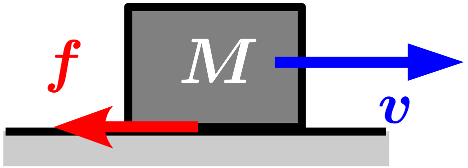
Nonsmooth Systems?

- Systems subject to discontinuous differential equations. A good example is ...
  - Coulomb friction





$$M\dot{v}\begin{cases} = -Fv/|v| & \text{if } v \neq 0 \\ \in [-F, F] & \text{if } v = 0 \end{cases}$$

- Mathematically very cumbersome.
- but very familiar in our daily lives.
- and useful for some control applications

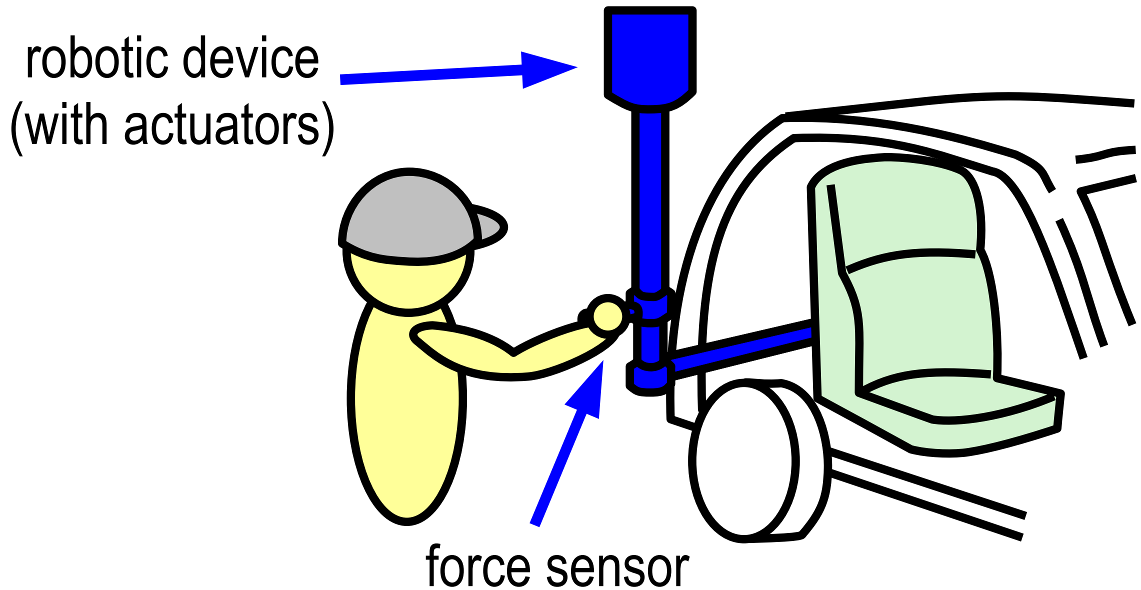
Today's Talk

1. about **natural** nonsmooth systems

2. about **artificial** nonsmooth systems

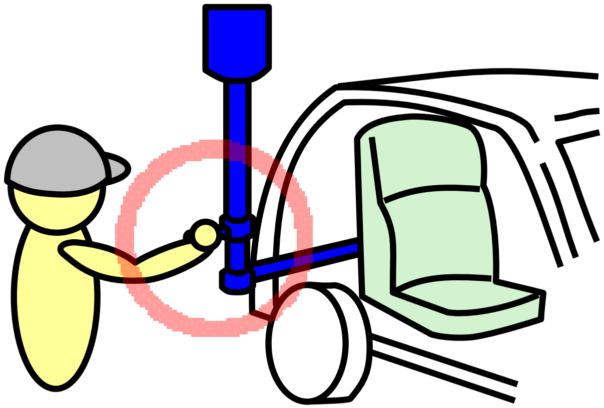
Simulation of Natural Nonsmooth Systems: Systems Subject to Coulomb Friction

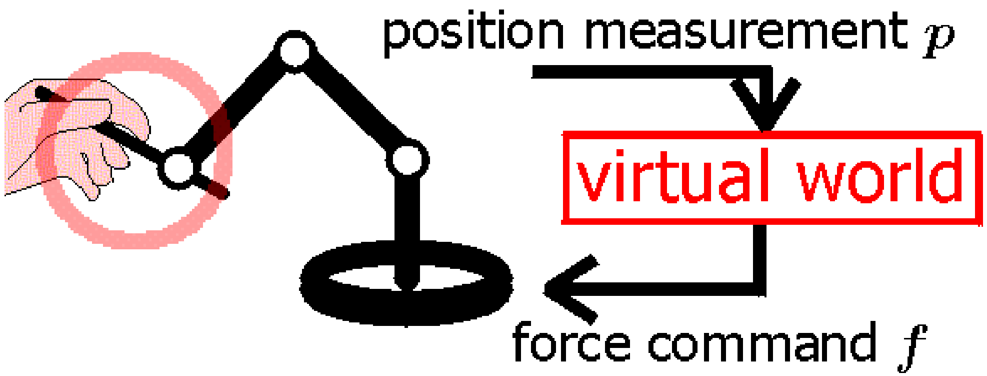
Motivation: Request from Automobile Industry



- Robotic assistance for manual fine positioning in assembling.
- Problem: What kind of resistive forces are appropriate for enhancing the performance ?

