Renormalization Group Effects on Rank Degenerate Matrices in the Type-I Seesaw Model

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with
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arXiv: in preparation

Outline

- Introduction
  - Massive Neutrinos
  - Type-I Seesaw Model
- 3-2 Type-I Seesaw
- Rank Degenerate, 3-3 Type-I Seesaw
- Conclusion

Introduction

- The Standard Model is our best theory for 3 fundamental forces of nature
  - However, it is not a complete model

the Standard Model formulates neutrinos as massless.
However, massive neutrinos are required to explain flavor oscillations

Introduction

- Can we have a single model that explains all of these questions?

Which type of mass do neutrinos have?

$\overline{\nu_R} m_D \nu_L$

Dirac Mass

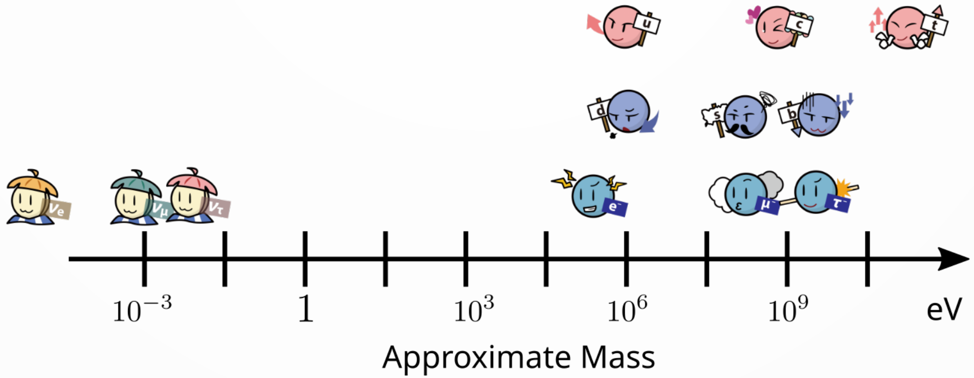
$\frac{1}{2} m_M \overline{\nu_L^c} \nu_L$

Majorana Mass

Is the lightest neutrino zero mass?

$(m_{\nu_e}^{(\mathrm{eff})})^2 < 1.1\mathrm{eV}^2$

Why does a large scale hierarchy exist?



PLANK, 2018
PDG, 2021

Type-I Seesaw Model

- Type-I Seesaw Model is an extension of the SM
  - Introduced to explain Neutrino masses

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_N$$

$$\mathcal{L}_N = \frac{1}{2}\overline{N_n} i\partial\!\!\!/ N_n - y_{\nu n}\overline{l_L}\widetilde{H}N_n - \frac{1}{2}\overline{N_n^C}M_{R\,n}N_n$$

new right-handed singlet neutrinos

neutral under SM gauge symmetries

[Minkowski, P. 1977]
[Yanagida, T. 1979]
[Gell-Mann, M. Etc 1979]
[Glashow, S. L. 1980]
[Mohapatra, R. N. 1980]
[Fukugita, M. & Yanagida, T. 1986]

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$$\mathcal{L}_N = \frac{1}{2}\overline{N_n} i\partial\!\!\!/ N_n - \underline{y_{\nu n}\overline{l_L}\widetilde{H}N_n} - \underline{\frac{1}{2}\overline{N_n^C}M_{R\,n}N_n}$$

SM-like Yukawa coupling to left-handed lepton doublets

Majorana mass

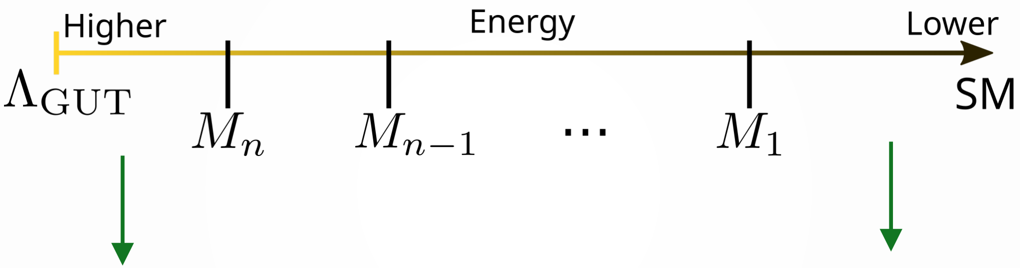
So we have both Dirac mass and Majorana mass

[Minkowski, P. 1977]
[Yanagida, T. 1979]
[Gell-Mann, M. Etc 1979]
[Glashow, S. L. 1980]
[Mohapatra, R. N. 1980]
[Fukugita, M. & Yanagida, T. 1986]

Type-I Seesaw Model

- The new right-handed neutrinos are heavy
  - Right-handed neutrino mass is close to GUT energy scales

We do not consider what occurs above the cut-off scale



High Energy
so,
'Full' Theory

Low Energy Experimental Regions
so,
Low Energy Effective Theory

Quantum Effects occur between energy regions

Apply Renormalization Group Methods (RGEs) to connect regions

Type-I Seesaw Model

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  - Right-handed neutrino mass is close to GUT energy scales

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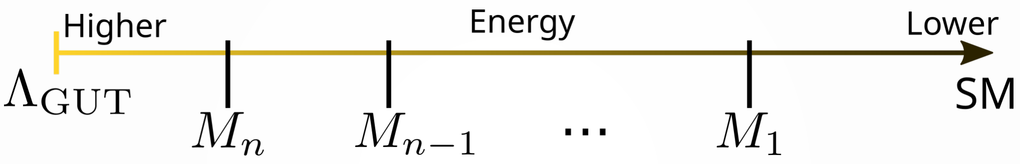
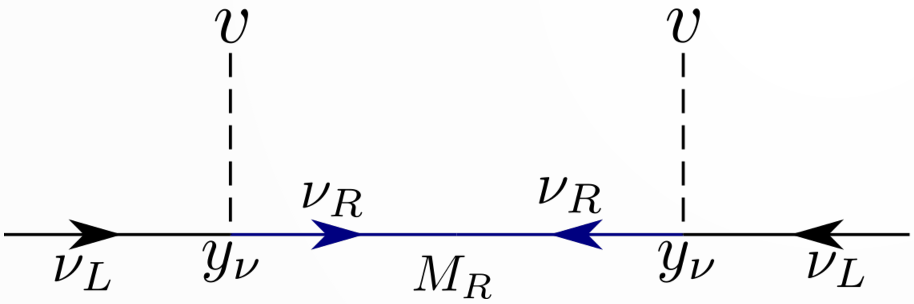
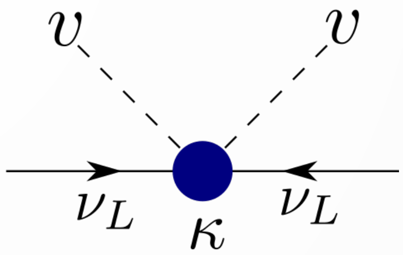


'Full' Theory

Low Energy Effective Theory





$$m_{eff}(\mu) = -\frac{v^2}{2} y_\nu(\mu) \frac{1}{M_R(\mu)} y_\nu^T(\mu)$$

$$m_{eff}(\mu) = -\frac{v^2}{4} \kappa(\mu)$$

[Weinberg, S. 1979]

