2nd IIT Bombay-Hiroshima workshop for Frontiers of Astro-Particle Physics
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Modified Theories of Gravity and their Phenomenological Consequences

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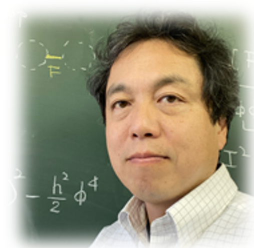

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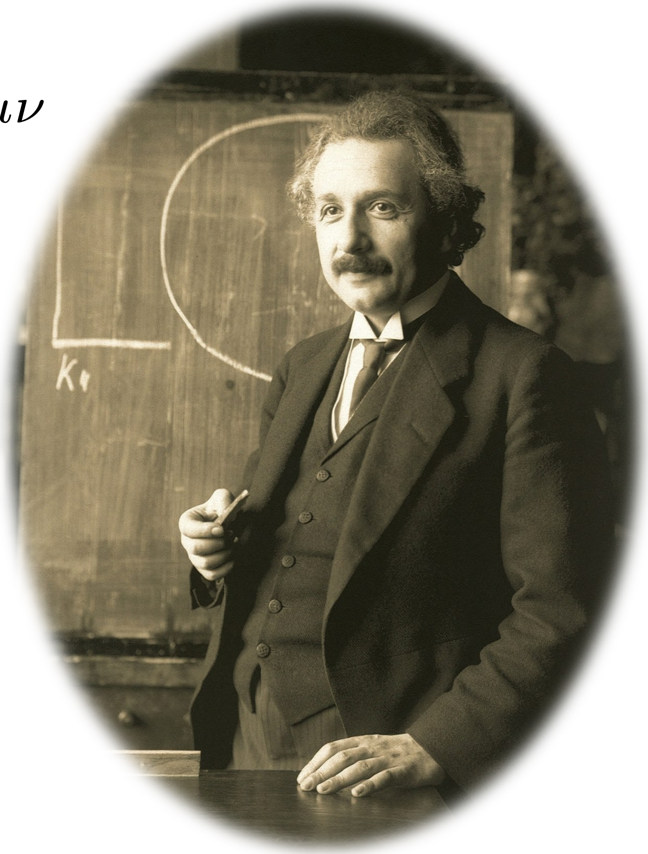
- Modified theories of Gravity
- Phenomenological consequences
- Conclusion

Introduction

Why?

Einstein's General Relativity

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



Why?

Einstein's General Relativity

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Phenomenological consequences
  Apsidal precession of the planet Mercury,
  Dense stars, Black holes,
  Gravitational lens,
  Gravitational wave,
  Expansion of the universe,
  …

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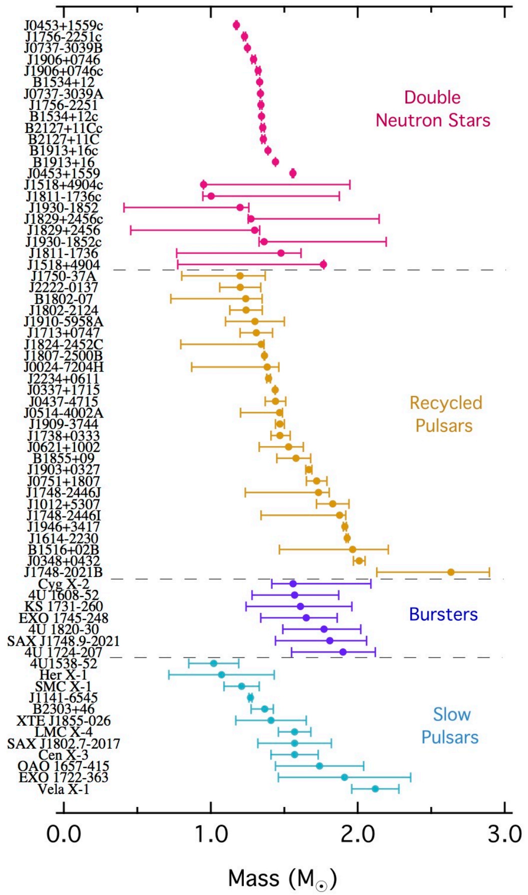
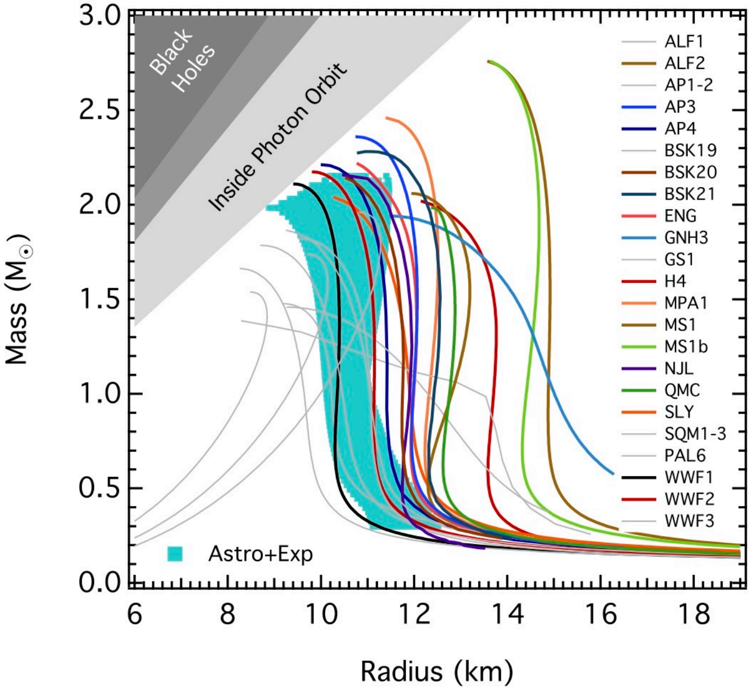
- Phenomenological consequences
  Apsidal precession of the planet Mercury,
  Dense stars, Black holes, ←Heavy neutron stars
  Gravitational lens, ←Dark matter
  Gravitational wave, ←Just started
  Expansion of the universe, ←Accelerated expansion
  … ←Small scale < 0.01mm

Heavy neutron stars





Ozel & Freire 2016

http://xtreme.as.arizona.edu/NeutronStars/

Why?