Haptic Feedback

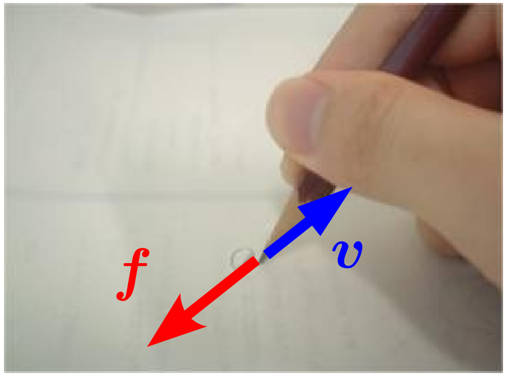


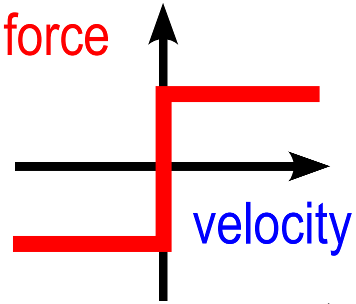


- To realize a prescribed position-force relation at the interface between a device and an operator contacting therewith.
- Applications:
  - Robotic assistance of manual tasks
  - Virtual reality (involving haptics)
    - surgery simulators, computer games, CAD interfaces, ...

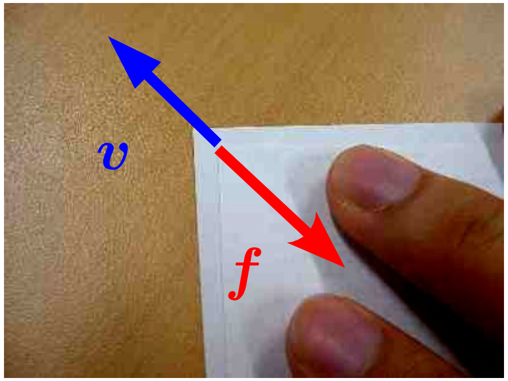
Idea: "Coulomb Friction"

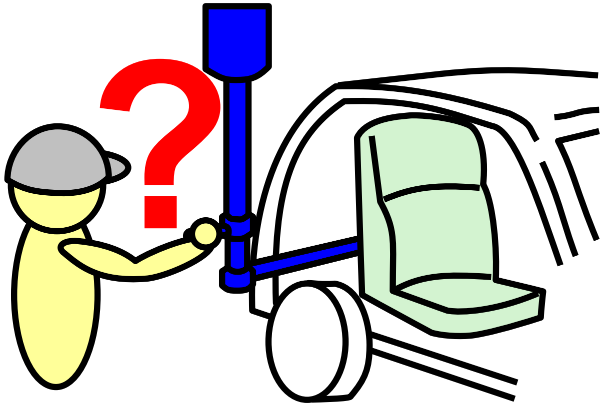
- It cancels external forces below static friction level. (Thus, effective for fine positioning.)
- Most familiar dynamics in our daily lives.