μ: renormalization scale

Type-I Seesaw Model

- For our work we are interested in the left-handed neutrino masses
  - Effective mass is renormalization scale dependent

$$m_{eff}(\mu) = -\frac{v^2}{4}\underline{\kappa(\mu)} - \frac{v^2}{2} y_\nu(\mu) \underline{\frac{1}{M_R(\mu)}} \underline{y_\nu^T(\mu)}$$

Neutrino Yukawa coupling

Weinberg Operator

Right-handed Majorana mass

[Antusch, S. Etc 2001]
[Antusch, S. Etc 2003]
[Antusch, S. Etc 2005]

Type-I Seesaw Model

$$m_{eff}(\mu) = -\frac{v^2}{4}\kappa(\mu) - \frac{v^2}{2} y_\nu(\mu)\frac{1}{M_R(\mu)} y_\nu^T(\mu)$$

- For our work we are interested in the left-handed neutrino masses
  - Focus on the one-loop RGEs for,

$$t \equiv \log \frac{M_{Rn}}{\mu}$$

Weinberg Operator

$$-16\pi^2 \frac{d\kappa(t)}{dt} = (y_\nu(t) y_\nu(t)^\dagger)\kappa(t) + \kappa(t)(y_\nu(t) y_\nu(t)^\dagger)^T + \alpha_\kappa(t)\kappa(t)$$

Neutrino Yukawa coupling

$$-16\pi^2 \frac{dy_\nu(t)}{dt} = \left(\frac{3}{2}(y_\nu(t) y_\nu(t)^\dagger) + \alpha_{y_\nu}(t)\right) y_\nu(t)$$

Right-handed Majorana mass

$$-16\pi^2 \frac{dM_R(t)}{dt} = \left(y_\nu(t)^\dagger y_\nu(t)\right)^T M_R(t) + M_R(t) y_\nu(t)^\dagger y_\nu(t)$$

[Antusch, S. Etc 2001]
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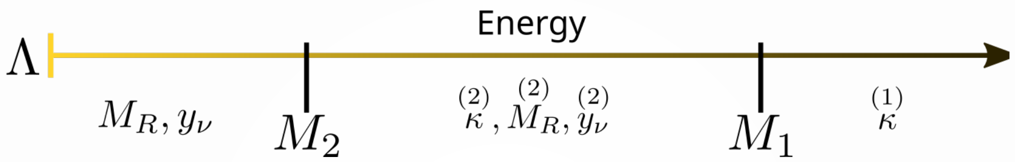
3-2 Type-I Seesaw

- We start with 3-2 as it is the simplest model still allowed by experiments



  - 2 Right-Handed neutrinos
- The zero mass holds for all energy scales as we will show

$$\text{Rank}[m_{eff}] = 2$$

$$\text{diag}[m_{eff}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

$\longrightarrow$

$$\text{Rank}[\overset{(1)}{m}_{eff}] = 2$$

$$\text{diag}[\overset{(1)}{m}_{eff}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \overset{(1)}{m}_2 & 0 \\ 0 & 0 & \overset{(1)}{m}_3 \end{pmatrix}$$

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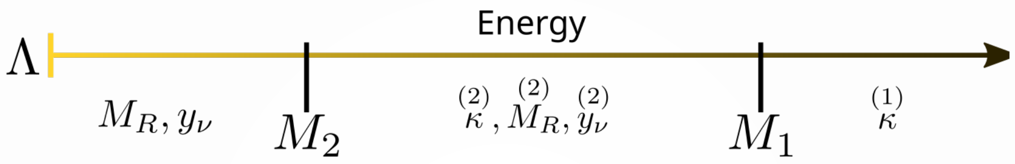


- Start with high-energy rank calculation

$$m_{eff}(0) = -\frac{v^2}{2} y_\nu(0) \frac{1}{M_R(0)} y_\nu^T(0)$$

Explicitly the Rank can be determined via two methods for a square, complex effective mass matrix

$$m_{eff} = U D_{eff} U^\dagger$$ decomposition of the matrix

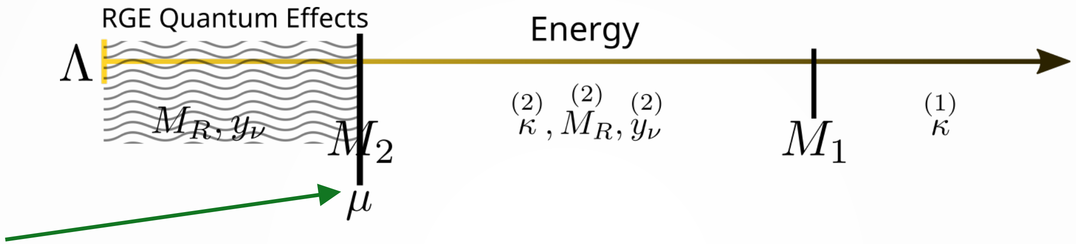
$$p_{eff} = \det(m_{eff}^\dagger m_{eff} - \lambda \mathbf{1})$$ Solving the characteristic polynomial

$$\mathrm{Rank}[m_{eff}] = 2$$

$$\longrightarrow \quad \mathrm{diag}[m_{eff}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

3-2 Type-I Seesaw

- How to calculate, start at energies equal to heaviest neutrino mass



Here we must integrate out the heaviest neutrino

Then we match the effective mass matrices

$$m_{eff}(0) = \overset{(2)}{m}_{eff}(0)$$

$$-\frac{v^2}{2} y_\nu(0) \frac{1}{M_R(0)} y_\nu^T(0) = -\frac{v^2}{4} \overset{(2)}{\kappa}(0) - \frac{v^2}{2} \overset{(2)}{y_\nu}(0) \frac{1}{\overset{(2)}{M_R}(0)} \left(\overset{(2)}{y_\nu}(0)\right)^T$$

Now we have new RGEs

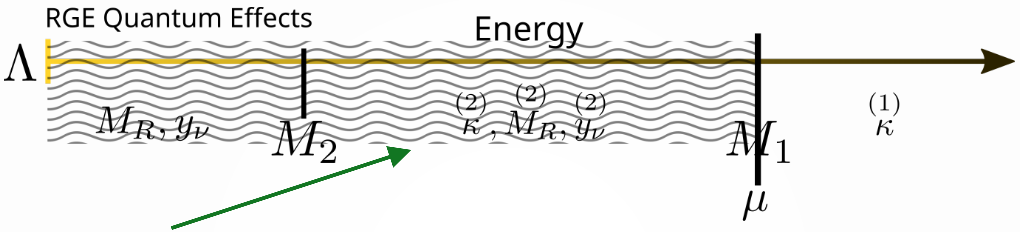
$$-16\pi^2 \frac{d\overset{(2)}{\kappa}(t)}{dt}$$

