Fronsdal Lagrangian extended with continuous spin parameter

Topics: Higher Spin Fields

2nd IITB-Hiroshima workshop online

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Based on the collaboration with I.L. Buchbinder and V. Krykhtin

Higher Spin theory: spin independent framework, including scalar, spinor, vector, gravity, etc

Plan of talk

I. Introduction
   i. Poincare symmetry as a space time symmetry. and possible particles
   ii. Construction for continuous spin field
II. Modifying massless theory
   i. Possible form of Lagrangian and gauge transformation
   ii. Determine coefficients of Lagrangian and gauge transformation recursively
III. Fronsdal like Lagrangian for continuous spin
   i. General form of Lagrangian
IV. Summary and tasks

I. Introduction Poincare symmetry

Symmetry → **Irreducible representation** ↔ **Elementary particle**

Spacetime symmetry：Poincare symmetry＝4D translation inv.+Lorentz inv.

Find all possible irr. representation for Poincare group.

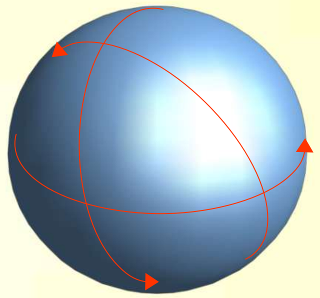
1-1. Represetation of Poincare group or algebra

Classified by little group, around 1940, by Wigner

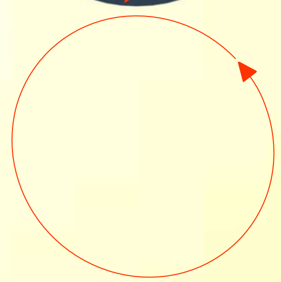
Representation of Poincare group may have 3 real parameters: mass m, spin s and continuous spin parameter μ

$O(3,1) \times T(4)$

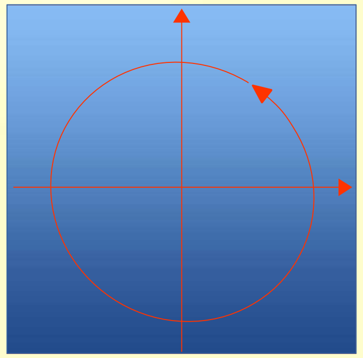
- $m \neq 0$ — $s = 1/2, 1, 3/2, 2, \cdots$: **massive discrete spin** ········· $O(3)$



- $m = 0$
  - $\mu = 0$ — $s = 1/2, 1, 3/2, 2, \cdots$: **massless discrete spin(helicity)** ········· $O(2)$



  - $\mu \neq 0$ — : **massless continuous “spin”** ········· $O(2) \times T(2)$



I. Introduction

Principle for constructing Lagrangian for continuous spin field

Irreducible representation ⟷ **Elementary particle**

Irr. rep of Poincare algebra is classified by two Casimir elements:
They are written by momentum P_μ and Pauli-Lubanski vector W_μ as P^2 and W^2

Dimension-full parameters m and μ(together with (half) integer s) appear as eigenvalues of these Casimir operators as follows.

1. Spin s field with mass m (no μ)

$$P^2\Psi = m^2\Psi \qquad W^2\Psi = m^2 s(s+1)\Psi$$

2. Continuous spin: m=0 and μ≠0.

$$P^2\Psi = 0 \qquad W^2\Psi = \mu^2\Psi$$

→ Regarded as Equations of motion ⇒ Find Lagrangian

II. Modifying massless theory

Ref: Metsaev, PLB 767(2017),PLB781(2018)

2-1. Fronsdal Lagrangian

= theory for massless(m=μ=0) arbitrary helicity filed(free part)

$$
\begin{aligned}
\mathcal{L}^0(\varphi) &= \varphi\Box\varphi \\
\mathcal{L}^1(A_\mu) &= A^\mu\Box A_\mu + \cdots = (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\
\mathcal{L}^2(h_{\mu\nu}) &= h_{\mu\nu}\Box h^{\mu\nu} + \cdots : \text{bi-linear part of Einsten Lagrangain} \\
\mathcal{L}^3(\phi_{\mu\nu\lambda}) &= \phi_{\mu\nu\lambda}\Box\phi^{\mu\nu\lambda} + \cdots : \text{ Gauge invariant form} \\
\mathcal{L}^4(\phi_{\mu\nu\alpha\beta}) &= \phi_{\mu\nu\alpha\beta}\Box\phi^{\mu\nu\alpha\beta} + \cdots : \text{ Gauge invariant form} \\
&\vdots \\
\mathcal{L}^n(\phi_{\mu_1\mu_2\cdots\mu_n}) &= \;: \text{ Fronsdal Lagragian: Gauge invariant form}
\end{aligned}
$$

$$\eta^{\alpha_1\alpha_2}\eta^{\alpha_3\alpha_4}\phi_{\alpha_1\cdots\alpha_n} = 0$$

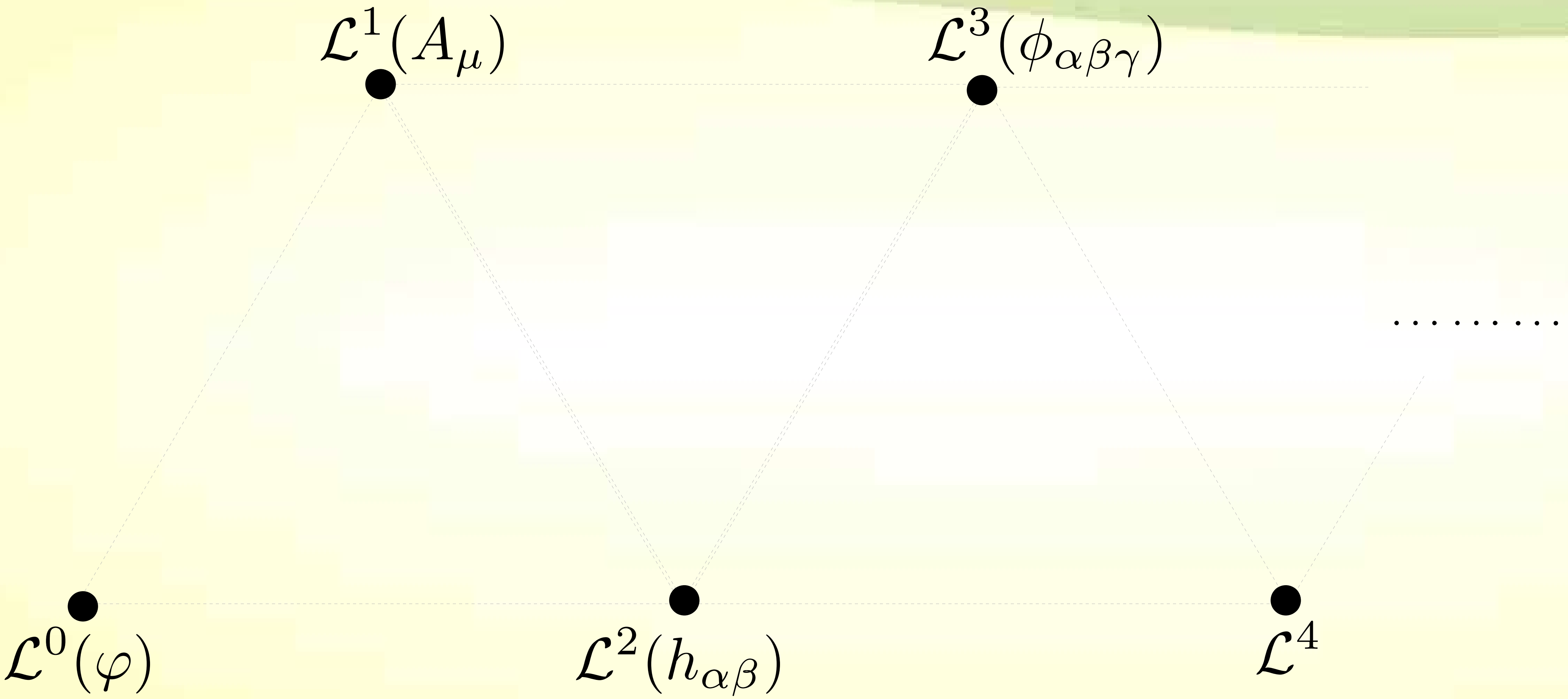
Each filed is independent, no mixing terms

