Chapter 2

Developed Program for Phase-Shift Analysis of Nucleon-Nucleon Scattering : PANN

Abstract. PANN is a partial-wave analysis program which can be used to determine the complex scattering amplitudes of elastic nucleon-nucleon scattering by fitting the experimental data on many kinds of observables. It is able to apply for an analysis in the wide energy region such as a few MeV $\sim$ 10 GeV. In PANN, the higher patial-wave amplitudes for the analysis in the low energy region ($<$ 1 GeV) are given by the one-pion exchange amplitude, and the ones in the high-energy region of several GeV are evaluated by the modified one-boson exchange amplitudes which is equivalent to Veneziano amplitudes. (Comp. Phys. Comm. **131** (2000), 225.)

2.1 Introduction

Phase-shift analysis (PSA) has constantly supplied a motivation for particle-scattering experiments and played an important role in the determination of hadron scattering amplitudes. Especially for resonance detection this method has been fruitful. Recently it has contributed to the study of dibaryon resonance[13, 23, 24, 25]. Hereafter we may expect these analyses to motivate spin-polarized experiments and be continuously in the forefront of research concerning the strong interaction. There will certainly be some problems to be solved involving PSA.

With PSA one can satisfy the unitarity condition for scattering amplitudes automatically and take account for spin effects without any assumptions more easily than in total amplitude analyses. On the other hand, some faults exist. The main fault is the difficulty that arises from the ever-increasing number of participating partial waves as the energy of the incident particle increases. This causes a doubt on its effectiveness at high energies. Nevertheless, the utility of the PSA of N-N scattering has been demonstrated by some groups in the region of a few GeV/c [23, 26, 27, 28]. Some ways for dealing with this difficulty have been proposed. They are, for example, the phase-band method suggested by Moravcsik[14] and the accelerated-convergence-expansion method proposed by Cutkosky[15]. As is well known, Cutkosky's method has had success for PSAs of π-N scattering at a few tens of GeV. Such success by some new methods is expected also in N-N scattering. The primitive PANN was developed by Matsuda and Watari[29] for the PSA of N-N scattering in the low energy region. Our expanded PANN described herein is proposed for a treatment of the above-mentioned fault and for a possible extension of the usual PSA to the PSA of N-N scattering in the high energy region. Our method of extension might be said to originate in the methodology proposed by Iwadare et al.[2] in the research of verification of the pion theory of the nuclear force. The program PANN has been applied to the analyses of Argonne data at 6 GeV/c[16, 17, 19] and 12 GeV/c[18], where the modified one-boson exchange (OBE) amplitudes have been included by the K-matrix method. In the program described herein, the S-matrix of the lower partial waves is either the one defined by Matsuda and Watari (MW-representation)[29] or the one by Arndt and Roper (VPI-representation)[30]. The user can choose either of those S-matrix representations. The relativistic Coulomb-amplitude is added according to sub-programs developed by Arndt.

In the next section, a brief formalism used in PANN program is described. In Section 2.3 some features of PANN are given and the structure of program is explained in Section 2.4. We give the input data format in Section 2.5. Our application plan of PANN is given in Section 2.6. Various expressions of observables are given in 2.7 Finally, we give an additional comment in Section 2.8.

2.2 Phase-shift analysis of nucleon-nucleon scattering

In the usual PSAs of N-N scattering at low energies, the peripheral part of the scattering amplitude is provided by the one-pion exchange (OPE) amplitude. In the analyses of N-N scattering at intermediate energies, one might consider using the one-boson exchange amplitudes instead of the OPE-amplitude according to Iwadare's way. Iwadare et al.[2] divided the inter-nucleon distance into three regions and estimated the reliability of the nuclear interaction in each region. And they have shown the validity of the OPE-potential in the outer region, which has since been the general view-point of the strong interaction. OBE-contributions are used in the intermediate region. OBE-amplitudes, however, have a t-dependence considerably different from that in the phenomena at high energies, which is the reason why OBE-amplitudes have not been used for a modified phase shift analysis. This difference of t-dependence is thought to be due to form-factor-like effects and/or Regge-pole type behavior. In the PANN program described herein, the modified OBE-amplitudes are used instead of the OPE-amplitude. They have been introduced by Kawasaki et al.[31] in the phenomenology of p-p scattering at high energies as the ones equivalent to Veneziano amplitudes. The OBE-amplitude for N-N scattering is modified as follows:

$$\frac{g^2}{m^2 - t} \to \frac{g^2}{m^2}\left(\frac{\Lambda^2}{\Lambda^2 - t/n}\right)^n, \tag{2.1}$$

where t is the squared momentum transfer and m the observed mass of the exchanged boson. g, Λ and n are the parameters for a particular boson, which are searched with the phase shifts

(δ, η) of low partial waves to determine the best fit to the experimental data (n is constrained as $10 \geq n \geq 1$ in the search). For the partial wave amplitude, this modification implies the replacement of the Legendre function of the second kind (Q_J):

$$Q_J(x_0) \rightarrow \frac{n\Lambda^2}{m^2}(x_0' - 1)^n \frac{(-1)^{n-1}}{(n-1)!} Q_J^{(n-1)}(x_0'), \tag{2.2}$$

$$x_0 = 1 + \frac{m^2}{2p^2}, \tag{2.3}$$

$$x_0' = 1 + \frac{n\Lambda^2}{2p^2}, \tag{2.4}$$

where $Q_J^{(n)}$ is the nth derivative of Q_J and p the c.m.s. momentum. This modification makes it possible to do the usual field-theoretical calculation of the N-N scattering-amplitude with the multi-pole propagator. We calculate the modified OBE-amplitudes with the well-known exchanged mesons: π, σ, ρ and ω[6].

