Heavy quark transport coefficients in viscous quark-gluon plasma

Adiba Shaikh

Dept. of Physics, Indian Institute of Technology Bombay

In collaboration with

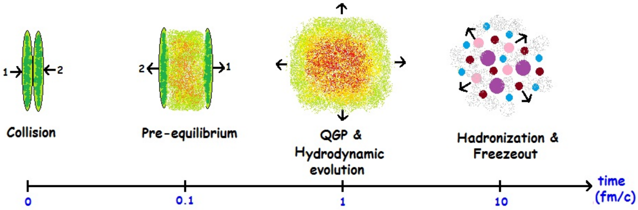
M. Kurian (McGill U)	S. K. Das (IIT Goa)	V. Chandra (IIT GN)	S. Dash B. K. Nandi (IIT Bombay)

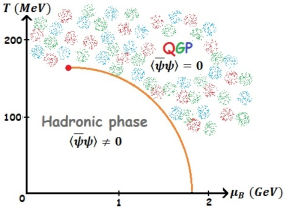
21 February 2023

Introduction

What is Quark-Gluon Plasma (QGP)?

- Deconfined phase of quarks, anti-quarks gluons as effective degrees of freedom.
- Exist at high temperature and/or high baryon density.
- E.g. in heavy-ion collision experiments at the time scale of $\sim \mathcal{O}(1$ fm/c).





Introduction

Why study heavy quarks (charm and bottom)?

- Heavy quark mass: $m_{HQ} >> \Lambda_{QCD}$ (~ 200 MeV)
- Small QCD coupling: perturbative expansion in α_s, non-relativistic treatment of bound state.
  $\alpha_s^2(m_c = 1.3 \text{ GeV}) \approx 0.3$
  $\alpha_s^2(m_b = 4.2 \text{ GeV}) \approx 0.1$

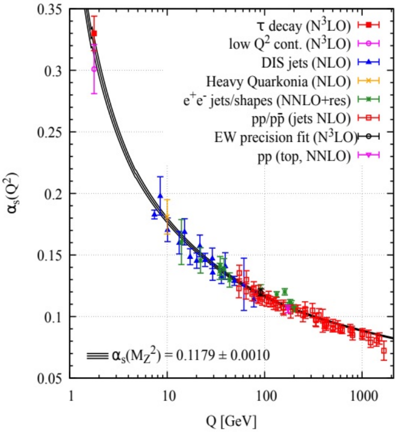


Image source: *pdg.lbl.gov/2019/reviews/rpp2019-rev-qcd*

Introduction & Motivation

What about heavy quarks in QGP?

- Hard energy scale : $m_{HQ} >> T$
- Formed in initial stage at time scale of $\sim 0.02 - 0.07$ fm
- Witness entire evolution of QGP : From creation to hadronization
- Important probes to study the QCD medium properties

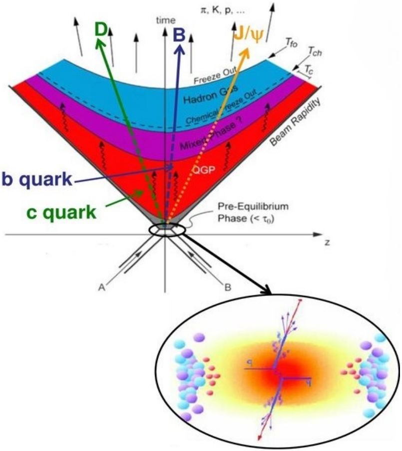


Image source: *F. Prino talk, EMMI-RRTF workshop, GSI, 2016*

Experimental Motivation

Nuclear Modification Factor

$$R_{AA}(p_T) = \frac{1}{N_{coll}} \frac{d^2N_{AA}/(dp_T dy)}{d^2N_{pp}/(dp_T dy)}$$