$\bullet\mathcal{L}^0(\varphi)$ $\bullet\mathcal{L}^1(A_\mu)$ $\bullet\mathcal{L}^2(h_{\alpha\beta})$ $\bullet\mathcal{L}^3(\phi_{\alpha\beta\gamma})$ $\cdots\cdots\cdots$

II. Modifying massless theory

2-1. Theory for massless(m=μ=0) arbitrary spin filed

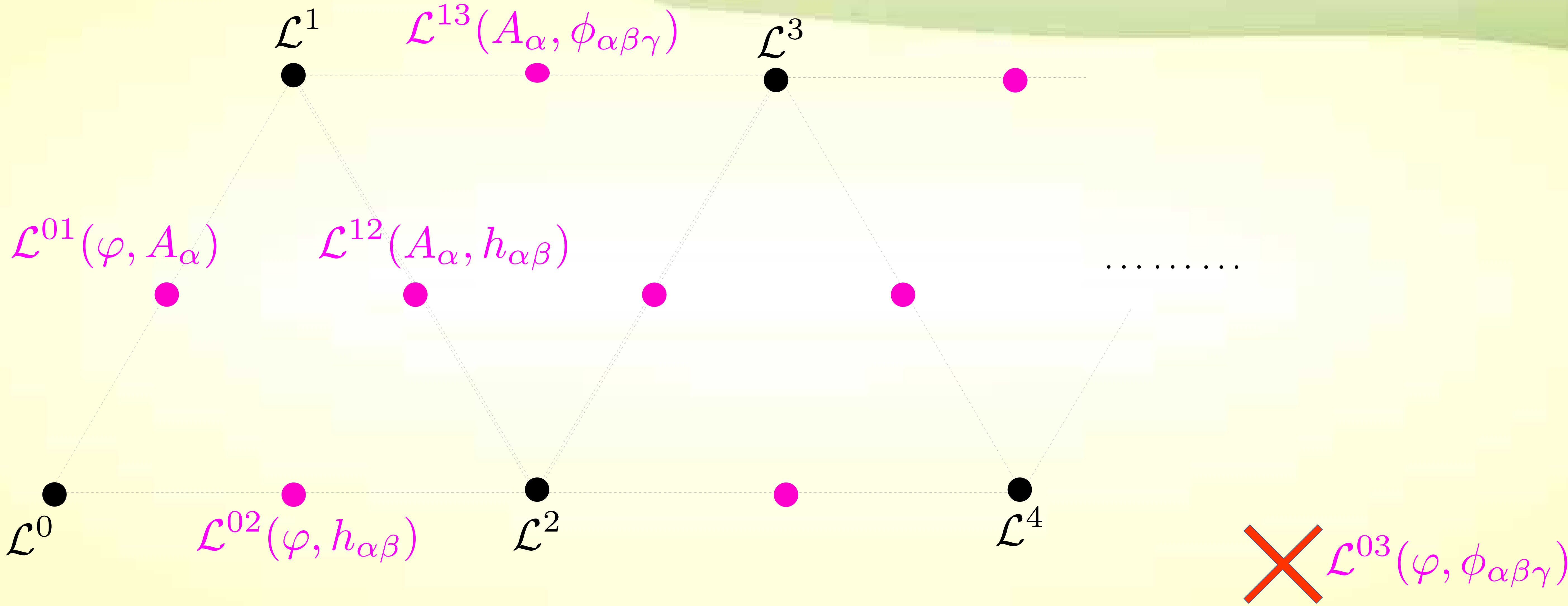
Introducing mixing terms



II. Modifying massless theory

2-2. Modifying Lagrangain(non-zero μ)

