Finite-size effect on phase structure in massive Gross–Neveu model

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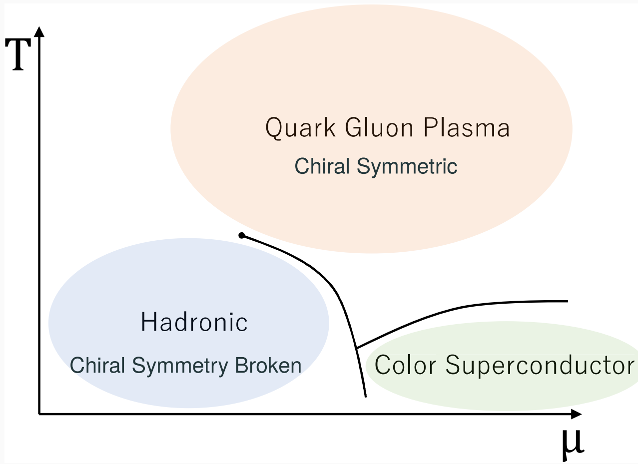
Outline: Introduction Model (Brief review) Result Summary

Introduction

Schematic QCD Phase Diagram



Introduction

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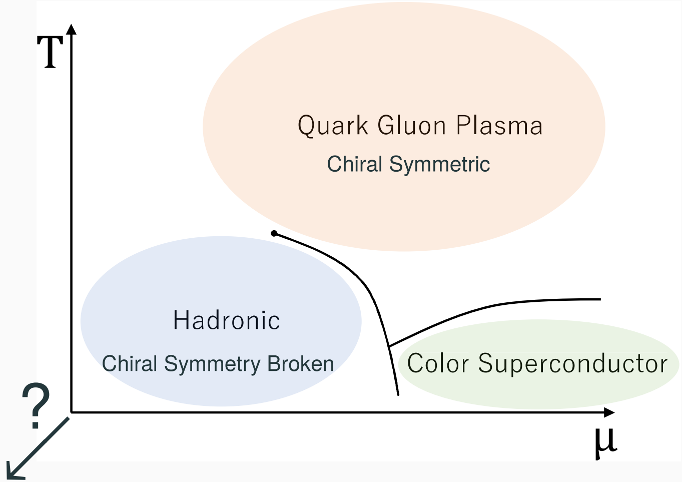

It is well known that the environment changes the QCD phase structure.

⟶ Boundary condition, Gravity, Magnetic field, Mass, Rotation, Volume, ...

Model

Gross–Neveu model D. J. Gross, and A. Neveu, Phys. Rev. D 10 (1974) 3235

Bare coupling

$$S = \int \mathrm{d}^D x \left[\bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) + \frac{\lambda_0}{2N} \left(\bar{\psi}(x)\psi(x)\right)^2\right]$$

The number of copies

$$\to \int \mathrm{d}^D x \left[\bar{\psi}(x) \left(i\gamma^\mu \partial_\mu - \sigma(x)\right) \psi(x) - \frac{N}{2\lambda_0} \sigma(x)^2\right]$$

Auxiliary field $\sigma(x) \sim -\frac{\lambda_0}{N}\bar{\psi}(x)\psi(x)$ (order parameter $\langle\sigma(x)\rangle$)

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Auxiliary field $\sigma(x) \sim -\frac{\lambda_0}{N} \bar{\psi}(x)\psi(x)$ (order parameter $\langle \sigma(x) \rangle$)

- Discrete chiral symmetry $\mathbb{Z}_2$: $\psi(x) \rightarrow \gamma^5 \psi(x)$
  $\rightarrow$ Spontaneous symmetry breaking of the chiral symmetry
- Asymptotic freedom
- Renormalizable ($D = 2$, and $D = 3$ in the leading order of the $1/N$ expansion)

Massive model

Bare mass m_0 (explicitly symmetry breaking term)

$$S = \int \mathrm{d}^D x \left[\bar{\psi}(x) \left(i\gamma^\mu \partial_\mu - m_0 - \sigma(x) \right) \psi(x) - \frac{N}{2\lambda_0} \sigma(x)^2 \right]$$

- Assumption: Homogeneous chiral condensate $\langle \sigma(x) \rangle = \mathrm{const.}$
- Evaluate the leading order of the $1/N$ expansion