The scattering amplitude is given by the partial wave amplitude and the modified OBE-amplitude as follows:

$$M = \sum_{\ell \leq \ell_m} f_\ell(\delta_\ell, \eta_\ell) + \sum_{\ell_m < \ell \leq \ell_x} f_\ell(\delta_\ell(\mathrm{OBEC}), \eta_\ell) + M_{\mathrm{OBEC}}(\ell > \ell_x). \tag{2.5}$$

Here $f_\ell(\delta_\ell, \eta_\ell)$ is the contribution from the real and the imaginary phase shifts (δ_ℓ, η_ℓ) for the orbital angular momentum ℓ. $\delta_\ell(\mathrm{OBEC})$ are the real phase shifts obtained from the modified OBE-amplitudes by the K-matrix method.

2.3 Features of Program

The features of PANN are as follows:

2.3.1 Coulomb amplitudes

The non-relativistic Coulomb amplitudes are usually used for PSA. The subroutine for relativistic Coulomb amplitudes are included in PANN which was calculated by Lechanoine et al.[32] based on the one-photon exchange interaction between nucleons, and developed by Arndt. It is needed for the extension of PSA to relativistic region.

2.3.2 New kinds of observables

Recent polarized-spin experiments of nucleon-nucleon scattering include the mixed spin observables of several usual ones. For instance, the experiments done by SATURNE II provide the following observables[33]. Here we call them as follows:

$$SAT1 = A_{00kk} + \delta A_{00sk} = A_{LL} + \delta A_{SL} \tag{2.6}$$

$$SAT2 = A_{00n0} + \delta A_{00sk} = P + \delta A_{SL} \tag{2.7}$$

$$SAT3 = K_{0sk0} + \alpha K_{0ss0} \beta K_{0kk0} = K_{LS} + \alpha K_{SS} + \beta K_{0kk0} \tag{2.8}$$

$$SAT4 = N_{0nkk} + \beta K_{0kk0} = H_{LLN} + \beta K_{LL} \tag{2.9}$$

$$SAT5 = N_{0snk} + \alpha K_{0ss0} = H_{NLS} + \alpha K_{SS} \tag{2.10}$$

$$\begin{aligned} SAT6 &= N_{0skn} + \alpha N_{0ssn} + \beta N_{0kkn} \\ &= H_{LNS} + \alpha H_{SNS} + \beta K_{LL} \end{aligned} \tag{2.11}$$

$$\begin{aligned} SAT7 &= K_{0nn0} + \alpha N_{0nsk} + \beta K_{0ks0} + \gamma N_{0knk} \\ &= H_{LNS} + \alpha H_{SLN} + \beta K_{SL} + \gamma H_{NLL} \end{aligned} \tag{2.12}$$

where α, β, γ and δ are the coefficients depending on the energy. The observables A_{ij} , K_{ij} and H_{ijk} are defined in Section 2.7. The definition of $A_{ijk\ell}$, $K_{ijk\ell}$ and $N_{ijk\ell}$ should be referred in Ref. [33]. The subroutine PRECES treats these mixed spin observables correctly.

2.3.3 Higher partial waves

In order to make it possible to perform the PSA in high energy region, the modified OBE amplitudes are used in PANN instead of the one-pion exchange amplitudes which are included in PSAs in low energy region. We can decrease drastically the number of free-searched parameters in PANN by using modified OBE amplitudes.

2.3.4 Representation of Scattering Amplitudes

Currently there are two ways to parameterize the scattering amplitudes of nucleon-nucleon scattering as follows:

MW-representation

The S-matrix with an orbital angular momentum ℓ and a total angular momentum J in the case $\ell = J$ is given by

$$S_{\ell,J} = \eta_{\ell,J} \exp(2i\delta_{\ell,J}), \tag{2.13}$$

where $\eta_{\ell,J}$ and $\delta_{\ell,J}$ are the reflection parameters and the phase shifts with the orbital angular momentum ℓ and the total angular momentum J.

For coupled state between $\ell = J \pm 1$ waves,

$$S_J = \begin{pmatrix} \sqrt{1-|\rho|^2}\eta_- e^{2i\delta_-} & i\rho\sqrt{\eta_-\eta_+}e^{i(\delta_-+\delta_+)} \\ i\rho\sqrt{\eta_-\eta_+}e^{i(\delta_-+\delta_+)} & \sqrt{1-|\rho|^2}\eta_+ e^{2i\delta_+} \end{pmatrix}. \tag{2.14}$$

Here $\delta_+ = \delta_{J+1,J}, \delta_- = \delta_{J-1,J}$, and $\eta_{+(-)}$ are the reflection parameters with $\ell = J+1(J-1)$, respectively. ρ is the mixing parameter.

VPI-representation

The partial wave S-matrix is given by

$$S = \frac{1+iK}{1-iK}, \tag{2.15}$$

for the singlet state and the uncoupled triplet state. K is represented as

$$K = \tan\delta + i\tan^2\rho. \tag{2.16}$$

For the coupled triplet states, the K-matrix is taken as

$$K = \begin{bmatrix} K_{r-} & K_{r0} \\ K_{r0} & K_{r+} \end{bmatrix} + i\begin{bmatrix} \tan^2\rho_- & \tan\rho_-\tan\rho_+ \\ \tan\rho_-\tan\rho_+ & \tan^2\rho_+ \end{bmatrix}, \tag{2.17}$$

$$K_{r\pm} = \frac{\sin(\delta_+ + \delta_-) \pm \cos(2\epsilon)\sin(\delta_+ - \delta_-)}{\cos(\delta_+ + \delta_-) + \cos(2\epsilon)\sin(\delta_+ - \delta_-)}, \tag{2.18}$$

$$K_{r0} = \frac{\sin(2\epsilon)}{\cos(\delta_+ + \delta_-) + \cos(2\epsilon)\cos(\delta_+ - \delta_-)}. \tag{2.19}$$

Here $\delta_\pm = \delta_{J\pm1,J}$, $\rho_\pm = \rho_{J\pm1,J}$ and ϵ is the mixing parameter.