Introducing mixing terms ●

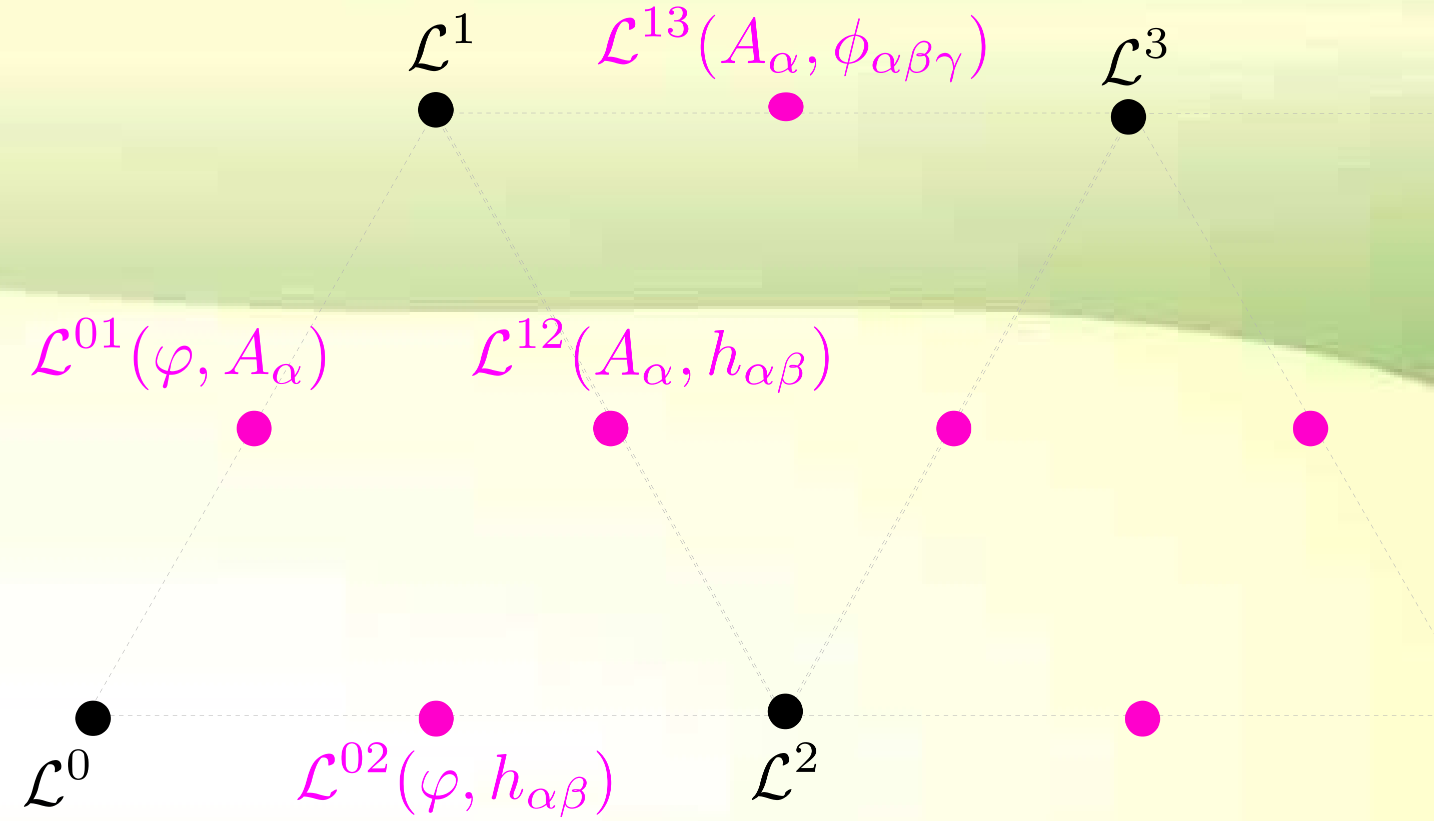


II. Modifying massless theory

2-2. Modifying Lagrangain(non-zero μ)

Introducing mixing terms ●

$$
\begin{aligned}
\mathcal{L}^{01}(\varphi, A_\alpha) &\sim \mu\varphi\partial^\alpha A_\alpha \\
\mathcal{L}^{02}(\varphi, h_{\alpha\beta}) &\sim \mu^2\varphi\eta^{\alpha\beta}h_{\alpha\beta} \\
\mathcal{L}^{12}(A_\lambda, h_{\alpha\beta}) &\sim \mu A^\alpha\partial^\beta h_{\alpha\beta}, \mu A^\alpha\partial_\alpha h^\gamma_\gamma \\
&\vdots
\end{aligned}
$$



Together with mass like terms ●

$$
\begin{aligned}
\mathcal{L}^0(\varphi) &= \varphi(\Box + \mu^2)\varphi \\
\mathcal{L}^1(A_\mu) &= A_\alpha(\Box + \mu^2)A^\alpha + \cdots \\
\mathcal{L}^2(h_{\mu\nu}) &= h_{\alpha\beta}(\Box + \mu^2)h^{\alpha\beta} + \cdots \\
&\vdots
\end{aligned}
$$

✕ $\mathcal{L}^{03}(\varphi, \phi_{\alpha\beta\gamma}) \sim \mu\varphi\partial^\alpha\eta^{\beta\gamma}\phi_{\alpha\beta\gamma}$

II. Modifying massless theory

2-3. gauge transformation(μ=0)

When $\mu = 0$ each Lagrangian is independently gauge invariant

$$\delta\mathcal{L}^0(\varphi) = 0 \qquad \delta\mathcal{L}^1(A_\mu) = 0 \qquad \delta\mathcal{L}^2(h_{\alpha\beta}) = 0 \qquad \delta\mathcal{L}^3(\phi_{\alpha\beta\gamma}) = 0 \qquad \cdots\cdots\cdots$$

$$\delta\varphi = 0 \qquad \delta A_\alpha = \partial_\alpha\lambda \qquad \delta h_{\alpha\beta} = \partial_\alpha\lambda_\beta \qquad \delta\phi_{\alpha\beta\gamma} = \partial_\alpha\lambda_{\beta\gamma}$$

$$\eta^{\alpha_1\alpha_2}\lambda_{\alpha_1\cdots\alpha_n} = 0$$

Number of index of gauge parameter is less than that of field by 1

$$\delta\varphi \qquad \delta A_\alpha \qquad \delta h_{\alpha\beta} \qquad \delta\phi_{\alpha\beta\gamma} \qquad \cdots\cdots\cdots$$

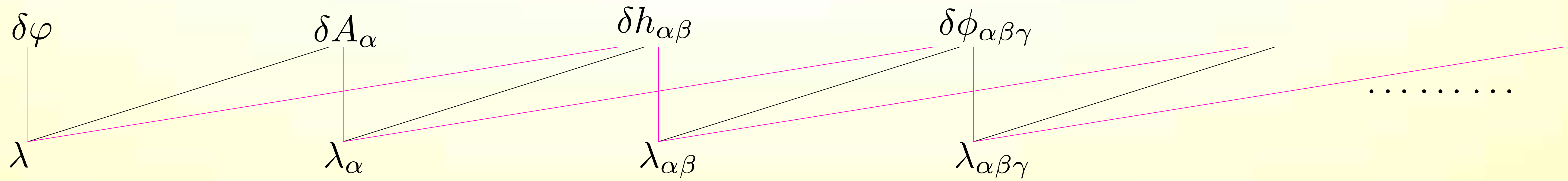
$$\lambda \qquad \lambda_\alpha \qquad \lambda_{\alpha\beta} \qquad \lambda_{\alpha\beta\gamma}$$

II. Modifying massless theory

2-4. Modifying gauge transformation(non-zero μ)

When $\mu \neq 0$ Possible modification of gauge transformation
(assume recovering standard gauge transformation when $\mu \to 0$)

$$
\begin{aligned}
\delta\varphi &= \mu\lambda \\
\delta A_\alpha &= \partial_\alpha\lambda + \mu\lambda_\alpha \\
\delta h_{\alpha\beta} &= \partial_\alpha\lambda_\beta + \mu(\eta_{\alpha\beta}\lambda + \lambda_{\alpha\beta}) \\
\delta\phi_{\alpha\beta\gamma} &= \partial_\alpha\lambda_{\beta\gamma} + \mu(\eta_{\alpha\beta}\lambda_\gamma + \lambda_{\alpha\beta\gamma}) \\
&\vdots
\end{aligned}
$$



