Chapter 4

Phase-Shift Analyses in Frontier Hadron-Physics

4.1 Developed Program for Phase-Shift Analysis of p-^{3}He Scattering : PAPH

Abstract. PAPH is a phase-shift analysis program which can be used to determine the complex scattering amplitudes of elastic p-^{3}He scattering by fitting the experimental data for many kinds of observables. It is developed by new representation of the S-matrix so as to make it possible to carry out the analysis both in the elastic and inelastic regions such as several MeV$\sim$ a few hundred MeV.
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4.1.1 Introduction

Since 1950's, the experiments on p-^{3}He scattering[111] and the phase-shift analysis(PSA)[112, 113] of their data have been performed with an interest in nuclei of mass number 4. Recently an experiment on ^{3}He$(d,p)^4$He reaction was performed[114], and its theoretical studies were progressed by Oryu et al.[115]. They used the multi-channel Faddeev-equations to carry out the theoretical analysis of this experiment. For carring out this project, a study of the p+n+^{3}He system is important. In order to analyze the p+n+^{3}He system, the precise determination of p-^{3}He and n-^{3}He phase shifts would be required in a wide range of incident-nucleon energies.

Recent accumulation of data in the elastic p-^{3}He scattering makes it available to carry out the single energy PSA at low energies. Moreover in the intermediate energy region there exist some energy points where the data on spin-correlation parameters were measured[116]. Usual PSAs of p-^{3}He scattering use the S-matrix representation introduced by Blatt and Biedenharn[117]. This representation can be used only in the energy region where the inelastic channels are not opened. On the other hand, the representation introduced by Stapp et al.[37], where the phase shifts are called as nuclear bar phase shifts, can be extended easily to the one of Matsuda and Watari[29] in which the inelastic effect is correctly treated.

In the next subsection, a brief formalism used in PAPH program is described. Some features of PAPH are given in subsection 4.1.3, and the structure of program is explained in subsection 4.1.4. We give the input data format in subsection 4.1.5. Finally, our application plan of PAPH and an additional comment are given in subsections 4.1.6 and 4.1.7.

4.1.2 Phase-shift analysis of p-^{3}He scattering

The scattered wave function of the two spin-1/2 particles is written as a vector

$$\Psi^{(n)} = \mathrm{e}^{i(\boldsymbol{k}_i\cdot\boldsymbol{r})}\chi^{(n)} + f^{(n)}\frac{\mathrm{e}^{ikr}}{r},$$
$$f^{(n)} = M\chi^{(n)}. \tag{4.1}$$

Here $\chi^{(n)}$ is a vector with four components representing the initial spin states n of two particles, and M is a 4×4 matrix operating on the initial spin state. $\boldsymbol{k}_i(k = |\boldsymbol{k}_i|)$ and $\boldsymbol{r}(r = |\boldsymbol{r}|)$ are the wave number in the initial state and the relative distance of two particles, respectively.

The general form of the M-matrix can be given by the following summation of six invariant-amplitudes[118],

$$M = \frac{1}{2}[a + b + (a-b)(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})(\boldsymbol{\sigma}_2\cdot\boldsymbol{n}) + (c+d)(\boldsymbol{\sigma}_1\cdot\boldsymbol{m})(\boldsymbol{\sigma}_2\cdot\boldsymbol{m})$$
$$+(c-d)(\boldsymbol{\sigma}_1\cdot\boldsymbol{l})(\boldsymbol{\sigma}_2\cdot\boldsymbol{l}) + e(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n} + f(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}] \tag{4.2}$$

subject to its invariance under space rotations, reflections and time reversal. The characters $\boldsymbol{l}$, $\boldsymbol{m}$ and $\boldsymbol{n}$ are the unit vectors defined by

$$\boldsymbol{l} = \frac{\boldsymbol{p}_f+\boldsymbol{p}_i}{|\boldsymbol{p}_f+\boldsymbol{p}_i|}, \qquad \boldsymbol{m} = \frac{\boldsymbol{p}_f-\boldsymbol{p}_i}{|\boldsymbol{p}_f-\boldsymbol{p}_i|}, \qquad \boldsymbol{n} = \frac{\boldsymbol{p}_i\times\boldsymbol{p}_f}{|\boldsymbol{p}_i\times\boldsymbol{p}_f|} \tag{4.3}$$

where $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$ are the momenta in the initial and final states in the center of mass system (c.m.s.), respectively.

