4.3 Phase-Shift Analysis of Elastic Σ^+-p Scattering

Abstract. The phase-shift analysis of elastic Σ^+-p scattering at an incident energy (Σ^+) of 12 MeV has been carried out. Two kinds of solutions have been found, for which the sign of the $^3S_1$ phase shift is different. It has been found that measurement of the polarization in Σ^+-p scattering could contribute to distinguishing among various models for YN scattering. (Submitted to Phys. Rev. C)

Hyperon-nucleon potentials have been provided by various groups[142, 143, 144, 145]. Especially, the Nijmegen potentials have been applied to hypernuclei. However, many questions have been raised. Generally speaking, vertex form factors and meson-meson correlations are not well known. This creates serious uncertainties in the potentials, which reflects the fact that the theory of the strong interaction is not yet established.

The Ehime group aims to build an effective model which would be useful for studies of hypernuclei and YN interactions, rather than a microscopic description[143, 144, 145]. One-boson-exchange potentials, 99A[144] and 00A[145], were presented for ΛN and ΣN systems. The two potentials reproduce equally well the present data of the Σ^+-p scattering. However, 99A generates a bound state in the $^3S_1$ wave, while 00A not. Thus, the nature of the forces in the two potentials is quite different. From the experimental point of view of hypernuclear physics, the existence of a bound state in the $I = 3/2$ channel is not excluded. Therefore, it is important to study the difference in other observables of Σ^+-p scattering generated by the two kinds of phase parameter sets suggested by the potentials.

On the other hand in nucleon-nucleon scattering the plentiful experimental data have been analyzed in terms of a phase shift analysis (PSA). Extension of the PSA to YN scattering is desirable. However, hyperon scattering is a very difficult experiment, and data from a polarized spin experiment has seldom been reported until now. Therefore, a phase shift analysis was not carried out. Scattering experiments involving a hyperon have been planned in some laboratories, and the possibility of a polarization experiment is high. We have examined what kind of spin observable is important for the determination of the YN scattering amplitudes and what kind of information could be obtained by using the PSA. In particular, in Σ^+-p scattering experimental data for differential and total cross sections exist at low energy and inelastic channel is not open. We performed a PSA for elastic Σ^+-p scattering at an Σ^+ incident energy $T_L^{\Sigma^+}$=12 MeV. The result of our analyis is reported below.

In Σ^+-p scattering, which involves non-identical particles, the invariant amplitudes are different from the ones in nucleon-nucleon scattering, and the M-matrix is given as a sum of 7 terms. We had previously perfomed a PSA for p-^{3}He non-identical particle scattering[146]. Our computer-code was expanded such that phase shift analysis of hyperon-nucleon scattering is practical. Because Σ^+-p scattering is an isospin 3/2 scattering process, the inelastic channel is only a result of particle production at $T_L^{\Sigma^+} \geq 147$ MeV. This is the threshold of the lowest inelastic channel. Therefore, all of the absorption coefficients are taken as 1 at $T_L^{\Sigma^+}$=12 MeV, since only elastic scattering is possible. The corresponding impact parameter b for $P(\ell = 1)$ wave is $b \sim 3.74$ fm $T_L^{\Sigma^+}$=12 MeV. It becomes 6.48 fm and 9.16 fm for $D(\ell = 2)$ and $F(\ell = 3)$ waves, respectively. A stabilized solution was not obtained in the preliminary analysis in which the D wave was included. Therefore, the number of the parameter was decreased, and partial waves with orbital angular momentum $\ell \leq 1$ were redetermined in

Table 4.3: Experimental data for elastic Σ^+-p scattering below $P_L^{\Sigma^+}$=4 GeV/c.

Obs.	$P_L^{\Sigma^+}$ (MeV/c)	Total number of data (events)	References
σ_t	100 - 1900	1 (10)	ST61[147]
	140 - 150	1 (4)	EI71[148]
	140 - 175	1 (9)	RU67[149]
	148 - 158	1	DO66[150]
	150 - 160	1 (13)	EI71
	158 - 168	1	DO66
	160 - 170	1 (35)	EI71
	168 - 178	1	DO66
	170 - 180	1 (69)	EI71
	500 - 1500	1 (10)	CH70[151]
	1500 - 2500	1 (8)	CH70
	2500 - 4000	1 (4)	CH70
$d\sigma/d\Omega$	148 - 178	6 (30)	DO66
	160 - 180	7 (156)	EI71
	300 - 600	2 (11)	AH99[152]

order to obtain a stabilized solution. The existing experimental data are summarized in Table 4.3. Eisele et al. (P_L = 170 MeV/c, T_L=12 MeV) is the most precise data for $d\sigma/d\Omega$. The polarization rate of the decay of the Σ^+ to $p\pi^0$ was also measured by Eisele et al., and they reported a value of P (polarization)=0.0 ± 0.16, and argued that the contribution of a P wave at this energy would be very small. The experimental data on the total cross section (σ_t) is plotted in Fig. 4.7. From Fig. 4.7 σ_t is around 89 mb at $P_L^{\Sigma^+}$= 170 MeV/c.