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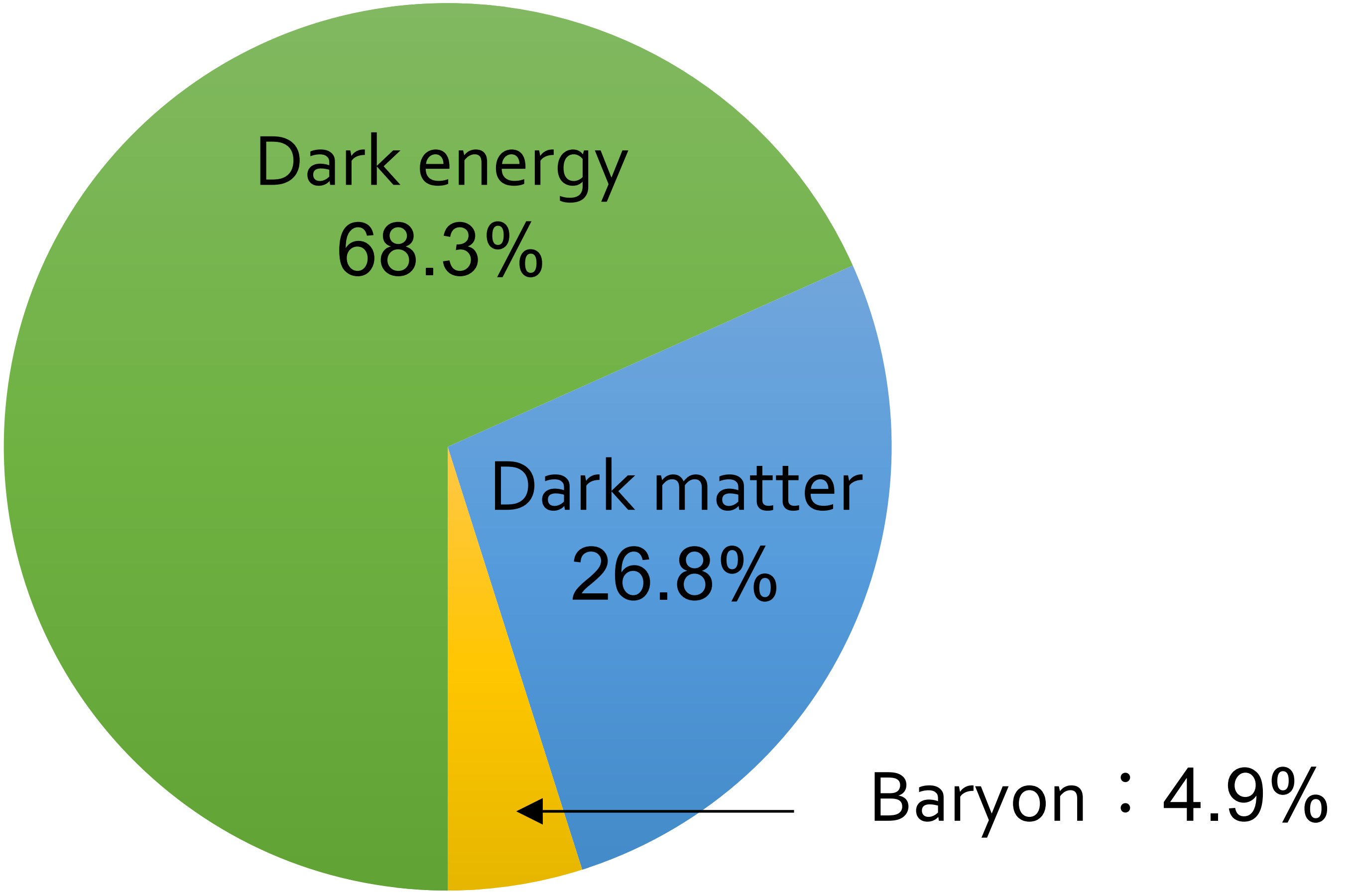
Dark matter and dark energy



How?

Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} (R - 2\Lambda) + S_{\mathrm{matter}}$$

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Modified Gravity

- Higher order
- Non-local
- Gauss-Bonnet
- Torsion
- …

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Particle physics

- Neutrino
- Axion
- Superpartner
- …

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Spacetime

- Extra dimensions
- D-brane
- …

Modified Theories of Gravity

Modification of the Einstein eq.

Einstein equation

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Modified Gravity

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F(R) gravity

- Extension of the Einstein Hilbert action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}(R - 2\Lambda) + S_{\mathrm{matter}}$$

$$\Downarrow$$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F(R) + S_{\mathrm{matter}}$$

S. Nojiri and S. D. Odintsov, Phys. Rept. 505, 59 (2011).

Starobinsky model

A. A. Starobinsky, 1979

- Action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} (R + \kappa^2 R^2) + S_{\mathrm{matter}}$$

- After the Weyl transformation of the metric tensor

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}} \tilde{R} + S_{\mathrm{matter}} + \int d^4x \sqrt{-\tilde{g}} \left[-\frac{1}{2} \tilde{g}^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$

