Study of Weak Basis Invariants, Hierarchy Limit, and Effective Theory in the Universal Seesaw Model

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in collaboration with

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(on going work)

Outline

- Introduction
- The model
- Weak-basis invariants
- WL-WR mixing, mass eigenvalue, and mixing angle
- Hierarchy limit
- Summary

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Introduction

- Standard Model (SM) is most successful theory incorporate the dynamics of sub-atomic particle

- However, SM cannot explain e.g, quark mass hierarchy

$$m_i = \frac{y_i v}{\sqrt{2}}; \quad i = u, d, c, s, t, b$$

- Universal seesaw model → smallness of up quark mass explained by the tiny ratio of $SU(2)_R$ breaking and $SU(2)$ singlet vector-like quark (VLQ) mass M_U

$$m_u = \frac{y_{uR} v_R y_{uL} v}{2M_U} = \frac{y_u v}{\sqrt{2}} \rightarrow y_u = \frac{y_{uR} v_R y_{uL}}{\sqrt{2} M_U}$$

$v \approx 246\ \mathrm{GeV}$

Quark mass (PDG)	Yukawa coupling
$m_u = 2.16\ \mathrm{MeV}$	$y_u \simeq 1.24 \times 10^{-5}$
$m_d = 4.67\ \mathrm{MeV}$	$y_d \simeq 2.68 \times 10^{-5}$
$m_s = 93.4\ \mathrm{MeV}$	$y_s \simeq 5.37 \times 10^{-4}$
$m_c = 1.27\ \mathrm{GeV}$	$y_c \simeq 7.30 \times 10^{-3}$
$m_b = 4.18\ \mathrm{GeV}$	$y_b \simeq 0.024$
$m_t = 172.69\ \mathrm{GeV}$	$y_t \simeq 0.99$

m_u, m_d, m_s from $\overline{\mathrm{MS}}$ at $\mu = 2\ \mathrm{GeV}$

m_c, m_b from $\overline{\mathrm{MS}}$ at $\mu = \overline{m}$

m_t from direct measurement

A. Davidson, K.C. Wali (1987); S. Rajpoot (1987); T. Morozumi, T. Satou, M. N. Rebelo, M. Tanimoto (1997)
Y. Kiyo, T. Morozumi, P Parada, M. N. Rebelo, M Tanimoto (1999)

Introduction

- Physical observables are basis-independent
- In quark sector, one have freedom to rephase the quark fields → Redefinition of CKM matrix

$$V'_{\alpha j} = e^{-i\phi_\alpha} V_{\alpha j} e^{i\phi_j}$$

- Example of CKM rephasing invariants → CP-odd Jarlskog invariant $J = \mathrm{Im}(V_{11} V_{22} V_{12}^* V_{21}^*)$
- Flavor matrices (Yukawa matrices) are basis-dependent → weak-basis transformation
- Our purpose:
  - obtain the weak-basis invariants quantities
  - mass eigenvalue, mixing matrix at some hierarchy limit

} One generation case of Quark sector of universal seesaw model

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The model

[T. Morozumi, A.S. Adam, Y. Kawamura, A.H.P, Y. Shimizu, K. Yamamoto arXiv:2211.02360]

We study the quark sector of the universal seesaw model with $SU(2)_L \times SU(2)_R \times U(1)_{Y'}$

Particle content: $\psi_L^i = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L : (\mathbf{2}, 1, 1/6)$ $\quad \psi_R^i = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_R : (1, \mathbf{2}, 1/6)$

$U_I : (1, 1, 2/3)$ $\quad D_I : (1, 1, -1/3)$

$\phi_L = \begin{pmatrix} \phi_L^+ \\ \phi_L^0 \end{pmatrix} : (\mathbf{2}, 1, 1/2)$ $\quad \phi_R = \begin{pmatrix} \phi_R^+ \\ \phi_R^0 \end{pmatrix} : (1, \mathbf{2}, 1/2)$

