# Polarization observables as a probe of anomalous gauge-Higgs couplings

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# Plan of the talk

- Motivation of the work
- Spin Density Matrix
- Helicity Amplitudes for the process
- Asymmetries and Sensitivities
- Summary

# Motivations of the work

- The discovery of the Higgs boson ($H$) with mass around 125 GeV at the LHC completes the particle spectrum of the Standard Model (SM).
- A precise measurement of the couplings of the Higgs to electroweak gauge bosons is needed to uncover the exact mechanism of EWSB.
- Obsevables like total cross section, angular distribution are required to study the couplings in experiments.
- $W, Z$ being a spin-1 particle provides additional observables.

- Various studies(Saavedra *etal.*PhysRevD.93.011301, Rahaman *etal.* Eur.Phys.J. C76 (2016) no.10, 539) show that angular asymmetries corresponding to different polarizations are useful to probe new physics.
- J.Nakamura (JHEP08(2017)008 )studies $ZZH$ at LHC using polarisations of the $Z$, S.Banerjee *et al.*,PhysRevD.100.115004 (2019)has studied $ZH$ coupling in the Higgstrahlung process in the dimension 6 SMEFT.

# Goal of The Work

- Study anomalous $ZZH$ vertex in the associated $ZH$ production at the $e^+e^-$ and LHC using the $Z$ polarization observables.

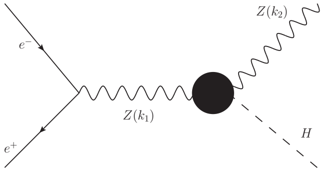



Figure: Feynman diagrams for $ZH$ production.

where the vertex $Z_\mu(k_1) \to Z_\nu(k_2)H$ takes the following Lorentz invariant structure

$$\Gamma^V_{\mu\nu} = \frac{g_w}{\cos\theta_w} m_z \left[ a_z g_{\mu\nu} + \frac{b_z}{m_z^2}\left(k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1.k_2\right) + \frac{\tilde{b}_z}{m_z^2}\epsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \right] \quad (1)$$

The form factors $a_z$, $b_z$ and $\tilde{b}_z$ are in general complex.

# Formalism

**Polarization Parameters of $Z$ boson**
The $2 \times 2$ density matrix for spin-1/2 system-

$$\rho = \frac{1}{2}I + \frac{1}{2}\boldsymbol{P}.\boldsymbol{\sigma} \tag{2}$$

where the Pauli matrices $\sigma$ serve the basis for this expansion and $\boldsymbol{P}$ is called the **spin- polarization vector** for the ensemble

$$\mathcal{P} = \langle \sigma \rangle = Tr(\rho \boldsymbol{\sigma})$$

For spin-1,the elements of $3 \times 3$ spin density matrix written as

$$\rho = \frac{1}{3}I + \frac{1}{2}\sum_{M=-1}^{M=1} \langle S_M \rangle^* S_M + \sum_{M=-2}^{M=2} \langle T_M \rangle^* T_M \tag{3}$$

where $S_0 = S_3$,$S_{\pm 1} = \mp \frac{1}{\sqrt{2}}(S_1 + iS_2)$ are the spin operators in spherical basis and $T_M$s are five rank 2 irreducible tensors built from $S_M$ .

# THE PRODUCTION AND DECAY DENSITY MATRICES

For a generic process $AB \to VX$, $V \to f\bar{f}'$. Total rate with $V$ being on-shell is given as

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_f} = \frac{2s+1}{4\pi}\sum_{\lambda,\lambda'} P(\lambda,\lambda')\Gamma(\lambda,\lambda') \quad (4)$$

Here $\sigma = \sigma_V BR(V \to f\bar{f}')$ is the total crossection for production of $V$. $P(\lambda,\lambda')(\lambda,\lambda' = \pm 1, 0)$ is the polarization density matrix for $V$ and in terms of a hermitian $3\times 3$ production density matrix given as

$$P(\lambda,\lambda') = \frac{1}{\sigma_v}\int \rho(\lambda,\lambda')d\Omega_v = \frac{1}{\sigma_v}\rho_T(\lambda,\lambda') \quad (5)$$

with $\sigma_v$ the production cross section of $V$ without decay. Matrix $P$ can be parametrized in terms of a vector $P = (P_x, P_y, P_z)$ and a rank 2 traceless,symmetric tensor $T_{ij}$ (E.Leader," Spin in particle physics")

# The production and decay density matrices

$$P(\lambda,\lambda^{'}) = \begin{bmatrix} \frac{1}{3}+\frac{P_z}{2}+\frac{T_{zz}}{\sqrt{6}} & \frac{P_x-iP_y}{2\sqrt{2}}+\frac{T_{xz}-iT_{yz}}{\sqrt{3}} & \frac{T_{xx}-T_{yy}-2iT_{yz}}{\sqrt{6}} \\ \frac{P_x+iP_y}{2\sqrt{2}}+\frac{T_{xz}+iT_{yz}}{\sqrt{3}} & \frac{1}{3}-\frac{2T_{zz}}{\sqrt{6}} & \frac{P_x-iP_y}{2\sqrt{2}}-\frac{T_{xz}-iT_{yz}}{\sqrt{3}} \\ \frac{T_{xx}-T_{yy}-2iT_{xy}}{\sqrt{6}} & \frac{P_x+iP_y}{2\sqrt{2}}-\frac{T_{xz}+iT_{yz}}{\sqrt{3}} & \frac{1}{3}-\frac{P_z}{2}+\frac{T_{zz}}{\sqrt{6}} \end{bmatrix} \quad (6)$$