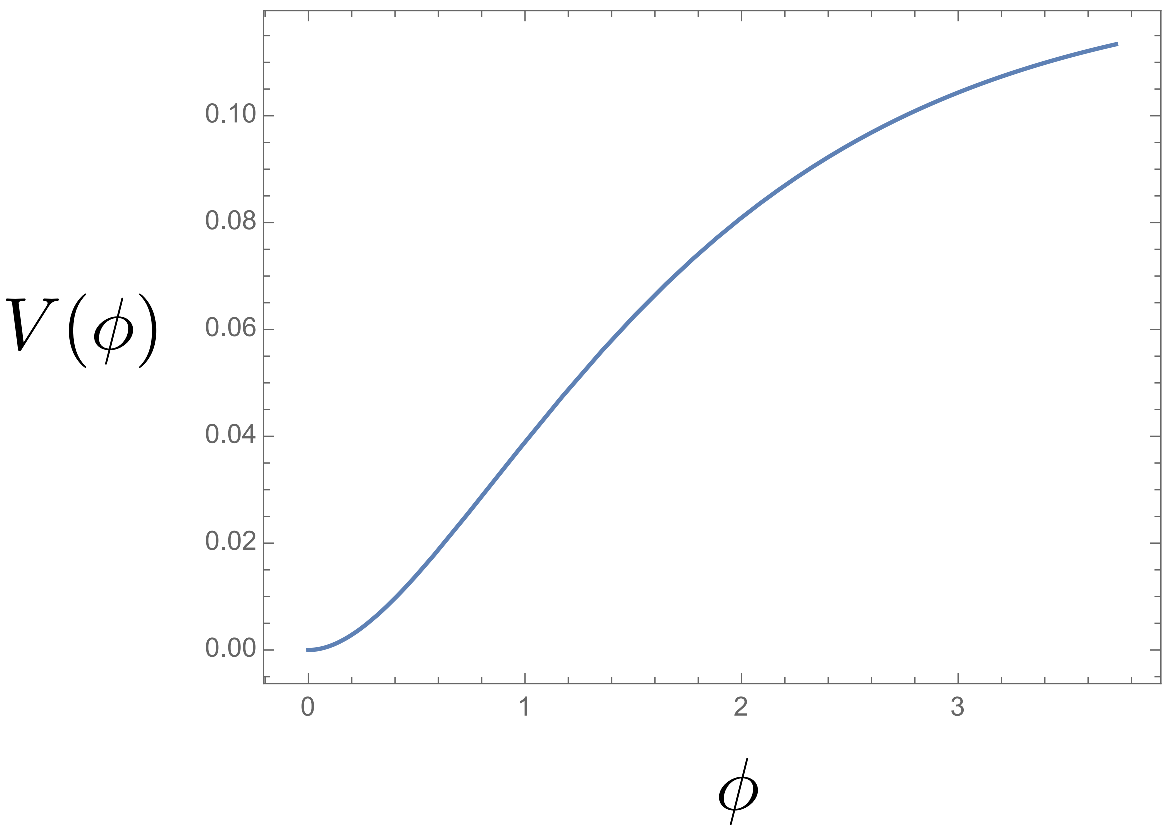
Scalaron

S. Nojiri and S. D. Odintsov, Phys. Rept. 505, 59 (2011).

Starobinsky model

A. A. Starobinsky, 1979

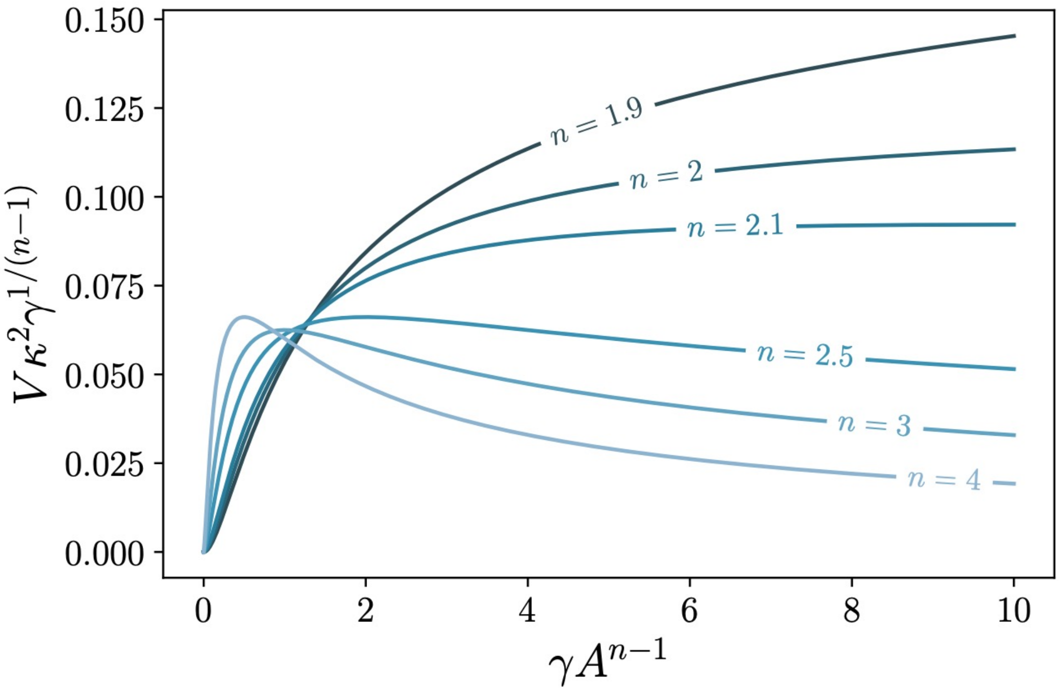
- Scalaron potential



More general cases

- Power law model

$$F(R) = R + \gamma R^n$$

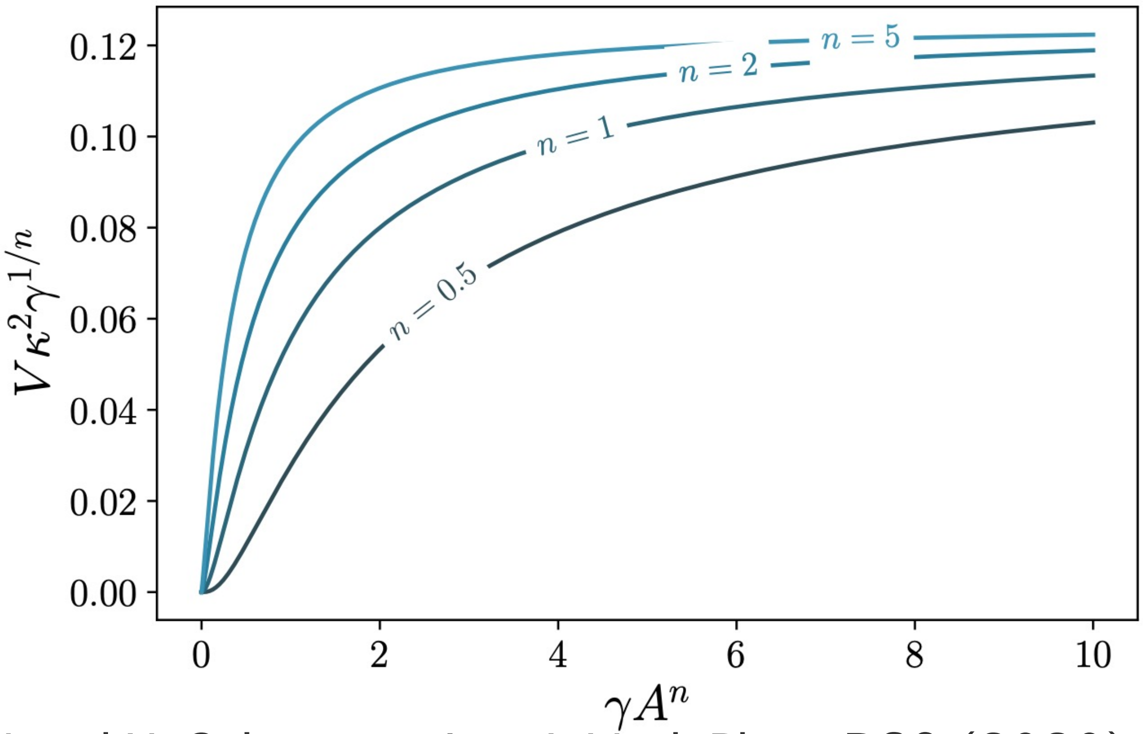


T. Inagaki and H. Sakamoto, Int. J. Mod. Phys. **D29** (2020)

More general cases

- Modified power law model

$$F(R) = R(1 + \gamma R^n)^{1/n}$$



T. Inagaki and H. Sakamoto, Int. J. Mod. Phys. **D29** (2020)

F(G) gravity

- Gauss-Bonnet invariant

$$G = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$

- Extension of the Einstein Hilbert action

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\mathrm{matter}}$$

$$\Downarrow$$

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R + F(G)) + S_{\mathrm{matter}}$$

S. Nojiri and S. D. Odintsov, Phys. Lett. B 631, (2005)

F(R, G) gravity

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$$\Downarrow$$

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}F(R, G) + S_{\mathrm{matter}}$$

Cartan formarism

- Vierbein

$$g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu$$

- Fermionic fields are described in a local Lorentz frame.
- Torsion tensor is naturally introduced.