$$-16\pi^2 \frac{d\overset{(2)}{y_\nu}(t)}{dt}$$

$$-16\pi^2 \frac{d\overset{(2)}{M_R}(t)}{dt}$$

3-2 Type-I Seesaw

- **Continue integrating energy until next heavy neutrino**



Solutions to the RGEs in this region are given as

$$-16\pi^2 \frac{d\overset{(2)}{\kappa}(t)}{dt} \qquad \overset{(2)}{\kappa}(t) = \exp\left[-\frac{1}{16\pi^2}\int_0^t \overset{(2)}{\alpha}_\kappa(s)ds\right]\overset{(2)}{W}(t)\overset{(2)}{\kappa}(0)\overset{(2)}{W}(t)^T$$

$$-16\pi^2 \frac{d\overset{(2)}{y_\nu}(t)}{dt} \longrightarrow \overset{(2)}{y_\nu}(t) = \exp\left[-\frac{1}{16\pi^2}\int_0^t \overset{(2)}{\alpha}_{y_\nu}(s)ds\right]\overset{(2)}{U}(t)\overset{(2)}{y_\nu}(0)$$

$$-16\pi^2 \frac{d\overset{(2)}{M_R}(t)}{dt} \qquad \overset{(2)}{M_R}(t) = \exp\left[-\frac{1}{8\pi^2}\int_0^t \overset{(2)}{y_\nu}(s)^\dagger \overset{(2)}{y_\nu}(s)ds\right]M_1(0)$$

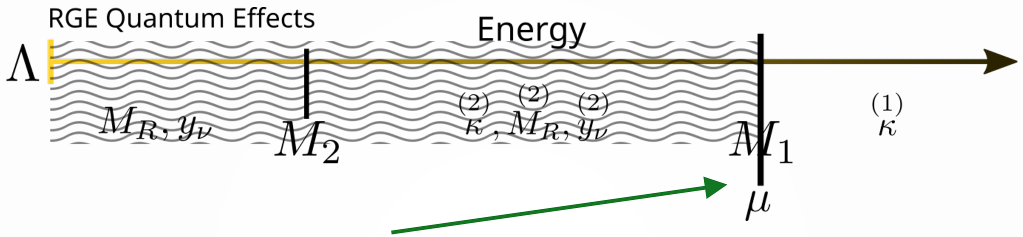
RGEs Matrix Kernel Solutions

$$\overset{(2)}{W}(t) \equiv T\exp\left[-\frac{1}{16\pi^2}\int_0^t \overset{(2)}{y_\nu}(s)\overset{(2)}{y_\nu}(s)^\dagger ds\right]$$

$$\overset{(2)}{U}(t) \equiv T\exp\left[-\frac{1}{16\pi^2}\int_0^t \frac{3}{2}\overset{(2)}{y_\nu}(s)\overset{(2)}{y_\nu}(s)^\dagger ds\right]$$

3-2 Type-I Seesaw

- Continue integrating energy until next heavy neutrino



Here we integrate the last heavy neutrino out

Then we match the effective mass matrices

$$\overset{(2)}{m}_{eff}(t_1) = \overset{(1)}{m}_{eff}(t_1)$$

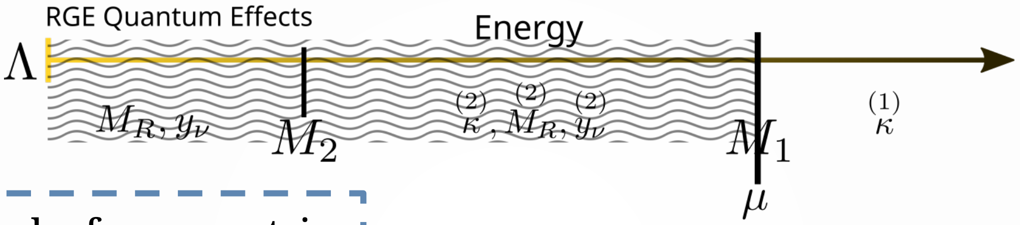
$$-\frac{v^2}{4}\overset{(2)}{\kappa}(t_1) - \frac{v^2}{2}\overset{(2)}{y_\nu}(t_1)\frac{1}{\overset{(2)}{M_R}(t_1)}\left(\overset{(2)}{y_\nu}(t_1)\right)^T = -\frac{v^2}{4}\overset{(1)}{\kappa}(t_1)$$

Now we only have the RGE

$$-16\pi^2\frac{d\overset{(1)}{\kappa}(t)}{dt}$$

3-2 Type-I Seesaw

- Solve for Rank of the low energy mass matrix



Now we compute the rank of mass matrix

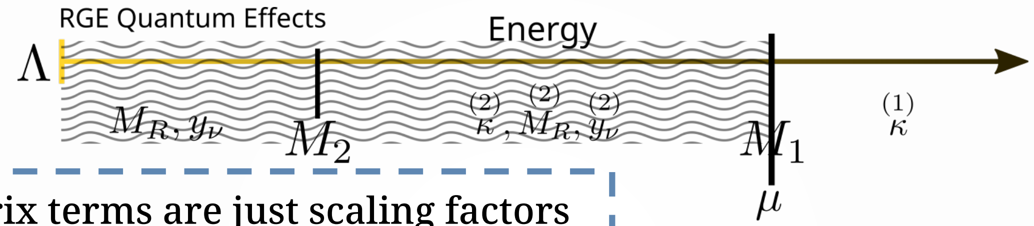
$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4}\overset{(1)}{\kappa}(t_1) = -\frac{v^2}{4}\overset{(2)}{y}_\nu(t_1)\frac{1}{\overset{(2)}{M}_R(t_1)}\overset{(2)}{y}_\nu(t_1)^T - \frac{v^2}{4}\overset{(2)}{\kappa}(t_1)$$

Sub in the solutions to the RGEs

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4}\overset{(2)}{y}_\nu(t_1)\frac{1}{\overset{(2)}{M}_R(t_1)}\overset{(2)}{y}_\nu(t_1)^T - \frac{v^2}{2}e^{-\frac{1}{16\pi^2}\int_0^{t_1} ds\,\overset{(2)}{\alpha}_\kappa(s)}\overset{(2)}{W}(t_1)y_{\nu 2}(0)\frac{1}{M_2(0)}y_{\nu 2}^T(0)\overset{(2)}{W}(t_1)^T$$

3-2 Type-I Seesaw

- Solve for Rank of the low energy mass matrix



Right handed mass matrix terms are just scaling factors

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4}\overset{(2)}{y_\nu}(t_1)\frac{1}{\overset{(2)}{M_R}(t_1)}\overset{(2)}{y_\nu}(t_1)^T - \frac{v^2}{2}e^{-\frac{1}{16\pi^2}\int_0^{t_1} ds\overset{(2)}{\alpha}_\kappa(s)}\overset{(2)}{W}(t_1)y_{\nu 2}(0)\frac{1}{M_2(0)}y_{\nu 2}^T(0)\overset{(2)}{W}(t_1)^T$$

$$\overset{(2)}{M_R}(t_1) = \exp\left[-\frac{1}{8\pi^2}\int_0^{t_1}\overset{(2)}{y_\nu}(s)^\dagger\overset{(2)}{y_\nu}(s)ds\right]M_1(0)$$