The decay density matrix with the interaction vertex $Vf\bar{f}:\gamma^{\mu}(c_L^f P_L + c_R^f P_R)$ in its rest frame is given by(Boudjema,JHEP 0907 (2009) 028)

$$\Gamma(\lambda,\lambda^{'}) = \begin{bmatrix} \frac{(1+\cos^2\theta+2\alpha\cos\theta}{4} & \frac{\sin\theta(\alpha+\cos\theta)e^{i\phi}}{2\sqrt{2}} & \frac{(1-\cos\theta^2)e^{2i\phi}}{4} \\ \frac{\sin\theta(\alpha+\cos\theta)e^{-i\phi}}{2\sqrt{2}} & \frac{\sin^2\theta}{2} & \frac{\sin\theta(\alpha-\cos\theta)e^{i\phi}}{2\sqrt{2}} \\ \frac{(1-\cos\theta^2)e^{-2i\phi}}{4} & \frac{\sin\theta(\alpha-\cos\theta)e^{-i\phi}}{2\sqrt{2}} & \frac{(1+\cos^2\theta-2\alpha\cos\theta}{4} \end{bmatrix} \quad (7)$$

$\alpha \rightarrow \frac{c_R^2-c_L^2}{c_R^2+c_L^2}$ for masless final state fermions, is the polarization analyser. The angles $\theta$ and $\phi$ are polar and azimuthal angles of the fermions defined in the rest frame of the $V$.

Therefore the angular distribution of the fermion in the rest frame of $V$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_f} = \frac{3}{8\pi}[(\frac{2}{3} - \frac{T_{zz}}{\sqrt{6}}) - P_z \cos\theta] \tag{8}$$

$$+\sqrt{\frac{3}{2}} T_{zz} \cos^2\theta + (-P_x + 2\sqrt{\frac{2}{3}} T_{xz} \cos\theta) \sin\theta \cos\phi$$

$$+(-P_y + 2\sqrt{\frac{2}{3}} T_{yz} \cos\theta) \sin\theta \sin\phi$$

$$+(\frac{T_{xx} - T_{yy}}{\sqrt{6}}) \sin^2\theta \cos 2\phi + \sqrt{\frac{2}{3}} T_{xy} \sin^2\theta \sin 2\phi$$

Two ways to estimate the various polarization parameters of $Z$.

1. At production level, by using the polarization matrix elements (Rahaman *etal.* Eur.Phys.J. C76 (2016) no.10, 539)

$$P_x = \frac{\{\sigma(+,0)+\sigma(0,+)\}+\{\sigma(0,-)+\sigma(-,0)\}}{\sqrt{2}\sigma}$$

$$P_y = \frac{-i\{[\sigma(0,+)-\sigma(+,0)]+[\sigma(-,0)-\sigma(0,-)]\}}{\sqrt{2}\sigma}$$

$$P_z = \frac{[\sigma(+,+)]-[\sigma(-,-)]}{\sigma}$$

$$\begin{aligned}
T_{xy} &= \frac{-i\sqrt{6}[\sigma(-,+) - \sigma(+,-)]}{4\sigma} \\
T_{xz} &= \frac{\sqrt{3}\{[\sigma(+,0) + \sigma(0,+)] - [\sigma(0,-) + \sigma(-,0)]\}}{4\sigma} \\
T_{yz} &= \frac{-i\sqrt{3}\{[\sigma(0,+) - \sigma(+,0)] - [\sigma(-,0) - \sigma(0,-)]\}}{4\sigma} \\
T_{xx} - T_{yy} &= \frac{\sqrt{6}[\sigma(-,+) + \sigma(+,-)]}{2\sigma} \\
T_{zz} &= \frac{\sqrt{6}}{2}\left\{\frac{[\sigma(+,+)] + [\sigma(-,-)]}{\sigma} - \frac{2}{3}\right\} \\
&= \frac{\sqrt{6}}{2}\left[\frac{1}{3} - \frac{\sigma(0,0)}{\sigma}\right]
\end{aligned}$$

Here $T_{xx}$ and $T_{yy}$ can be separately calculated by using the tracelessness property of $T_{ij}$. $\sigma(i,j)$ is the integral of $\rho(i,j)$ and $\sigma$ is the total production cross section given by the sum of the helicity fractions over the phase space

$$\sigma = \sigma(0,0) + \sigma(+,+) + \sigma(-,-) \tag{9}$$

2. At decay level, by using partial integration of the differential distribution (**equation**(4)) and then constructing various asymmetries.

$$A_x = \frac{3\alpha P_x}{4} \equiv \frac{\sigma(\cos\phi > 0) - \sigma(\cos\phi < 0)}{\sigma(\cos\phi > 0) + \sigma(\cos\phi < 0)}$$

$$A_y = \frac{3\alpha P_y}{4} \equiv \frac{\sigma(\sin\phi > 0) - \sigma(\sin\phi < 0)}{\sigma(\sin\phi > 0) + \sigma(\sin\phi < 0)}$$

$$A_z = \frac{3\alpha P_z}{4} \equiv \frac{\sigma(\cos\theta^{>} 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)}$$

$$A_{xz} = \frac{-2}{\pi}\sqrt{\frac{2}{3}}T_{xz} \equiv \frac{\sigma(\cos\theta\cos\phi < 0) - \sigma(\cos\theta\cos\phi > 0)}{\sigma(\cos\theta\cos\phi > 0) + \sigma(\cos\theta\cos\phi < 0)}$$