The partial wave expansion of matrices M for p-^{3}He scattering is given by,

$$M_{ss_zs's'_z} = C(\theta)\delta_{ss'}\delta_{s_zs'_z} + 4\pi\sum_{\ell,\ell',J}\sqrt{\frac{2\ell'+1}{4\pi}}Y_\ell^{s'_z-s_z}(\theta,\phi)$$
$$\times C_{\ell s}(J,s'_z,s'_z-s_z,s_z)C_{\ell's'}(J,s'_z,0,s_z)h^J_{\ell s,\ell's'}. \tag{4.4}$$

Here s, s' are the total spin of the proton and ^{3}He in the final and initial states respectively, and s_z, s'_z are their z-components. θ and ϕ are the scattering angle in the c.m.s. and the azimuthal angle. $C(\theta)$ is the Coulomb amplitude. $h^J_{\ell s,\ell's'}$ is the p-^{3}He scattering amplitude which has the following relation to the S-matrix,

$$h^J_{\ell s,\ell's'} = \frac{1}{2ip}(S^J_{\ell s,\ell's'} - 1)\exp\{i(\Phi_\ell + \Phi_{\ell'})\},$$

where Φ_ℓ is the coulomb phase shift, and ℓ, ℓ' are the orbital angular momenta in the final and initial states, respectively, J the total angular momentum. p is the momentum in the c.m.s. $C_{\ell s}$ is the Clebsch-Gordan coefficient.

The M-matrix elements for p-^{3}He scattering are described explicitly as follows:

$$M_{0000} = C(\theta) + \sum_\ell (2\ell+1) h_\ell P_\ell(\theta), \tag{4.5}$$

$$M_{1111} = C(\theta) + \sum_\ell \left\{ \frac{\ell+2}{2} h_{\ell,\ell+1} + \frac{2\ell+1}{2} h_{\ell,\ell} + \frac{\ell-1}{2} h_{\ell,\ell-1} \right.$$
$$\left. -\frac{1}{2}\sqrt{(\ell+1)(\ell+2)}\, h^{\ell+1} - \frac{1}{2}\sqrt{(\ell-1)\ell}\, h^{\ell-1} \right\} P_\ell(\theta), \tag{4.6}$$

$$M_{1010} = C(\theta) + \sum_\ell \left\{ (\ell+1) h_{\ell,\ell+1} + \ell h_{\ell,\ell-1} + \sqrt{(\ell+1)(\ell+2)}\, h^{\ell+1} \right.$$
$$\left. +\sqrt{(\ell-1)\ell}\, h^{\ell-1} \right\} P_\ell(\theta), \tag{4.7}$$

$$M_{1011} = \sum_\ell \left\{ -\frac{\ell+2}{\sqrt{2}(\ell+1)} h_{\ell,\ell+1} + \frac{2\ell+1}{\sqrt{2}\ell(\ell+1)} h_{\ell,\ell} + \frac{\ell-1}{\sqrt{2}\ell} h_{\ell,\ell-1} \right.$$
$$\left. +\sqrt{\frac{\ell+2}{2(\ell+1)}}\, h^{\ell+1} - \sqrt{\frac{\ell-1}{2\ell}}\, h^{\ell-1} \right\} P_\ell^1(\theta), \tag{4.8}$$

$$M_{1110} = \sum_\ell \left\{ \frac{1}{\sqrt{2}} h_{\ell,\ell+1} - \frac{1}{\sqrt{2}} h_{\ell,\ell-1} + \sqrt{\frac{\ell+2}{2(\ell+1)}}\, h^{\ell+1} \right.$$
$$\left. -\sqrt{\frac{\ell-1}{2\ell}}\, h^{\ell-1} \right\} P_\ell^1(\theta), \tag{4.9}$$

$$M_{111-1} = \sum_\ell \left\{ \frac{1}{2(\ell+1)} h_{\ell,\ell+1} - \frac{2\ell+1}{2\ell(\ell+1)} h_{\ell,\ell} + \frac{1}{2\ell} h_{\ell,\ell-1} \right.$$
$$\left. -\frac{1}{2\sqrt{(\ell+1)(\ell+2)}} h^{\ell+1} - \frac{1}{2\sqrt{(\ell-1)\ell}} h^{\ell-1} \right\} P_\ell^2(\theta), \tag{4.10}$$

$$M_{1000} = \sum_\ell \frac{2\ell+1}{\sqrt{2\ell(\ell+1)}} h^{ST} P_\ell^1(\theta). \tag{4.11}$$

Here, h_ℓ represents the singlet amplitude, $h_{\ell,J}$ represents the triplet amplitude, h^J represents the transition amplitude of triplet states between $J = \ell+1$ and $J = \ell-1$, and h^{ST} represents the transition amplitude between singlet and triplet states with $J = \ell$.