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}$$

T. W. B. Kibble (1961), D. W. Sciama(1962)

Modification of the Einstein eq.

Einstein equation

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Particle physics

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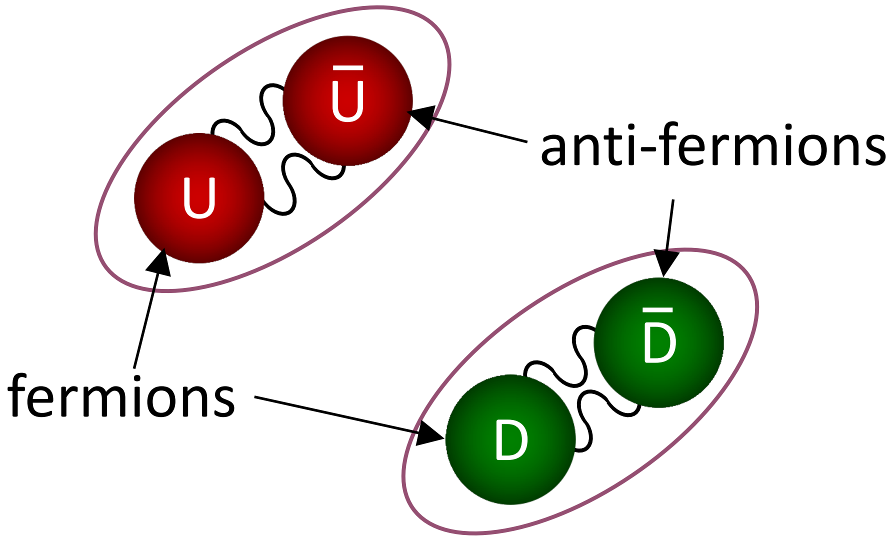
Gauged NJL model

- Low energy effective theory of light pseudo-scalar mesons constructed by quarks and anti-quarks
- We scale up the model from the QCD scale to the inflation scale



T. I., S. D. Odintsov and H. Sakamoto, Astr. Space Sci. (2015),
T. I., S. D. Odintsov and H. Sakamoto, Nucl. Phys. B (2017),
T. I., S. D. Odintsov and H. Sakamoto, Europhys. Lett. (20170.

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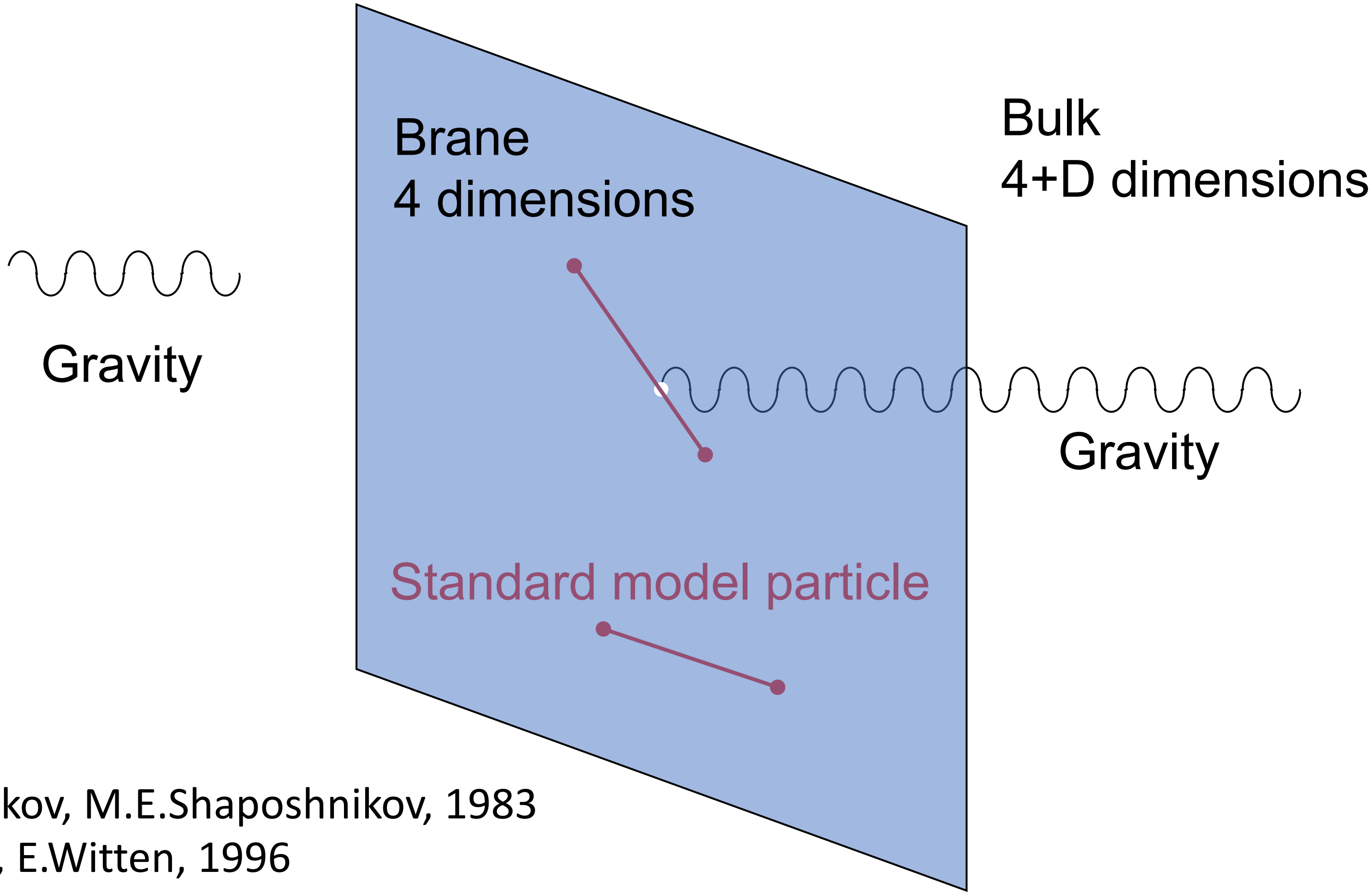
Brane world



V.A.Rubakov, M.E.Shaposhnikov, 1983
P.Horava, E.Witten, 1996
N.Arkani-Hamed, S.Dimopoulos, G.Dvali, 1998
L.Randall, R.Sundrum, 1999

Phenomenological consequences

Origin of accelerated expansion

- In a homogeneous and isotropic spacetime
  $$ds^2 = c^2 dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$
- Sources of energy density

Radiation	$a(t) \propto t^{1/2}$
Matter	$a(t) \propto t^{2/3}$
Potential energy	$a(t) \propto \exp(\alpha t)$
Cosmological constant	