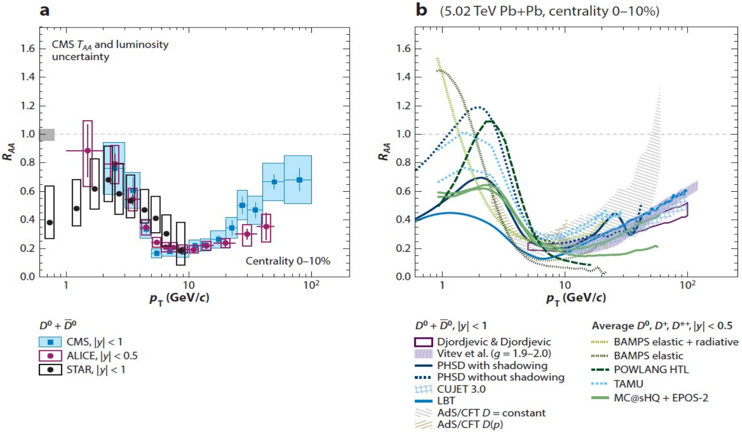
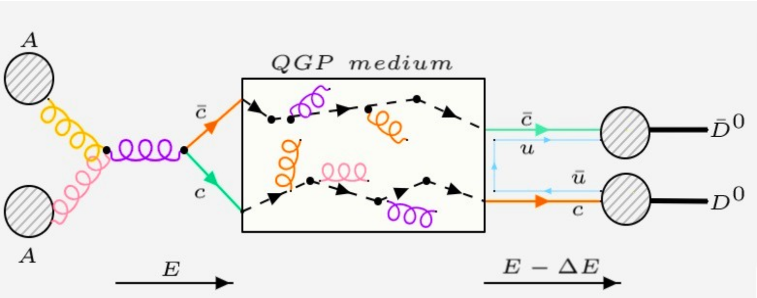


Image source: *Annu. Rev. Nucl. Part. Sci. 2019.69:417-445*

HQ transport

- Charm quark moving in QGP loose its energy.
  1. Collision ($2 \rightarrow 2$): Elastic scattering with the medium constituents
  2. Radiation ($2 \rightarrow 3$): Inelastic scattering with medium induced gluon emission

- Transport coefficients in small momentum transfer limit.
  1. **Drag** $\Longrightarrow$ Resistance to the motion by plasma particles i.e. light quarks (u, d, s), anti-quarks $(\bar{u}, \bar{d}, \bar{s})$ and gluons (g).
  2. **Diffusion** $\Longrightarrow$ In momentum space along transverse and longitudinal direction.



HQ transport

Collisional energy loss

- Non-equilibrated heavy quark traversing equilibrated plasma.
- Brownian motion of heavy quark within QGP medium
- *Boltzmann transport equation* for the phase space density $f(\mathbf{x}, \mathbf{p}, t)$ of the heavy quark.
- Homogeneous plasma ($\partial f/\partial \mathbf{x} = \mathbf{0}$) with no external force ($\mathbf{F} = 0$)

B. Svetitsky, Phys. Rev. D, 37(9), 1988

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{E_p}\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}\right) f(\mathbf{x}, \mathbf{p}, t) = \left(\frac{\partial f}{\partial t}\right)_{col} \implies \frac{\partial f(\mathbf{p}, t)}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{col}$$

- Landau's soft scattering approximation (small momentum transfer) $\implies$ *Fokker-Planck equation*

$$\frac{\partial f}{\partial t} \approx \frac{\partial}{\partial p_i}\left(A_i(\mathbf{p})f + \frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p})]f\right)$$

HQ transport

Collisional energy loss

- A_i and B_{ij} depends only on the initial momentum $(\mathbf{p})$

  $\rightarrow$ **Drag:**

$$A_i = p_i A(p^2)$$

$$\boxed{A(p^2) = \frac{p_i A_i}{p^2} = \langle \mathbf{1} \rangle - \frac{\langle \mathbf{p} \cdot \mathbf{p}' \rangle}{p^2}}$$

$\rightarrow$ **Diffusion:** $B_{ij} \equiv \frac{1}{2}\langle (p - p')_i (p - p')_j \rangle$

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) B_0(p^2) + \left(\frac{p_i p_j}{p^2} \right) B_1(p^2)$$

Transverse $\rightarrow$ $\boxed{B_0(p^2) = \frac{1}{2}\left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) B_{ij} = \frac{1}{4}\left[\langle p'^2 \rangle - \frac{\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle}{p^2} \right]}$