In PAPH for the PSA, the following Coulomb amplitude $C(\theta)$ and phase shift Φ_ℓ are used. The Coulomb amplitude $C(\theta)$ is given by

$$C(\theta) = -\frac{n}{p(1-\cos\theta)} e^{-in\ln(1-\cos\theta)/2}, \tag{4.12}$$

where n is defined by a relative velocity v as $n = \dfrac{2e^2}{\hbar v}$.

The Coulomb phase shift is given by

$$\Phi_\ell = \sum_{x=1}^{\ell} \tan^{-1}\left(\frac{n}{x}\right). \tag{4.13}$$

The six coefficients a, b, c, d, e and f of the M-matrix in Eq. (4.2) have the following relations to the matrix elements given in Eqs. (4.5) ~ (4.11),

$$\begin{aligned}
a &= \frac{1}{2}(M_{1010} + M_{1111} - M_{1-111}),\\
b &= \frac{1}{2}(M_{0000} + M_{1111} + M_{1-111}),\\
c &= \frac{1}{2}(-M_{0000} + M_{1111} + M_{1-111}),\\
d &= -\frac{1}{\sqrt{2}\sin\theta}(M_{1110} + M_{1011}),\\
e &= \frac{i}{\sqrt{2}}(M_{1110} - M_{1011}),\\
f &= -i\sqrt{2}M_{1000}.
\end{aligned} \tag{4.14}$$

The differential cross section $d\sigma/d\Omega$, the proton analyzing power A_{y0}, the ^{3}He analyzing power A_{0y} and the spin-correlation coefficients $A_{yy}, A_{xz}, A_{zx}, A_{xx}, A_{zz}$ are defined by means of the above six parameters as follows[118]:

$$\begin{aligned}
d\sigma/d\Omega(=\sigma) &= \frac{1}{2}(|a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 + |f|^2),\\
\sigma A_{y0} &= \mathrm{Re}(a^*e + b^*f),\\
\sigma A_{0y} &= \mathrm{Re}(a^*e - b^*f),\\
\sigma A_{yy} &= \frac{1}{2}(|a|^2 - |b|^2 - |c|^2 + |d|^2 + |e|^2 - |f|^2),\\
\sigma A_{xz} &= -\mathrm{Re}(a^*d)\sin\theta + \mathrm{Im}(c^*f) - \mathrm{Im}(d^*e)\cos\theta,\\
\sigma A_{zx} &= -\mathrm{Re}(a^*d)\sin\theta - \mathrm{Im}(c^*f) - \mathrm{Im}(d^*e)\cos\theta,\\
\sigma A_{xx} &= \mathrm{Re}(a^*d)\cos\theta + \mathrm{Re}(b^*c) - \mathrm{Im}(d^*e)\sin\theta,\\
\sigma A_{zz} &= -\mathrm{Re}(a^*d)\cos\theta + \mathrm{Re}(b^*c) + \mathrm{Im}(d^*e)\sin\theta,\\
\sigma A_{\ell\ell} &= -\mathrm{Re}(a^*d - b^*c),\\
\sigma A_{m\ell} &= -\mathrm{Im}(d^*e - c^*f),\\
\sigma A_{\ell m} &= -\mathrm{Im}(d^*e + c^*f),\\
\sigma A_{mm} &= \mathrm{Re}(a^*d + b^*c),\\
\sigma R &= \mathrm{Re}(a^*b - e^*f + c^*d)\cos\frac{\theta}{2} - \mathrm{Im}(a^*f + b^*e)\sin\frac{\theta}{2},\\
\sigma A &= -\mathrm{Re}(a^*b + c^*d - e^*f)\sin\frac{\theta}{2} - \mathrm{Im}(a^*f + b^*e)\cos\frac{\theta}{2},\\
\sigma D &= \frac{1}{2}(|a|^2 + |b|^2 - |c|^2 - |d|^2 + |e|^2 + |f|^2),\\
\sigma R' &= \mathrm{Re}(a^*b - c^*d - e^*f)\sin\frac{\theta}{2} + \mathrm{Im}(a^*f + b^*e)\cos\frac{\theta}{2},
\end{aligned} \tag{4.15}$$