We selected the energy point $T_L^{\Sigma^+}$= 12 MeV ($P_L^{\Sigma^+}$=170 MeV/c) to perform the PSA of elastic Σ^+-p scattering considering the situation of the experimental data. The number of data used in the phase shift analysis was 8, including the data of Eisele et al. for the differential cross section and of Dosch et al. for the total cross section.

In the Ehime model by Tominaga[144] and Ueda[145], two possibilities were indicated for the $\delta(^3S_1)$ phase shift, and they are respectively about 170(deg)[144] and -10(deg)[145]. There is a difference of π in the phase shift, and there seems to be no difference in the representation of the experimental data. However, a difference appears in the spin observables involving the mixing parameter, since the S-matrix for coupled waves between $\ell = J-1$ and $\ell = J+1$ in the spin triplet states[146] are given by,

$$S_J = \begin{pmatrix} \sqrt{1-|\rho_J^+|^2}\exp(2i\delta_-) & i\rho_J^+\exp\{i(\delta_-+\delta_+)\} \\ i\rho_J^+\exp\{i(\delta_-+\delta_+)\} & \sqrt{1-|\rho_J^+|^2}\exp(2i\delta_+) \end{pmatrix}. \tag{4.43}$$

Here ρ_J^+ is the mixing parameter for the coupled channels of the spin triplet waves. δ_- and δ_+ are the phase shifts for $\ell = J-1$ and $\ell = J+1$. J and ℓ are the total and orbital angular momentum, respectively. The difference of π in the phase shift for the $^3S_1(\ell = J-1)$ wave influences only the off-diagonal elements in this equation.

In YN scattering, the spin singlet state also couples to the spin triplet state in the case

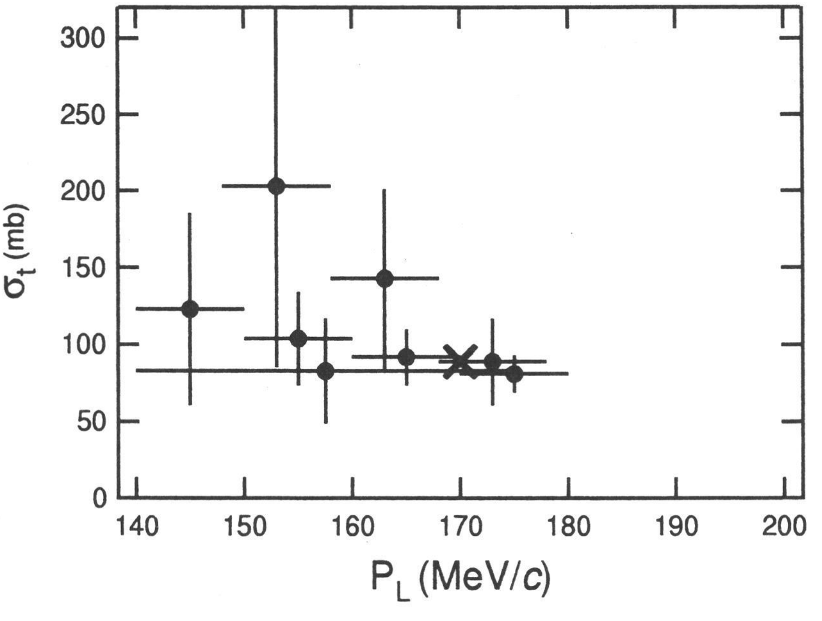

Figure 4.7: The total cross section of elastic Σ^+-p scattering[147, 148, 149, 150, 151]. The cross ($\times$) shows the values predicted by the present PSA