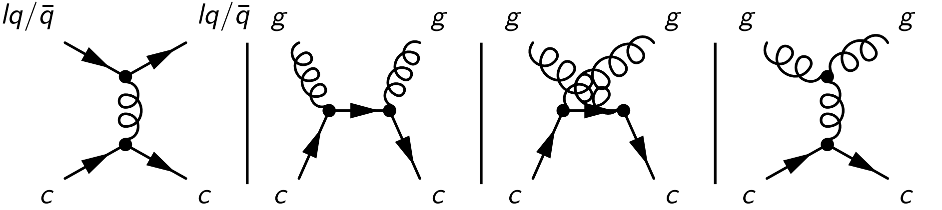
Longitudinal $\rightarrow$ $\boxed{B_1(p^2) = \left(\frac{p_i p_j}{p^2} \right) B_{ij} = \frac{1}{2}\left[\frac{\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle}{p^2} - 2\langle \mathbf{p} \cdot \mathbf{p}' \rangle + p^2 \langle \mathbf{1} \rangle \right]}$

HQ transport

Collisional energy loss

- $HQ(p) \;+\; lq/l\bar{q}/g(q) \rightarrow HQ(p') \;+\; lq/l\bar{q}/g(q')$

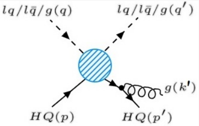
$$\langle F(p)_{col}\rangle = \frac{1}{16(2\pi)^5 E_p \gamma_{HQ}} \int \frac{d^3q}{E_q} \int \frac{d^3q'}{E_{q'}} \int \frac{d^3p'}{E_{p'}} \sum |\mathcal{M}|^2_{2\rightarrow 2}\; \delta(p+q-p'-q') \times f(E_q)\;(1 \pm f(E_q'))\; F(p)$$



HQ transport coefficients

Radiative Energy Loss

- $HQ(p) \; + \; lq/l\bar{q}/g(q) \to HQ(p') \; + \; lq/l\bar{q}/g(q') \; + \; g(k')$
- Soft gluon emission by the charm quark induced by the QGP medium (after scattering by light quarks, antiquarks and gluons).
- Radiated soft gluon momentum: $k' = (E_{k'}, \mathbf{k}'_\perp, k'_z) \to 0$,



$$|\mathcal{M}|^2_{2\to 3} = |\mathcal{M}|^2_{2\to 2} * \frac{12 g_s^2}{{k'_\perp}^2} \left(1 + \frac{m^2_{HQ}}{s} e^{2y_{k'}}\right)^{-2}$$

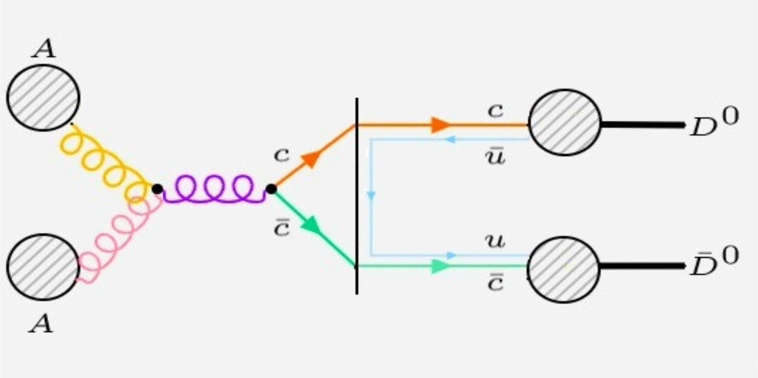
R. Abir et al., Phys. Rev. D 85, 054012 (2012)

$$\langle F(p)_{rad}\rangle = \langle F(p)_{col}\rangle \times \int \frac{d^3k'}{(2\pi)^3 E_{k'}} \frac{12 g_s^2}{{k'_\perp}^2} \left(1 + \frac{m^2_{HQ}}{s} e^{2y_{k'}}\right)^{-2} \delta(p + q - p' - q' - k') \times (1 + f(E_{k'}))\, \theta(E_p - E_{k'})\, \theta(\tau - \tau_f)$$