$$\sigma A' = \mathrm{Re}(a^*b - c^*d - e^*f)\cos\frac{\theta}{2} - \mathrm{Im}(a^*f + b^*e)\sin\frac{\theta}{2},$$
$$\sigma R_t = \mathrm{Re}(a^*c - b^*d)\sin\frac{\theta}{2} + \mathrm{Im}(c^*e - d^*f)\cos\frac{\theta}{2},$$
$$\sigma A_t = \mathrm{Re}(a^*c - b^*d)\cos\frac{\theta}{2} - \mathrm{Im}(c^*e - d^*f)\sin\frac{\theta}{2},$$
$$\sigma D_t = \frac{1}{2}(|a|^2 - |b|^2 + |c|^2 - |d|^2 + |e|^2 - |f|^2),$$
$$\sigma R_t' = \mathrm{Re}(a^*c + b^*d)\cos\frac{\theta}{2} - \mathrm{Im}(c^*e + d^*f)\sin\frac{\theta}{2},$$
$$\sigma A_t' = -\mathrm{Re}(a^*c + b^*d)\sin\frac{\theta}{2} - \mathrm{Im}(c^*e + d^*f)\cos\frac{\theta}{2}.$$

4.1.3 Features of program

Blatt-Biedenharn and Matsuda-Watari representations

In the PSA of p-^{3}He scattering only Blatt-Biedenharn representation (BB) has been used so far. Recently Mefford and Landau[119] gave an expression by the representation of Stapp et al.[37]. We choose the representation which was given by Matsuda and Watari (MW)[29]. The MW-representation can be used in both elastic and inelastic regions. PAPH covers the energy region where the inelastic channel is open.

In the MW-representation, the S-matrices are as follows:

For uncoupled waves,

$$S_{\ell J} = \eta_{\ell J}\exp\left(2i\bar{\delta}^N_{\ell J}\right). \tag{4.16}$$

where $\eta_{\ell J}$ and $\bar{\delta}^N_{\ell J}$ are the reflection parameters and the phase shifts with the orbital angular momentum ℓ and the total angular momentum J.

For coupled waves,

$$S_J = \begin{pmatrix} \sqrt{1-|\rho_J|^2}\,\eta_+\exp(2i\delta_+) & i\rho_J\sqrt{\eta_+\eta_-}\exp\{i(\delta_+ + \delta_-)\} \\ i\rho_J\sqrt{\eta_+\eta_-}\exp\{i(\delta_+ + \delta_-)\} & \sqrt{1-|\rho_J|^2}\,\eta_-\exp(2i\delta_-) \end{pmatrix}. \tag{4.17}$$

Here $\delta_+ = \delta_{J+1,J}, \delta_- = \delta_{J-1,J}$, and $\eta_\pm$ are the reflection parameters with $\ell = J+1(J-1)$, respectively. ρ_J is the mixing parameter. In p-^{3}He scattering, we have the coupled S-matrix between the singlet triplet states, where $\delta_\pm$ and $\eta_\pm$ are the phase shifts and the reflection parameters of singlet and triplet states, respectively.

In the elastic region, this representation is equivalent to that of Stapp et al.. On the other hand, the relations between the phase-shift parameters in the MW- and BB-representations are given by

$$\delta_-^N + \delta_+^N = \bar{\delta}_-^N + \bar{\delta}_+^N$$

$$\sin(\bar{\delta}_-^N - \bar{\delta}_+^N) = \frac{1}{\tan 2\epsilon}\frac{\rho}{\sqrt{1-\rho^2}} \tag{4.18}$$

$$\sin(\delta_-^N - \delta_+^N) = \frac{\rho}{\sin 2\epsilon},$$

where δ_i^N, ϵ are the phase shift of i-wave and its mixing parameter in the BB-representation, respectively.