Physical Mass of heavy neutrino after RGE effects,
Just a number

Physical Mass of heavy neutrino,
Just a number

3-2 Type-I Seesaw

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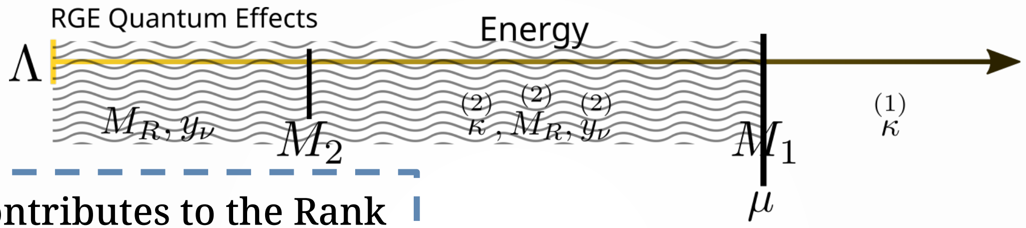


Only the Yukawa RGEs contributes to the Rank

$$\overset{(1)}{m}_{eff}(t_1) = -X\,\overset{(2)}{U}(t_1)y_{\nu 1}(0)y^T_{\nu 1}(0)\overset{(2)}{U}(t_1)^T - Z\,\overset{(2)}{W}(t_1)y_{\nu 2}(0)y^T_{\nu 2}(0)\overset{(2)}{W}(t_1)^T$$

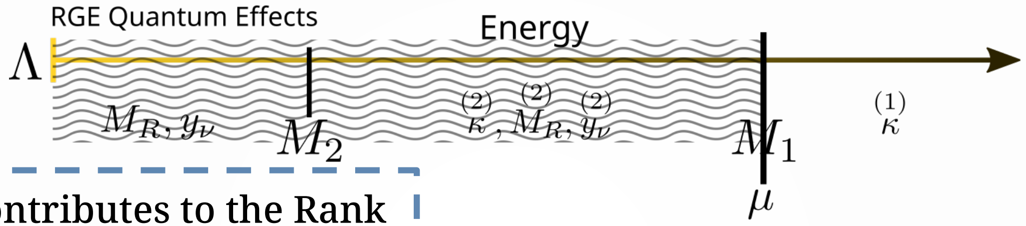
RGE Kernels act like linear transformations on the Yukawa vectors

$$\overset{(1)}{m}_{eff}(t_1) = -X\,y'_{\nu 1}(0)y'_{\nu 1}(0)^T - Z\,y'_{\nu 2}(0)y'_{\nu 2}(0)^T$$

Combination of 2 transformed Yukawa vectors construct the mass matrix, suggests Rank is 2

3-2 Type-I Seesaw

- **Solve for Rank of the low energy mass matrix**



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$$\overset{(1)}{m}_{eff}(t_1) = -X\, y'_{\nu 1}(0) y'_{\nu 1}(0)^T - Z\, y'_{\nu 2}(0) y'_{\nu 2}(0)^T$$

Explicitly the Rank can be determined via two methods for a square, complex effective mass matrix

$$\overset{(1)}{m}_{eff} = U \overset{(1)}{D}_{eff} U^\dagger$$

decomposition of the matrix

$$\overset{(1)}{p}_{eff} = \det\left(\overset{(1)}{m}_{eff}{}^\dagger \, \overset{(1)}{m}_{eff} - \lambda \mathbf{1} \right)$$

Solving the characteristic polynomial

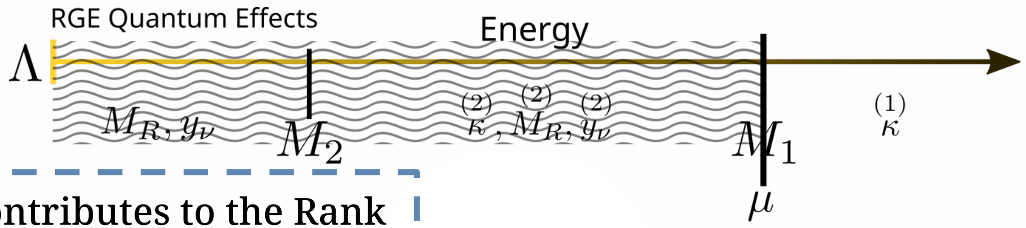
→

$$\mathrm{diag}\left[\overset{(1)}{m}_{eff}(t_1)\right] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \overset{(1)}{m}_2(t_1) & 0 \\ 0 & 0 & \overset{(1)}{m}_3(t_1) \end{pmatrix}$$

$$\mathrm{Rank}\left[\overset{(1)}{m}_{eff}(t_1)\right] = 2$$

3-2 Type-I Seesaw

- **Solve for Rank of the low energy mass matrix**



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$$\mathrm{Rank}\left[\overset{(1)}{m}_{eff}(t_1)\right] = 2$$

$\longrightarrow$

$$\mathrm{Rank}[m_{eff}] = 2$$

$$m_{eff} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Rank is the same at Low and High Energies

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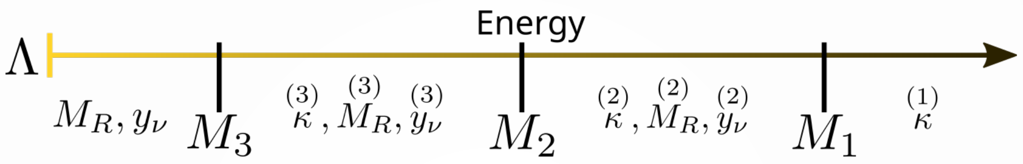
3-3 Type-I Seesaw

- The 3-3 model is the next simplest model



  - 3 Right-handed neutrinos
- We construct a Rank degenerate mass matrix at the high energy scale

$$m_{eff} = -\frac{v^2}{2} y_\nu \frac{1}{M_R} y_\nu^T \longrightarrow \mathrm{Rank}[m_{eff}] = 2$$

3-3 Type-I Seesaw

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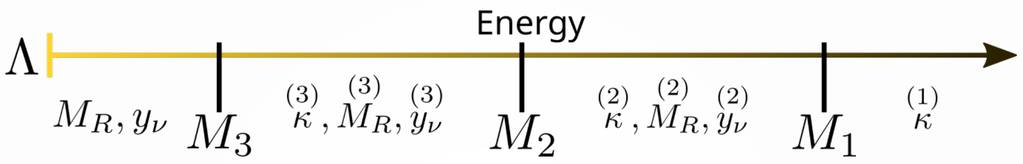

  - 3 Right-handed neutrinos
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$$m_{eff} = -\frac{v^2}{2} y_\nu \frac{1}{M_R} y_\nu^T \longrightarrow \text{Rank}[m_{eff}] = 2$$

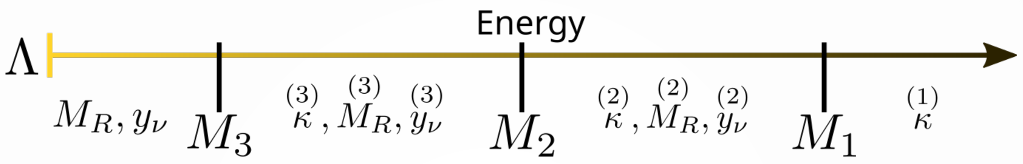
We can achieve that by constructing the Yukawa matrix,

$$y_\nu = \begin{pmatrix} y_{11} & y_{12} & ay_{11} + by_{12} \\ y_{21} & y_{22} & ay_{21} + by_{22} \\ y_{31} & y_{32} & ay_{31} + by_{32} \end{pmatrix}$$