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Effective potential

$$V_D(\sigma) = \frac{\sigma^2}{2\lambda_0} - \int \frac{\mathrm{d}^D k}{(2\pi)^D} \mathrm{tr} \ \ln(k_\mu \gamma^\mu - m_0 - \sigma)$$

$$\rightarrow \frac{\sigma^2 - 2m_0\sigma}{2\lambda_0} - \int \frac{\mathrm{d}^D k}{(2\pi)^D} \mathrm{tr} \ \ln(k_\mu \gamma^\mu - \sigma)$$

SHIFT $\sigma \rightarrow \sigma - m_0$, and drop the constant term m_0^2/λ_0 A. Barducci, et al. (1995)

Renormalization

Effective potential

$$\frac{\operatorname{tr} I}{(4\pi)^{D/2}}\Gamma\left(1-\frac{D}{2}\right) \sim \frac{1}{D-2} \text{ for } D \to 2$$

$$V_D(\sigma) = \frac{\sigma^2 - 2m_0\sigma}{2\lambda_0} - \frac{C_D}{D}\left(\sigma^2\right)^{D/2}$$

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Renormalization procedures with a renormalization scale μ_r

$$\left.\frac{\partial^2 V_D(\sigma)}{\partial\sigma^2}\right|_{\sigma=\mu_r} = \frac{\mu_r^{D-2}}{\lambda}, \qquad \left.\frac{\partial V_D(\sigma)}{\partial\sigma}\right|_{\sigma=0} = -\frac{\mu_r^{D-2}}{\lambda} m$$

λ: Renormalized coupling

m: Renormalized mass parameter

Renormalization of the coupling and the mass in $2 \le D < 4$

with no divergence for $D \to 2$

Dynamically generated fermion mass

Renormalized effective potential

$$V_D(\sigma) = \frac{1}{2}\left(\frac{1}{\lambda} + C_D(D-1)\right)\sigma^2\mu_r^{D-2} - \frac{m}{\lambda}\sigma\mu^{D-2} - \frac{C_D}{D}\left(\sigma^2\right)^{D/2}$$

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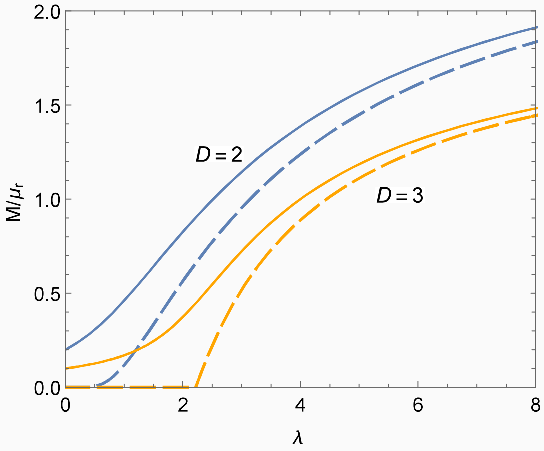
The gap equation

$$\left.\frac{\partial V_D(\sigma)}{\partial\sigma}\right|_{\sigma=M} = 0$$

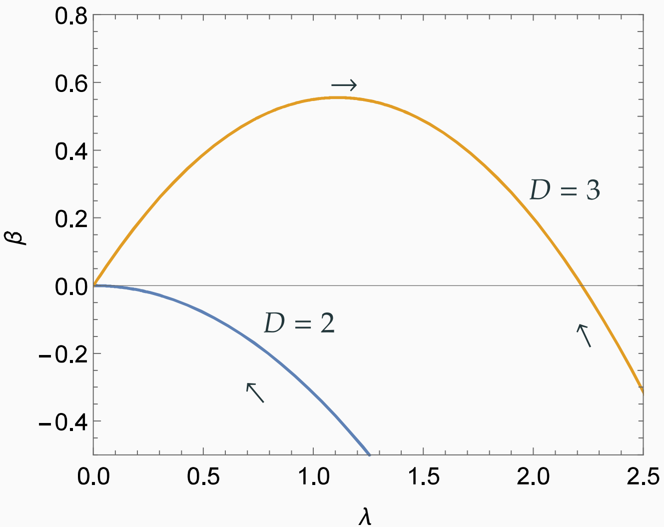
gives the dynamically generated fermion mass, M.