Helicity amplitudes

The helicity amplitudes which are concerned for p-^{3}He scattering are as follows:

$$\begin{aligned}
\Phi_1 &= <++|M|++>, \\
\Phi_2 &= <--|M|++>, \\
\Phi_3 &= <+-|M|+->, \\
\Phi_4 &= <+-|M|-+>, \\
\Phi_5 &= <++|M|+->, \\
\Phi_6 &= <++|M|-+>,
\end{aligned} \tag{4.19}$$

$$\begin{aligned}
\Phi_1 &= \frac{1}{2}(-\sqrt{2}\sin\theta M_{1110} + \cos\theta M_{1010} + M_{0000}), \\
\Phi_2 &= \frac{1}{2}(-\sqrt{2}\sin\theta M_{1110} + \cos\theta M_{1010} - M_{0000}), \\
\Phi_3 &= \frac{1}{2}(1+\cos\theta)M_{1111} + \frac{\sqrt{2}}{2}\sin\theta M_{1011} + \frac{1}{2}(1-\cos\theta)M_{111-1}, \\
\Phi_4 &= \frac{1}{2}(1+\cos\theta)M_{111-1} - \frac{\sqrt{2}}{2}\sin\theta M_{1011} + \frac{1}{2}(1-\cos\theta)M_{1111}, \\
\Phi_5 &= -\frac{1}{2}\sin\theta M_{1010} - \frac{1}{\sqrt{2}}\cos\theta M_{1110} - \frac{1}{\sqrt{2}}M_{1100}, \\
\Phi_6 &= \frac{1}{2}\sin\theta M_{1010} + \frac{1}{\sqrt{2}}\cos\theta M_{1110} - \frac{1}{\sqrt{2}}M_{1100}.
\end{aligned} \tag{4.20}$$

Here $+(-)$ are the helicity $+1/2(-1/2)$ and M is the M-matrix.

Forward amplitudes

At the forward angle (θ=0), only the amplitudes Φ_1, Φ_2 and Φ_3 contribute to forward observables. If we measure six kinds of observables at θ=0, we can determine the forward amplitudes completely.

The total cross section:

$$\sigma_t = 2\sqrt{\pi}\mathrm{Im}[\Phi_1(0) + \Phi_3(0)]. \tag{4.21}$$

The real to imaginary ratio:

$$\alpha = \mathrm{Re}[\Phi_1(0) + \Phi_3(0)]/\mathrm{Im}[\Phi_1(0) + \Phi_3(0)]. \tag{4.22}$$

The cross section difference in the longitudinal spin states:

$$\Delta\sigma_L = \sigma(\overset{\rightarrow}{\leftarrow}) - \sigma(\overset{\rightarrow}{\rightarrow}) = 4\sqrt{\pi}\mathrm{Im}[\Phi_1(0) - \Phi_3(0)]. \tag{4.23}$$

The cross section difference in the transverse spin states:

$$\Delta\sigma_T = \sigma(\uparrow\downarrow) - \sigma(\uparrow\uparrow) = -4\sqrt{\pi}\mathrm{Im}\Phi_2(0). \tag{4.24}$$

We can use the following observables ($\mathrm{Re}F_2$, $\mathrm{Re}F_3$) given by the dispersion relations[35, 36] in the analysis by PAPH.

The real part of Φ_2:

$$\mathrm{Re}F_2 = \frac{P_L}{\sqrt{\pi}}\mathrm{Re}\Phi_2(0), \tag{4.25}$$

The real part of $\Phi_1 - \Phi_3$:

$$\mathrm{Re}F_3 = \frac{P_L}{\sqrt{\pi}}\mathrm{Re}[\Phi_1(0) - \Phi_3(0)], \tag{4.26}$$

where P_L is the momentum in the laboratory system (L.S.).

For the analysis in the inelastic region, the reflection parameters $\eta_{\ell J}$ are determined by the experimental data on the inelastic cross section,

$$\begin{aligned}\sigma_r = \frac{\pi}{4p^2}\sum_J (2J+1)\ \Big\{ & 1 - (1 - |\rho_J^{(T)}|^2)\ \eta_{J+1J}^2 - |\rho_J^{(T)}|^2\ \eta_{J+1J}\eta_{J-1J} \\ & + 1 - (1 - |\rho_J^{(T)}|^2)\ \eta_{J-1J}^2 - |\rho_J^{(T)}|^2\ \eta_{J+1J}\eta_{J-1J} \\ & + 1 - (1 - |\rho_J^{(S)}|^2)\ \eta_{JJ}^2 - |\rho_J^{(S)}|^2\ \eta_{JJ}\eta_J \\ & + 1 - (1 - |\rho_J^{(S)}|^2)\ \eta_J^2 - |\rho_J^{(S)}|^2\ \eta_{JJ}\eta_J \Big\}.\end{aligned} \tag{4.27}$$

Here, η_J is the reflection parameter of the spin singlet state, $\eta_{\ell J}$ is the reflection parameter of the spin triplet state, $\rho_J^{(T)}$ is the mixing parameter between $J = \ell \pm 1$ states, and $\rho_J^{(S)}$ the mixing parameter between the singlet and triplet states.