$$f = F\mathrm{sgn}(v)$$

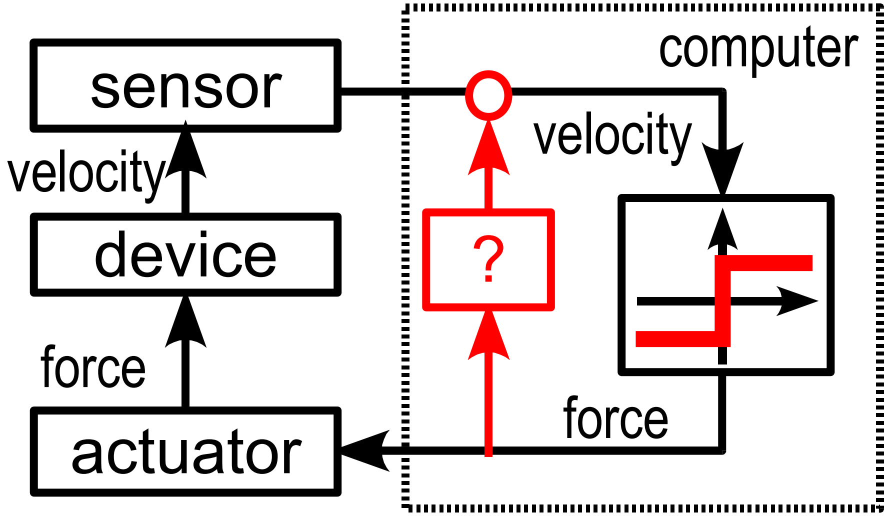


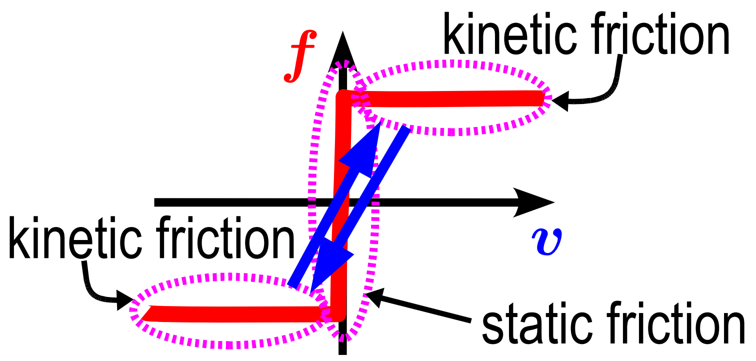


- But producing such force-velocity relation by digital control is very challenging!!

Why Is Coulomb Friction Problematic?

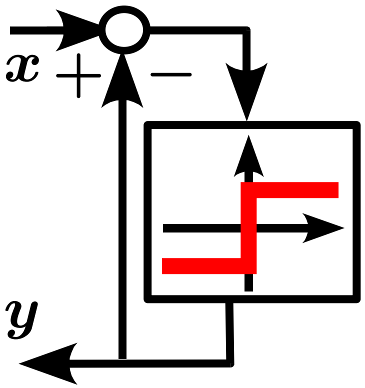
- In discrete time, the force is determined according to the relative velocity.
- Due to the latency, chattering (repeated zero-velocity crossings) occurs.





- Idea: put a delayless feedback in the loop to remove chattering.

Delayless Feedback Including Signum?

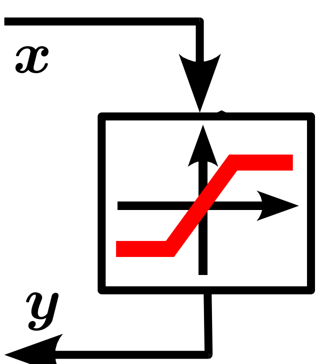


$$y = \mathrm{sgn}(x - y)$$

$$\text{where } \mathrm{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ [-1, 1] & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$$\iff (y = 1 \wedge x - y > 0)$$
$$\vee(-1 < 0 < 1 \wedge x - y = 0)$$
$$\vee(y = -1 \wedge x - y < 0)$$

equivalent



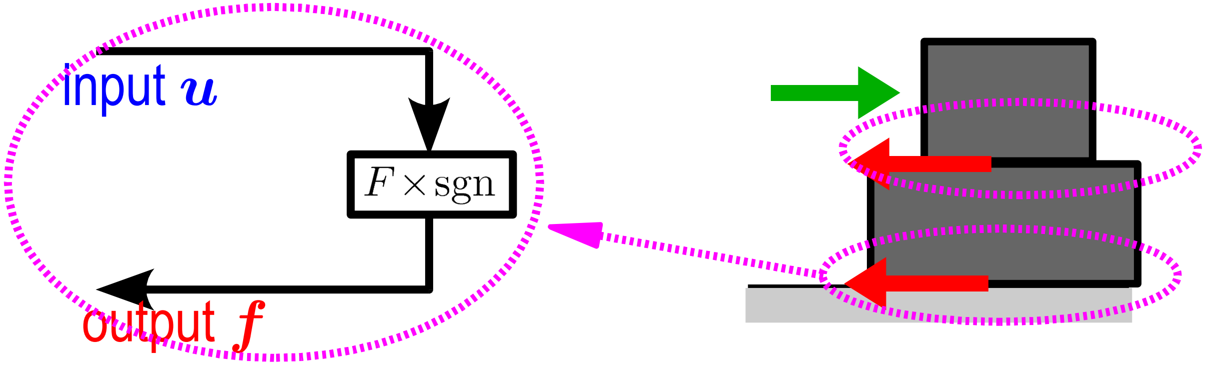
$$\iff y = \mathrm{sat}(x)$$

$$\text{where } \mathrm{sat}(x) = \begin{cases} 1 & \text{if } x > 1 \\ x & \text{if } x \in [-1, 1] \\ -1 & \text{if } x < -1 \end{cases}$$