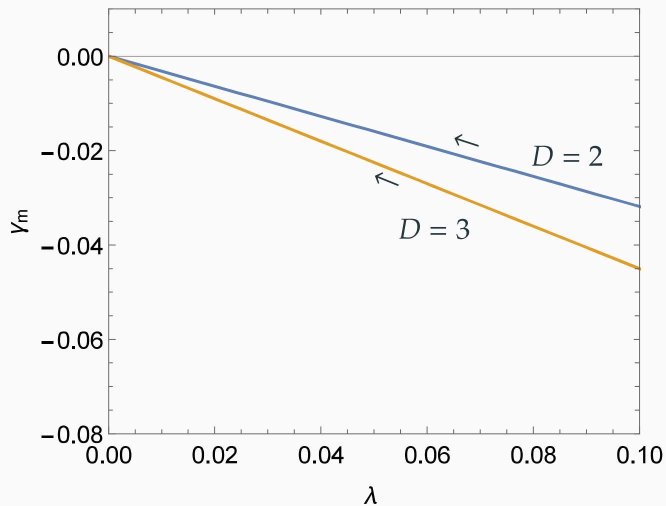


Running coupling and mass

Renormalization group equation

$$\frac{\mathrm{d}V_D(\sigma;\lambda,m,\mu_r)}{\mathrm{d}\mu_r} = \mu_r\frac{\partial V_D(\sigma;\lambda,m,\mu_r)}{\partial \mu_r} + \beta(\lambda)\frac{\partial V_D(\sigma;\lambda,m,\mu_r)}{\partial \lambda} + m\gamma_m(\lambda)\frac{\partial V_D(\sigma;\lambda,m,\mu_r)}{\partial m} = 0$$





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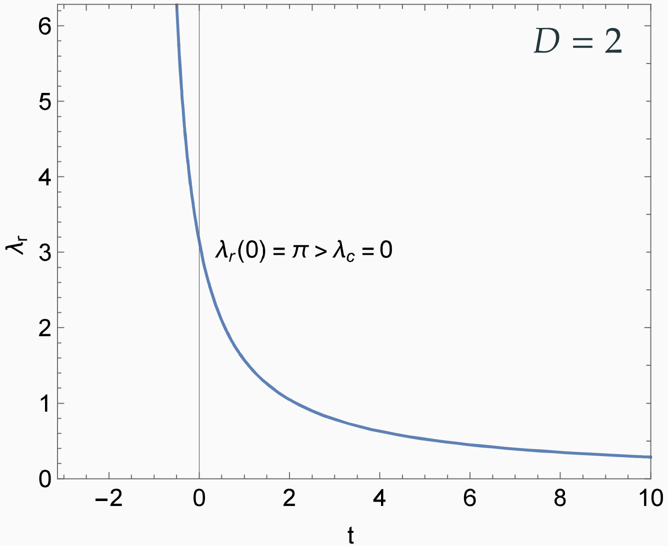
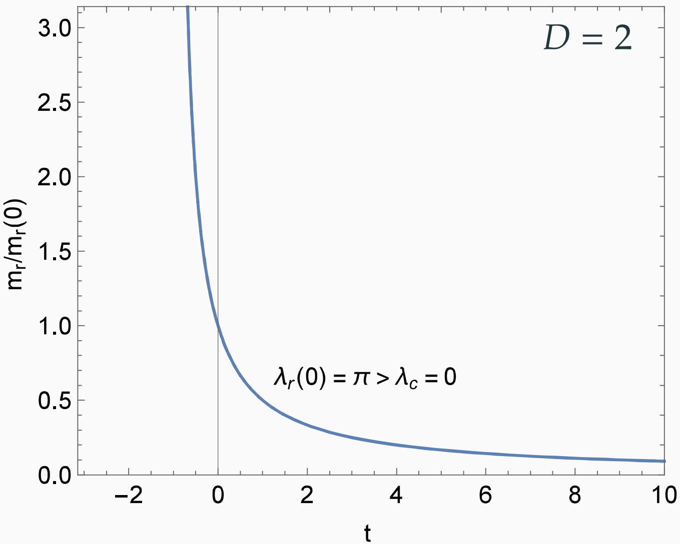
$t = \ln(\mu_r/\mu_{r,0})$