S. Mazumdar *et al.*, Phys. Rev. D 89, 014002 (2014)

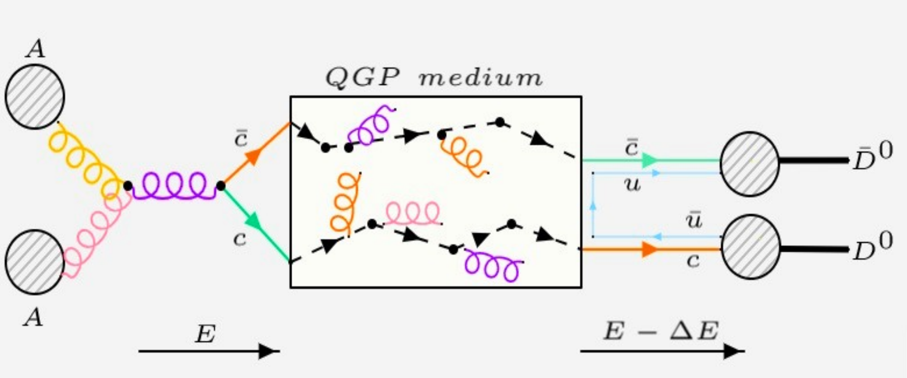
No QGP

Charm quark transport in vacuum



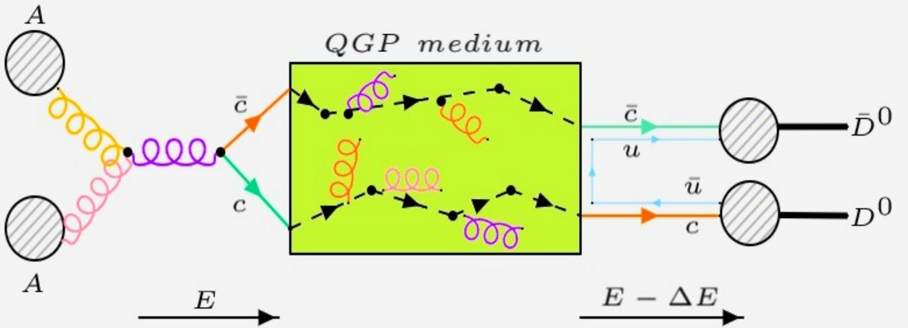
Ideal QGP

Charm quark transport in non-interacting medium



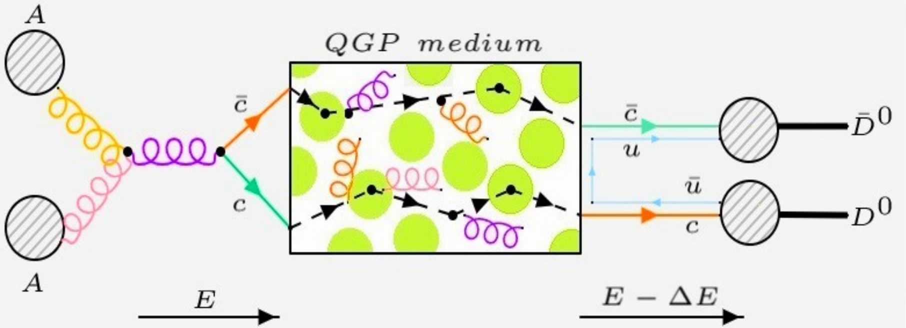
Deviation from Ideal case

Charm quark transport within thermally interacting QGP



Deviation from Ideal case

Charm quark transport with thermally interacting QGP (EQPM)



[i] Thermal medium interaction: EQPM

- **EQPM**: **E**ffective fugacity **Q**uasi-**P**article **M**odel
- In-medium interactions of QGP encoded into particle $\rightarrow$ **quasiparticle** $\rightarrow$ Model based on mapping of the EoS with lattice QCD EoS

V. Chandra et al., Phys. Rev. C, 76 (054909), 2007

- Temperature dependence of effective fugacity $z_{k=q,g}$ is parametrized using (2+1) flavor LQCD EoS at T_c =170 MeV as follows,

$$z_k = a_k \ \exp\left\{-\frac{b_k}{(T/T_c)^2} - \frac{c_k}{(T/T_c)^4} - \frac{d_k}{(T/T_c)^6}\right\}$$

- Introduction of temperature dependent effective fugacity z_k in the distribution functions of quasiparticles $k \equiv (lq, l\bar{q}, g)$.