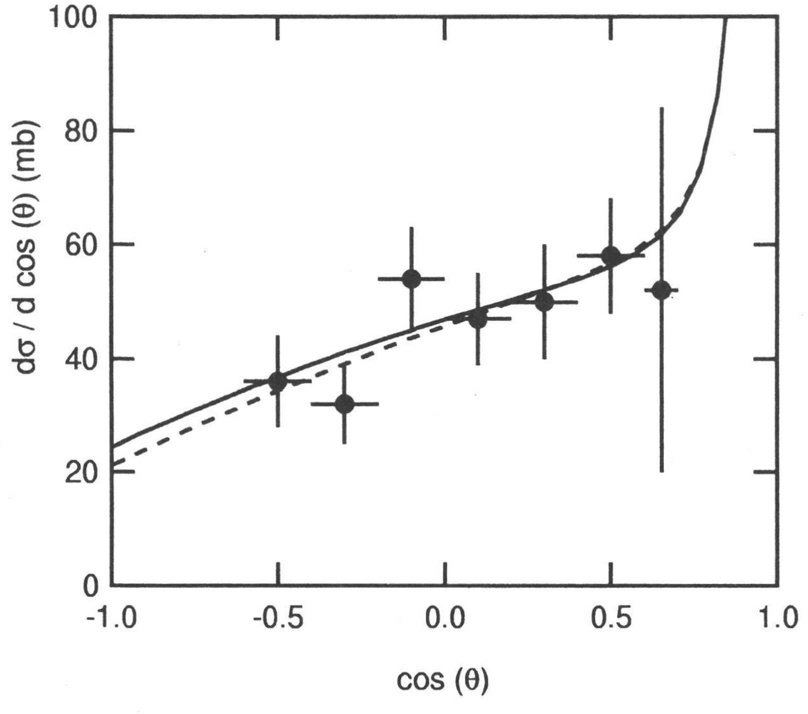

Figure 4.8: The $d\sigma/d\cos(\theta)$ of elastic Σ^+-p scattering at T_L= 12 MeV[152]. The solid and dotted lines show the values predicted by Sols. α and β, respectively.

Table 4.4: The solutions determined by the present PSA.

Partial Waves	δ (deg) and $\rho_J^{\pm}$	
	(α)	(β)
$^1S_0$	26.97	26.01
$^3S_1$	172.84	−7.42
$^3P_0$	−3.72	1.09
$^1P_1$	1.88	3.07
$^3P_1$	−0.17	−0.99
$^3P_2$	−0.05	−0.26
ρ_1^-	−0.2501	−0.2501
ρ_1^+	0.1774	0.1774
χ^2	2.96	2.59

of $\ell = J$ as follows,

$$S_J^{ST} = \begin{pmatrix} \sqrt{1-|\rho_J^-|^2}\exp(2i\delta_J) & i\rho_J^- \exp\{i(\delta_J+\delta_{J,J})\} \\ i\rho_J^- \exp\{i(\delta_J+\delta_{J,J})\} & \sqrt{1-|\rho_J^-|^2}\exp(2i\delta_{J,J}) \end{pmatrix}, \tag{4.44}$$

where ρ_J^- is the mixing parameter for the coupled channels of the spin singlet and triplet states. δ_J and $\delta_{J,J}$ are the phase shifts of the spin singlet and triplet states with $\ell = J$. The difference of π in the phase shift for the $^3S_1$ wave does not influence S_J^{ST} since there is no spin singlet state which couples to the $^3S_1$ wave.

In the present PSA, we searched solutions where $\delta(^3S_1)$ became close to the value suggested by the Ehime model. Two kinds of solutions (α and β) were obtained. By fixing the mixing parameters as the values of Sol. α, Sol. β was obtained. The solutins obtained are given in Table 4.4. Here, $\chi^2 = \sum_{i,j}\{(\theta_{i,j}^{ex} - \theta_{i,j}^{th})/\Delta\theta_{i,j}^{ex}\}^2$, where $\theta_{i,j}^{ex}$ is the experimental datum for observable i from the jth experiment, with experimental error $\Delta\theta_{ij}^{ex}$, and θ_{ij}^{th} is its theoretical value.

The corresponding predicted values of various spin observables are given in Figs. 4.8, 4.9 and 4.10. Here, we took the the mixing parameters in the Sols. α and β as the same values to examine how the difference of the $\delta(^3S_1)$ appears in the spin observables. In A_{yy} and D_{NN} the mixing parameters ρ_J^- are included only in the form of $|\rho_J^-|^2$, and there is no effect from the difference of the sign. Such a consideration is useful to examine the influence of $\delta(^3S_1)$, because the uncertainty of the sign can be disregarded in the observables. There are large differences in A_{yy}, A_{mm}, A, D_{NN} and D_{LS} between Sols. α and β. The experiments of A_{0y}, D_{NN}, D_{LS}, D_{SL}, D_{SS} and D_{LL}, in which the polarization quantity of target and recoil protons are measured, would be performed more precisely than other experiments in which the polarization of Σ^+ has to be detected.