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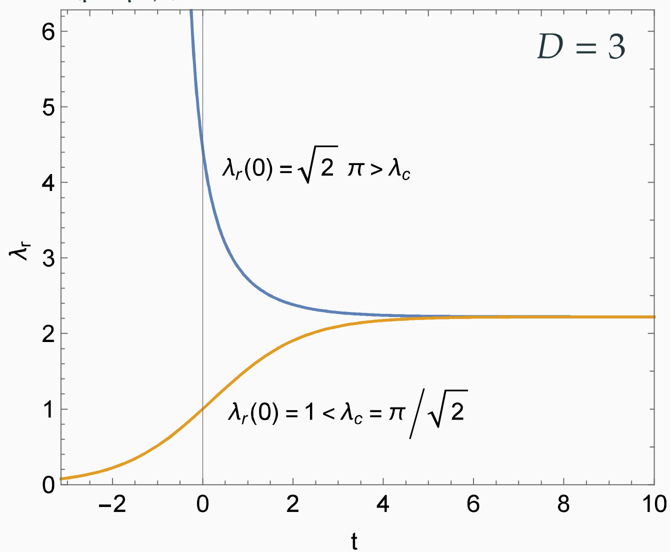

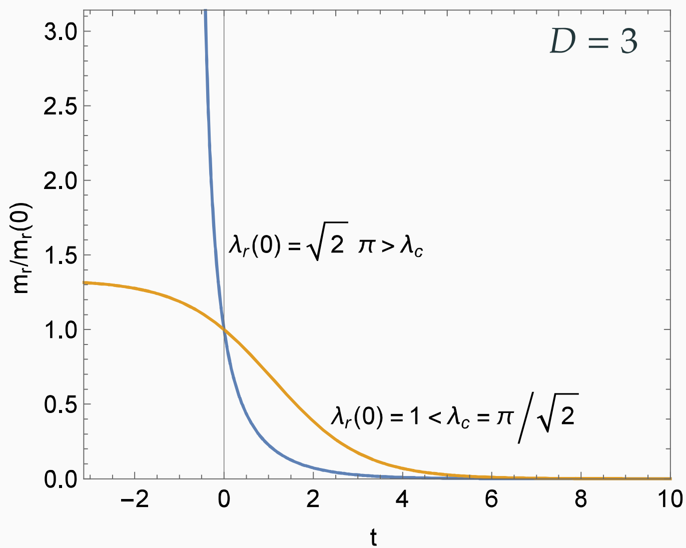

Setup: Finite size, temperature, and chemical potential

Renormalized effective potential

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$\ln\det$ term does NOT include explicitly the mass term!

$\longrightarrow$ Can apply easily the extensions to the massive model.

(finite size, temperature, and chemical potential)

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(finite size, temperature, and chemical potential)

- Background spacetime: $S^1 \times S^1$, $\mathbb{R} \times S^1 \times S^1$ finite size and finite temperature
- U(1)-valued boundary condition with the length of the circle, L:

$$\psi\left(x^1, \ldots, x^{D-1} + L, x^D\right) = e^{-2i\pi\delta}\psi\left(x^1, \ldots, x^{D-1}, x^D\right)$$

- Chemical potential (finite density): $\mu\psi(x)^\dagger\psi(x)$

Setup

Do **the minimum (or extrema) search** for six parameters: $\lambda, m, L, \delta, \beta, \mu$

Plot phase diagrams by using

- Dynamically generated fermion mass M
- Susceptibility $\chi_M = \frac{\partial M}{\partial m}$
- Extrema of the renormalized effective potential

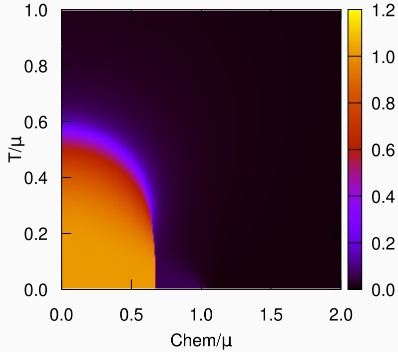
Here we set the coupling λ so that $M/\mu_r = 1$

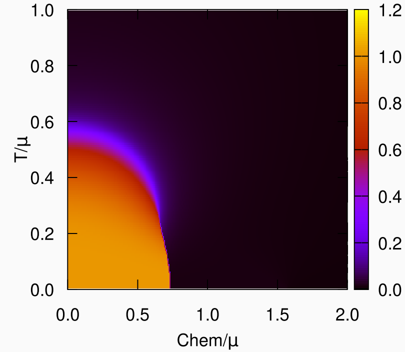
under the trivial condition: $m = \mu = 0$ and $L, \beta \to \infty$