$$
\begin{aligned}
A_{yz} &= \frac{2}{\pi}\sqrt{\frac{2}{3}}T_{yz} \equiv \frac{\sigma(\cos\theta\sin\phi > 0) - \sigma(\cos\theta\sin\phi < 0)}{\sigma(\cos\theta\sin\phi > 0) + \sigma(\cos\theta\sin\phi < 0)} \\
A_{x^2-y^2} &= \frac{1}{\pi}\sqrt{\frac{2}{3}}(T_{xx} - T_{yy}) \equiv \frac{\sigma(\cos 2\phi > 0) - \sigma(\cos 2\phi < 0)}{\sigma(\cos 2\phi > 0) + \sigma(\cos 2\phi < 0)} \\
A_{xy} &= \frac{2}{\pi}\sqrt{\frac{2}{3}}T_{xy} \equiv \frac{\sigma(\sin 2\phi > 0) - \sigma(\sin 2\phi < 0)}{\sigma(\sin 2\phi > 0) + \sigma(\sin 2\phi < 0)} \\
A_{zz} &= \frac{3}{8}\sqrt{\frac{3}{2}}T_{zz} \equiv \frac{\sigma(\sin 3\theta > 0) - \sigma(\sin 3\theta < 0)}{\sigma(\sin 3\theta > 0) + \sigma(\sin 3\theta < 0)}
\end{aligned}
\tag{10}
$$

# Helicity Amplitudes for $e^-(p_1) + e^+(p_2) \to Z^\alpha(k_2) + H(k)$

In the limit of massless initial states

$$
\begin{aligned}
M(-,+,+) &= \frac{g_W^2 m_Z \sqrt{s}}{\cos^2\theta_W((s-m_Z^2)+i\Gamma_Z m_Z)} \frac{(c_V+c_a)}{2}\left[1-\frac{\sqrt{s}}{m_Z^2}(E_Z b_Z + i\tilde{b_Z} P_Z)\right]\frac{(1-\cos\theta)}{\sqrt{2}} \\
M(-,+,-) &= \frac{g_W^2 m_Z \sqrt{s}}{\cos^2\theta_W((s-m_Z^2)+i\Gamma_Z m_Z)} \frac{(c_V+c_a)}{2}\left[1-\frac{\sqrt{s}}{m_Z^2}(E_Z b_Z - i\tilde{b_Z} P_Z)\right]\frac{(1+\cos\theta)}{\sqrt{2}} \\
M(-,+,0) &= \frac{g_W^2 \sqrt{s}}{\cos^2\theta_W((s-m_Z^2)+i\Gamma_Z m_Z)} \frac{(c_V+c_a)}{2}\left[E_Z-\sqrt{s}b_Z\right]\sin\theta \\
M(+,-,+) &= \frac{g_W^2 m_Z \sqrt{s}}{\cos^2\theta_W((s-m_Z^2)+i\Gamma_Z m_Z)} \frac{(c_V-c_a)}{2}\left[-1+\frac{\sqrt{s}}{m_Z^2}(E_Z b_Z + i\tilde{b_Z} P_Z)\right]\frac{(1+\cos\theta)}{\sqrt{2}} \\
M(+,-,-) &= \frac{g_W^2 m_Z \sqrt{s}}{\cos^2\theta_W((s-m_Z^2)+i\Gamma_Z m_Z)} \frac{(c_V-c_a)}{2}\left[-1+\frac{\sqrt{s}}{m_Z^2}(E_Z b_Z - i\tilde{b_Z} P_Z)\right]\frac{(1-\cos\theta)}{\sqrt{2}} \\
M(+,-,0) &= \frac{g_W^2 \sqrt{s}}{\cos^2\theta_W((s-m_Z^2)+i\Gamma_Z m_Z)} \frac{(c_V-c_a)}{2}\left[E_Z-\sqrt{s}b_Z\right]\sin\theta
\end{aligned}
$$

where the first two entries in $M$ denote the helicities $+1/2$ and $-1/2$ of the electron and positron respectively

$\sqrt{s}$ = total center of mass energy , $C_v = -0.5 + \sin^2\theta_w$, $C_a = -0.5$ where $\theta_w$ is the weak mixing angle.

we adopt the following representations for the polarization vectors of $Z$

$$\varepsilon_\mu(s = \pm 1) = \mp\frac{1}{\sqrt{2}}(0, -\cos\theta, \mp i, \sin\theta) \tag{11}$$

$$\varepsilon_\mu(s = 0) = \frac{1}{m_z}(\mid p_z \mid, -E_z\sin\theta, 0, -E_z\cos\theta) \tag{12}$$

where $E_z$,$\mid p_z \mid$ are the energy and momentum of the $Z$ respectively,with $\theta$ being the polar angle made by $Z$ with respect to the $e^-$ coming along the positive $z$ axis.

# Production Density Matrix Elements

The density matrix elements derived from the helicity amplitudes, to linear order in couplings $b_z, \tilde{b}_z$ are