Clearly the 3rd column is a linear combination of the first 2 columns $\longrightarrow$ $\text{Rank}[m_{eff}] = 2$

3-3 Type-I Seesaw

- The 3-3 model is the next simplest model



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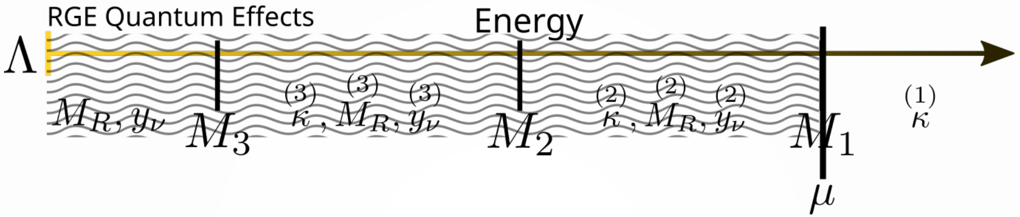
$$y_\nu = \begin{pmatrix} y_{11} & y_{12} & a y_{11} + b y_{12} \\ y_{21} & y_{22} & a y_{21} + b y_{22} \\ y_{31} & y_{32} & a y_{31} + b y_{32} \end{pmatrix}$$

In terms of matrix invariants of the Characteristic Polynomial, importantly the lowest order term is zero $\longrightarrow$ $\text{Rank}[m_{eff}] = 2$

$$p_{eff} = \det\left(m_{eff}^\dagger m_{eff} - \lambda \mathbf{1}\right) = \sum_{k=0}^{3} I_k \lambda^k$$

3-3 Type-I Seesaw

- Same as the 3-2 Model, we integrate down to the low energy scale



After integrating out the last right-handed neutrino, we use the matching to solve for the effective mass

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4}\overset{(1)}{\kappa}(t_1) = -\frac{v^2}{4}\overset{(2)}{y}_\nu(t_1)\frac{1}{\overset{(2)}{M}_R(t_1)}\overset{(2)}{y}_\nu(t_1)^T - \frac{v^2}{4}e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds\,\overset{(2)}{\alpha}_\kappa(s)}\overset{(2)}{W}(t_1-t_2)\overset{(2)}{\kappa}(t_2)\overset{(2)}{W}(t_1-t_2)^T$$

3-3 Type-I Seesaw

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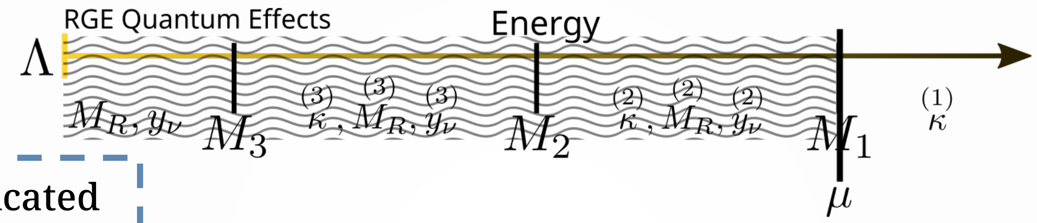

Becomes quite complicated

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4} e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds\,\alpha^{(2)}_{y_\nu}(s)} e^{-\frac{2}{16\pi^2}\int_{t_2}^{t_1} ds\,\alpha^{(3)}_{y_\nu}(s)} \overset{(2)}{U}(t_1-t_2)\overset{(3)}{U}(t_2)\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^*_{11}+\overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^*_{21}\right)\frac{1}{\overset{(2)}{M}_R(t_1)}\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^*_{11}+\overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^*_{21}\right)\overset{(3)}{U}(t_2)^T\overset{(2)}{U}(t_1-t_2)^T$$

$$-\frac{v^2}{4} e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds\,\alpha^{(2)}_{\kappa}(s)} e^{-\frac{1}{16\pi^2}\int_{0}^{t_2} ds\,\alpha^{(3)}_{\kappa}(s)} \overset{(2)}{W}(t_1-t_2)\overset{(3)}{W}(t_2)\overset{(3)}{y}_{\nu 3}(0)\frac{1}{\overset{(3)}{M}_3(0)}\overset{(3)}{y}_{\nu 3}(0)^T\overset{(3)}{W}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T$$

$$-\frac{v^2}{4} e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds\,\alpha^{(2)}_{\kappa}(s)} e^{-\frac{2}{16\pi^2}\int_{t_2}^{t_1} ds\,\alpha^{(3)}_{y_\nu}(s)} \overset{(2)}{W}(t_1-t_2)\overset{(3)}{U}(t_2)\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^*_{12}+\overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^*_{22}\right)\frac{1}{\overset{(3)}{M}_{D2}(t_2)}\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^*_{12}+\overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^*_{22}\right)\overset{(3)}{U}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T$$

3-3 Type-I Seesaw

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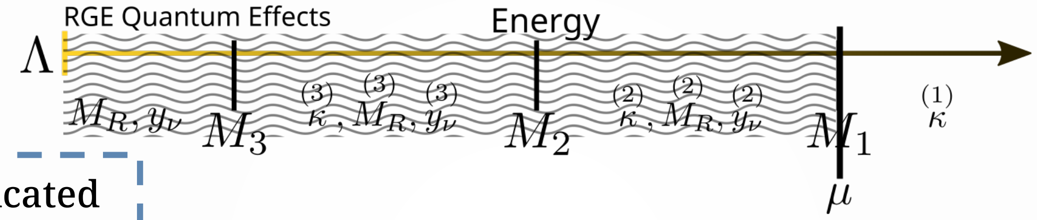


Becomes quite complicated

$$\overset{(1)}{m}_{eff}(t_1) = -\frac{v^2}{4} e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds \overset{(2)}{\alpha}_{y_\nu}(s)} e^{-\frac{2}{16\pi^2}\int_{t_2}^{t_1} ds \overset{(3)}{\alpha}_{y_\nu}(s)} \overset{(2)}{U}(t_1 - t_2) \overset{(3)}{U}(t_2) \left(\overset{(3)}{y}_{\nu 1}(0) \overset{(3)}{V}{}^*_{11} + \overset{(3)}{y}_{\nu 2}(0) \overset{(3)}{V}{}^*_{21} \right) \frac{1}{\overset{(2)}{M}_R(t_1)} \left(\overset{(3)}{y}_{\nu 1}(0)^T \overset{(3)}{V}{}^*_{11} + \overset{(3)}{y}_{\nu 2}(0)^T \overset{(3)}{V}{}^*_{21} \right) \overset{(3)}{U}(t_2)^T \overset{(2)}{U}(t_1 - t_2)^T$$