$$f_k^0 = \frac{z_k \exp\{-\beta\, E_k\}}{1 \pm z_k \exp\{-\beta\, E_k\}}$$

[i] Thermal medium interaction: EQPM

- Quasiparticle dispersion relation: $\tilde{q_k}^{\mu} = q_k^{\mu} + \delta\omega_k\, u^{\mu}$
- Collective excitations of quasi-partons: $\delta\omega_k = T^2\,\partial_T\{\ln(z_k)\}$

$$\omega_{q/g} = E_{q/g} + T^2\,\partial_T\{\ln(z_k)\}$$

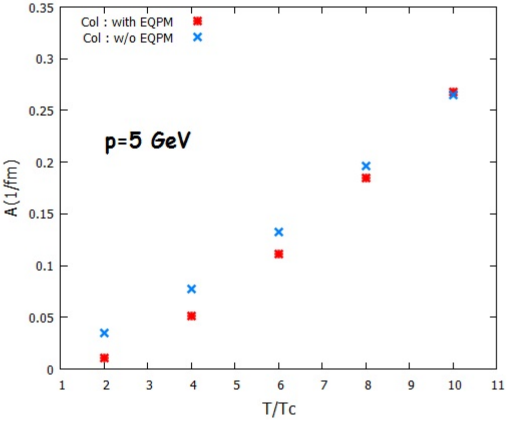
- Effective strong coupling constant $\alpha_{s(eff)}$ is introduced through EQPM based Debye mass.

S. Mitra et al., Phys. Rev. D, 96 (094003), 2017

- The effective QCD coupling constant in EQPM is,

$$\alpha_{s(eff)}(T) = \alpha_s(T)\frac{\left\{\frac{2N_c}{\pi^2}\mathrm{PolyLog}[2, z_g] - \frac{2N_f}{\pi^2}\mathrm{PolyLog}[2, -z_q]\right\}}{\left\{\frac{N_c}{3} - \frac{N_f}{6}\right\}}$$

[i] Thermal medium interaction: EQPM



[ii] Viscous hydrodynamic corrections

- Energy-momentum tensor for the dissipative (viscous) hydrodynamics,

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- Near local thermal equilibrium, the quasi-parton distribution function becomes,

$$f_k = f_k^0 + \delta f_k \text{ where } \frac{\delta f_k}{f_k^0} << 1$$

- Viscous hydrodynamic evolution of QGP to study its transport properties with EQPM using covariant kinetic theory approach.

S. Bhadury et al., J. Phys. G 47 (2020) 8, 085108

- Solving the effective Boltzmann equation for quasiparticles with RTA and using Chapman-Enskog iterative method,

$$\tilde{q}_k^{\mu}\, \partial_{\mu} f_k(x, \tilde{q}_k) + F_k^{\mu}\, (u \cdot \tilde{q}_k)\, \partial_{\mu}^{(q)} f_k = -\,(u \cdot \tilde{q}_k)\, \frac{\delta f_k}{\tau_R}$$

[ii] Viscous hydrodynamic corrections

- Considering boost-invariant Bjorken (longitudinal) expansion of QGP.
  $\rightarrow$ In the fluid rest frame $\implies$ $u^{\mu} \equiv (1, \overrightarrow{0})$
- Milne coordinates (τ, x, y, η_s):
  - $\tau = \sqrt{t^2 - z^2} \rightarrow$ proper time
  - $\eta_s = tanh^{-1}(z/t) \rightarrow$ space time rapidity
  - $g^{\mu\nu} = (1, -1, -1, -1/\tau^2) \rightarrow$ metric tensor
  - $\theta = 1/\tau$, $\Pi = -\zeta/\tau$ and $\pi^{\mu\nu}\sigma_{\mu\nu} = 4\eta/3\tau^2$
    with η and ζ representing the shear viscosity and bulk viscosity respectively.
- First order shear and bulk viscous corrections to the quasiparticle distribution function.