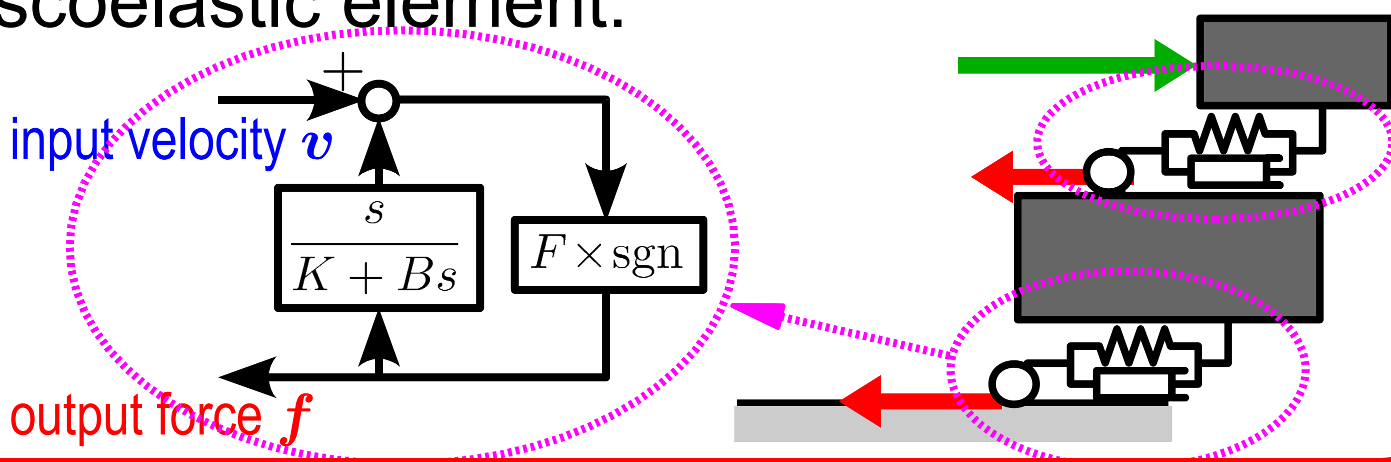
(unit saturation function)

Solution

- Rigid, discontinuous friction between objects: difficult to simulate.

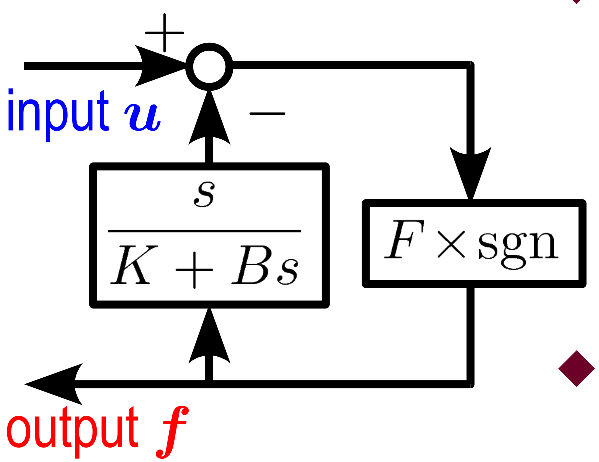


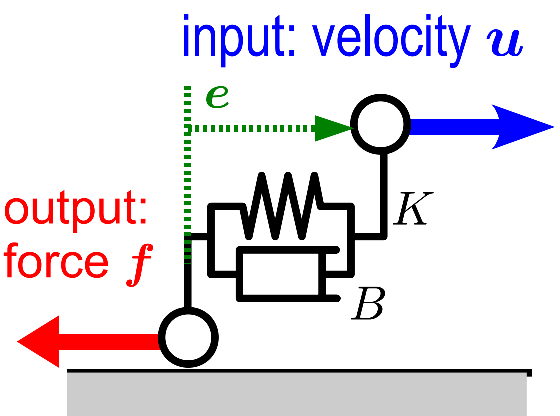
- Put a delayless feedback as a virtual viscoelastic element.



[Kikuuwe et al.: IEEE Trans. on Robotics, 2006]

Derivation of Simulation Algorithm





- step 1: continuous-time representation
  - differential algebraic equations

$$f = Ke + B\dot{e}$$
$$f = F\mathrm{sgn}(u - \dot{e})$$

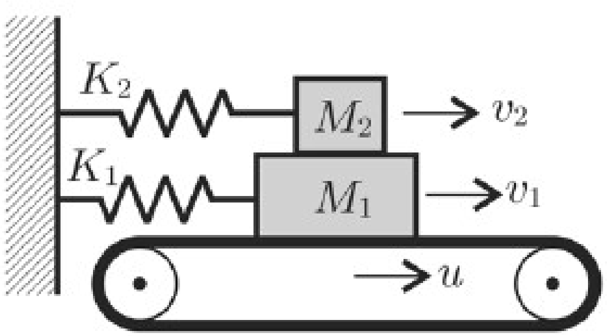
- step 2: discrete-time representation
  - backward Euler; algebraic equations

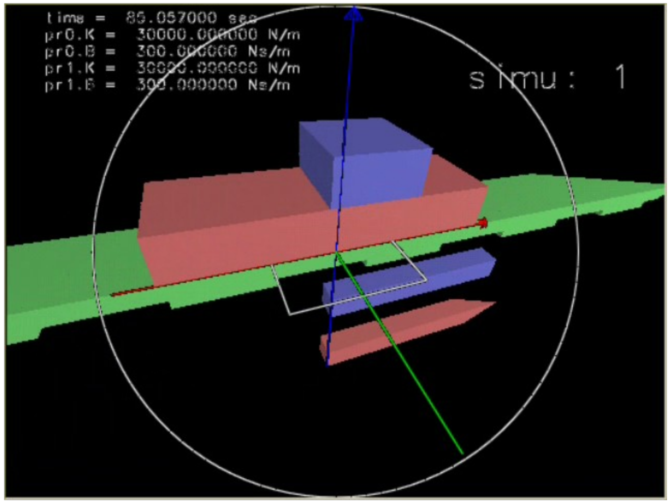
$$f_k = Ke_k + B(e_k - e_{k-1})/T$$
$$f_k = F\mathrm{sgn}(u_k - (e_k - e_{k-1})/T)$$