II. Modifying massless theory

2-5. Determing coefficients by gauge invariance

Idea

$$0 = \delta\mathcal{L} = \sum_{m,n} \phi_m X_{mn} \lambda_n$$

$$\rightarrow \sum_n X_{mn}\lambda_n = 0 \quad \text{....For each m}$$

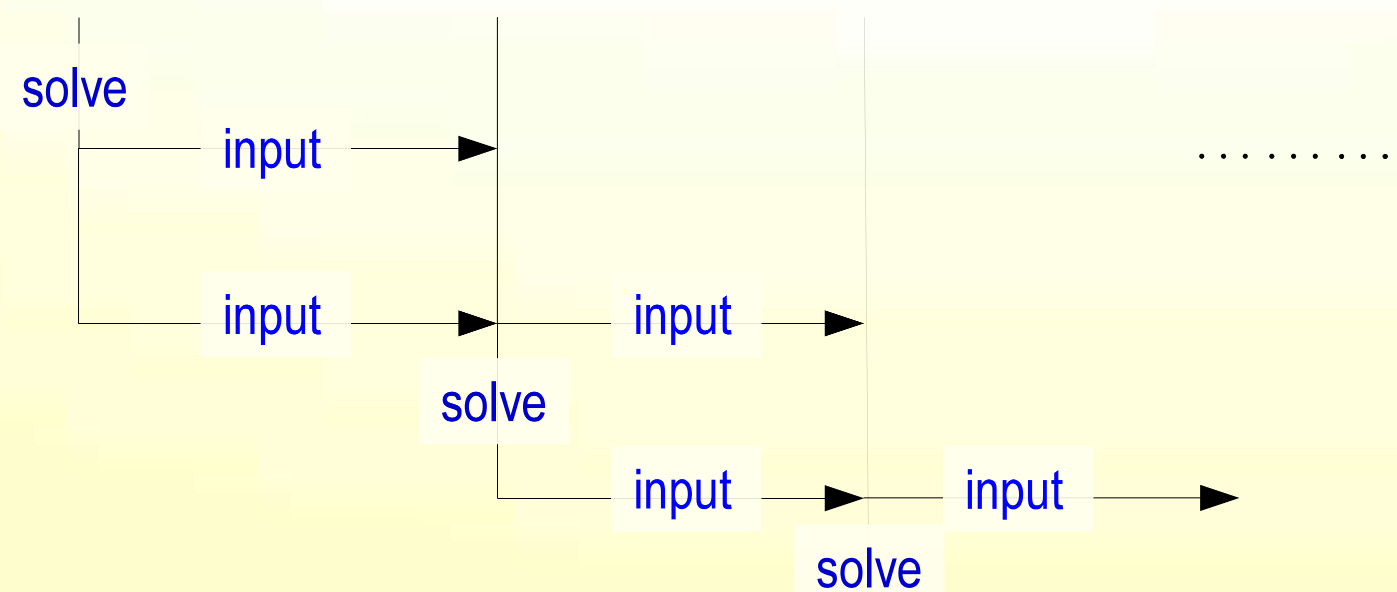
$$\rightarrow X_{mn} = 0 \quad \text{....For each } m, n$$

m,n : rank of tensor
m, n=0,1,2,..

We will solve $X_{mn} = 0$, starting from scalar field recursively.

Then coefficients in Lagrangian and gauge transformation will be determined.

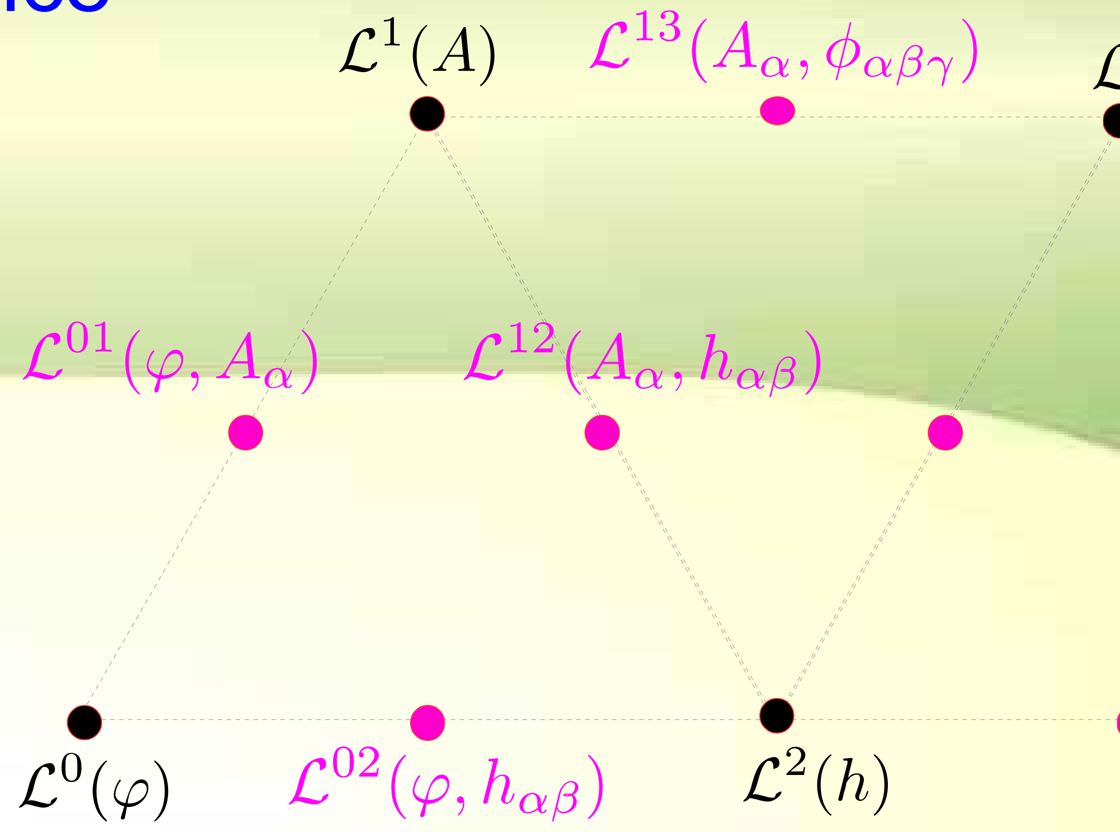
scalar(m=0) $X_{0n} = 0$ | vector(m=1) $X_{1n} = 0$ | tensor(m=2) $X_{2n} = 0$



II. Modifying massless theory

2-5. Determing coefficients by gauge invariance

Gauge transformation of scalar(n=0) part



Step1. Pick up terms including φ from total Lagrangian:

$$
\begin{aligned}
\mathcal{L}^{(0)}(\varphi) &= \varphi\partial^2\varphi + a_0\mu^2\varphi^2 \\
\mathcal{L}^{(0,1)}(\varphi, A) &= -\mu b_1 \varphi\partial_\alpha A^\alpha \\
\mathcal{L}^{(0,2)}(\varphi, h) &= \mu^2 d_2 \varphi \bar{h}
\end{aligned}
$$

a_0, b_1, d_2 will be determined from gauge invariance

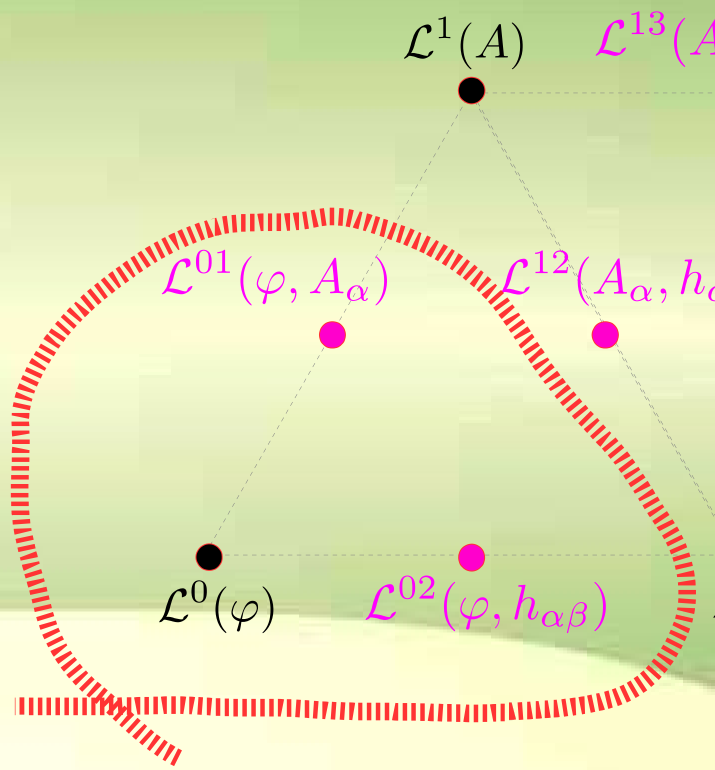
Step2. Pick up gauge transformation of fields those are included in above terms:

$$
\begin{aligned}
\delta\varphi &= \mu s_0 \lambda \\
\delta A_\alpha &= \partial_\alpha \lambda + \mu s_1 \lambda_\alpha \\
\delta h_{\alpha\beta} &= \partial_\alpha \lambda_\beta + \mu s_2 \lambda_{\alpha\beta} + \mu t_2 \eta_{\alpha\beta} \lambda
\end{aligned}
$$

s_0, s_1, s_2, t_2 will be determined from gauge invariance

II. Modifying massless theory

2-5. Determing coefficients by gauge invariance



Gauge transformation of scalar(n=0) part

Step3. Pick up terms including φ **from** $\delta\mathcal{L}^0, \delta\mathcal{L}^{01}, \delta\mathcal{L}^{02}$ **and sum up**

$$
\begin{aligned}
\delta\mathcal{L}^{(0)}(\varphi) &= \delta\left\{\varphi\partial^2\varphi + a_0\mu^2\varphi^2\right\} \longrightarrow \varphi \times 2\left(\partial^2 + a_0\mu^2\right)\delta\varphi \\
\delta\mathcal{L}^{(0,1)}(\varphi, A) &= \delta\left\{-\mu b_1\varphi\partial_\alpha A^\alpha\right\} \longrightarrow -\varphi \times \mu b_1\partial_\alpha\delta A^\alpha \\
\delta\mathcal{L}^{(0,2)}(\varphi, h) &= \delta\left\{\mu^2 d_2\varphi\bar{h}\right\} \longrightarrow \varphi \times \mu^2 d_2\delta\bar{h}
\end{aligned}
$$

Gauge invariance require

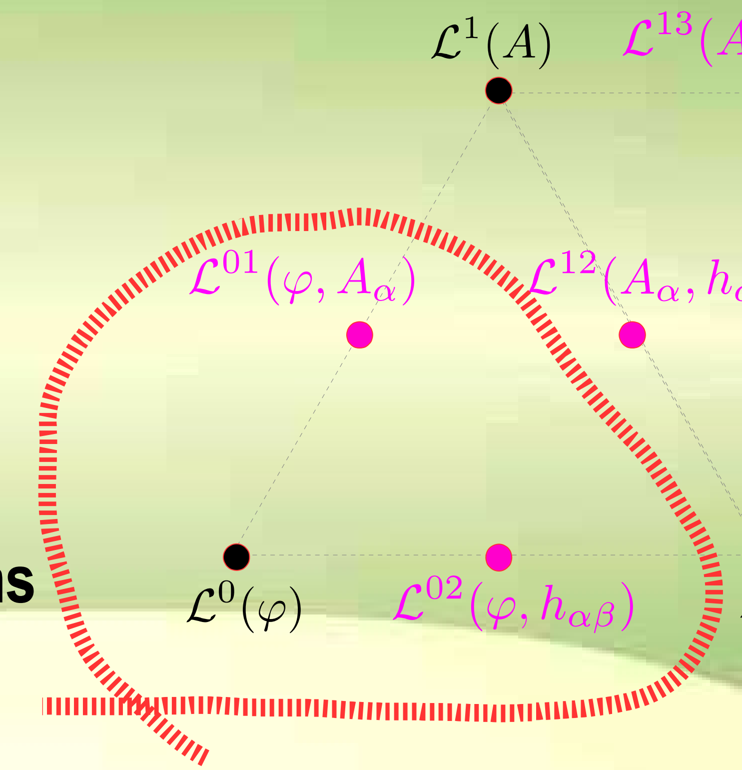
$$
\delta\left(\mathcal{L}^{\text{total}}\Big|_{\varphi\text{ included}}\right)\Big|_{O(\varphi^1)} = 0 \quad\Rightarrow\quad \delta\mathcal{L}^{(0)}(\varphi) + \delta\mathcal{L}^{(0,1)}(\varphi, A) + \delta\mathcal{L}^{(0,2)}(\varphi, h)\ \Big|_{O(\varphi^1)} = 0
$$

$$
\Downarrow
$$

$$
2\left(\partial^2 + a_0\mu^2\right)\delta\varphi - \mu b_1\partial_\alpha\delta A^\alpha + \mu^2 d_2\delta\bar{h} = 0
$$

II. Modifying massless theory

2-5. Determing coefficients by gauge invariance



Gauge transformation of scalar(n=0) part

Step4. Input gauge transformation and arrange by independent gauge parameters terms

$$2\left(\partial^2 + a_0\mu^2\right)\delta\varphi - \mu b_1 \partial_\alpha \delta A^\alpha + \mu^2 d_2 \delta\bar{h} = 0$$

$$\begin{aligned} \delta\varphi &= \mu s_0 \lambda \\ \delta A_\alpha &= \partial_\alpha \lambda + \mu s_1 \lambda_\alpha \\ \delta h_{\alpha\beta} &= \partial_\alpha \lambda_\beta + \mu s_2 \lambda_{\alpha\beta} + \mu t_2 \eta_{\alpha\beta}\lambda \end{aligned}$$

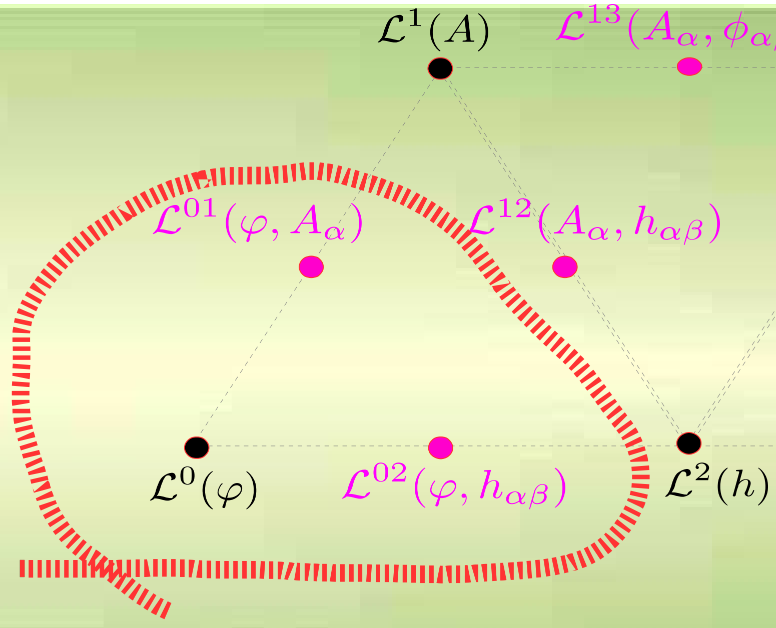
$$\mu(2s_0 - b_1)\partial^2\lambda + \mu^3(2s_0 a_0 + D t_2 d_2)\lambda + \mu^2(d_2 - s_1 b_1)\partial^\alpha \lambda_\alpha = 0$$