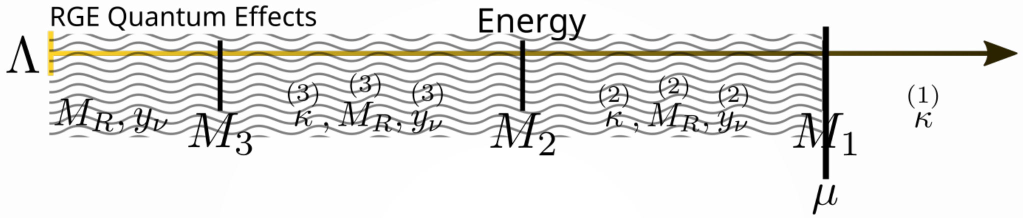
$$- \frac{v^2}{4} e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds \overset{(2)}{\alpha}_{\kappa}(s)} e^{-\frac{1}{16\pi^2}\int_{0}^{t_2} ds \overset{(3)}{\alpha}_{\kappa}(s)} \overset{(2)}{W}(t_1 - t_2) \overset{(3)}{W}(t_2) \overset{(3)}{y}_{\nu 3}(0) \frac{1}{\overset{(3)}{M}_3(0)} \overset{(3)}{y}_{\nu 3}(0)^T \overset{(3)}{W}(t_2)^T \overset{(2)}{W}(t_1 - t_2)^T$$

$$- \frac{v^2}{4} e^{-\frac{1}{16\pi^2}\int_{t_2}^{t_1} ds \overset{(2)}{\alpha}_{\kappa}(s)} e^{-\frac{2}{16\pi^2}\int_{t_2}^{t_1} ds \overset{(3)}{\alpha}_{y_\nu}(s)} \overset{(2)}{W}(t_1 - t_2) \overset{(3)}{U}(t_2) \left(\overset{(3)}{y}_{\nu 1}(0) \overset{(3)}{V}{}^*_{12} + \overset{(3)}{y}_{\nu 2}(0) \overset{(3)}{V}{}^*_{22} \right) \frac{1}{\overset{(3)}{M}_{D2}(t_2)} \left(\overset{(3)}{y}_{\nu 1}(0)^T \overset{(3)}{V}{}^*_{12} + \overset{(3)}{y}_{\nu 2}(0)^T \overset{(3)}{V}{}^*_{22} \right) \overset{(3)}{U}(t_2)^T \overset{(2)}{W}(t_1 - t_2)^T$$

But these are all scaling factors

3-3 Type-I Seesaw

- Same as the 3-2 Model, we integrate down to the low energy scale



Scaling factors do not contribute to the Rank, only RGE kernel solutions and Yukawa vectors are important

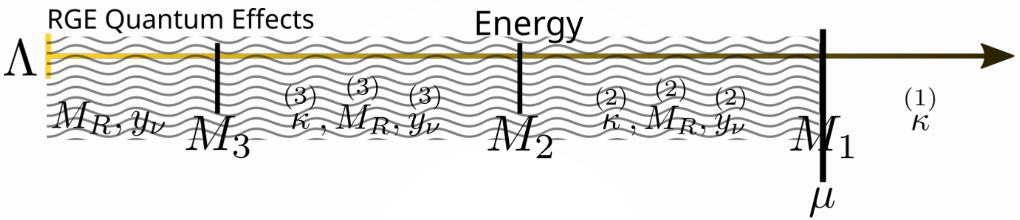
$$\overset{(1)}{m}_{eff}(t_1) = -A\left[\overset{(2)}{U}(t_1-t_2)\overset{(3)}{U}(t_2)\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^*_{11} + \overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^*_{21}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^*_{11}(0) + \overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^*_{21}(0)\right)\overset{(3)}{U}(t_2)^T\overset{(2)}{U}(t_1-t_2)^T\right]$$

$$-B\left[\overset{(2)}{W}(t_1-t_2)\overset{(3)}{W}(t_2)\overset{(3)}{y}_{\nu 3}(0)\overset{(3)}{y}_{\nu 3}(0)^T\overset{(3)}{W}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T\right]$$

$$-C\left[\overset{(2)}{W}(t_1-t_2)\overset{(3)}{U}(t_2)\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^*_{12} + \overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^*_{22}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^*_{12} + \overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^*_{22}\right)\overset{(3)}{U}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T\right]$$

3-3 Type-I Seesaw

- Same as the 3-2 Model, we integrate down to the low energy scale



Scaling factors do not contribute to the Rank, only RGE kernel solutions and Yukawa vectors are important

$$\overset{(1)}{m}_{eff}(t_1) = -A\left[\overset{(2)}{U}(t_1-t_2)\overset{(3)}{U}(t_2)\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^{*}_{11}+\overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^{*}_{21}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^{*}_{11}(0)+\overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^{*}_{21}(0)\right)\overset{(3)}{U}(t_2)^T\overset{(2)}{U}(t_1-t_2)^T\right]$$

$$-B\left[\overset{(2)}{W}(t_1-t_2)\overset{(3)}{W}(t_2)\overset{(3)}{y}_{\nu 3}(0)\overset{(3)}{y}_{\nu 3}(0)^T\overset{(3)}{W}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T\right]$$

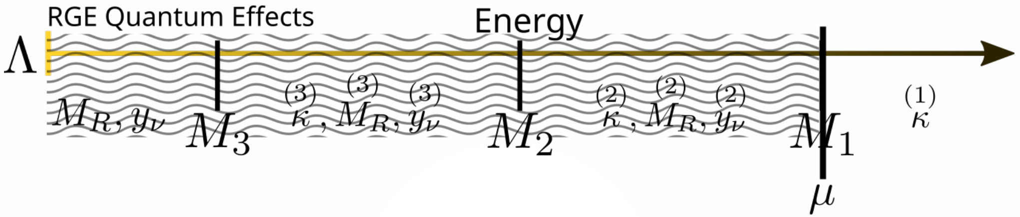
$$-C\left[\overset{(2)}{W}(t_1-t_2)\overset{(3)}{U}(t_2)\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^{*}_{12}+\overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^{*}_{22}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^{*}_{12}+\overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^{*}_{22}\right)\overset{(3)}{U}(t_2)^T\overset{(2)}{W}(t_1-t_2)^T\right]$$

The RGE kernels are 3 different linear transformations to be applied to the Yukawa vectors

$$\mathbf{T}_{UU} \equiv \overset{(2)}{U}(t_1-t_2)\overset{(3)}{U}(t_2) \qquad \mathbf{T}_{WW} \equiv \overset{(2)}{W}(t_1-t_2)\overset{(3)}{W}(t_2) \qquad \mathbf{T}_{WU} \equiv \overset{(2)}{W}(t_1-t_2)\overset{(3)}{U}(t_2)$$

3-3 Type-I Seesaw

- Same as the 3-2 Model, we integrate down to the low energy scale



General Solution has 3 transformations and 3 unique Yukawa vectors

$$\overset{(1)}{m}_{eff}(t_1) = -A\left[\underline{\mathbf{T}_{UU}}\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^*_{11} + \overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^*_{21}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^*_{11}(0) + \overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^*_{21}(0)\right)\mathbf{T}^T_{UU}\right]$$