$$\begin{aligned}
\sigma(\pm,\pm) &= \frac{2(1-P_L\bar{P}_L)g^4 m_Z^2 s}{3\cos^4\theta_W (s-m_Z^2)^2}(c_V^2+c_A^2-2P_L^{\rm eff}c_Vc_A) \\
&\times \left[1-2(\text{Re } b_Z \mp \beta_Z \text{Im } \tilde{b}_Z)\frac{E_Z\sqrt{s}}{m_Z^2}\right] \\
\sigma(0,0) &= \frac{2(1-P_L\bar{P}_L)g^4 E_Z^2 s}{3\cos^4\theta_W (s-m_Z^2)^2}(c_V^2+c_A^2-2P_L^{\rm eff}c_Vc_A) \\
&\times \left[1-2\text{Re } b_Z\frac{\sqrt{s}}{E_Z}\right] \\
\sigma(\pm,\mp) &= \frac{(1-P_L\bar{P}_L)g^4 m_Z^2 s}{3\cos^4\theta_W (s-m_Z^2)^2}(c_V^2+c_A^2-2P_L^{\rm eff}c_Vc_A) \\
&\times \left[1-2(\text{Re } b_Z \pm i\beta_Z \text{Re } \tilde{b}_Z)\frac{E_Z\sqrt{s}}{m_Z^2}\right] \\
\sigma(\pm,0) &= \frac{(1-P_L\bar{P}_L)\pi g^4 m_Z E_Z s}{4\sqrt{2}\cos^4\theta_W (s-m_Z^2)^2}\left[(2c_Vc_A-P_L^{\rm eff}(c_V^2+c_A^2))\right] \\
&\times \left[1-\text{Re } b_Z\sqrt{s}\frac{(E_Z^2+m_Z^2)}{E_Z m_Z^2}-i\sqrt{s}\frac{E_Z}{m_Z^2}\left(\text{Im } b_Z\beta_Z^2 \pm \tilde{b}_Z\beta_Z)\right)\right]
\end{aligned}$$

In the above equations, $P_L^{\rm eff} = (P_L - \bar{P}_L)/(1-P_L\bar{P}_L)$. $\beta_Z = \frac{|\vec{p_Z}|}{E_Z}$.

# SM Asymmetries at $\sqrt{s} = 500$ GeV

We consider longitudinal polarizations of $P_L = \pm 0.8$ and $\bar{P}_L = \pm 0.3$ which would be available in the collider.

| Observable | $P_L = 0$ $\bar{P}_L = 0$ | $P_L = -0.8$ $\bar{P}_L = 0.3$ |
|---|---|---|
| $\sigma$ (in fb) | 58.04 | 81.56 |
| $A_x$ | $-0.012$ | $-0.071$ |
| $A_{x^2-y^2}$ | 0.035 | 0.035 |
| $A_{zz}$ | $-0.251$ | $-0.251$ |

Table: The total production cross section (in fb) of $Z$ and the non-zero angular asymmetries in the SM for unpolarized and polarized beams at $\sqrt{s} = 500$ GeV.

# SM Asymmetries at $\sqrt{s} = 250$ GeV

| Observable | $P_L = 0$<br>$\bar{P}_L = 0$ | $P_L = -0.8$<br>$\bar{P}_L = 0.3$ |
|---|---|---|
| $\sigma$ (in fb) | 243.67 | 342.24 |
| $A_x$ | $-0.014$ | $-0.085$ |
| $A_{x^2-y^2}$ | 0.092 | 0.092 |
| $A_{zz}$ | $-0.05$ | $-0.05$ |

Table: The total production cross section (in fb) of $Z$ and the non-zero angular asymmetries in the SM for unpolarized and polarized beams at $\sqrt{s} = 250$ GeV.

# BSM Asymmetries at $\sqrt{s} = 500$ GeV

| Observable | Coupling | $P_L = 0$ <br> $\bar{P}_L = 0$ | $P_L = -0.8$ <br> $\bar{P}_L = 0.3$ |
|---|---|---|---|
| $\sigma$ (in fb) | Re $b_z$ | $-559.5$ | $-785.88$ |
| $A_x$ | Re $b_z$ | $+0.081$ | $+0.497$ |
| $A_y$ | Re $\tilde{b}_z$ | $-0.157$ | $-0.958$ |
| $A_z$ | Im $\tilde{b}_z$ | $-0.668$ | $-0.668$ |
| $A_{xy}$ | Re $\tilde{b}_z$ | $+0.948$ | $+0.948$ |
| $A_{yz}$ | Im $b_z$ | $+0.412$ | $+2.521$ |
| $A_{xz}$ | Im $\tilde{b}_z$ | $-0.444$ | $-2.720$ |
| $A_{x^2-y^2}$ | Re $b_z$ | $-0.685$ | $-0.685$ |
| $A_{zz}$ | Re $b_z$ | $-2.421$ | $-2.421$ |

Table: Anomalous contribution to cross section (in fb) and angular asymmetries for unpolarized and polarized beams at $\sqrt{s} = 500$ GeV for unit values of the relevant couplings.

# BSM Asymmetries at $\sqrt{s} = 250$ GeV

| Observable | Coupling | $P_L = 0$ $\bar{P}_L = 0$ | $P_L = -0.8$ $\bar{P}_L = 0.3$ |
|---|---|---|---|
| $\sigma$ (in fb) | Re $b_z$ | $-1400.17$ | $-1966.55$ |
| $A_x$ | Re $b_z$ | $-0.0022$ | $-0.014$ |
| $A_y$ | Re $\tilde{b}_z$ | $-0.026$ | $-0.158$ |
| $A_z$ | Im $\tilde{b}_z$ | $-0.242$ | $-0.242$ |
| $A_{xy}$ | Re $\tilde{b}_z$ | $+0.344$ | $+0.344$ |
| $A_{yz}$ | Im $b_z$ | $+0.041$ | $+0.253$ |
| $A_{xz}$ | Im $\tilde{b}_z$ | $-0.073$ | $-0.448$ |
| $A_{x^2-y^2}$ | Re $b_z$ | $-0.082$ | $-0.082$ |
| $A_{zz}$ | Re $b_z$ | $-0.289$ | $-0.289$ |

Table: Anomalous contribution to cross section (in fb) and angular asymmetries for unpolarized and polarized beams at $\sqrt{s} = 250$ GeV for unit values of the relevant couplings.