In PANN, it is possible to perform the PSA by means of either MW- or VPI-representations, and the obtained solution by VPI-representation is automatically converted to the one by MW-representation.

2.3.5 Forward Amplitudes

The helicity amplitudes which are concerned in nucleon-nucleon scattering are

$$\begin{aligned} \Phi_1 &= <++|M|++>, \\ \Phi_2 &= <--|M|++>, \\ \Phi_3 &= <+-|M|+->, \\ \Phi_4 &= <+-|M|-+>, \\ \Phi_5 &= <++|M|+->, \end{aligned} \tag{2.20}$$

where $+(-)$ are the helicity $+1/2(-1/2)$ and M is a M-matrix[34].

$$\begin{aligned} \Phi_1 &= \frac{1}{2}(M_{ss} + \cos\theta_c M_{00} - \sqrt{2}\sin\theta_c M_{10}), \\ \Phi_2 &= \frac{1}{2}(-M_{ss} + \cos\theta_c M_{00} - \sqrt{2}\sin\theta_c M_{10}), \\ \Phi_3 &= \frac{1}{2}[(1+\cos\theta_c)M_{11} + (1-\cos\theta_c)M_{1-1} + \sqrt{2}\sin\theta_c M_{01}], \\ \Phi_4 &= \frac{1}{2}[(1-\cos\theta_c)M_{11} + (1+\cos\theta_c)M_{1-1} - \sqrt{2}\sin\theta_c M_{01}], \\ \Phi_5 &= \frac{1}{2}(-\sin\theta_c M_{11} + \sqrt{2}\cos\theta_c M_{01} + \sin\theta_c M_{1-1}), \\ &= \frac{1}{2}(-\sin\theta_c M_{00} - \sqrt{2}\cos\theta_c M_{10}), \end{aligned} \tag{2.21}$$

where θ_c is the scattering angle in the c.m.s..

In the forward direction ($\theta_c = 0$), only Φ_1, Φ_2 and Φ_3 are not zero. It means that the forward amplitudes are completely determined by measuring the six kinds of observables at $\theta_c = 0$.

In our program PANN, the seven observables ($\sigma_t, \sigma_r, \Delta\sigma_T, \Delta\sigma_L, \alpha$[35], Re$F_2$[36] and Re$F_3$[36]) can be treated as experimental data.

$$\sigma_t = 2\sqrt{\pi}\mathrm{Im}[\Phi_1(0) + \Phi_3(0)], \tag{2.22}$$

$$\sigma_r = \frac{\pi}{p^2}\sum_{\ell,J}(2\ell+1)(1-|\eta_{\ell,J}|^2), \tag{2.23}$$

$$\Delta\sigma_T = -4\sqrt{\pi}\mathrm{Im}\Phi_2(0), \tag{2.24}$$

$$\Delta\sigma_L = 4\sqrt{\pi}\mathrm{Im}[\Phi_1(0) - \Phi_3(0)], \tag{2.25}$$

$$\alpha = \mathrm{Re}[\Phi_1(0) + \Phi_3(0)]/\mathrm{Im}[\Phi_1(0) + \Phi_3(0)], \tag{2.26}$$

$$\mathrm{Re}F_2 = \frac{P_L}{\sqrt{\pi}}\mathrm{Re}\Phi_2(0), \tag{2.27}$$

$$\mathrm{Re}F_3 = \frac{P_L}{\sqrt{\pi}}\mathrm{Re}[\Phi_1(0) - \Phi_3(0)], \tag{2.28}$$

where P_L is the momentum in the laboratory system.

2.4 Structure of Program

The program PANN is written in the FORTRAN77 language (or called VS FORTRAN on IBM computers). It is composed of the main program and 29 subprograms, the relation of which is shown in Fig. 2.1, as follows:

- MAIN: Reads the input data for operating PANN and the starting values for χ^2-minimizing search, controls the whole program and prints the obtained results.
- EXPERM: Reads the experimental data, arranges them, calculates the associated Legendre polynomials at each data point and prints the used experimental data.
- VPIDTF: Reads the experimental data by the input data format of the VPI (Virginia Polytechnic Institute)-group.
- TANGA: Converts the differential cross section from mb/(GeV/c)^2 unit to mb/sr units. One can read the differential cross section data in either of the units.
- TOTAL: Calculates the forward observables.
- CROSE: Calculates the other observables. See section 2.7.
- PHASE: Calculates the partial-wave amplitudes from the supported phase-shift parameters.
- SUMSQ: Calculates the χ^2 value, which is defined as

$$\chi^2 = \sum_{ij}[\frac{\theta_{ij}^{th} - n_j\theta_{ij}^{ex}}{n_j\Delta\theta_{ij}^{ex}}]^2 + \sum_j[\frac{1-n_j}{\Delta n_j}]^2, \tag{2.29}$$

or

$$\chi^2 = \sum_{ij}[\frac{N_j\theta_{ij}^{th} - \theta_{ij}^{ex}}{\Delta\theta_{ij}^{ex}}]^2 + \sum_j[\frac{1-N_j}{\Delta n_j}]^2, \tag{2.30}$$