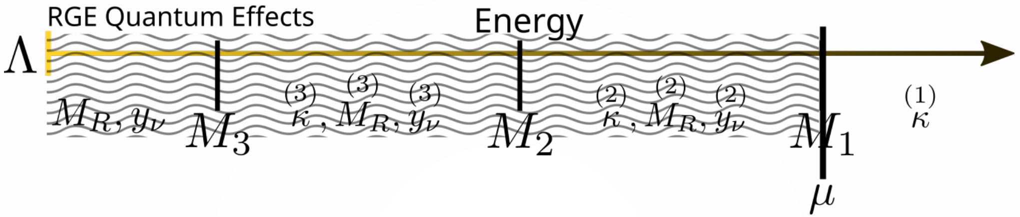
$$-B\left[\underline{\mathbf{T}_{WW}}\overset{(3)}{y}_{\nu 3}(0)\overset{(3)}{y}_{\nu 3}(0)^T\mathbf{T}^T_{WW}\right]$$

$$-C\left[\underline{\mathbf{T}_{WU}}\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^*_{12} + \overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^*_{22}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^*_{12} + \overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^*_{22}\right)\mathbf{T}^T_{WU}\right]$$

The transformed Yukawa vectors combine to form 3 unique eigenvectors for the mass matrix

3-3 Type-I Seesaw

- Same as the 3-2 Model, we integrate down to the low energy scale



In our case, we take an initial condition to reduce the unique Yukawa vectors to 2

$$\overset{(3)}{y}_{\nu 3}(0) \equiv a\overset{(3)}{y}_{\nu 1}(0) + b\overset{(3)}{y}_{\nu 2}(0)$$

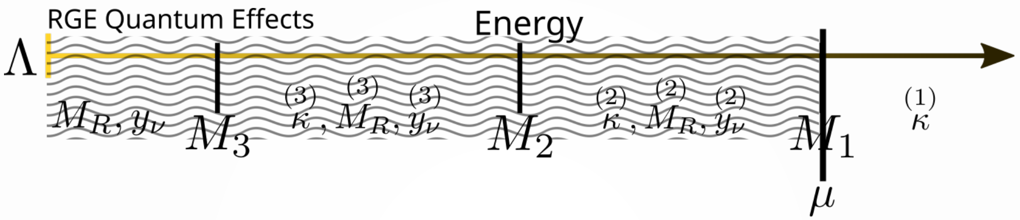
Recall our condition lead to a Rank degenerate mass matrix, with a Rank of 2 at the 'Full Theory' energy scale →

$$\mathrm{diag}[m_{eff}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

$$\mathrm{Rank}[m_{eff}] = 2$$

3-3 Type-I Seesaw

- **Same as the 3-2 Model, we integrate down to the low energy scale**



$$y^{(3)}_{\nu 3}(0) \equiv a y^{(3)}_{\nu 1}(0) + b y^{(3)}_{\nu 2}(0)$$

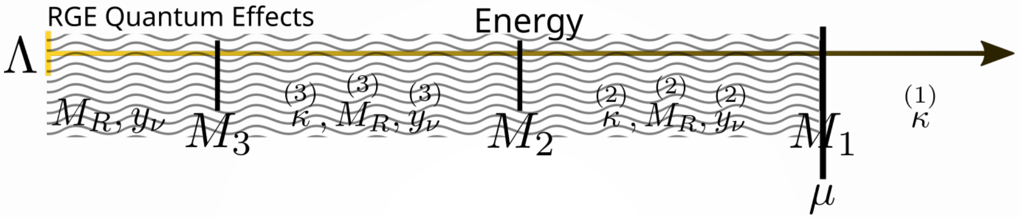
In our case, we take an initial condition to reduce the unique Yukawa vectors to 2

$$\begin{aligned} m^{(1)}_{eff}(t_1) = &- A\left[\underline{\mathbf{T}_{UU}}\left(y^{(3)}_{\nu 1}(0)V^{(3)*}_{11} + y^{(3)}_{\nu 2}(0)V^{(3)*}_{21}\right)\left(y^{(3)}_{\nu 1}(0)^T V^{(3)*}_{11}(0) + y^{(3)}_{\nu 2}(0)^T V^{(3)*}_{21}(0)\right)\mathbf{T}^T_{UU}\right] \\ &- B\left[\underline{\mathbf{T}_{WW}}\left(a y^{(3)}_{\nu 1}(0) + b y^{(3)}_{\nu 2}(0)\right)\left(a y^{(3)}_{\nu 1}(0)^T + b y^{(3)}_{\nu 2}(0)^T\right)\mathbf{T}^T_{WW}\right] \\ &- C\left[\underline{\mathbf{T}_{WU}}\left(y^{(3)}_{\nu 1}(0)V^{(3)*}_{12} + y^{(3)}_{\nu 2}(0)V^{(3)*}_{22}\right)\left(y^{(3)}_{\nu 1}(0)^T V^{(3)*}_{12} + y^{(3)}_{\nu 2}(0)^T V^{(3)*}_{22}\right)\mathbf{T}^T_{WU}\right] \end{aligned}$$

Different from the 'Full Theory' case, we now have 3 transformations to consider

3-3 Type-I Seesaw

- Same as the 3-2 Model, we integrate down to the low energy scale



$$\overset{(3)}{y}_{\nu 3}(0) \equiv a\overset{(3)}{y}_{\nu 1}(0) + b\overset{(3)}{y}_{\nu 2}(0)$$

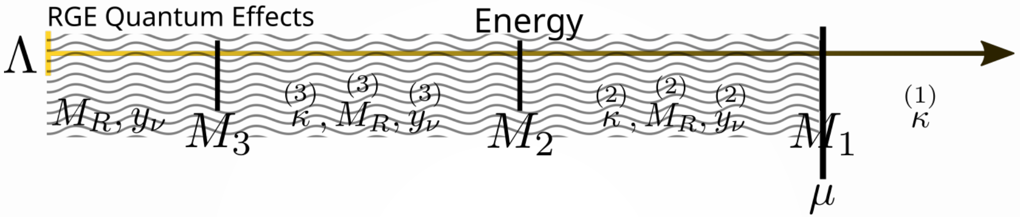
After transformations we have 3 unique transformed Yukawa vectors

$$\begin{aligned}\overset{(1)}{m}_{eff}(t_1) = &- A\left[\mathbf{T}_{UU}\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^{*}_{11} + \overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^{*}_{21}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^{*}_{11}(0) + \overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^{*}_{21}(0)\right)\mathbf{T}^T_{UU}\right]\\ &- B\left[\mathbf{T}_{WW}\left(a\overset{(3)}{y}_{\nu 1}(0) + b\overset{(3)}{y}_{\nu 2}(0)\right)\left(a\overset{(3)}{y}_{\nu 1}(0)^T + b\overset{(3)}{y}_{\nu 2}(0)^T\right)\mathbf{T}^T_{WW}\right]\\ &- C\left[\mathbf{T}_{WU}\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V}{}^{*}_{12} + \overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V}{}^{*}_{22}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V}{}^{*}_{12} + \overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V}{}^{*}_{22}\right)\mathbf{T}^T_{WU}\right]\end{aligned}$$

$$\mathbf{T}_{UU} \equiv \overset{(2)}{U}(t_1 - t_2)\overset{(3)}{U}(t_2) \qquad \mathbf{T}_{WW} \equiv \overset{(2)}{W}(t_1 - t_2)\overset{(3)}{W}(t_2) \qquad \mathbf{T}_{WU} \equiv \overset{(2)}{W}(t_1 - t_2)\overset{(3)}{U}(t_2)$$