In conclusion, we have carried out the phase shift analysis of elastic Σ^+-p scattering at $T_L^{\Sigma^+}$=12 MeV using the data on total and differential cross sections. Here, analysis was done

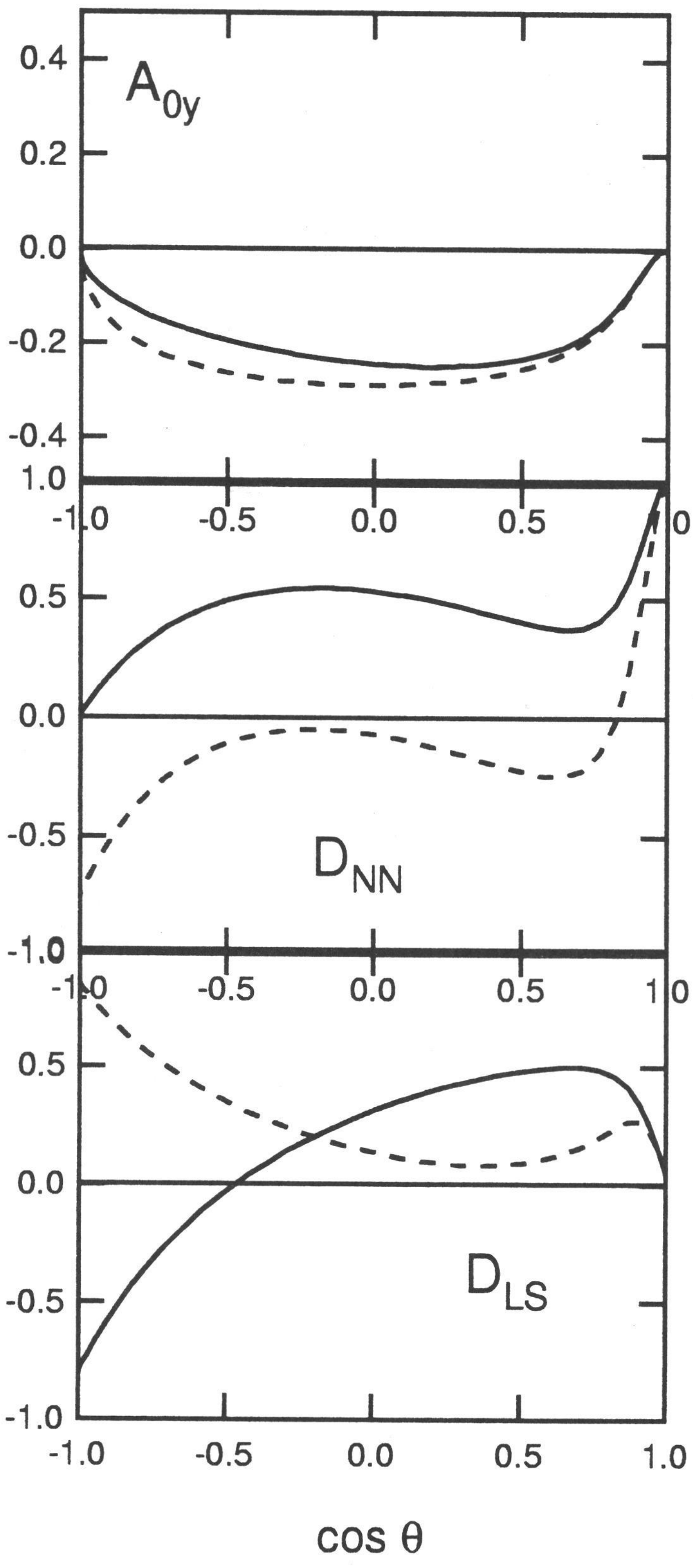

Figure 4.9: The spin observables (A_{0y}, D_{NN} and D_{LS}) of elastic Σ^+-p scattering at T_L=12 MeV predicted by present PSA. The solid and dotted lines show the values predicted by Sols. α and β, respectively.