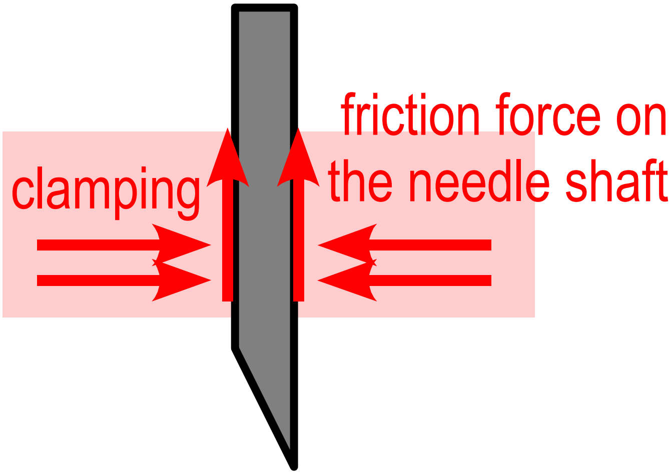
- step 3: Algorithm (discrete-time)
  - solution to the above

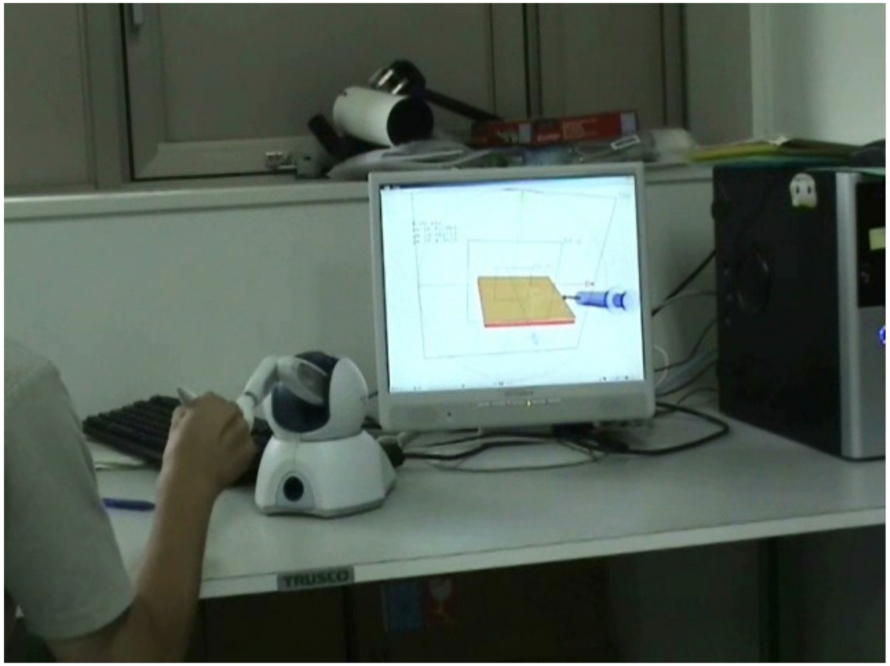
$$f_k := F\mathrm{sat}(((KT + B)u_k + Ke_{k-1})/F)$$
$$e_k := (Be_{k-1} + Tf_k)/(KT + B)$$

Applications





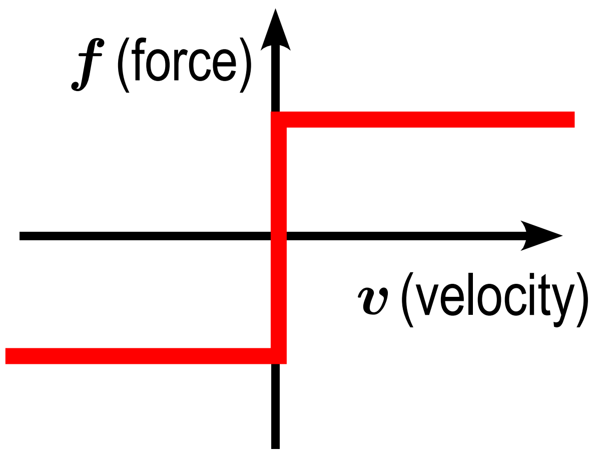




Implementation of Artificial Nonsmooth Systems: "Proxy-based Sliding Mode Control"