3-3 Type-I Seesaw

- Same as the 3-2 Model, we integrate down to the low energy scale



The 3 unique transformed Yukawa vectors combine to create 3 eigenvalues for the effective mass matrix

$$\begin{aligned}\overset{(1)}{m}_{eff}(t_1) = &- A\left[\mathbf{T}_{UU}\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V^*_{11}} + \overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V^*_{21}}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V^*_{11}}(0) + \overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V^*_{21}}(0)\right)\mathbf{T}^T_{UU}\right] \\ &- B\left[\mathbf{T}_{WW}\left(a\overset{(3)}{y}_{\nu 1}(0) + b\overset{(3)}{y}_{\nu 2}(0)\right)\left(a\overset{(3)}{y}_{\nu 1}(0)^T + b\overset{(3)}{y}_{\nu 2}(0)^T\right)\mathbf{T}^T_{WW}\right] \\ &- C\left[\mathbf{T}_{WU}\left(\overset{(3)}{y}_{\nu 1}(0)\overset{(3)}{V^*_{12}} + \overset{(3)}{y}_{\nu 2}(0)\overset{(3)}{V^*_{22}}\right)\left(\overset{(3)}{y}_{\nu 1}(0)^T\overset{(3)}{V^*_{12}} + \overset{(3)}{y}_{\nu 2}(0)^T\overset{(3)}{V^*_{22}}\right)\mathbf{T}^T_{WU}\right]\end{aligned}$$

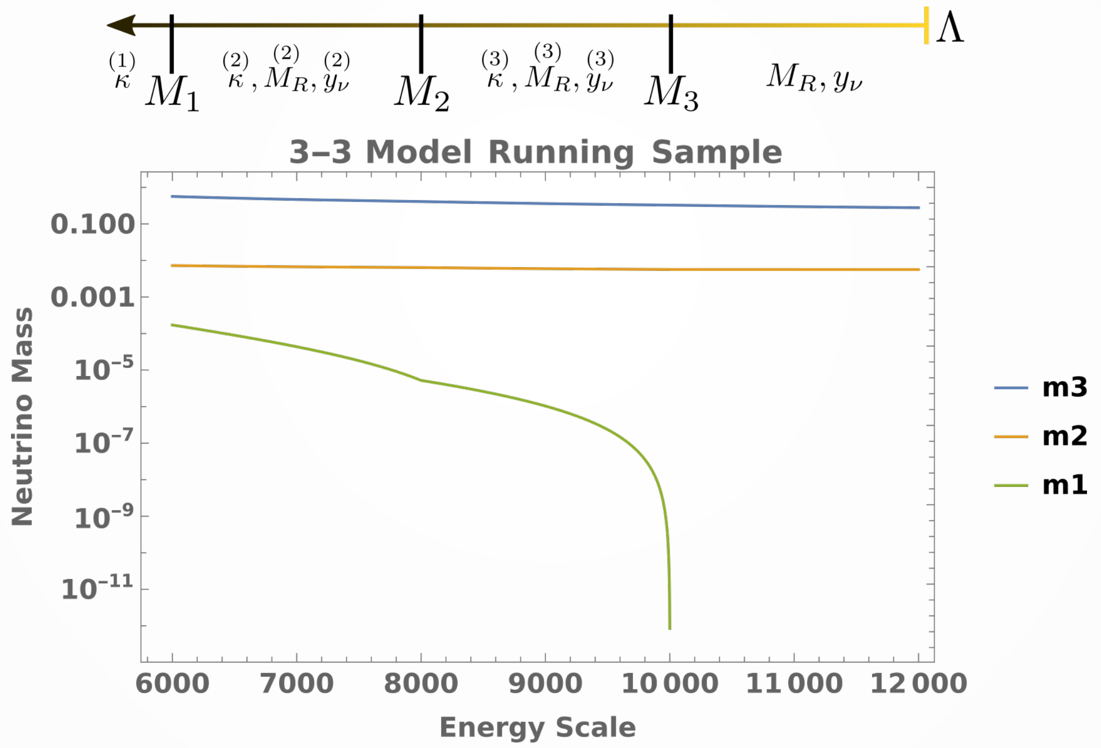
$$\mathrm{diag}[\overset{(1)}{m}_{eff}(t_1)] = \begin{pmatrix} m_1(t_1) & 0 & 0 \\ 0 & m_2(t_1) & 0 \\ 0 & 0 & m_3(t_1) \end{pmatrix}$$

$$\mathrm{Rank}[\overset{(1)}{m}_{eff}(t_1)] = 3$$

In general it is no longer Rank degenerate and the lightest neutrino is massive!

3-3 Type-I Seesaw

Sample plot of RGE running in 3-3 Model



Outline

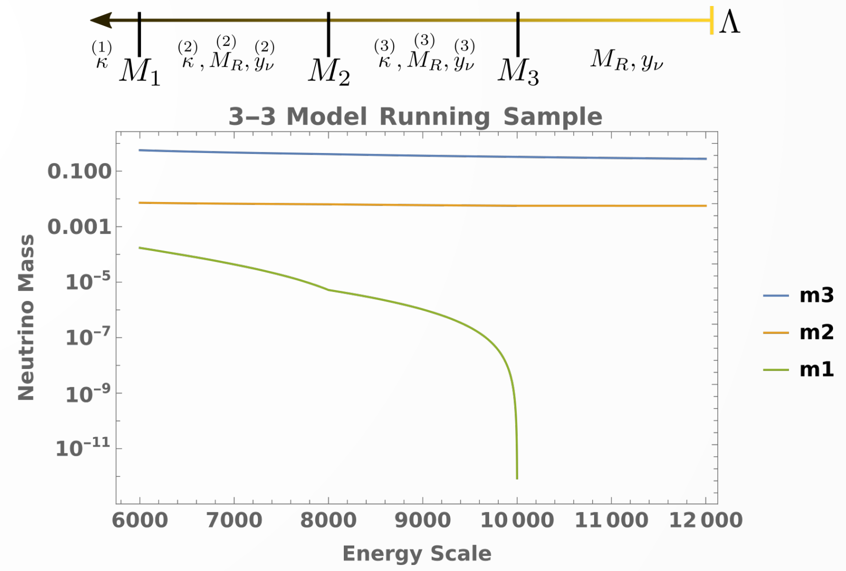
- Introduction
  - Massive Neutrinos
  - Type-I Seesaw Model
- 3-2 Type-I Seesaw
- Rank Degenerate, 3-3 Type-I Seesaw
- Conclusion

Conclusion

- We have shown the RGE effects on a Rank degenerate Yukawa matrix in the Type-I seesaw model

- We find it is possible to have a massless neutrino at high energies become massive at low energies in 3-3 Model

- This is contrary to usual case where a massless neutrino is always massless (i.e. 3-2 model)



Thank you!

Questions?