# Sensitivities at $\sqrt{s} = 500$ GeV

| Observable | Coupling | Limit ($\times 10^{-3}$) for | |
|---|---|---|---|
| | | $P_L = 0$, $\bar{P}_L = 0$ | $P_L = -0.8$, $\bar{P}_L = 0.3$ |
| $\sigma$ | Re $b_z$ | 3.32 | 2.8 |
| $A_x$ | Re $b_z$ | 394 | 54.2 |
| $A_y$ | Re $\tilde{b}_z$ | 204 | 28.2 |
| $A_z$ | Im $\tilde{b}_z$ | 47.9 | 40.4 |
| $A_{xy}$ | Re $\tilde{b}_z$ | 33.7 | 28.5 |
| $A_{yz}$ | Im $b_z$ | 77.7 | 10.7 |
| $A_{xz}$ | Im $\tilde{b}_z$ | 72.0 | 9.93 |
| $A_{x^2-y^2}$ | Re $b_z$ | 46.7 | 39.4 |
| $A_{zz}$ | Re $b_z$ | 12.8 | 10.8 |

TABLE: $1\sigma$ limit obtained from various leptonic asymmetries for unpolarized and polarized beams at $\sqrt{s} = 500$ GeV.

# Sensitivities at $\sqrt{s} = 250$ GeV

| Observable | Coupling | Limit ($\times 10^{-3}$) for $P_L = 0$, $\bar{P}_L = 0$ | Limit ($\times 10^{-3}$) for $P_L = -0.8$, $\bar{P}_L = 0.3$ |
|---|---|---|---|
| $\sigma$ | Re $b_z$ | 1.36 | 1.15 |
| $A_x$ | Re $b_z$ | 3480 | 478 |
| $A_y$ | Re $\tilde{b}_z$ | 303 | 41.7 |
| $A_z$ | Im $\tilde{b}_z$ | 32.3 | 27.2 |
| $A_{xy}$ | Re $\tilde{b}_z$ | 22.7 | 19.2 |
| $A_{yz}$ | Im $b_z$ | 189 | 26.1 |
| $A_{xz}$ | Im $\tilde{b}_z$ | 107 | 14.7 |
| $A_{x^2-y^2}$ | Re $b_z$ | 94.5 | 80.2 |
| $A_{zz}$ | Re $b_z$ | 26.8 | 22.8 |

TABLE: $1\sigma$ limit obtained from various leptonic asymmetries for unpolarized and polarized beams at $\sqrt{s} = 250$ GeV.

# Asymmteries with Kinematical cuts

1. $E_f \geq 10 GeV$ for each outgoing charged lepton.
2. $5^\circ \leq \theta_0 \leq 175^\circ$ for each outgoing charged lepton to remain away from the beam pipe.

It is observed that these cuts lead to a less than 1% change in all the observables including the total cross section.

# Asymmetries and Sensitivities at LHC

$$q(p_1) + \bar{q}(p_2) \rightarrow Z^{\alpha}(p) + H(k)$$

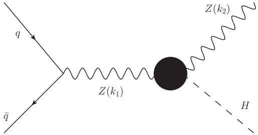



Figure: Feynman diagrams for $ZH$ production.

where the vertex $Z_{\mu}(k_1) \rightarrow Z_{\nu}(k_2)H$ takes the following Lorentz invariant structure

$$\Gamma^{V}_{\mu\nu} = \frac{g_w}{\cos\theta_w} m_z \left[ a_z g_{\mu\nu} + \frac{b_z}{m_z^2}\left(k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1.k_2\right) + \frac{\tilde{b}_z}{m_z^2}\epsilon_{\mu\nu\alpha\beta}k_1^{\alpha}k_2^{\beta}\right]$$

The form factors $a_z$, $b_z$ and $\tilde{b}_z$ are in general complex.

# Production Density Matrix Elements

The density matrix elements derived from the helicity amplitudes, to quadratic order in couplings $b_z, \tilde{b}_z$ are

$$
\begin{aligned}
\rho(\pm,\pm) &= \frac{g^4 m_Z^2 s}{8\cos^4\theta_W(\hat{s}-m_Z^2)^2}\left[(c_V+c_A)^2(1\mp\cos\theta)^2\right. \\
&\left.+(c_V-c_A)^2(1\pm\cos\theta)^2\right]\left[1-2(\mathrm{Re}\ b_Z \mp \beta_Z \mathrm{Im}\ \tilde{b}_Z)\frac{E_Z\sqrt{\hat{s}}}{m_Z^2}\right. \\
&+\frac{E_Z^2\hat{s}}{m_Z^4}|b_Z|^2 \mp \frac{2E_Z P_Z \hat{s}}{m_Z^4}(\mathrm{Im}\ \tilde{b}_Z\ \mathrm{Re}\ b_Z - \mathrm{Im}\ b_Z\ \mathrm{Re}\ \tilde{b}_Z) \\
&\left.+\frac{P_Z^2\hat{s}}{m_Z^4}|\tilde{b}_Z|^2\right] \\
\rho(0,0) &= \frac{g^4 E_Z^2 s}{2\cos^4\theta_W(s-m_Z^2)^2}\sin^2\theta\,(c_V^2+c_A^2)\left[1-2\mathrm{Re}\ b_Z\frac{\sqrt{s}}{E_Z}\right. \\
&\left.+\frac{\hat{s}}{E_Z^2}|b_Z|^2\right]
\end{aligned}
$$