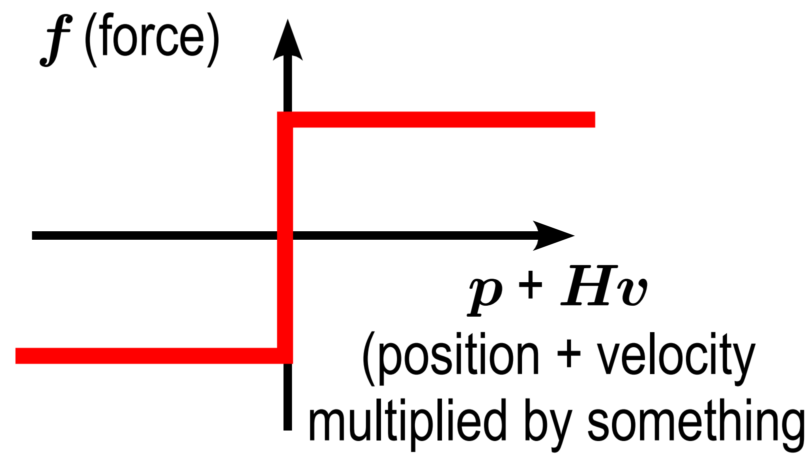
Motivation

- Coulomb Friction



$$f = \mathrm{sgn}(v)$$

- What happens?



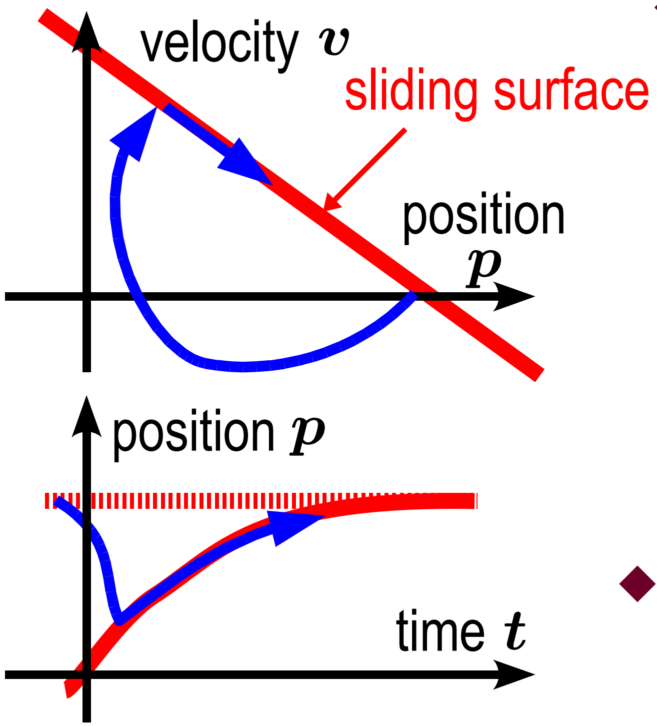
$$f = \mathrm{sgn}(p + Hv)$$

◆ Physical characteristic that does not exist in nature but can be produced by digital control

Sliding Mode Control ?

Sliding Mode Control

- A "*nonsmooth*" control scheme.
  - The actuator force is "discontinuously" switched on a "sliding surface" in the state space.



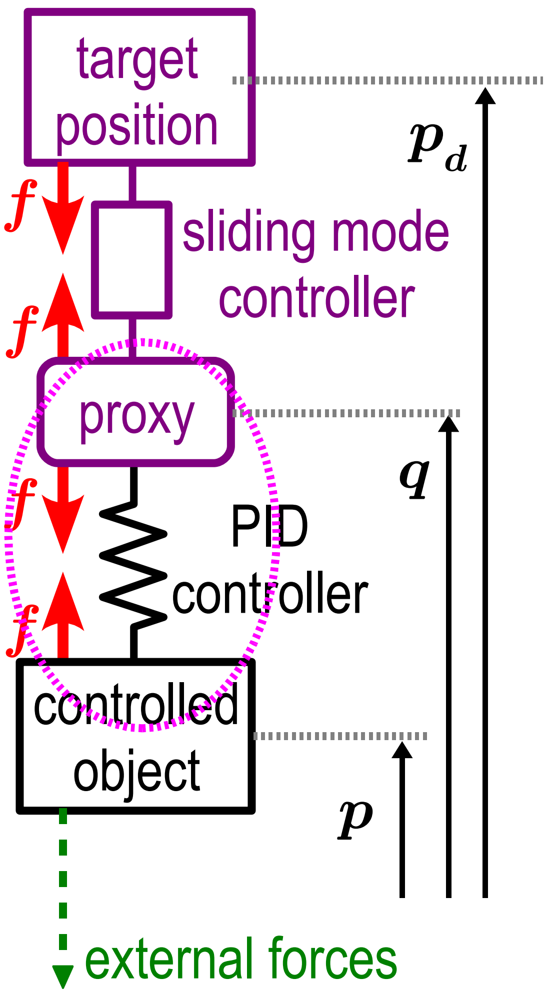
- Simplest Example:

$$f = F\operatorname{sgn}(p_d - p - H\dot{p})$$

  - F : upper bound of force
  - H : time constant
  - exponentially converges to the target position with time constant H
- In discrete time, however, there is chattering due to the discontinuity!!