$$\delta f_k = f_k^0(1 \pm f_k^0)\{\phi_k(bulk)^{(1)} + \phi_k(shear)^{(1)}\}$$

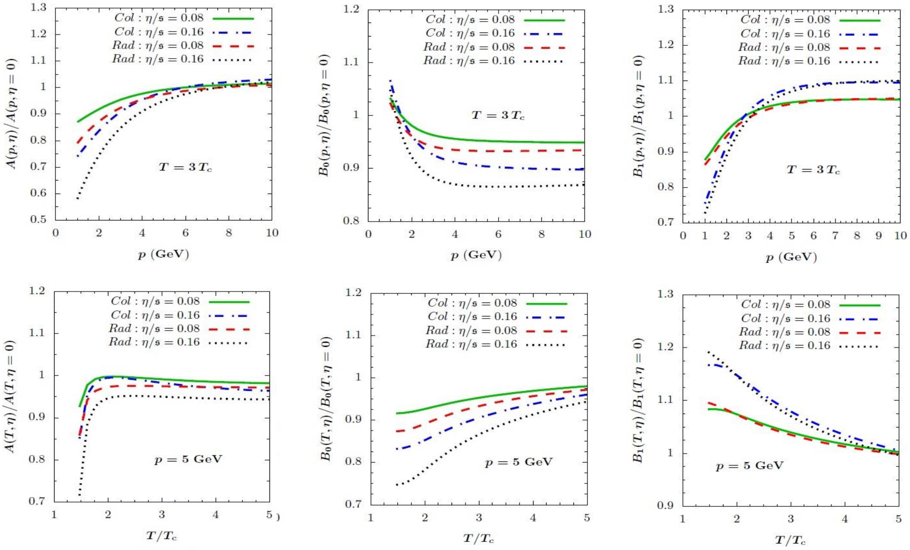
$$\phi_k(bulk)^{(1)} = \frac{s}{\beta_{\Pi}\omega_k T\tau}\left(\frac{\zeta}{s}\right)\left[\omega_k^2 c_s^2 - \frac{|\overrightarrow{q_k}|^2}{3} - \omega_k \delta\omega_k\right]$$

$$\phi_k(shear)^{(1)} = \frac{s}{\beta_{\pi}\omega_k T\tau}\left(\frac{\eta}{s}\right)\left[\frac{|\overrightarrow{q_k}|^2}{3} - (q_k)_z^2\right]$$

A. Shaikh *et al.*, **Phys. Rev. D 104, 034017 (2021)**

Results for **shear** viscous correction (i. Transport Coefficients)

$$N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$

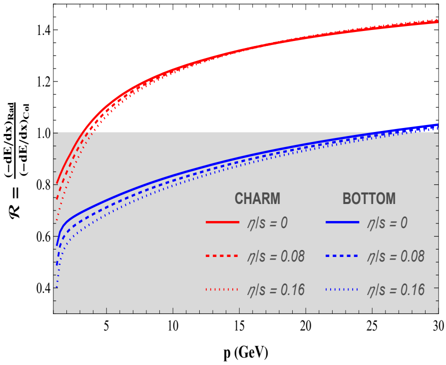


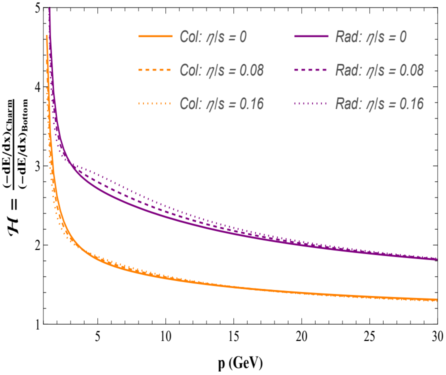
Results for **shear** viscous correction (ii. Energy Loss)