Result: Dynamically generated fermion mass on the μ-T plane

$L\mu_r = 6.0$

$m/\mu_r = 0.01$





Periodic B. C. $\delta = 0.0$ Antiperiodic B. C. $\delta = 0.5$

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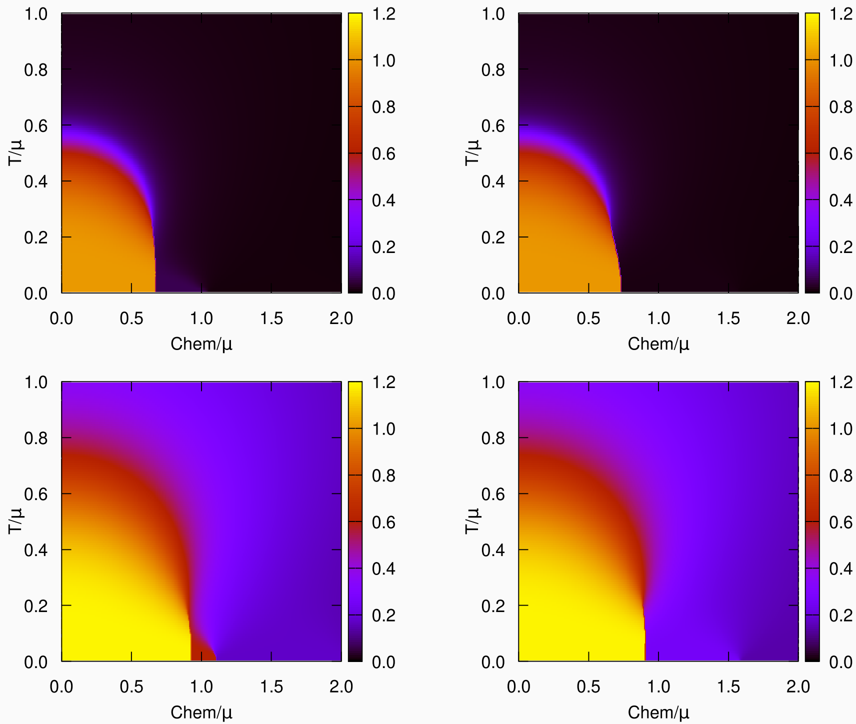
$m/\mu_r = 0.01$

$m/\mu_r = 0.2$



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Black line: 1st-order

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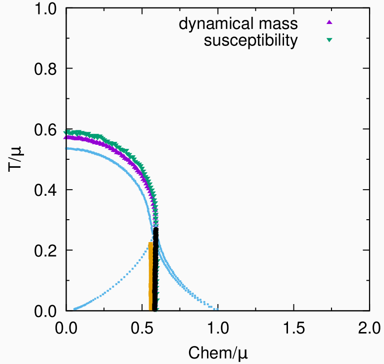