[Kikuuwe & Fujimoto, ICRA2006]

“Proxy-based Sliding Mode Control”



- step 1: continuous-time representation
  - differential algebraic equations

$$f = Ke + B\dot{e} + L\int e dt \qquad (e = q - p)$$

$$f = F\mathrm{sgn}(p_d - q + H(\dot{p}_d - \dot{q}))$$

- step 2: discrete-time representation
  - algebraic equations $\left(a = \int e dt\right)$

$$f_k = La_k + K\nabla a_k/T + B\nabla^2 a_k/T^2$$

$$f_k = F\mathrm{sgn}(p_{d,k} - q_k + H\nabla(p_{d,k} - q_k)/T)$$

- step 3: Discrete-time representation
  - solution to the above

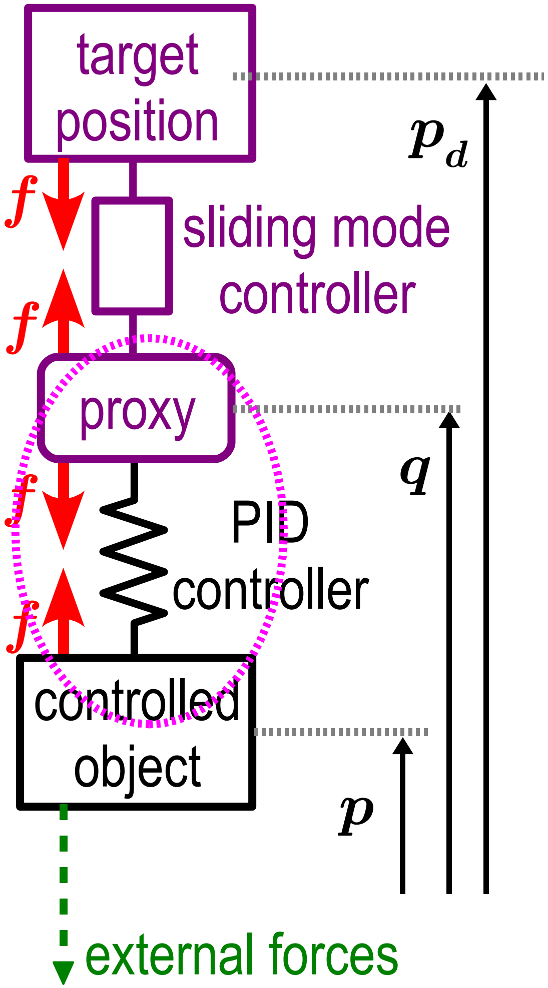
$$\sigma_k := p_{d,k} - p_k + H(\nabla p_{d,k}/T - \nabla p_k/T)$$

$$f_k^* := \frac{B+KT+LT^2}{H+T}\sigma_k + \frac{KH-B+LT(2H+T)}{(H+T)T}a_{k-1} - \frac{KH-B+LTH}{(H+T)T}a_{k-}$$