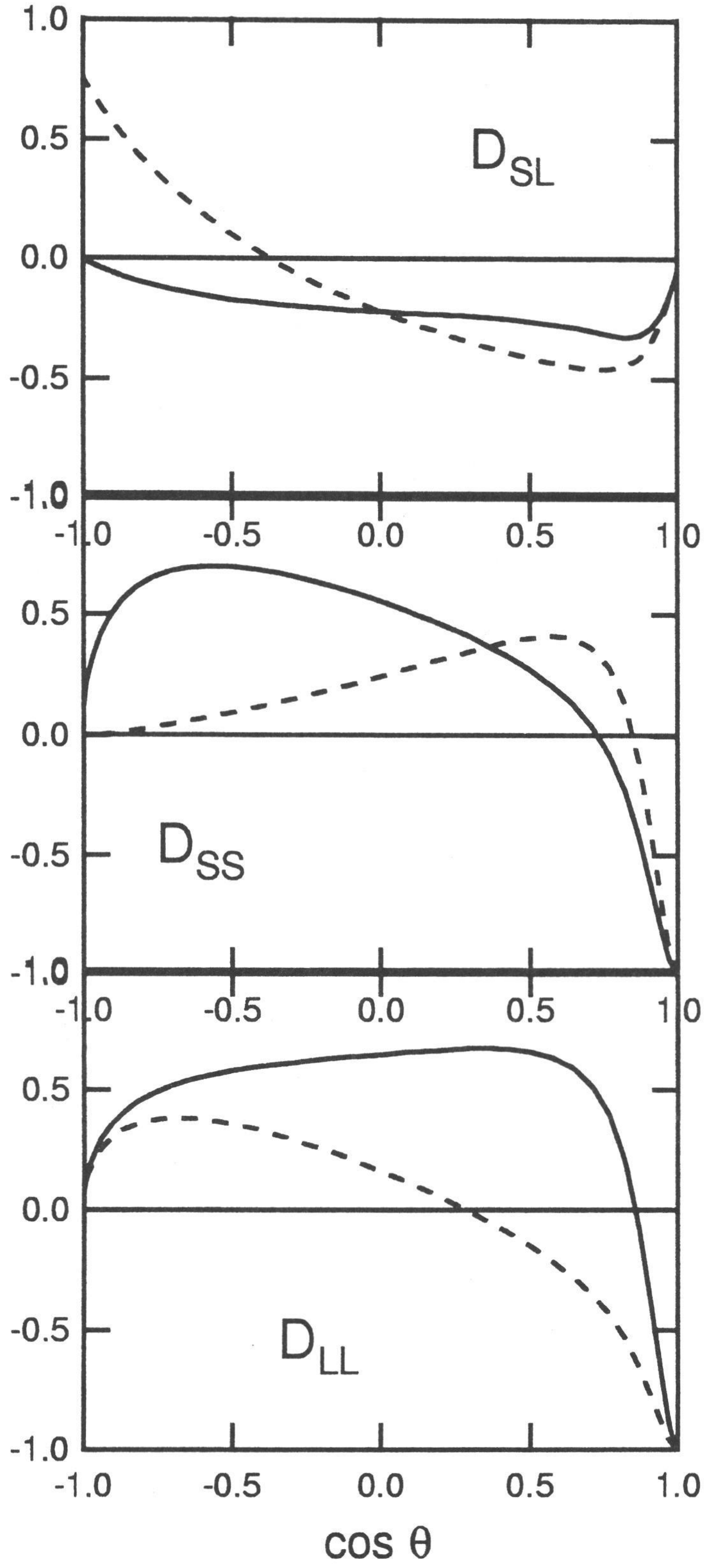

Figure 4.10: The spin observables (D_{SL}, D_{SS} and D_{LL}) of elastic Σ^+-p scattering at T_L=12 MeV predicted by present PSA. The solid and dotted lines show the values predicted by Sols. α and β, respectively.

without any approximation on the mass differences of particles unlike other model analyses. The phase shift of the S and P waves were determined, and the values of various spin observables were calculated by two kinds of solutions. For the determination of $\delta(^3S_1)$, it has been found that measurements of observables which were influenced largely by mixing parameters were very useful. The phase shift analysis of elastic Σ^+-p scattering becomes more interesting in the energy region where the P and D waves have large contributions. The experiment of KEK[152] corresponds to $T_L^{\Sigma^+} \sim 80$ MeV, and the contributions from these waves are to be expected. A more detailed discussion about $\delta(^3S_1)$ will be possible if the data on spin observables become available in this energy region.

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