$$\rho(\pm,\mp) = \frac{g^4 m_Z^2 s}{4\cos^4\theta_W(\hat{s}-m_Z^2)^2}\sin^2\theta\,(c_V^2+c_A^2)$$
$$\times\Big[1-2(\mathrm{Re}\ b_Z \pm i\beta_Z \mathrm{Re}\ \tilde{b}_Z)\frac{E_Z\sqrt{s}}{m_Z^2}+\frac{E_Z^2\hat{s}}{m_Z^4}|b_Z|^2 \pm i\frac{2E_Z P_Z\hat{s}}{m_Z^4}$$
$$(\mathrm{Im}\ \tilde{b}_Z\ \mathrm{Im}\ b_Z+\mathrm{Re}\ b_Z\ \mathrm{Re}\ \tilde{b}_Z)-\frac{2P_Z^2\hat{s}}{m_Z^4}|\tilde{b}_Z|^2\Big] \quad (13)$$

$$\rho(\pm,0) = \frac{g^4 m_Z E_Z s}{4\sqrt{2}\cos^4\theta_W(\hat{s}-m_Z^2)^2}\sin\theta$$
$$\times\left[(c_V+c_A)^2(1\mp\cos\theta)-(c_V-c_A)^2(1\pm\cos\theta)\right]$$
$$\times\Big[1-\mathrm{Re}\ b_Z\sqrt{\hat{s}}\frac{(E_Z^2+m_Z^2)}{E_Z m_Z^2}-i\sqrt{\hat{s}}\frac{E_Z}{m_Z^2}\left(\mathrm{Im}\ b_Z\ \beta_Z^2\pm\tilde{b}_Z\beta_Z\right)$$
$$\mp\frac{\hat{s}}{m_Z^2}|b_Z|^2\pm\frac{\hat{s}P_Z}{m_Z^2E_Z}(\mathrm{Im}\ b_Z+i\mathrm{Re}\ b_Z)(\mathrm{Re}\ \tilde{b}_Z+i\mathrm{Im}\ \tilde{b}_Z)\Big] \quad (14)$$

where $\beta_z = \frac{|\vec{p_z}|}{E_z}$.

$\sqrt{\hat{s}}$ is the partonic c.m. energy, and $c_V = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$ denoting the vector coupling and $c_A = \frac{1}{2}$ corresponds to axial vector couplings of the $Z$ to up-type quarks. Similarly for down-type quarks the vector and axial couplings are given by $c_V = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$ and $c_A = -\frac{1}{2}$ respectively.
we adopt the following representations for the polarization vectors of $Z$

$$\varepsilon_\mu(s = \pm 1) = \mp\frac{1}{\sqrt{2}}(0, -\cos\theta, \mp i, \sin\theta) \tag{15}$$

$$\varepsilon_\mu(s = 0) = \frac{1}{m_z}(\mid p_z \mid, -E_z\sin\theta, 0, -E_z\cos\theta) \tag{16}$$

where $E_z$,$\mid p_z \mid$ are the energy and momentum of the $Z$ respectively,with $\theta$ being the polar angle made by $Z$ with respect to the quark coming along the positive $z$ axis.

# Polarization asymmetries

$$\sigma = 0.067294\ (7506.45\ |\tilde{b}_Z|^2 + 12420.4\ |b_Z|^2 - 4486.85\ \text{Re}\ b_Z + 665.87) \tag{17}$$

$$A_x = \frac{0.012\ \text{Re}\ b_Z - 0.019\ |b_Z|^2 - 0.002}{0.604\ |\tilde{b}_Z|^2 + |b_Z|^2 - 0.361\ \text{Re}\ b_Z + 0.054} \tag{18}$$

$$A_y = \frac{0.024\ \text{Im}\ \tilde{b}_Z\ \text{Im}\ b_Z + \text{Re}\ \tilde{b}_Z\ (0.024\ \text{Re}\ b_Z - 0.011)}{|\tilde{b}_Z|^2 + 1.655\ |b_Z|^2 + (\text{Re}\ \tilde{b}_Z)^2 - 0.598\ \text{Re}\ b_Z + 0.089} \tag{19}$$

$$A_z = \frac{\text{Im}\ \tilde{b}_Z\ (1976.66\ \text{Re}\ b_Z - 232.627) - 1976.66\ \text{Im}\ b_Z\ \text{Re}\ \tilde{b}_Z}{7506.45\ |\tilde{b}_Z|^2 + 12420.4\ |b_Z|^2 - 4486.85\ \text{Re}\ b_Z + 665.87} \tag{20}$$

$$A_{xy} = \frac{\text{Re}\ \tilde{b}_Z\ (0.044 - 0.374\ \text{Re}\ b_Z) - 0.374\ \text{Im}\ \tilde{b}_Z\ \text{Im}\ b_Z}{|\tilde{b}_Z|^2 + 1.655\ |b_Z|^2 - 0.598\ \text{Re}\ b_Z + 0.089} \tag{21}$$

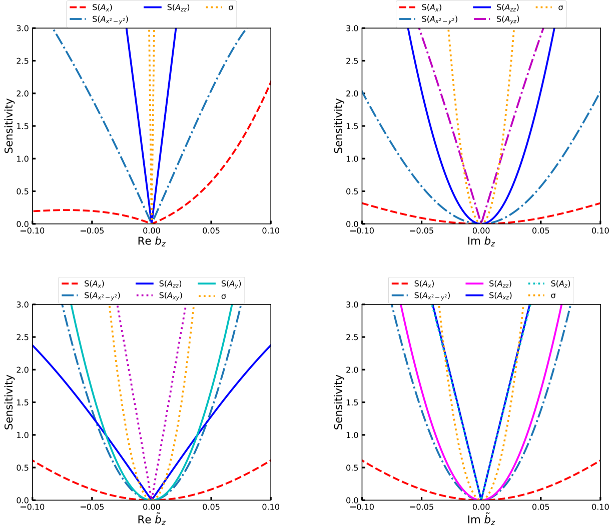
$$A_{yz} = \frac{190.164 \text{ Im } b_Z}{7506.45\ |\tilde{b}_Z|^2 + 12420.4\ |b_Z|^2 - 4486.85 \text{ Re } b_Z + 665.87} \tag{22}$$