In our program PAPH, the seven observables ($\sigma_t, \sigma_r, \Delta\sigma_T, \Delta\sigma_L, \alpha$, $\mathrm{Re}F_2$ and $\mathrm{Re}F_3$) can be inputted as experimental data.

4.1.4 Structure of Program

The program PAPH is written in the FORTRAN77 language (or called VS FORTRAN on IBM computers).

- MAIN: Reads the input data for operating PAPH and the starting values for χ^2-minimizing search, controls the whole program and prints the obtained results.
- EXPERM: Reads the experimental data, arranges them, calculates the associated Legendre polynomials at each data point and prints the used experimental data.
- VPIDTF: Reads the experimental data by the input data format of the VPI (Virginia Polytechnic Institute)-group.
- TANGA: Converts the differential cross section from mb/(GeV/c)^2 unit to mb/sr unit. One can read the differential cross section data in either of the unit.

- TOTAL: Calculates the forward observables.
- CROSE: Calculates the other observables.
- PHASB: Calculates the partial-wave amplitudes from the supported phase-shift parameters.
- SUMSQ: Calculates the χ^2 value, which is defined as

$$\chi^2 = \sum_{ij} \left[\frac{\theta^{\rm th}_{ij} - n_j \theta^{\rm ex}_{ij}}{n_j \Delta\theta^{\rm ex}_{ij}} \right]^2 + \sum_j \left[\frac{1 - n_j}{\Delta n_j} \right]^2, \tag{4.28}$$

  where $\theta^{\rm th}_{ij}$ is the theoretical value of an observable i, and $\theta^{\rm ex}_{ij}$, $\Delta\theta^{\rm ex}_{ij}$ are its experimental value and error by the jth experiment, respectively. n_j and Δn_j are the experimental renormalization parameter and the systematic error of the jth experiment, which can be searched in the χ^2-minimization, if desired.

- MATRIX: Calculates the M-matrix from the partial-wave amplitudes.
- WATARI: Calculates WATARI's parameters.
- BLATT: Subroutine for calculation of amplitudes by using the phase shift parameters in the BB-representation.
- POWELL & INV: χ^2-minimization program which was written by Y. Oyanagi with Powell's modified gradient method.
- DATAST: Generates a data file to be read for starting the 2nd search from the parameters obtained by the 1st search.
- KINEMA: Calculates the kinematics of p-^{3}He system.
- NRCOUL: Calculates the non-relativistic Coulomb amplitudes.
- PHOUT: Output of MW-parameters[29] and Hoshizaki's parameters[34]
- STAPP: The S matrix is calculated by the Stapp's representation[37].
- TCALL: Calculates the amplitudes $2iT = S - 1$ by using MW-representation.
- BLOCK DATA: Gives the values of Planck constant, light velocity and 1eV $= 1.602177\times 10^{-12}$ erg. In PAPH, all calculations are done in the natural unit and the conversion of the calculated results to the usual unit is given by the c.g.s. unit.

4.1.5 Input data format

The symbols for parameters and their functions for each read statement are explained in this subsection. The reading formats are shown in the square brackets. The input-data file must be formatted in the sequence shown in the following:

(1) The 1st read statement:

IWW, IAMP, NCV, ICOUL [4I2]

IWW=1: MW-representation is used in PAPH; IWW=2: BB-representation is used in PAPH.

IAMP=0: executes the χ^2-minimizing search; =1: calculates the scattering amplitudes and the observables by using the current phase shifts.

NCV=0: prints the amplitudes of the three groups; VPI & SU, Kyoto Univ. and Hiroshima Univ.. If NCV$\neq$0, they are not printed. So NCV=0 must be used when IAMP=1.

ICOUL=0: Coulomb-correction; =1: Coulomb amplitude is suppressed; =2: Coulomb correction is excluded.

(2) The 2nd read statement:

EMP, HEM [2F10.0]

EMP: nucleon mass which is taken as 938.272 MeV if defaulted.