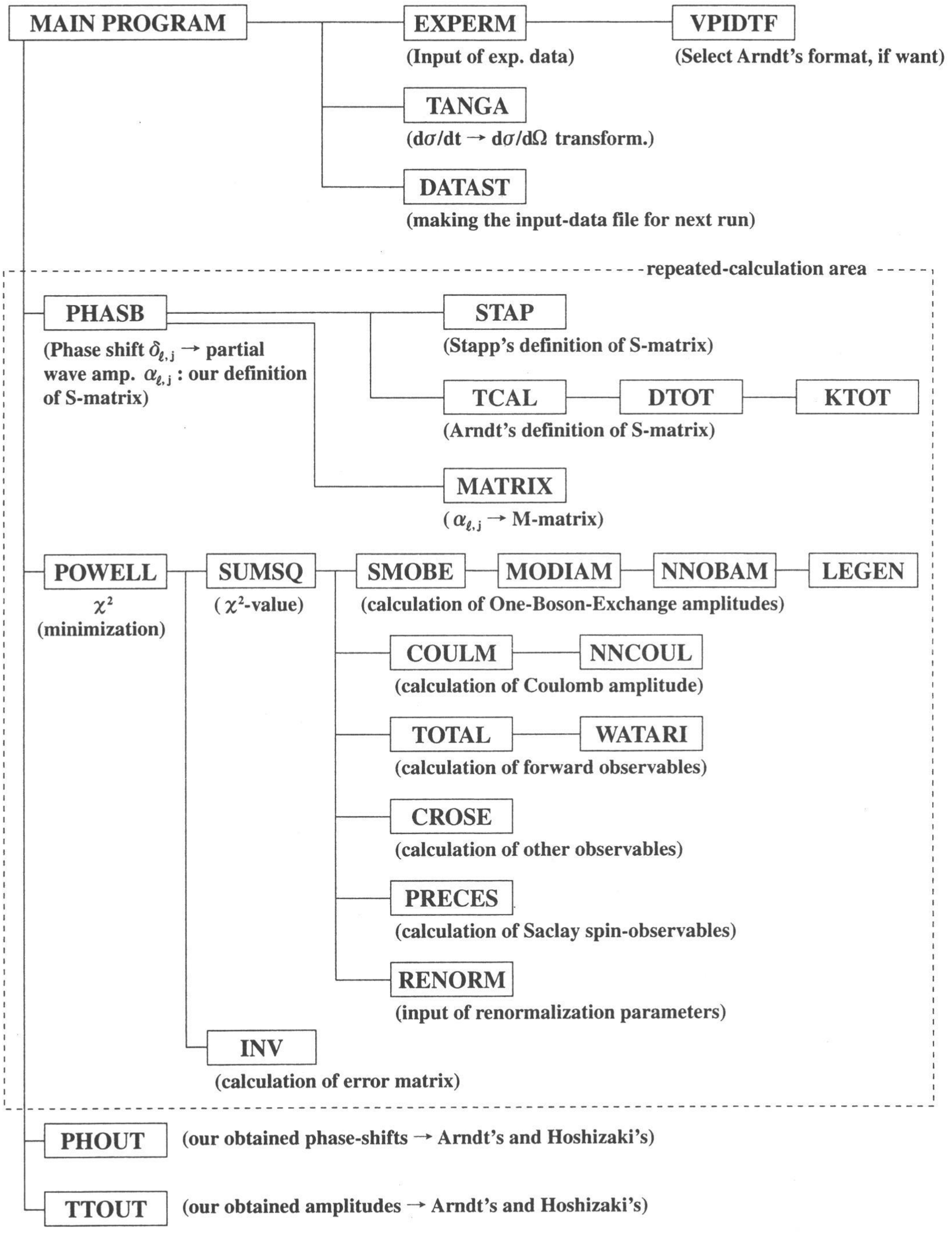

Figure 2.1: Relations among the subroutines used in PANN

where θ_{ij}^{th} is the theoretical value of an observable i, and θ_{ij}^{ex}, $\Delta\theta_{ij}^{ex}$ are its experimental value and error by the jth experiment, respectively. n_j and Δn_j are the experimental renormalization parameter and the systematic error of the jth experiment, which can be searched in the χ^2-minimization, if desired. N_j is the renormalization parameter of the calculated observable corresponding to datum with the systematic error Δn_j of the experiment. In this program, N_j can be determined only at the minimum point of the χ^2-valley. The way of using these parameters varies among the different analyses. One can choose either Eq. (2.29) or (2.30) in operating the PANN.

- RENORM: Calculates the parameter N_j at the χ^2-minimum point.
- MATRIX: Calculates the M-matrix from the partial wave amplitudes.
- LEGEN: Calculates the Legendre functions of the second kind (Q_J) and their nth derivatives $Q_J^{(n)}$.
- NNOBAM: Calculates the partial-wave OBE-amplitudes of N-N scattering with pseudoscalar boson (π), scalar boson (σ) and vector bosons ρ, ω.
- SMOBE: Calculates the OBE-amplitudes for the exchanged bosons ($\pi, \sigma, \rho, \omega$) and calculates their M-matrix.
- MODIAM: Puts the multi-pole parameters n, Λ of the OBE-amplitudes into the subprogram NNOBAM.
- WATARI: Calculates the parameters in MW-representation.
- PHOUT: Calculates Hoshizaki's parameters[34].
- TTOUT: Converts the scattering amplitudes between Arndt's, Hoshizaki's and ours. Print the calculated observables and the helicity amplitudes.
- STAPP: Calculates the S-matrix by Stapp's definition[37]. This subroutine is used only for an analysis of the N-N scattering data in the elastic region.
- TCAL: Connects the MW-representation and the VPI-representation.
- TCALL: Calculates the amplitudes $2iT = S - 1$ by using MW-representation.
- DTOT & KTOT: Calculates the S-matrix by the K-matrix form.
- COULM: Connects this program to the subprogram NNCOUL which was written by R.A. Arndt.
- NNCOUL: Calculates the relativistic Coulomb-amplitude for N-N scattering.
- NRCOUL: Calculates the non-relativistic Coulomb-amplitude for N-N scattering.
- POWELL & INV: χ^2-minimization program which was written by Oyanagi[38] with Powell's modified gradient method.