$$A_{xz} = \frac{\text{Im } \tilde{b}_Z\ (0.0404 \text{ Re } b_Z - 0.019) - 0.0404 \text{ Im } b_Z \text{ Re } \tilde{b}_Z}{0.604\ |\tilde{b}_Z|^2 + |b_Z|^2 - 0.361 \text{ Re } b_Z + 0.054} \tag{23}$$

$$A_{x^2-y^2} = \frac{-0.297\ |\tilde{b}_Z|^2 + 0.019 \text{ Re } b_Z - 0.005}{|\tilde{b}_Z|^2 + 1.655\ |b_Z|^2 - 0.598 \text{ Re } b_Z + 0.089} + 0.138 \tag{24}$$

$$A_{zz} = \frac{0.074\ |\tilde{b}_Z|^2 + 0.068 \text{ Re } b_Z - 0.019}{|\tilde{b}_Z|^2 + 1.655\ |b_Z|^2 - 0.598 \text{ Re } b_Z + 0.089} + 0.113 \tag{25}$$

# Sensitivities at $\sqrt{s} = 14$ TeV and $\int \mathcal{L}dt = 1000$ fb$^{-1}$



| Observable | Coupling | Limit ($\times 10^{-3}$) |
| --- | --- | --- |
| $\sigma$ | $\lvert \text{Re } b_Z \rvert$ | 0.70 |
| $\sigma$ | $\lvert \text{Im } b_Z \rvert$ | 15.9 |
| $A_{xy}$ | $\lvert \text{Re } \tilde{b}_Z \rvert$ | 9.54 |
| $A_{xz}$, $A_z$ | $\lvert \text{Im } \tilde{b}_Z \rvert$ | 13.3 |

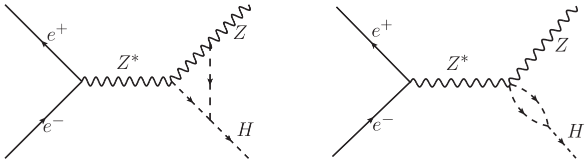
TABLE: The best $1\sigma$ limit on couplings and the corresponding observables at $\sqrt{s} = 14$ TeV

(K.Rao, S.D.Rindani, P.Sarmah, Nucl.Phys.B 964 115317 (2021))

- The CMS Collabotration(PhysRevD.100.112002) obtains bounds on CP conserving and CP violating $ZZH$ couplings which in our notation translate to $|\mathrm{Re} b_Z/a_Z| < 0.058$ and $|\mathrm{Re} \tilde{b}_Z/a_Z| < 0.078$ at $\sqrt{s} = 13$ TeV,$\int \mathcal{L}dt = 35.9$ fb$^{-1}$

# Use of $Z$ polarization in $e^+e^- \to ZH$ to measure the triple-Higgs coupling

(K. Rao, S.D.Rindani, P. Sarmah, B. Singh, arxiv: 2109.11134)




$\lambda_3$ deviates from its SM value,

$$\lambda_3 = \lambda_3^{SM}(1+\kappa). \tag{26}$$

Aim is to show that a particular CPT odd asymmetry, $A_{yz}$ sensitive to the loop-level contribution of the triple-Higgs coupling.

$$A_{yz} = \frac{2}{\pi}\sqrt{\frac{2}{3}}T_{yz} \equiv \frac{\sigma(\cos\theta^* \sin\phi^* > 0) - \sigma(\cos\theta^* \sin\phi^* < 0)}{\sigma(\cos\theta^* \sin\phi^* > 0) + \sigma(\cos\theta^* \sin\phi^* < 0)} \tag{27}$$

$$A_{yz} = \left( \frac{2c_V c_A - P_L^{\rm eff}(c_V^2 + c_A^2)}{4(c_V^2 + c_A^2 - 2P_L^{\rm eff} c_V c_A)} \right) \left( \frac{|\vec{k}_Z|^2 \sqrt{s}}{(E_Z^2 + m_Z^2) m_Z} \right) \left( \frac{{\rm Im}\ (a_Z^* b_Z)}{|a_Z|^2} \right). \quad (28)$$

where

$$ {\rm Im}\ b_Z = m_Z^2 {\rm Im}\ \mathcal{F}_2,\ \mathcal{F}_2(k_1^2, k_2^2) = \frac{\lambda_3^{\rm SM}(1+\kappa)}{(4\pi)^2} 12(C_1 + C_{11} + C_{12}). \quad (29)$$

The asymmetry proportional to Im $b_Z$ that gets contribution at one-loop from the triple-Higgs coupling.

# Result

| Collider | c.m. energy (Gev) | $10^4 \times A_{yz}$ unpolarized beams | polarized beams | Luminosity ($ab^{-1}$) | Limit unpolarized beams | polarized beams |
|---|---|---|---|---|---|---|
| CEPC | 240 | −0.159 | −0.625 | 10 | 506 | 105 |
| CEPC | 240 | −0.159 | −0.625 | 20 | 358 | 74.4 |
| CLIC | 380 | −2.88 | −10.6 | 0.5 | 124 | 31.0 |
| FCC | 240 | −0.159 | | 10 | 506 | |
| FCC | 250 | −0.314 | | 5 | 362 | |
| FCC | 365 | −2.64 | | 1.5 | 78.2 | |
| ILC | 250 | −0.314 | −1.23 | 2 | 573 | 119 |
| ILC | 250 | −0.314 | −1.23 | 5 | 362 | 75.3 |
| ILC | 350 | −2.39 | −9.38 | 30 | 19.4 | 4.03 |
| ILC | 500 | −4.00 | −15.7 | 4 | 31.6 | 6.57 |
| ILC | 500 | −4.00 | −15.7 | 10 | 20.0 | 4.16 |
| ILC | 500 | −4.00 | −15.7 | 30 | 11.5 | 2.40 |

TABLE: Values of the asymmetry $A_{yz}$ for $\kappa = 1$ and 1 $\sigma$ limits on $\kappa$ from $A_{yz}$ at various colliders with different energies and luminosities.