HEM: ^{3}He mass which is taken as 2808.41 MeV.

(3) The 3rd read statement:

T, LX, IPHASE, IANG, ITEST, IWRITE, IDATA [F10.0, 6I2]

T: kinetic energy of incident nucleon in the L. S. in MeV.

LX: the boundary value of the orbital angular momentum ℓ to which the scattering amplitude is calculated with the partial wave amplitude f_ℓ.

IPHASE: the number of fixed parameters which are excluded from the searched parameters, so that it gives the number of phase-shift parameters read in by the 8th read statement.

IANG: =1 for input of the phase-shift parameters in degrees and =0 for input of them in radians.

ITEST: If it is not taken as 0, the final χ^2-value and the intermediate χ^2-value at each cycle are printed out.

IWRITE: =0 for printing the complete information about the obtained result, and =1 for a partial print.

IDATA: If it is not equal to 0, the subprogram DATAST is called to generate a data file for the next search.

(4) The 4th read statement:

IUDX, IATL, ITOTAL, IOUT, IOUT1 [I3, 4I2]

IUDX: equals to the total number of experimental groups of the used data.

IATL: specification of the unit for the differential cross section, i.e. IATL=0 for mb/str and IATL=1 for mb/(GeV)^2.

ITOTAL: =1 for an analysis with some forward data, and =0 for one without forward

data.
IOUT: =0 to print out the numbered experimental data which are needed for investigation of the M-value of data, and $\neq 0$ for no printout of them. The first summation in Eq. (28), i.e. $[(\theta_{ij} - \theta_{ij}^{ex}/\Delta\theta_{ij}^{ex}]^2$ is called as M-value.
IOUT1: =0 to print out the used data-file and $\neq 0$ for no printout of them.

(5) The 5th read statement

The 5th, 6th and 7th read statements are for reading the experimental data. In PAPH program, the input data format is same as the one used in SAID maintained by VPI-group. In this format, the data obtained by different experiments are divided into the different groups of data:

T, ND, AN(1−4), C(1−5) [F9.3,I4,4F7.3,2(1X,A4),1X,5A4]

T: kinetic energy of incident proton in the L. S. for each experimental data group.
ND: total number of data provided by each experimental groups.
AN(1): systematic error of the experiments.
AN(2−4): not used in the present version of the PAPH program.
C(2): gives the kind of observables of the experimental data where the four columns are used, that is read by A4 type using four characters. The correspondence between the symbols in PAPH and the observables is as follows:

DSG = $d\sigma/d\Omega$, AY0=proton analyzing power, A0Y = ^{3}He analyzing power, R, A, RP, AP, D, DT, RT, RTP, AT, ATP = R, A, R', A', D, D_t, R_t, R'_t, A_t, A'_t (Wolfenstein parameters), AXX, AXZ, AZX, AZZ, AYY = A_{xx}, A_{xz}, A_{zx}, A_{zz}, A_{yy}, ALL, AML, ALM, AMM = $A_{\ell\ell}$, $A_{m\ell}$, $A_{\ell m}$, A_{mm} (observables defined in the c.m.s.), SGTR, SGT, SGTT, SGTL, ALFA, REF2, REF3 = σ_r, σ_t, $\Delta\sigma_T$, $\Delta\sigma_L$, α, Re$F2$, Re$F3$ (forward observables).
C(1), C(3−5): some remarks for the experiments.

(6) The 6th read statement

COM(1−18) [18A4]

COM: comments for the each experimental data group. References of the experimental data are written here.

(7) The 7th read statement:

ANGL, EXD, DEXD [3(F7.0, F10.0, F7.0), 8X]

ANGL: the scattering angle (θ_c) in degrees if IATL=0, or the squared momentum transfer ($-t$) in (GeV/c)^2 unit if IATL=1.
EXD, DEXD: experimental value of the observable and its experimental error, respectively.