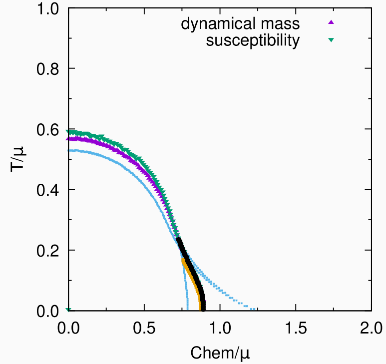

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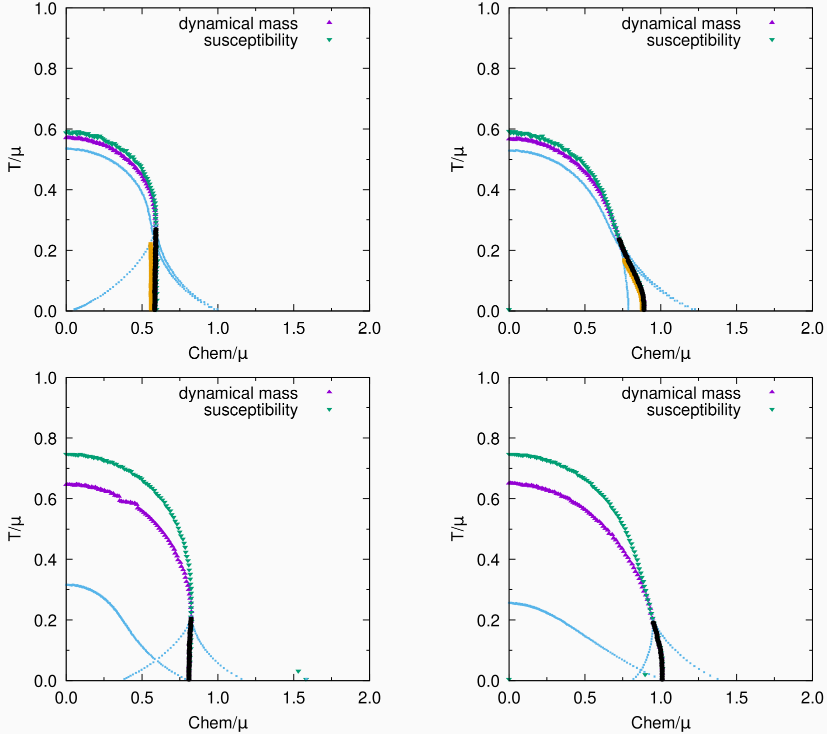

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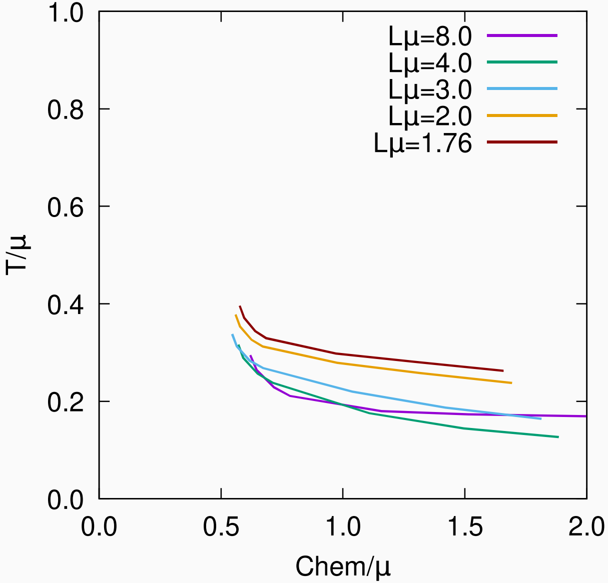
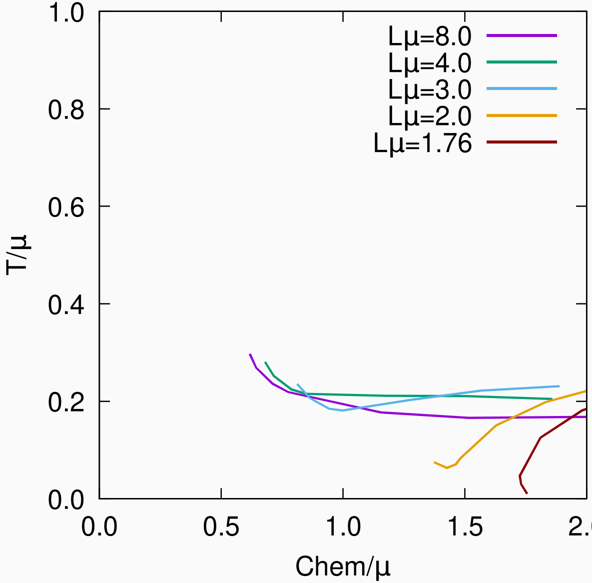
Result: Critical end-point

$1.0 \times 10^{-3} \leq m/\mu_r \leq 1.6$



Periodic boundary condition $\delta = 0.0$



Antiperiodic boundary condition $\delta = 0.5$

Summary

- Analyzed **the Massive Gross–Neveu model** at finite temperature and chemical potential.
- Compactified a spacial direction to S^1, and assigned the $\mathrm{U}(1)$-valued boundary condition to the fermionic field.
- Evaluated the effective potential in the leading order of the $1/N$ expansion under the assumption the chiral condensate is homogeneous.

With the renormalized coupling and mass for $2 \leq D < 4$, we showed the finite-size effect on the phase diagrams on the μ-T plane. And also, we showed the behavior of the critical end-point with the change of the mass for each the size of the system and the boundary condition.