- DATAST: Generates an input-data file to be read for starting the 2nd search from the parameters obtained by the 1st search, which is needed for an analysis of high-energy data in the present status of computer ability. It may be unavoidable to do some repeated search and some artificial operations because of the sharp diffraction pattern of the differential cross section.
- BLOCK DATA: Gives the values of Planck constant, light velocity and 1 eV=1.602177 $\times 10^{-12}$ erg. In PANN, all calculations are done in the natural unit and the conversion of calculated results to the usual unit is given by the c.g.s. unit.
- PRECES: Calculates mixed spin observables measured at Saclay.

2.5 Input Data Format

The symbols for parameters and their functions in each read statement are explained in this section. The reading formats are shown in the square brackets. The input-data file must be formatted in the sequence shown in the following:

(1) The 1st read statement:

IWV, ICR, IAMP, NCV, ICOUL [5I2]

IWV=0: VPI-representation is used in PANN; =1: MW-representation is used in PANN.
ICR=0: relativistic Coulomb amplitudes are used;
=1: non-relativistic Coulomb amplitudes are used.
IAMP=0: execution of a partial wave analysis;
=1: calculation of the scattering amplitudes and the observables by using the current phase shifts and/or boson parameters without χ^2-minimizing search.
NCV=0: prints the amplitudes of the three groups; VPI, Kyoto Univ. and Hiroshima Univ. If NCV=0, they are not printed. So NCV=0 must be used when IAMP=1.
ICOUL=0: Coulomb-correction is done;
=1: Coulomb amplitude is suppressed;
=2: Coulomb correction is excluded.

(2) The 2nd read statement:

EMP, BM (1-4) [5F10.0]
EMP: nucleon mass which is taken as 938.272 MeV if defaulted.
BM(1): pion mass which is taken as 134.98 MeV if defaulted.
BM(2) and BM(3): vector boson masses in MeV.
BM(4): scalar boson mass in MeV.

(3) The 3rd read statement:

T,NP,LX,IPHASE,IAN,IOBE, JR, ITEST, IWRITE, IDATA [F10.0, 9I2]

T: kinetic energy of incident nucleon in the laboratory system in MeV.
NP=1: an analysis of *p-p* scattering;
=2: an analysis of *n-p* scattering.
LX: the boundary value ℓ_x of the orbital angular momentum ℓ to which the scattering amplitude is calculated with the partial wave amplitude f_ℓ as seen in the expression Eq. (2.5). If it is defaulted, LX=9 is taken automatically, and if LX>35, LX=35 is taken.
IPHASE: the number of fixed parameters which are excluded from the searched parameters, so that it gives the number of phase-shift parameters and boson parameters which are read in by the 8th read statement.
IANG=1: input of the phase-shift parameters in degrees;
=0: input of them in radians.
IOBE: specification of the method of calculation of the real phase shifts δ_ℓ(OBEC) by the modified oneboson exchange model, i.e. IOBE=0 for K-matrix calculation and =1 for Born approximation.
JR: parameter for grouping the experimental data. It is not needed usually, so should be equal to 0.
ITEST: If it is not taken as 0, the final χ^2-value is not only printed out, but also the intermediate χ^2-values are printed out at each cycle.
IWRITE=0: printing the complete information about the obtained result;
=1: a partial print of the obtained result.
IDATA: If it is not equal to 0, the subprogram DATAST is called to generate a data file for the next search.

(4) The 4th read statement:

IUDX, IATL, ITOTAL, IOUT, IOUT1, NRM [I3, 5I2]

IUDX: total number of experimental groups of the used data.
IATL: specification of the unit for the differential cross section, i.e. IATL=0 for mb/sr and IATLD=1 for mb/(GeV)^2.
ITOTAL=1: an analysis with some forward data;
=0: an analysis without forward data.
IOUT: =0 to print out the numbered experimental data which are needed for investigation of the M-value of data, and $\neq 0$ for no printout of them. Each term of the 1st summation in Eq. (2.29) or (2.30) i.e. $[(\theta_i^{th} - \theta_i^{ex})/\Delta\theta_i^{ex}]^2$ is called as M-value.
IOUT1: =0 to print out the used data-file of VPI and $\neq 0$ for no printout of them.
NRM: If you want to carry out the renormalization (N_J in Eq. (2.30)) of the calculated observables at χ^2 minimum point, put NRM as $\neq 0$.

(5) The 5th read statement.

The 5th, 6th and 7th read statements are for reading the experimental data. In PAŊN program, the input data format is same as the one used in SAID maintained by VPI-group. In this format, the data given by different experiments are divided into the different groups of data:

T, ND, AN(1-4), C(1-5) [F9.3, I4, 4F7.3, 2(1X, A4), 1X, 3A4]

T: kinetic energy of incident nucleon in the laboratory system for each experimental data group.
ND: total number of data provided by each experimental data groups.
AN(1): systematic error of the experiments.
AN(2-4): coefficients used to calculate the mixed spin observables measured by SATURNE II[33] (for instance, SAT1, SAT2, etc.).
C(2): gives the kind of observables of the experimental data where the four columns are used, that is read by A4 type using four characters. The correspondence between the symbols in PANN and the observables is as follows:

DSG=$d\sigma/dt$, P=polarization, R, A, RP, AP= R, A, R', A' (Wolfenstein parameters), ALS, ANN, ASS, ASL, ALL, DNN, DSS, DSL, DLS, DLL, KSS, KSL, KLS, KLL, KNN, DSSP, DSLP = A_{LS}, A_{NN}, A_{SS}, A_{SL}, A_{LL}, D_{NN}, D_{SS}, D_{SL}, D_{LS}, D_{LL}, K_{SS}, K_{SL}, K_{LS}, K_{LL}, K_{NN}, $D_{SS'}$, $D_{SL'}$ (two spin correlation parameters), SAT1, SAT2, SAT3= $SAT1, SAT2, SAT3$ (mixed spin observables of two spin correlation parameters), CQKN= C_{qkn} (observable defined in the c.m.s.[34]), HSNS, HLNL, HSNL, HLNS, HNSS, HLSN, HNLS, HSSN, HSLN, HLLN, HNSL, HNLL = H_{SNS}, H_{LNL}, H_{SNL}, H_{LNS}, H_{NSS}, H_{LSN}, H_{NLS}, H_{SSN}, H_{SLN}, H_{LLN}, H_{NSL}, H_{NLL} (three spin correlation parameters), SAT4, SAT5, SAT6, SAT7, SAT8= $SAT4$, $SAT5$, $SAT6$, $SAT7$, $SAT8$ (mixed spin observables of three spin correlation parameters), DSN, CRN0, CRU2, CIN1, CIN2, CIN0= D_{SN}, $CRN0$, $CRU2$, $CIN1$, $CIN2$, $CIN0$ (constraints), SGTR, SGT, SGTT, SGTL, ALFA, REF2, REF3= σ_r, σ_t, $\Delta\sigma_T$, $\Delta\sigma_L$, ReF_2, ReF_3 (forward observables).

C(1), C(3-5): some remarks for the experiments.

(6) The 6th read statement:

COM(1-18) [18A4]

COM: comments for the each experimental data group. References of the experimental data are written here.

(7) The 7th read statement:

ANGL, EXD, DEXD [3(F7.0, F10.0, F7.0), 8X]

ANGL: scattering angle θ_c in degree unit if IATL=0, or the squared momentum transfer $(-t)$ in $(\text{GeV}/c)^2$ unit if IATL=1.
EXD, DEXD: experimental value of the observable and its experimental error, respectively.

(8) The 8th read statement:

IQ, NQ, LQ, JQ, QEL [4(2I1, 2I2, 1X, F9.0)]

IQ=0 and 1 mean for QEL to be the real phase shift δ and the imaginary one η (or ρ in the case of the VPI representation), respectively. In this case, NQ=1 for the spin-singlet state and NQ=3 for the spin-triplet state. LQ and JQ are the orbital and the total angular momentum of phase shift QEL, respectively. IQ=2 means QEL is the boson-nucleon coupling parameter g. In this case, JQ=1 (g_P) for the pseudoscalar boson, JQ=2 (g_V) and 3 (f_V) for the vector bosons and JQ=4 (g_S) for the scalar boson. NQ and LQ have no meanings. IQ=3 means that (1.0 + QEL) is renormalization parameters n_i multiplied by the experimental data and their errors in Eq. (2.29). In this case, (NQ, LQ) shows the kind of observables, and JQ its data group. IQ=4 means that QEL is the auxiliary mass parameter A of the boson in GeV unit, which must be equal to the observed boson mass BM if you want the unmodified OBE-amplitude. IQ=5 means that QEL is the pole-multiplicity of the modified OBE-amplitude, which must be taken as 1 if you want the unmodified OBE-amplitude. Here for IQ=4 and 5, NQ, LQ and JQ have the same meanings as those of the case IQ=2, respectively. The number of QEL parameters has to be equal to IPHASE in the 3rd read statement. The values of QEL given here are fixed in the χ^2-minimizing search.

(9) The 9th read statement:

NFIRST, LANG, NOTE, LOS. NCUT, DELTA, ERROR, ISTART [I3, 2I2, I1, I4, 2F8.0, 1X, I1]

NFIRST: the total number of varied parameters in χ^2-minimizing.
LANG: specification of the unit of phase-shift parameters inputted in the next read statement, i.e. LANG=0 for radians and =1 for degrees.
NOTE: the number of the comment cards which are read in by the 11th read statement.
LOS: If LOS$\neq$0, the theoretical values of observables calculated by the obtained solution are not printed out.
NCUT: the upper limit of the iteration of χ^2-minimizing search in the subroutine POWELL.
DELTA: this gives the step size in the gradient of χ^2 - space, i.e. (DMAX−DMIN)/DELTA. If it is left out, DELTA=500.0 is taken. Here DMAX and DMIN are the maximum and the minimum values of the searched domain of parameters, respectively and read in by the next read statement.
ERROR: this gives the uncertainty C of χ^2-value at χ^2-minimum point where C = ERROR$\times$(N_e $-N_p$), N_e is the total number of experimental data, and N_p the total number of searched parameters. If it is left out, it is taken as 0.0001.
ISTART=0: an execution of the χ^2-minimizing search and no outputs of the calculated obervables for connecting with the other program of X-Y plotting;
ISTART=1: a searching and plotting;
ISTART=2: no searching and no plotting;
ISTART=3: no searching and plotting.

(10) The 10th read statement:

IR, NR, LD, JD, DEL, DMAX, DMIN [3(2I1, 2I2, F10.0, 2F4.0]

IR, NR, LD and JD have the same means as IQ, NQ, LQ and JQ in the 8th read statement, respectively.
DEL: starting values of the varied parameter.
DMAX, DMIN: the maximum and the minimum values of the searched domain of DEL. If they are not given, they are taken as (180., 180.) for the phase-shift parameters in de-

grees, (10., 10.) for the boson-nucleon coupling parameters, (10., 0.) for the auxiliary mass parameters, (10., 1.) for the pole-multiplicity parameters and (0.3, 0.3) for the experimental renormalization parameters. The total number of DEL parameters has to be equal to NFIRST in the 9th read statement.

(11) The 11th read statement:

COMMENT [9A8]

You can write some comment statements about your analysis in the number of lines specified by the parameter NOTE in the 9th read statement.

2.6 Application Plans of PANN

See Refs.[13, 21, 27, 39] for the detailed applications of PANN to the analyses of N-N scattering. The following applications are prepared in future by dividing the energy region to three parts as,

[T_L = 10 MeV $\sim$ 800 MeV]

There are plentiful data for N-N scattering in this energy region. Therefore we can determine the pion-nucleon coupling constants precisely, that is now important for a study of the hadron dynamics. The determination by using PANN is now under progress.