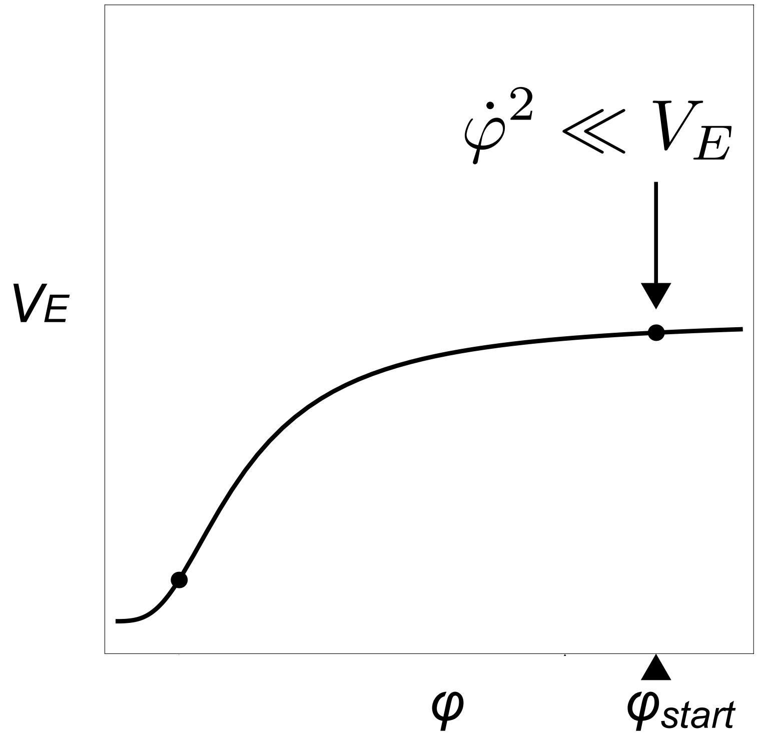
Quasi de-Sitter expansion

- Friedman equation

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{2}\dot{\varphi}^2 + V_E$$

- Assumption $\dot{\varphi}^2 \ll V_E$

$$a(t+\Delta t) \sim a(t)e^{\sqrt{\frac{V_E}{3}}\Delta t}$$



A. D. Linde, Contemp. Concepts Phys. 5, 1 (1990).

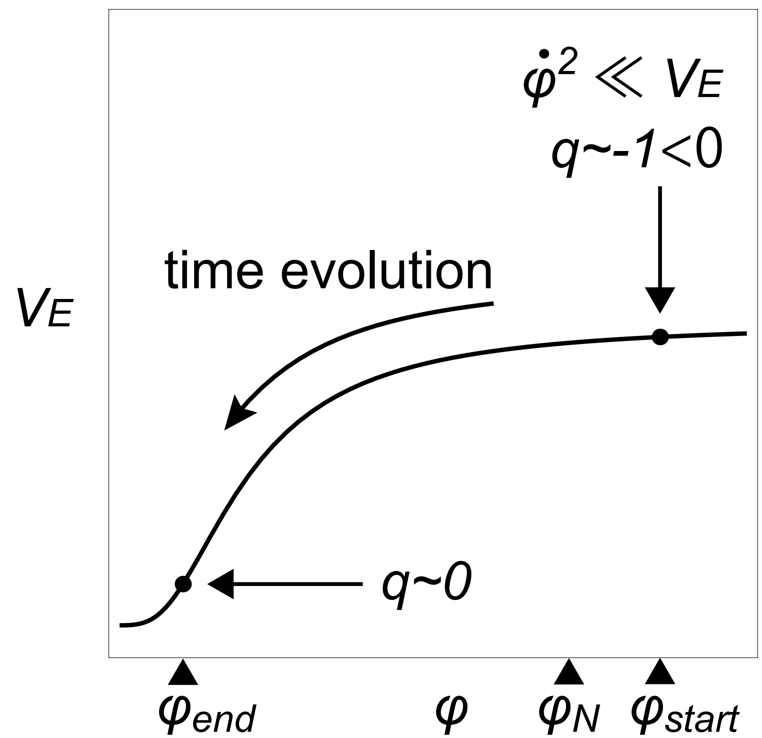
Exit from Inflation

- Equation of motion

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} = -\frac{\partial V_E}{\partial \varphi}$$