Note: $\eta^{\alpha\beta}\lambda_{\alpha\beta} \equiv 0$

$$\begin{aligned} 2s_0 - b_1 &= 0 \\ 2s_0 a_0 + D t_2 d_2 &= 0 \\ d_2 - s_1 b_1 &= 0 \end{aligned}$$

II. Modifying massless theory

2-5. Determining coefficients by gauge invariance



Gauge transformation of scalar(n=0) part

Step5. Solve vanishing coefficients equations under traceless gauge parameters

$$
\begin{aligned}
b_1 &= 2s_0 \\
d_2 &= 2s_0 s_1 \\
t_2 &= -\frac{a_0}{D s_1}
\end{aligned}
$$

Lagrangian and gauge transformation given:

Continuous spin theory: scalar part

$$
\begin{aligned}
\mathcal{L}^{(0)}(\varphi) &= \varphi\partial^2\varphi + \mu^2\varphi^2 \\
\mathcal{L}^{(0,1)}(\varphi, A) &= -2\mu\varphi\partial_\alpha A^\alpha \\
\mathcal{L}^{(0,2)}(\varphi, h) &= 2\mu^2\varphi\bar{h}
\end{aligned}
$$

$$\eta^{\alpha\beta}\lambda_{\alpha\beta} = 0$$

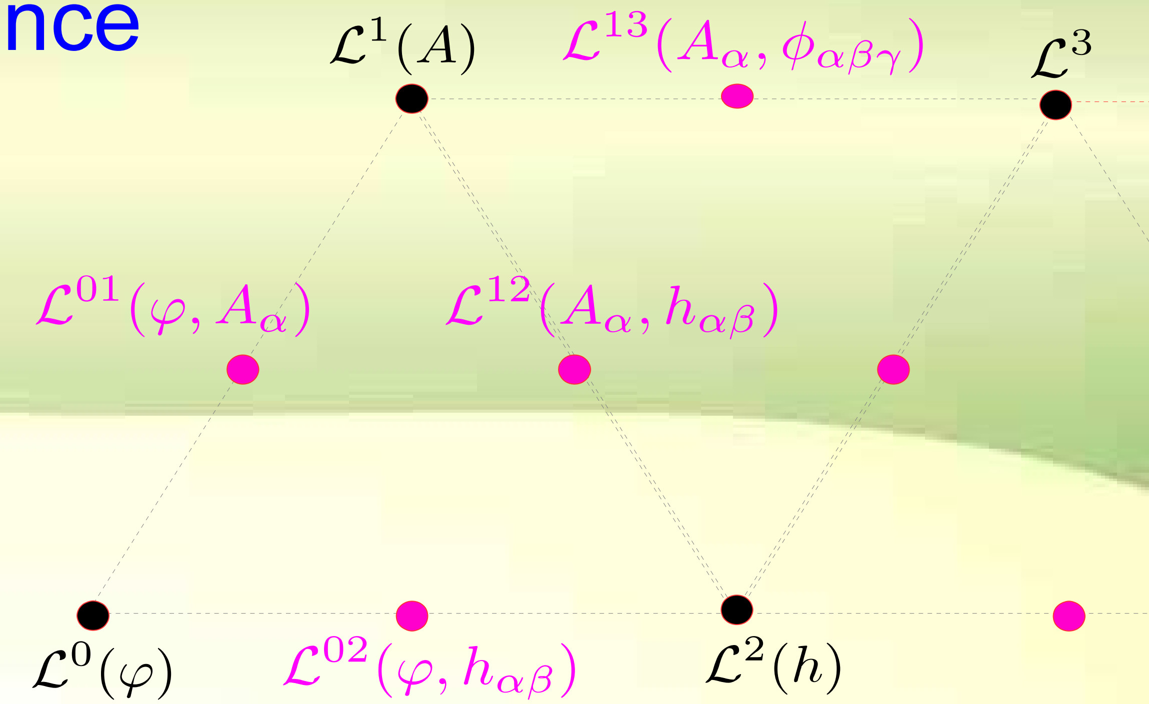
$$
\begin{aligned}
\delta\varphi &= \mu\lambda \\
\delta A_\alpha &= \partial_\alpha\lambda + \mu\lambda_\alpha \\
\delta h_{\alpha\beta} &= \partial_\alpha\lambda_\beta + \mu\lambda_{\alpha\beta} - \mu\frac{1}{D}\eta_{\alpha\beta}\lambda
\end{aligned}
$$

temporary chosen as $s_0 = s_1 = 1$

$$\delta\mathcal{L}^{(0)} + \delta\mathcal{L}^{(0,1)} + \delta\mathcal{L}^{(0,2)} \sim A^\alpha(\cdots)_\alpha + h^{\alpha\beta}(\cdots)_{\alpha\beta}$$ must be canceled by variation of other terms of Lagrangian

II. Modifying massless theory

2-5. Determing coefficients by gauge invariance



Gauge transformation of vector(n=1) part

$$
\begin{aligned}
\mathcal{L}^{(1)}(A) &= A_\alpha(\partial^2 + a_1\mu^2 A^\alpha) + (\partial\cdot A)^2 \\
\mathcal{L}^{(0,1)}(\varphi, A) &= -\mu b_1 \varphi \partial_\alpha A^\alpha \\
\mathcal{L}^{(1,2)}(A, h) &= -\mu b_2 A_\alpha \partial_\beta h^{\alpha\beta} - \mu c_2 A_\alpha \partial^\alpha \bar{h} \\
\mathcal{L}^{(1,3)}(A, \phi) &= \mu^2 d_3 A_\alpha \bar{\phi}^\alpha
\end{aligned}
$$

$$
\begin{aligned}
\delta\varphi &= \mu s_0 \lambda \\
\delta A_\alpha &= \partial_\alpha \lambda + \mu s_1 \lambda_\alpha \\
\delta h_{\alpha\beta} &= \partial_\alpha \lambda_\beta + \mu s_2 \lambda_{\alpha\beta} + \mu t_2 \eta_{\alpha\beta}\lambda \\
\delta \phi_{\alpha\beta\gamma} &= \partial_\alpha \lambda_{\beta\gamma} + \mu s_3 \lambda_{\alpha\beta\gamma} + \mu t_3 \eta_{\alpha\beta}\lambda_\gamma
\end{aligned}
$$

Note: $\eta^{\alpha\beta}\lambda_{\alpha\beta} = \eta^{\alpha\beta}\lambda_{\alpha\beta\gamma} \equiv 0$