# Summary

- We study the angular asymmetries from the eight polarization parameters of Z boson at the $e^+e^-$ and LHC.
- We see that most of the $1\sigma$ limits are of the order of a few times $10^{-3}$ for 500 GeV $e^+e^-$ colliders and find that beams with opposite polarization provides better limits on the couplings.
- For LHC at c.m energy $\sqrt{s} = 14$ TeV with integrated luminosity $\int \mathcal{L}dt = 1000$ fb$^{-1}$ could provide a limit on the CP conseving couplings Re$b_z$ in the interval $[-0.7, 0.7] \times 10^{-3}$ and Im$b_z$ in the interval $[-15.9, 15.9] \times 10^{-3}$.
- CP violating couplings,Re$\tilde{b}_z$ and Im$\tilde{b}_z$ get a best bound of $|\text{Re}\tilde{b}_z| \leq 9.54 \times 10^{-3}$ and $|\text{Im}\tilde{b}_z| \leq 13.3 \times 10^{-3}$respectively.
- In case of triple Higgs coupling, high luminosity of at least 30 fb$^{-1}$ is needed to constrain anomalous triple-Higgs coupling even at the level of 200% to 400%. The better limit of about 240% requires a c.m. energy of 500 GeV.

# Back up

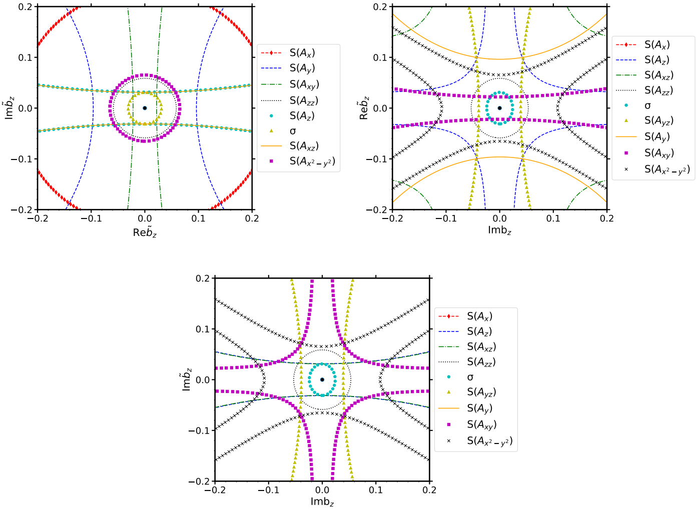



Figure: 1σ sensitivity contours for cross section and asymmetries obtained by varying

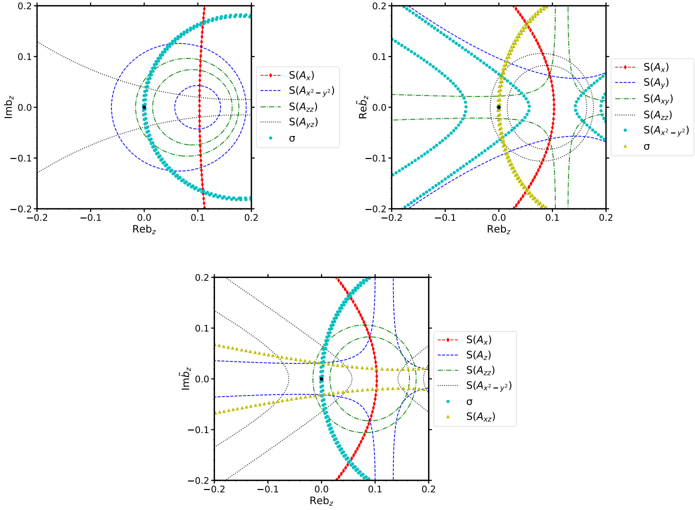



FIGURE: $1\sigma$ sensitivity contours for cross-section and asymmetries obtained by varying two parameters simultaneously.

# Sensitivities at $\sqrt{s} = 14$ TeV LHC

| Observable | Coupling | Limit ($\times 10^{-3}$) |
|---|---|---|
| $\sigma$ | Re $b_Z$ | 0.70 |
| $A_x$ | Re $b_Z$ | 136 |
| $A_y$ | Re $\tilde{b}_Z$ | 37.9 |
| $A_z$ | Im $\tilde{b}_Z$ | 13.5 |
| $A_{xy}$ | Re $\tilde{b}_Z$ | 9.53 |
| $A_{yz}$ | Im $b_Z$ | 16.5 |
| $A_{xz}$ | Im $\tilde{b}_Z$ | 13.3 |
| $A_{x^2-y^2}$ | Re $b_Z$ | 24.4 |
| $A_{zz}$ | Re $b_Z$ | 6.88 |

TABLE: $1\sigma$ limit obtained from cross section and various leptonic asymmetries calculated upto linear order in couplings at $\sqrt{s} = 14$ TeV with integrated luminosity $\int \mathcal{L} dt = 1000$ fb$^{-1}$.