- Deceleration parameter

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} \to 0$$



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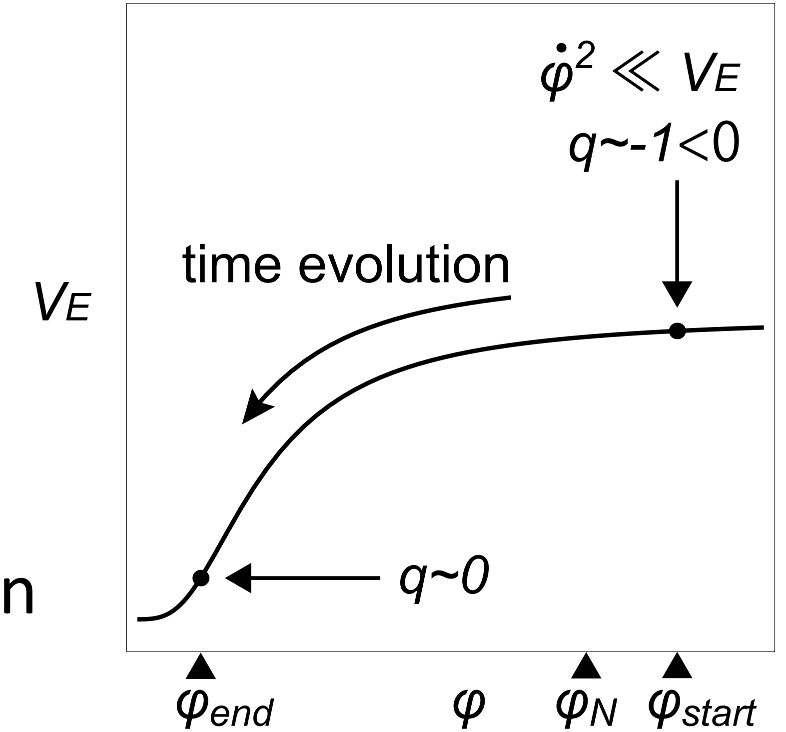
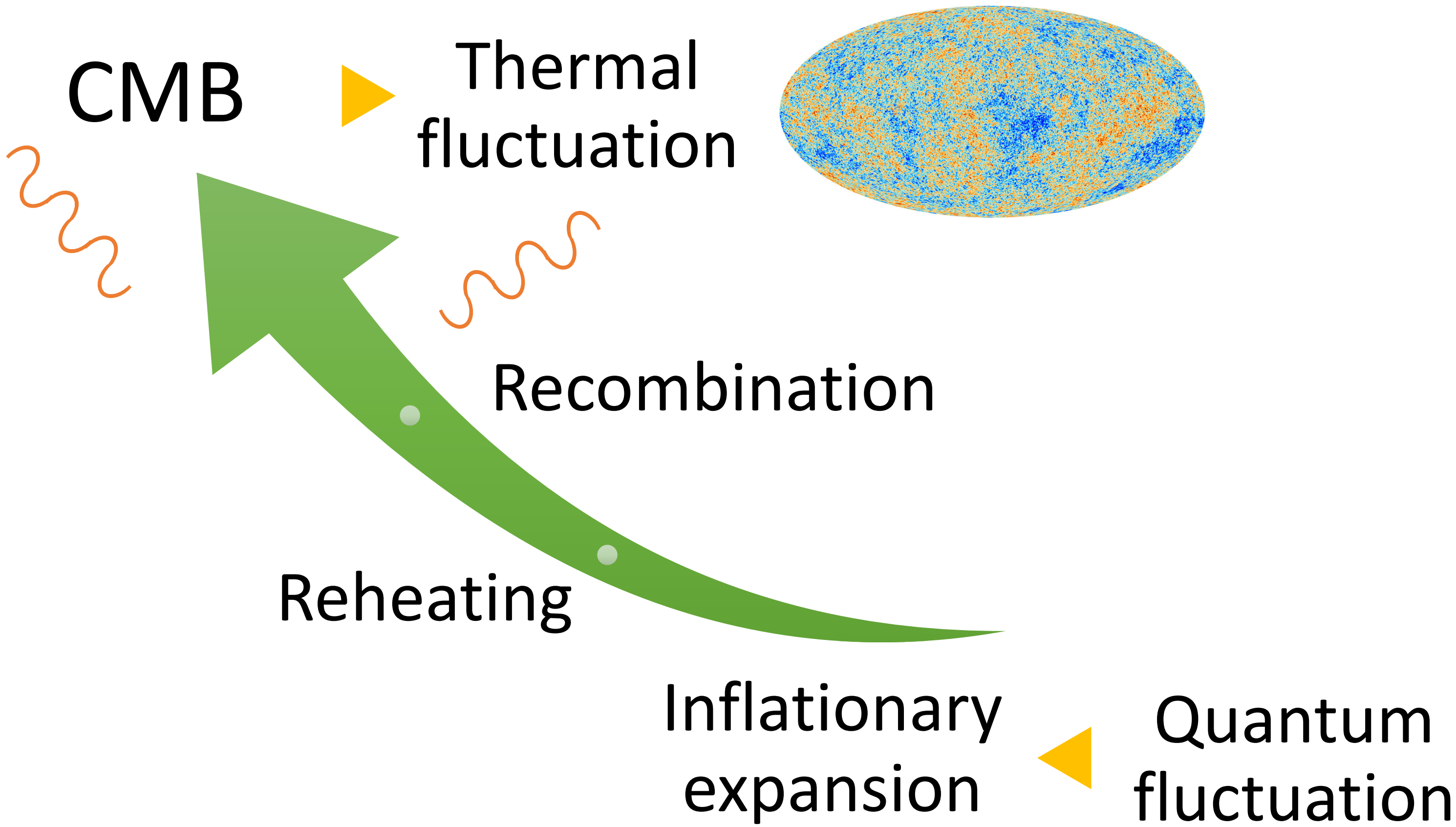
Modified gravity: Scalaron
gNJL: Composite scalar