$\sum \delta\mathcal{L}'s$ ↓

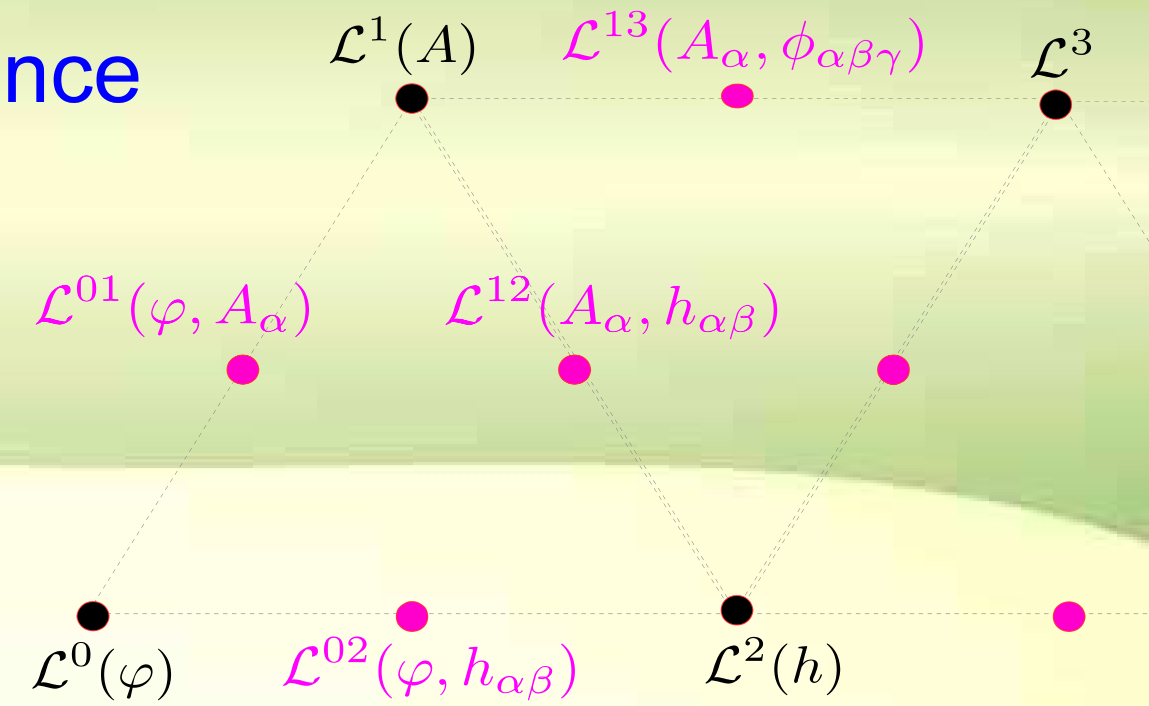
$$
\begin{aligned}
&\mu A^\alpha \left(2s_1 - \tfrac{1}{2}b_2\right)\partial^2\lambda_\alpha \\
&+\mu^2 A^\alpha (2a_1 + s_0 b_1 - (b_2 + c_2 D)t_2)\partial_\alpha \lambda \\
&+\mu^2 A^\alpha \left(\tfrac{2}{3}d_3 - b_2 s_2\right)(\partial\cdot\lambda)_\alpha \\
&+\mu^3 A^\alpha \left(2s_1 a_1 + \tfrac{D+2}{3}d_3 t_3\right)\lambda_\alpha
\end{aligned}
= 0
$$

⇒

$$
\begin{aligned}
2s_1 - \tfrac{1}{2}b_2 &= 0 \\
2s_1 + \tfrac{1}{2}b_2 + c_2 &= 0 \\
2a_1 + s_0 b_1 - (b_2 + c_2 D)d_3 t_2 &= 0 \\
\tfrac{2}{3}d_3 - b_2 s_2 &= 0 \\
2s_1 a_1 + \tfrac{D+2}{3}d_3 t_3 &= 0
\end{aligned}
$$

II. Modifying massless theory

2-5. Determing coefficients by gauge invariance



Gauge transformation of scalar(n=0) and vector(n=1) part

Parameters are determined as

$$
\begin{aligned}
a_0 &= 1 \quad , & a_1 &= \frac{D-2}{D} \\
b_1 &= 2 & b_2 &= 4 \\
 & & c_2 &= -4 \\
d_2 &= 2 & d_3 &= 6 \\
t_2 &= -\frac{1}{D} & t_3 &= -\frac{D-2}{D(D+2)}
\end{aligned}
$$

temporary chosen as

$a_0 = s_0 = s_1 = s_2 = 1$

Continuous spin theory: scalar and vector part

$$
\begin{aligned}
\mathcal{L}^{(0)}(\varphi) &= \varphi\partial^2\varphi + \mu^2\varphi^2 \\
\mathcal{L}^{(1)}(A) &= A_\alpha\left(\partial^2 + \frac{D-2}{D}\mu^2\right)A^\alpha + (\partial\cdot A)^2 \\
\mathcal{L}^{(0,1)}(\varphi, A) &= -2\mu\varphi\partial_\alpha A^\alpha \\
\mathcal{L}^{(0,2)}(\varphi, h) &= 2\mu^2\varphi\bar{h} \\
\mathcal{L}^{(1,2)}(A, h) &= 4\mu A_\alpha(\partial^\alpha\bar{h} - \partial_\beta h^{\alpha\beta}) \\
\mathcal{L}^{(1,3)}(A, \phi) &= 6\mu^2 A_\alpha\bar{\phi}^\alpha
\end{aligned}
$$

$$
\eta^{\alpha\beta}\lambda_{\alpha\beta} = 0
$$

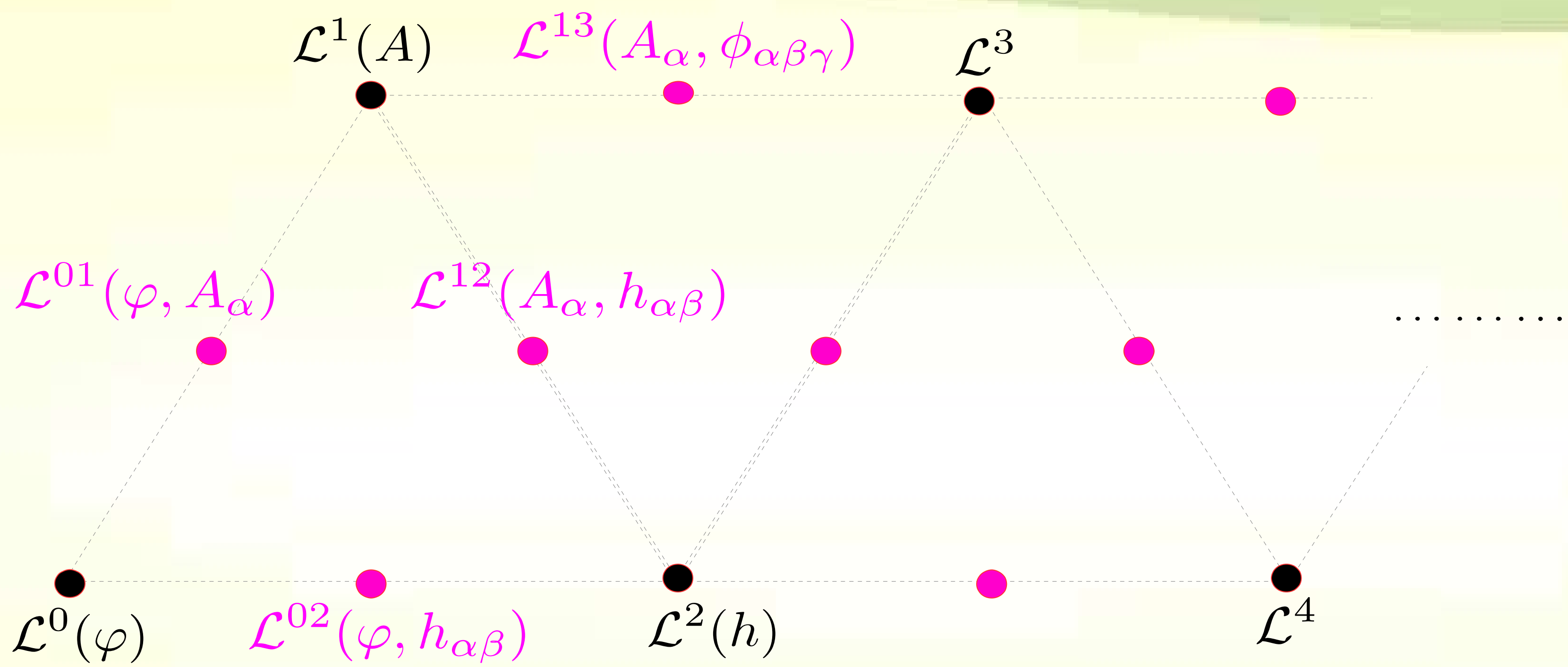
$$
\eta^{\alpha\beta}\lambda_{\alpha\beta\gamma} = 0
$$