[T_L = 0.5 $\sim$ 2.7 GeV]

Recently COSY provided a plentiful and precise $d\sigma/d\Omega$ data in this energy region[40]. PANN can provide the best solutions of phase-shifts and reflection parameters by using their data, the energy dependence of which would determine the spin-parity, mass and width of a dibaryon[13, 27, 39].

[T_L = 3 $\sim$ 11 GeV]

From our successive studies of N-N scattering by PANN, the catastrophic energy dependence of the spin-orbit interaction was found in this energy region[21]. This result was obtained by analyzing the data in wide energy region (T_L=1$\sim$11 GeV). It is very interesting to make clear the origin of this catastrophe.

2.7 Observables for Nucleon-Nucleon Scattering

We summarize various expressions of observables for the elastic nucleon-nucleon scattering. In this list, the meaning of spin-correlation parameters is clear by the bracket symbol$(i, j \rightarrow i', j')$ originally due to Thomas[41] where four characters i, j, i' and j' denote the spin directions of the beam, target, scattered and recoil nucleons, respectively, and L, N and S indicate the longitudinal, the normal to the scattering plane and the $N \times L$ directions, 0 implying the non-observation of spin-states.

$$d\sigma/dt = (0,0;0,0) = \frac{1}{2}[|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2], \tag{2.31}$$

$$P = (0,N;0,0) = (N,0;0,0) = \mathrm{Im}[(\Phi_1 + \Phi_2 + \Phi_3 - \Phi_4)^*\Phi_5]/(d\sigma/dt), \tag{2.32}$$

$$D = (N,0;N,0) = \{\mathrm{Re}(\Phi_1^*\Phi_3^* - \Phi_2^*\Phi_4^*) + 2|\Phi_5|^2\}/(d\sigma/dt), \tag{2.33}$$

$$\begin{aligned} R &= (S,0;S,0) \\ &= \{-\mathrm{Re}[\Phi_5^*(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)]\sin\theta_S + \mathrm{Re}(\Phi_1^*\Phi_3 + \Phi_2^*\Phi_4)\cos\theta_S\}/(d\sigma/dt), \end{aligned} \tag{2.34}$$

$$\begin{aligned} R' &= (S,0;L,0) \\ &= \{-\mathrm{Re}[\Phi_5^*(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)]\cos\theta_S - \mathrm{Re}(\Phi_1^*\Phi_3 + \Phi_2^*\Phi_4)\sin\theta_S\}/(d\sigma/dt), \end{aligned} \tag{2.35}$$

$$\begin{aligned} A &= (L,0;S,0) \\ &= \{\mathrm{Re}[\Phi_5^*(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)]\cos\theta_S + \frac{1}{2}(|\Phi_1|^2 - |\Phi_2|^2 + |\Phi_3|^2 - |\Phi_4|^2)\sin\theta_S\}/(d\sigma/dt), \end{aligned} \tag{2.36}$$

$$\begin{aligned} A' &= (L,0;L,0) \\ &= \{-\mathrm{Re}[\Phi_5^*(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)]\sin\theta_S + \frac{1}{2}(|\Phi_1|^2 - |\Phi_2|^2 + |\Phi_3|^2 - |\Phi_4|^2)\cos\theta_S\}/(d\sigma/dt), \end{aligned} \tag{2.37}$$

$$A_{\mathrm{NN}} = (\mathrm{N},\mathrm{N};0,0) = [\mathrm{Re}(\Phi_1^*\Phi_2 - \Phi_3^*\Phi_4) + 2|\Phi_5|^2]/(d\sigma/dt), \tag{2.38}$$

$$A_{\mathrm{SS}} = (\mathrm{S},\mathrm{S};0,0) = \mathrm{Re}(\Phi_1^*\Phi_2 + \Phi_3^*\Phi_4)/(d\sigma/dt), \tag{2.39}$$

$$A_{\mathrm{SL}} = (\mathrm{S},\mathrm{L};0,0) = \mathrm{Re}[(\Phi_1 + \Phi_2 - \Phi_3 + \Phi_4)^*\Phi_5]/(d\sigma/dt), \tag{2.40}$$

$$A_{\mathrm{LL}} = (\mathrm{L},\mathrm{L};0,0) = \frac{1}{2}[-|\Phi_1|^2 - |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2]/(d\sigma/dt), \tag{2.41}$$

$$D_{\mathrm{NN}} = (0,N;0,N) = \{\mathrm{Re}(\Phi_1^*\Phi_3 - \Phi_2^*\Phi_4) + 2|\Phi_5|^2\}/(d\sigma/dt), \tag{2.42}$$

$$\begin{aligned} D_{\mathrm{SS}} &= (0,S;0,S) \\ &= \{-\sin\theta_R \mathrm{Re}[(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)^*\Phi_5] - \cos\theta_R \mathrm{Re}(\Phi_1^*\Phi_3 + \Phi_2^*\Phi_4)\}/(d\sigma/dt), \end{aligned} \tag{2.43}$$

$$\begin{aligned} D_{\mathrm{SL}} &= (0,S;0,L) \\ &= \{-\sin\theta_R \mathrm{Re}(\Phi_1^*\Phi_3 + \Phi_2^*\Phi_4) + \cos\theta_R \mathrm{Re}[(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)^*\Phi_5\}/(d\sigma/dt), \end{aligned} \tag{2.44}$$

$$\begin{aligned} D_{\mathrm{LS}} &= (0,L;0,S) \\ &= \{\frac{1}{2}\sin\theta_R[|\Phi_1|^2 - |\Phi_2|^2 + |\Phi_3|^2 - |\Phi_4|^2] - \cos\theta_R \mathrm{Re}[(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)^*\Phi_5\}/(d\sigma/dt), \end{aligned}$$