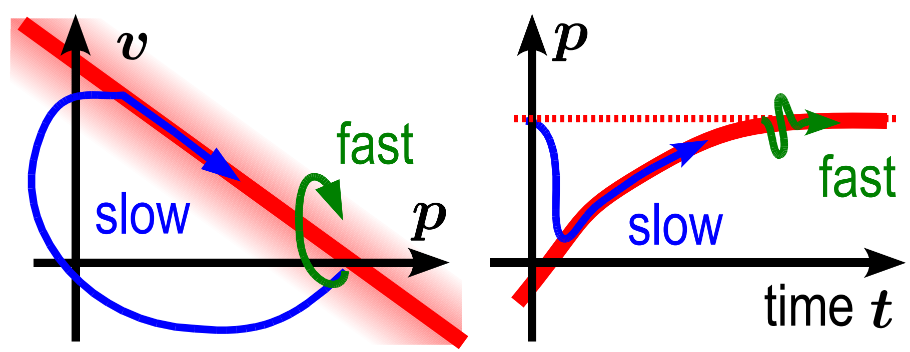
$$f_k := F\mathrm{sat}(f_k^*/F)$$

$$a_k := \frac{(2B+KT)a_{k-1} - Ba_{k-2} + T^2 f_k}{B+KT+LT^2}.$$

Benefit of PSMC



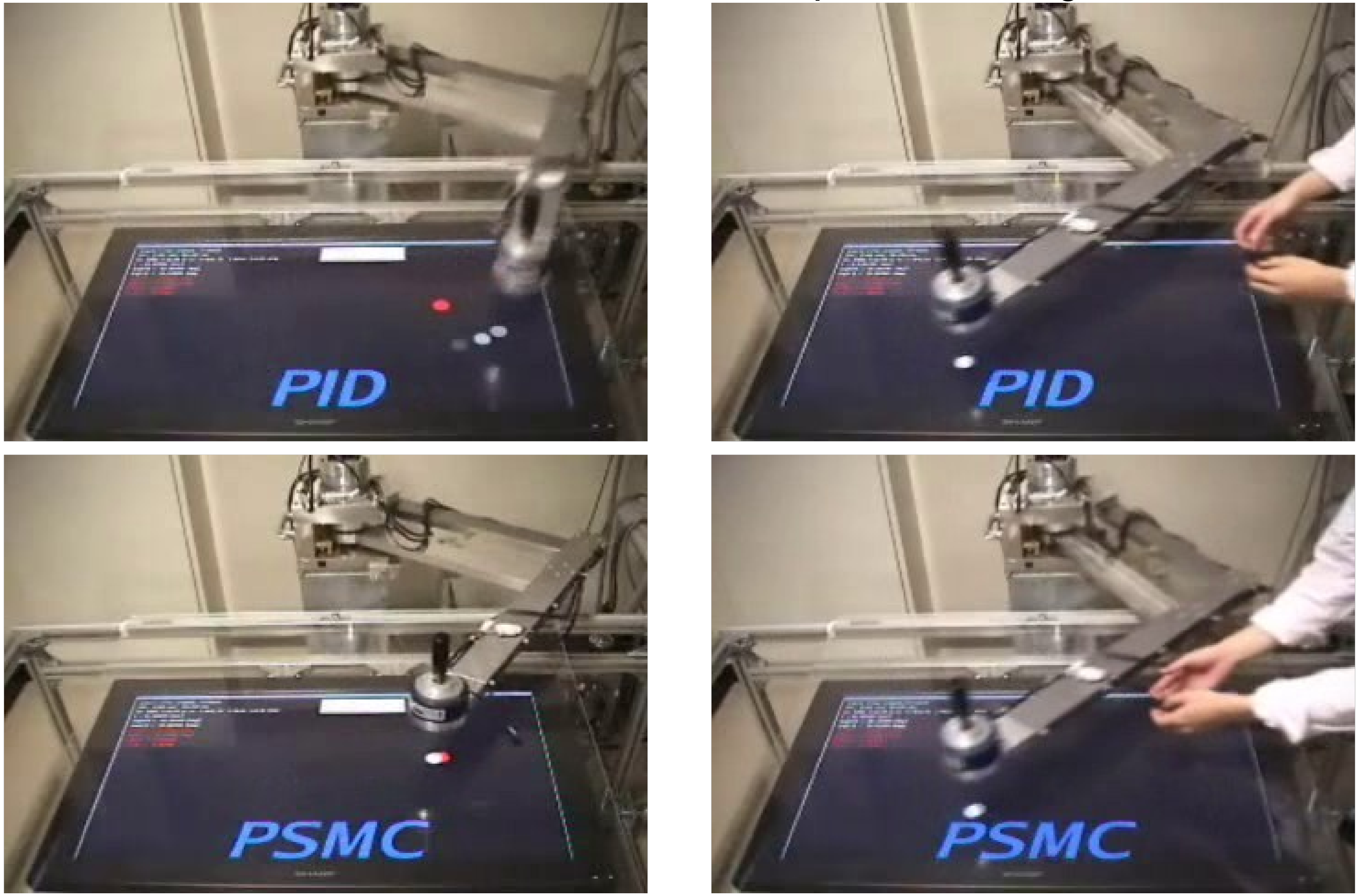
- PSMC is as accurate as but much safer than PID!!
  - Before saturation, equivalent to PID control.
    - fast response to small positional error
  - After saturation, SMC-like response is dominant
    - moderate recovery from large positional error without overshoots.



PID-C vs PSMC

Response to Step Input

Response to Large Disturbance



- PSMC is as accurate as but safer than PID

Voice of Users

[Van Damme et al., 2007]
@ Vrije Univ. Brussel

2007 IEEE International Conference on
Robotics and Automation
Roma, Italy, 10-14 April 2007

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Proxy-Based Sliding Mode Control of a Manipulator Actuated by Pleated Pneumatic Artificial Muscles

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Abstract— Recently, Kikuuwe and Fujimoto have introduced Proxy-Based Sliding Mode Control. It combines responsive and accurate tracking during normal operation with smooth, slow recovery from large position errors that can sometimes occur after abnormal events. The method can be seen as an extension to both conventional PID control and sliding mode control.

In this paper, Proxy-Based Sliding Mode Control is used to control a 2-DOF planar manipulator actuated by Pleated Pneumatic Artificial Muscles (PPAMs). The principal advantage of this control method is increased safety for people interacting with the manipulator.

Two different forms of Proxy-Based Sliding Mode Control were implemented on the system, and their performance was experimentally evaluated. Both forms performed very well with respect to safety. Good tracking was also obtained, especially

Fig. 1. The manipulator. The series arrangement of Pleated Pneumatic Artificial Muscles is clearly visible.

Conclusions

- Introduced an algorithm for producing Coulomb friction characteristics in discrete-time controller.
- Introduced a new position control scheme, "Proxy-based sliding mode control."

Future (Current Ongoing) Projects

- Utilization of friction algorithms
  - Driving simulator with realistic steering resistance.
- Extension of PSMC for safer, human-friendly robots.
  - Low-cost robot equipped with standardized gear transmissions.