A. D. Linde, Contemp. Concepts Phys. 5, 1 (1990).

Evidence for Inflation



Quantum fluctuations

$$\varphi + \delta\varphi$$
$$\rightarrow \mathcal{P}_s(k)$$

Scalar type fluctuation
Origin: quantum fluctuation of scalar field

Tensor type fluctuation
Origin: quantum fluctuation of space-time

$$g^{\mu\nu} + \delta h^{\mu\nu}$$
$$\rightarrow \mathcal{P}_t(k)$$

Observed CMB fluctuations

- Rescaled scalar type fluctuation

$$\mathcal{P}_s(k) \equiv A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

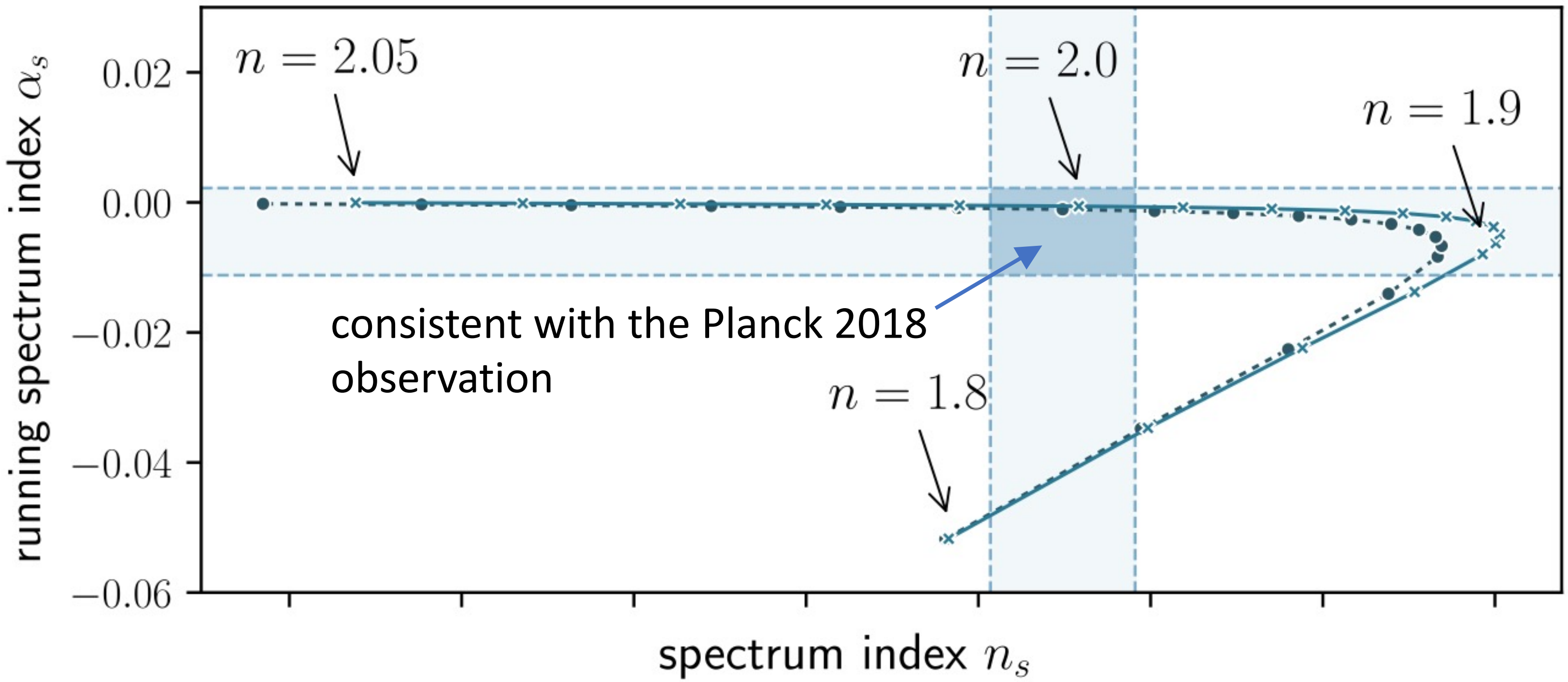
- Rescaled tensor type fluctuation

$$\mathcal{P}_t(k) \equiv A_t \left(\frac{k}{k_0}\right)^{n_t}$$

- Tensor to scalar ratio

$$r \equiv \frac{\mathcal{P}_t(k)}{\mathcal{P}_s(k)}$$

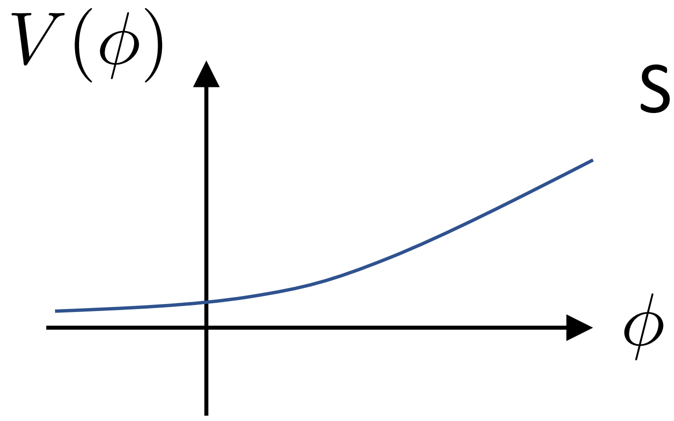
Power law model $F(R) = R + \gamma R^n$



T. Inagaki and H. Sakamoto, Int. J. Mod. Phys. **D29** (2020)

Screening mechanism

- Modified theories of Gravity should coincide with the general relativity around us. Some screening mechanism are proposed.
- Chameleon mechanism:



Scalaron potential

J. Khoury and A. Weltman, 2004,
P. Brax, C. van de Bruck, A. C. Davis and D. J. Shaw, 2008,
T. Katsuragawa and S. Matsuzaki, 2017

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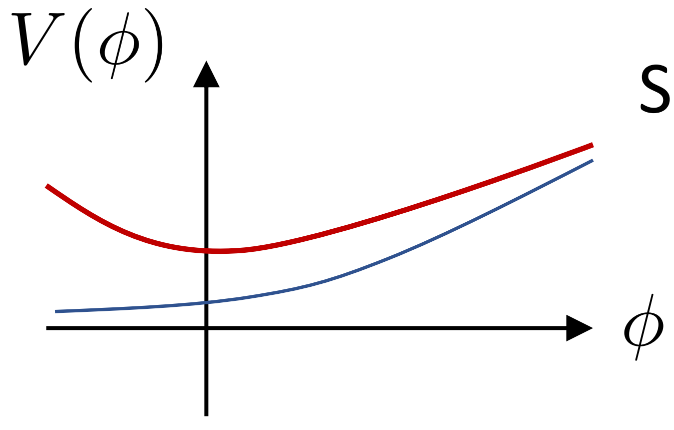

Scalaron potential + matter

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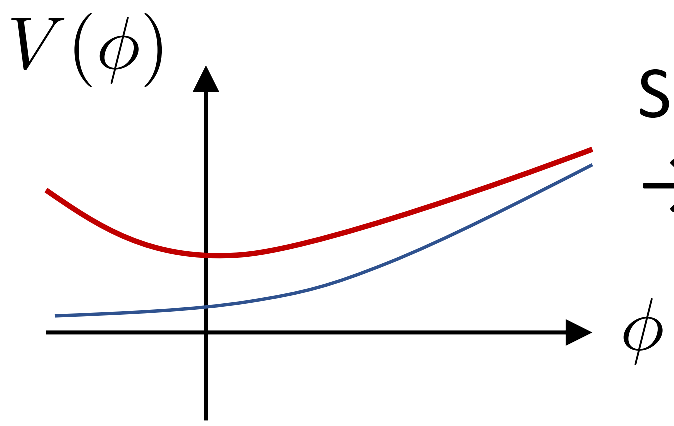
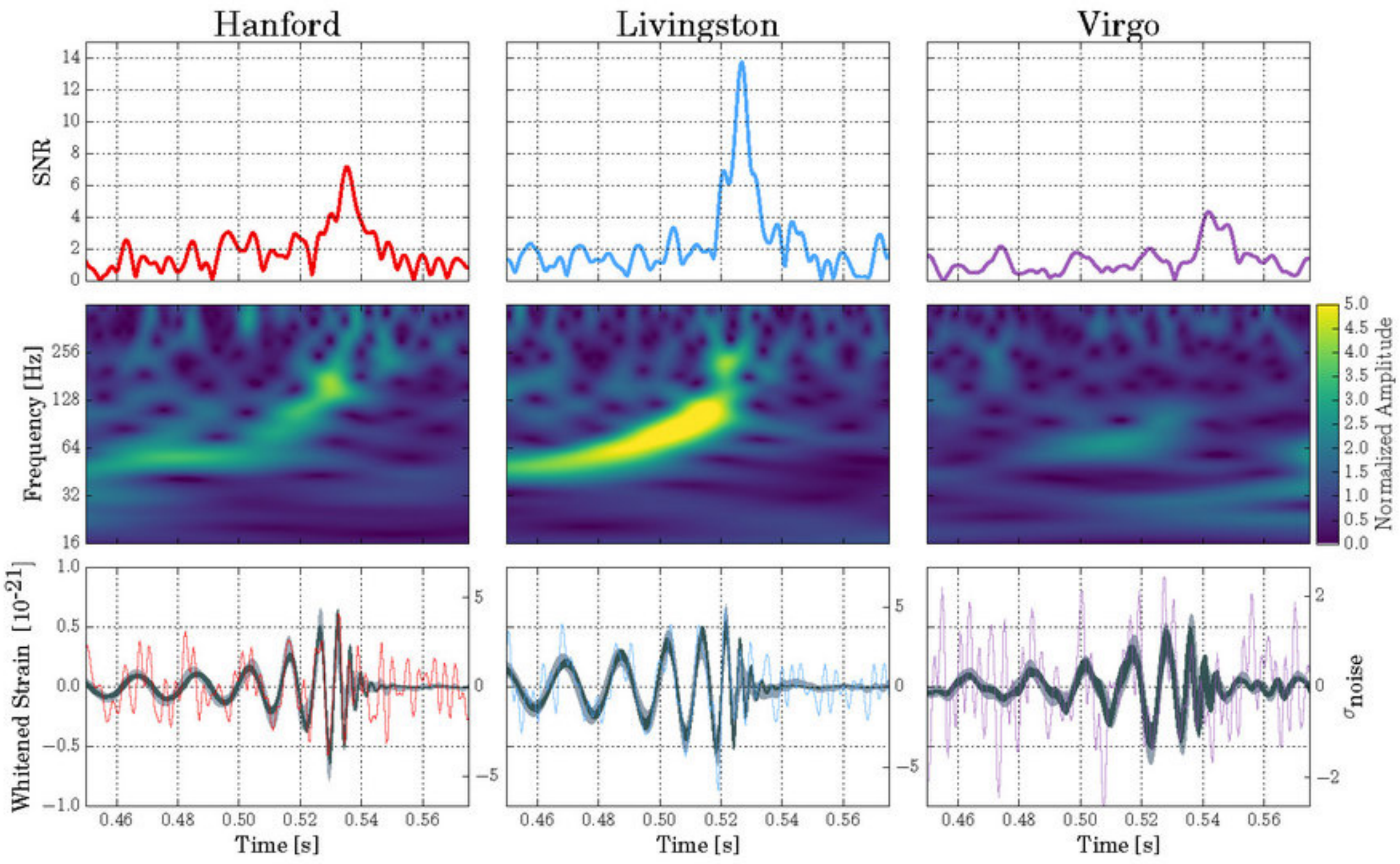