$$D_{\mathrm{LL}} = (0,\mathrm{L};0,\mathrm{L}) = \\ \{-\sin\theta_R \mathrm{Re}[(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)^*\Phi_5] \\ -\frac{1}{2}\cos\theta_R[|\Phi_1|^2 - |\Phi_2|^2 + |\Phi_3|^2 - |\Phi_4|^2]\}/(d\sigma/dt), \tag{2.45}$$

$$K_{\mathrm{NN}} = (N,0;0,N) \\ = \{-\mathrm{Re}(\Phi_1^*\Phi_4 - \Phi_2^*\Phi_3) + 2|\Phi_5|^2\}/(d\sigma/dt), \tag{2.46}$$

$$K_{\mathrm{SS}} = (S,0;0,S) \\ = \{\sin\theta_R \mathrm{Re}[(\Phi_1 - \Phi_2 - \Phi_3 - \Phi_4)^*\Phi_5] - \cos\theta_R \mathrm{Re}(\Phi_1^*\Phi_4 + \Phi_2^*\Phi_3)\}/(d\sigma/dt), \tag{2.47}$$

$$K_{\mathrm{SL}} = (S,0;0,L) \\ = \{-\sin\theta_R \mathrm{Re}[\Phi_1^*\Phi_4 + \Phi_2^*\Phi_3] - \cos\theta_R \mathrm{Re}[(\Phi_1 - \Phi_2 - \Phi_3 - \Phi_4)^*\Phi_5]\}/(d\sigma/dt), \tag{2.48}$$

$$K_{\mathrm{LS}} = (L,0;0,S) \\ = \{-\frac{1}{2}\sin\theta_R\{[(|\phi_1|^2 - |\phi_2|^2 - |\phi_3|^2 + |\phi_4|^2)] + \cos\theta_R \mathrm{Re}[(\Phi_1 - \Phi_2 - \Phi_3 - \Phi_4)^*\Phi_5]\}/(d\sigma/dt), \tag{2.49}$$

$$K_{LL} = (L,0;0,L) \\ = \{\sin\theta_R \mathrm{Re}[(\phi_1 - \phi_2 - \phi_3 - \phi_4)^*\phi_5] + \frac{1}{2}\cos\theta_R[(|\phi_1|^2 - |\phi_2|^2 - |\phi_3|^2 + |\phi_4|^2)]\}/(d\sigma/dt) \tag{2.50}$$

$$H_{\mathrm{SNS}} = (S,N;0,S) \\ = \{-\sin\theta_R \mathrm{Im}(\Phi_1^*\Phi_2 + \Phi_3^*\Phi_4) + \cos\theta_R \mathrm{Im}[(\Phi_1 - \Phi_2 - \Phi_3 - \Phi_4)^*\Phi_5]\}/(d\sigma/dt), \tag{2.51}$$

$$H_{\mathrm{NSS}} = (N,S;0,S) \\ = \{\sin\theta_R \mathrm{Im}(\Phi_1^*\Phi_2 - \Phi_3^*\Phi_4) - \cos\theta_R \mathrm{Im}[(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)^*\Phi_5]\}/(d\sigma/dt), \tag{2.52}$$

$$H_{\mathrm{NLS}} = (N,L;0,S) \\ = \{\sin\theta_R \mathrm{Im}[(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)^*\Phi_5] - \cos\theta_R \mathrm{Im}(\Phi_1^*\Phi_4 + \Phi_2^*\Phi_3)\}/(d\sigma/dt), \tag{2.53}$$

$$H_{\mathrm{LNS}} = (L,N;0,S) \\ = \{-\sin\theta_R \mathrm{Im}[(\Phi_1 - \Phi_2 + \Phi_3 + \Phi_4)^*\Phi_5] - \cos\theta_R \mathrm{Im}(\Phi_1^*\Phi_3 + \Phi_2^*\Phi_4)\}/(d\sigma/dt), \tag{2.54}$$

$$H_{\mathrm{LNL}} = (L,N;0,L) \\ = \{-\sin\theta_R \mathrm{Im}(\Phi_1^*\Phi_3 + \Phi_2^*\Phi_4) - \cos\theta_R \mathrm{Im}[(\Phi_1 - \Phi_2 - \Phi_3 - \Phi_4)^*\Phi_5]\}/(d\sigma/dt), \tag{2.55}$$

$$H_{\mathrm{SNL}} = (S,N;0,L) \\ = \{-\sin\theta_R \mathrm{Im}[(\Phi_1 - \Phi_2 - \Phi_3 - \Phi_4)^*\Phi_5] + \cos\theta_R \mathrm{Im}(\Phi_1^*\Phi_2 + \Phi_3^*\Phi_4)\}/(d\sigma/dt), \tag{2.56}$$

(2.57)

$$H_{\mathrm{SSN}} = (S,S;0,N)$$

$$= \mathrm{Im}[(\Phi_1 + \Phi_2 - \Phi_3 + \Phi_4)^* \Phi_5]/(d\sigma/dt), \tag{2.58}$$

$$\begin{aligned} H_{\mathrm{LSN}} &= (L, S; 0, N) \\ &= -\mathrm{Im}(\Phi_1^* \Phi_3 - \Phi_2^* \Phi_4)/(d\sigma/dt), \end{aligned} \tag{2.59}$$

$$\begin{aligned} H_{\mathrm{SLN}} &= (S, L; 0, N) \\ &= \mathrm{Im}(\Phi_1^* \Phi_4 - \Phi_2^* \Phi_3)/(d\sigma/dt), \end{aligned} \tag{2.60}$$

where θ_S and θ_R are the scattering and recoil angle in the laboratory system, respectively.

2.8 Additional Comment

In order to obtain a reasonable solution of PSA by using this program PANN, the user needs not only substantial database to do it, but also enough knowedge of nuclear physics. One could not automatically obtain any reasonable solutions. As posted up in SAID presented by R.A. Arndt, one's profanity should be always noticed.