$$
\begin{aligned}
\delta\varphi &= \mu\lambda \\
\delta A_\alpha &= \partial_\alpha\lambda + \mu\lambda_\alpha \\
\delta h_{\alpha\beta} &= \partial_{(\alpha}\lambda_{\beta)} + \mu\lambda_{\alpha\beta} - \mu\frac{1}{D}\eta_{\alpha\beta}\lambda \\
\delta\phi_{\alpha\beta\gamma} &= \partial_{(\gamma}\lambda_{\alpha\beta)} + \mu\lambda_{\alpha\beta\gamma} - \mu\frac{D-2}{D(D+2)}\eta_{(\beta\gamma}\lambda_{\alpha)}
\end{aligned}
$$

II. Modifying massless theory

2-5. Determining coefficients by gauge invariance

Gauge transformation of higher helicity(n=0,1,2,3,...)



All parameters a_n, b_n, c_n, d_n, t_n will be written by s_n. (s_n may also be fixed by D and n)

All gauge parameters are traceless:

$$\eta^{\alpha\beta}\lambda_{\alpha\beta}=\eta^{\alpha\beta}\lambda_{\alpha\beta\gamma}=\eta^{\alpha\beta}\lambda_{\alpha\beta\gamma\delta}=\cdots=0$$

III. Fronsdal like Lagrangian for continuous spin

3-1. Lagrangian: general form for continuous spin field given by “BRST” method

(L. Buchbinder and V. Krykhtin, H.T.)

$$[a_\alpha, a^\dagger_\beta] = \eta_{\alpha\beta} \qquad l_1 := -i\partial^\alpha a_\alpha \qquad l_2 := -\frac{1}{2}a^\alpha a_\alpha \qquad |\phi\rangle = \sum_{n=0}^{\infty} a^{\dagger\alpha(n)}|0\rangle \frac{1}{\sqrt{n!}}\phi_{\alpha(n)} \qquad \phi_{\alpha(n)} := \phi_{(\alpha_1\ldots\alpha_n)}(x)$$

divergence op. trace op.

$$N = a^{\dagger\alpha}a_\alpha \qquad l_1\phi \sim \partial_\alpha\phi^{\alpha\cdots} \qquad l_2\phi \sim \eta_{\alpha\beta}\phi^{\alpha\beta\cdots} \qquad |\lambda\rangle = \sum_{n=0}^{\infty} a^{\dagger\alpha(n)}|0\rangle \frac{1}{\sqrt{n!}}\lambda_{\alpha(n)} \qquad a^{\dagger\alpha(n)} := a^{\dagger\alpha_1}\ldots a^{\dagger\alpha_n}$$

$$\mathcal{L}_{\mu=0} = \langle\phi|\begin{pmatrix}1 & -l_2^\dagger\end{pmatrix}\begin{pmatrix}\partial^2 + l_1^\dagger l_1, & -l_1^\dagger l_1^\dagger \\ -l_1 l_1, & -2\partial^2 + l_1^\dagger l_1\end{pmatrix}\begin{pmatrix}1 \\ -l_2\end{pmatrix}|\phi\rangle \ , \qquad \delta|\phi\rangle = l_1^\dagger|\lambda\rangle$$

Fronsdal Lagrangian

$$l_1 \to L_{(N)} := l_1 + \mu s_N$$

: Divergence op. replaced with constant

$$\mathcal{L} = \langle\phi|\begin{pmatrix}1 & -l_2^\dagger\end{pmatrix}\begin{pmatrix}\partial^2 + L^\dagger_{(N)}L_{(N)}, & -L^\dagger_{(N)}L^\dagger_{(N+1)} \\ -L_{(N+1)}L_{(N)}, & -2\partial^2 + L^\dagger_{(N+2)}L_{(N+2)}\end{pmatrix}\begin{pmatrix}1 \\ -l_2\end{pmatrix}|\phi\rangle \ , \qquad \delta|\phi\rangle = \left(L^\dagger_{(N)} - \mu l_2^\dagger t_N\right)|\lambda\rangle$$

Continuous spin Lagrangian

$$s_N = -\frac{1}{\sqrt{2(N+D/2-1)}}$$

$$t_N = \; = -2{s_N}^2 s_{N+1}$$

$$||$$

$$\begin{pmatrix}\partial^2 + l_1^\dagger l_1 & -l_1^{\dagger 2} \\ -l_1^2 & -2\partial^2 + l_1^\dagger l_1\end{pmatrix} + \mu\left\{\begin{pmatrix}s_N & 0 \\ -2s_{N+1} & s_{N+2}\end{pmatrix} l_1 + h.c.\right\} + \mu^2\begin{pmatrix}s_N^2 & -s_N s_{N+1} \\ -s_N s_{N+1} & s_{N+2}^2\end{pmatrix}$$

Note: s,t are slightly differently defined from previous slides

IV. Summary and Next

What we did

- **Starting from Fronsdal theory(μ=0) for all helicity fields,
  we add mixing term to sum of Fronsdal Lagrangian and gauge transformation with coefficients.**
- **By demanding non-trivial gauge invariance, coefficients are recursively determined
  from lower helicity fields(n=0) to higher(n=1,2,..). We demonstrated this method for n=0,1,(2).**
- **We also show general form of Lagrangian given by BRST method.
  It is simply given, from Fronsdal Lagrangian, by adding divergence operator with n dependent constant**

What we are interested for next

- **Diagonalization of different helicity fields and 1loop effective action**
- **Fermionic case**
- **supersymmetric case (in superspace?)**
- **ADS spacetime case**
- **Cubic or quartic interaction**
- **Application to modified gravity and cosmology**