(8) The 8th read statement:

IQ, NQ, LQ, JQ, QEL [4(2I1, 2I2, 1X, F9.0)]

IQ= 0 and 1 mean for QEL to be the real phase shift δ and the imaginary one η, respectively. In this case, NQ=1 for the spin-singlet state and NQ=3 for the spin-triplet state. LQ and JQ are the orbital and the total angular momentum of phase shift QEL, respectively. IQ=3 means (1.0 + QEL) is renormalization parameters n_j

in Eq. (4.28) multiplied by the experimental data and their errors. In this case, (NQ, LQ) shows the kind of observables, and JQ its data group. The numbering of data groups must start with 0. The correspondence between the number (NQ, LQ) and the observables in PAPH is as follows:

$d\sigma/d\Omega$=1, A_{n0}=2 , A_{0n}=3, A_{ll}=4, A_{ml}=5, A_{lm}=6, A_{mm}=7, A_{xx}=8, A_{xz}=9, A_{zx}=10, A_{yy}=11, A_{zz}=12, R_T=13, R'_T=14, A_T=15, A'_T=16, D_T=17, R=18, R'=19, A=20, A'=21, D=22.
The number of QEL parameters has to be equal to IPHASE in the 3rd read statement. The values of QEL given here are fixed in the χ^2-minimizing search.

(9) The 9th read statement:

NFIRST, LANG, NOTE, J0, NCUT, DELTA, ERROR, ISTART [I3, 2I2, 1X, I2, I4, 2F8.0, 1X, I1]

NFIRST: the total number of varied parameters in χ^2-minimizing search.

LANG: specification of the unit of phase-shift parameters inputted in the next read statement, i.e. LANG=0 for radians and =1 for degrees.

NOTE: the number of the comment cards which are read in by the 11th read statement.

J0: boundary value of total angular momentum J to which the real part of the phase shifts are searched in χ^2-minimization.

NCUT: upper limit of the iteration of χ^2-minimizing search in the subroutine POWELL.

DELTA: this gives the step size in the gradient of χ^2-space, i.e. (DMAX - DMIN) / DELTA. If it is left out, DELTA=500.0 is taken. Here DMAX and DMIN are the maximum and the minimum values of the searched domain of parameters, respectively and read in by the next read.

ERROR: this gives the uncertainty C of χ^2-value at χ^2-minimum point where C=ERROR *$(N_e - N_p)$, N_e is the total number of experimental data, and N_p the total number of searched parameters. If it is left out, it is taken as 0.0001.

ISTART: = 0 for an execution of the χ^2-minimizing search and no outputs of the calculated obervables for connecting with the other program of X-Y plotting; =1: for a searching and plotting; =2: for no searching and no plotting; =3: for no searching and plotting.

(10) The 10th read statement:

IR, NR, LD, JD, DEL, DMAX, DMIN [3(2I1, 2I2, F10.0, 2F4.0)]

IR, NR, LD and JD have the same means as IQ, NQ, LQ and JQ in the 8th read statement, respectively.

DEL: starting values of the varied parameters.

DMAX, DMIN: the maximum and the minimum values of the searched domain of DEL. If they are not given, they are taken as (180., −180.) for the phase-shift parameters in degrees and (0.3, −0.3) for the experimental renormalization parameters. The total number of DEL parameters has to be equal to NFIRST in the 9th read statement.

(11) The 11th read statement:

COMMENT [9A8]

You can write some comment statements about your analysis in the number of lines specified by the parameter NOTE in the 9th read statement.

4.1.6 Application plans of PAPH

(1) The determination of p-^{3}He scattering amplitudes by PSA with incident proton energies in the laboratory system of $T_L < 10$ MeV. In this low energy region, several groups[112, 113] have already carried out PSA of p-^{3}He scattering so far. We compare our obtained solution of PAPH in the BB-representation with those of the previous works, and determine the scattering amplitudes in the MW-representation.

(2) The determination of p-^{3}He scattering amplitudes by PSA at the intermediate energies $T_L = 15 \sim 500$ MeV.

We carry out the PSA of p-^{3}He scattering with use of PAPH by the MW-representation in the intermediate energy region where the inelastic channels are opened.

(3) The development of a PSA program to determine the n-^{3}He scattering amplitudes.

Not only the p-^{3}He amplitudes but also the n-^{3}He amplitudes are needed to study the d-^{3}He scattering with the $p + n + ^3$He system. We will make the extended PAPH program possible to carry out the PSA of n-^{3}He scattering with incident neutron energies from several MeV to a few hundred MeV.

4.1.7 Additional Comment

In order to obtain a reasonable solution of PSA by using this program PAPH, the user needs not only substantial database to do it, but also enough knowledge of nuclear physics. One could not automatically obtain any reasonable solutions. As posted up in SAID presented by R. A. Arndt, one's profanity should be always noticed.