$$\boxed{N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}}$$

$$\boxed{m_b = 4.2\,\text{GeV}}$$

Differential energy loss: $-\dfrac{dE}{dx} = p\,A(p,T)$

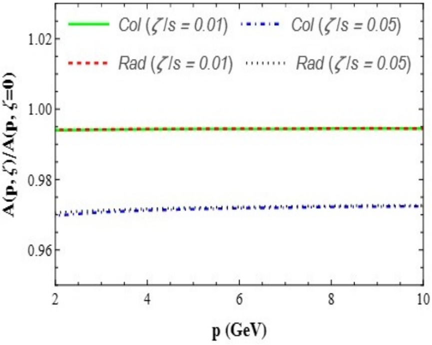


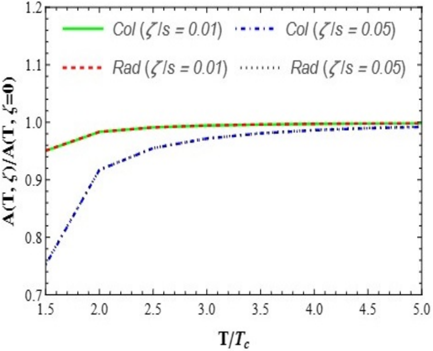


A. Shaikh, S. Dash, B. K. Nandi [arXiv: 2302.02235]

Results for **bulk** viscous correction

$$\boxed{N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}}$$





A.Shaikh *et al.*, **PoS CHARM2020 (2021) 060**

Second order shear viscous correction

$$\delta f(shear) = f^0(1 \pm f^0)\{\phi^{(1)} + \phi^{(2)}\}$$

General Case

$$\begin{aligned}
\phi^{(2)} = \frac{\beta}{\beta_\pi}\Big[&\frac{5}{14\beta_\pi(u\cdot p)}p^\alpha p^\beta \pi_\alpha^\gamma \pi_{\beta\gamma} \\
&- \frac{\tau_\pi}{(u\cdot p)}p^\alpha p^\beta \pi_\alpha^\gamma \omega_{\beta\gamma} - \frac{(u\cdot p)}{70\beta_\pi}\pi^{\alpha\beta}\pi_{\alpha\beta} \\
&+ \frac{6\tau_\pi}{5}p^\alpha \dot{u}^\beta \pi_{\alpha\beta} - \frac{\tau_\pi}{5}p^\alpha(\nabla^\beta \pi_{\alpha\beta}) \\
&- \frac{\tau_\pi}{2(u\cdot p)^2}p^\alpha p^\beta p^\gamma(\nabla_\gamma \pi_{\alpha\beta}) \\
&+ \frac{3\tau_\pi}{(u\cdot p)^2}p^\alpha p^\beta p^\gamma \pi_{\alpha\beta}\dot{u}_\gamma \\
&- \frac{\tau_\pi}{3(u\cdot p)}p^\alpha p^\beta \pi_{\alpha\beta}\theta \\
&+ \frac{\beta + (u\cdot p)^{-1}}{4(u\cdot p)^2\beta_\pi}(p^\alpha p^\beta \pi_{\alpha\beta})^2
\end{aligned}$$

For Bjorken (1D) expansion

$(\omega_{\mu\nu} = \dot{u}_\mu = 0)$

$$\begin{aligned}
\phi^{(2)} = \frac{s^2}{T\beta_\pi^2\tau^2}\left(\frac{\eta}{s}\right)^2\Big[&-\left(\frac{10}{63}\right)\frac{|\vec{q}|^2+3q_z^2}{E} \\
&-\left(\frac{4}{105}\right)E + \left(\frac{4}{15}\right)E - \left(\frac{4}{3}\right)\frac{q_z^2}{E} \\
&-\left(\frac{2}{3}\right)\frac{|\vec{q}|^2-3q_z^2}{3E} \\
&+\left(\frac{1}{T}+\frac{1}{E}\right)\left(\frac{|\vec{q}|^2-3q_z^2}{3E}\right)^2
\end{aligned}$$

C. Chattopadhyay *et al.*, Phys. Rev. C 91, 024917 (2015)

Conclusion

1. Heavy quark transport coefficients is studied in a **viscous QCD** medium including the **collisional and radiative processes**.
2. The thermal medium interactions are incorporated using **EQPM** and the **first-order shear and bulk viscous corrections** are included in the distribution function of the quasiparticles.
3. **Shear viscous corrections** are substantial for slow-moving HQ ($p \approx 1 - 2$ GeV at $T = 3T_c$) where the increase in η/s decreases the drag coefficient (exception: B_0 vs p and B_1 vs T).
4. **Bulk viscous corrections** are prominent near transition temperature ($T \approx 1.5T_c$).
5. **Radiative dominance** of energy loss mechanism for charm quark occurs at almost one order of magnitude less in initial momentum as compared with the bottom quark.
6. The effect of the **second-order viscous corrections** on the HQ transport coefficients is in progress.

ありがとう ございます

($\mathscr{T}hank\ \ you$!)