Scalaron potential + matter
→ Scalaron becomes heavy and screened out.
Dark matter candidate??

T. Inagaki, Y. Matsuo and H. Sakamoto, Int. J. Mod. Phys. **D28,** 2019,
T. Katsuragawa, S. Matsuzaki and E. Senaha, 2019

Gravitational waves



Gravitational wave signatures detected by each observatory
LIGO/Caltech/MIT/LSC 2017

Gravitational waves

- General relativity:
  two massless tensor modes
- F(R) gravity:
  two massless tensor + one massive scalar mode

$$[\Box - m_{F(R)}^2]\delta\Phi = 0, \quad m_{F(R)}^2 = \frac{1}{3}\left(\frac{F'(\tilde{R})}{F''(\tilde{R})} - \tilde{R}\right)$$

Y. S. Myung, 2016,
Y. Gong and S. Hou, 2018,
F. Moretti, F. Bombacigno and G. Montani, 2019,
T. Katsuragawa, T. Nakamura, T. Ikeda and S. Capozziello, 2019.

F(G) gravity case

- Einstein equation in De Sitter background

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu},$$

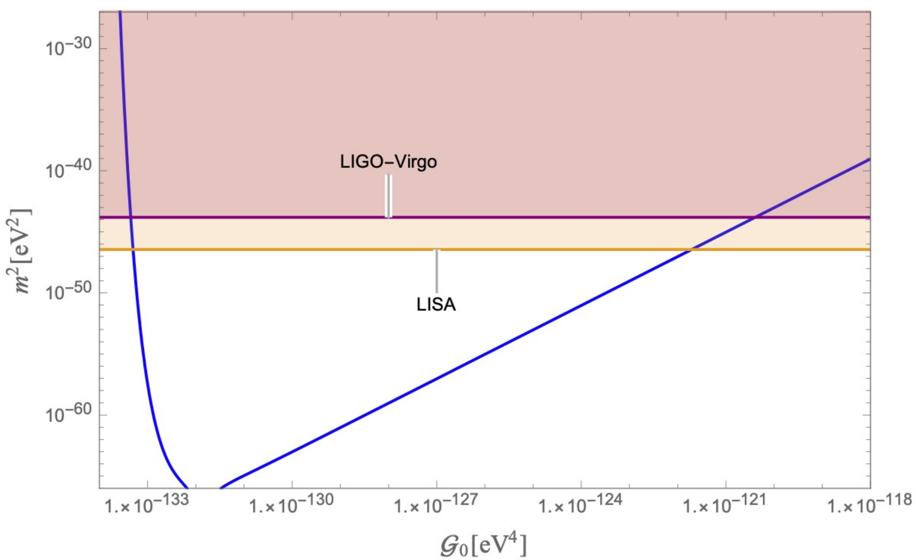
We obtain

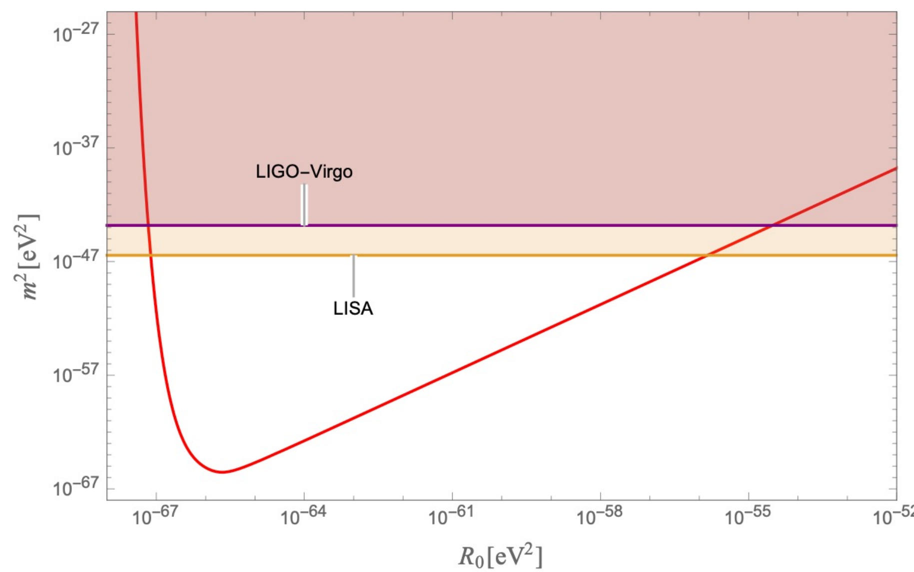
$$\frac{M_{\rm pl}^2}{4}\left[\Box h^T_{\mu\nu} - \frac{\tilde{R}}{6} h^T_{\mu\nu}\right] - \frac{\tilde{R}}{3}\left[\nabla_\mu \nabla_\nu - \frac{m^2_{F(\mathcal{G})}}{4}\tilde{g}_{\mu\nu}\right] F''(\tilde{\mathcal{G}})\delta\mathcal{G} = 0$$

Massless tensor modes   Massive scalar mode

T. Inagaki and M. Taniguchi, Int. J. Mod. Phys. **D29** (2020)

Massive scalar mode





$$F(R) = R - 2\Lambda(1 - e^{-R/R_0})$$

$$F(\mathcal{G}) = -\frac{M_{\rm pl}^2}{2} 2\Lambda(1 - e^{-\mathcal{G}/\mathcal{G}_0})$$

T. Inagaki and M. Taniguchi, Int. J. Mod. Phys. **D29** (2020)

Conclusion

Concluding remarks

- Alternative theory of the gravity is necessary.
- There are many possibilities.

Modification of the Einstein eq.

Einstein equation

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Concluding remarks

- Alternative theory of the gravity is necessary.
- There are many possibilities.
- To restrict the possibilities
  - Phenomenological consequences
  - Theoretical constraints