$i = 1, 2, 3; I = 1, 2, 3$

$Q = I_{3_L} + I_{3_R} + Y'$

$\langle \phi_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_L \end{pmatrix}$

$\langle \phi_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_R \end{pmatrix}$

The Lagrangian of the model,

$$\mathcal{L}_{\text{Doublets}} = \sum_{i=1}^{3} \overline{\psi_{Li}} \left(i\partial\!\!\!/ - g_L W\!\!\!/_L - g_1 \frac{1}{6} B\!\!\!/_1 \right) \psi_{Li} + \sum_{i=1}^{3} \overline{\psi_{Ri}} \left(i\partial\!\!\!/ - g_R W\!\!\!/_R - g_1 \frac{1}{6} B\!\!\!/_1 \right) \psi_{Ri}$$

$$\mathcal{L}_{\text{VLQ}} = \sum_{I=1}^{3} \overline{U_I} \left(i\partial\!\!\!/ - g_1 B\!\!\!/_1 \frac{2}{3} - M_{U_I} \right) U_I + \sum_{I=1}^{3} \overline{D_I} \left(i\partial\!\!\!/ + g_1 B\!\!\!/_1 \frac{1}{3} - M_{D_I} \right) D_I$$

$$\mathcal{L}_{\text{VLQ-Doublets}} = -y^u_{LiJ} \overline{\psi_{iL}} \tilde{\phi}_L U_J - y^u_{RiJ} \overline{\psi_{iR}} \tilde{\phi}_R U_J - y^d_{LiJ} \overline{\psi_{iL}} \phi_L D_J - y^d_{RiJ} \overline{\psi_{iR}} \phi_R D_J - h.c.$$

The model

Mass hierarchy:

M_U, M_D, M_C, M_S, M_B

$v_R \simeq M_T$

Z_R, W_R

v_L

SM

Seesaw mechanism works for u, d, c, s, b

$$m_{u(d)} = \frac{y_{u(d)R} v_R y_{u(d)L} v_L}{2M_{U(D)}}$$

We will study the one generation case

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Weak-basis invariants

[A.H.P &T. Morozumi arXiv:2211.02359]

The lagrangian of the model:

$$\mathcal{L} = -\overline{\psi_L} m_{uL} U - \overline{\psi_R} m_{uR} U - \overline{\psi_L} m_{dL} D - \overline{\psi_R} m_{dR} D - (h.c.)$$
$$- \overline{U_L} M_U U_R - \overline{D_L} M_D D_R - (h.c.)$$

$$m_{uL} = \frac{y_{uL} v_L}{\sqrt{2}} \qquad m_{dL} = \frac{y_{dL} v_L}{\sqrt{2}}$$
$$m_{uR} = \frac{y_{uR} v_R}{\sqrt{2}} \qquad m_{dR} = \frac{y_{dR} v_R}{\sqrt{2}}$$

Define following weak-basis transformation (WBT) on doublet quarks and VL quarks in this model as follow,

$$\psi'_L = e^{i\theta_{V_L}} \psi_L, \qquad \psi'_R = e^{i\theta_{V_R}} \psi_R, \qquad U' = e^{i\theta_U} U, \qquad D' = e^{i\theta_D} D$$

The Lagrangian will unchanged if the quark and VLQ mass have to transform as,

$$m'_{uL} = e^{i(\theta_{V_L} - \theta_U)} m_{uL}, \qquad m'_{uR} = e^{i(\theta_{V_R} - \theta_U)} m_{uR}, \qquad m'_{dL} = e^{i(\theta_{V_L} - \theta_D)} m_{dL}, \qquad m'_{dR} = e^{i(\theta_{V_R} - \theta_D)} m_{dR},$$
$$M'_U = M_U, \qquad M'_D = M_D$$

Denoting:

$$z_1 = e^{i\theta_{V_L}}, z_2 = e^{i\theta_{V_R}}, z_3 = e^{i\theta_U}, z_4 = e^{i\theta_D}$$
$$q_1 = m_{uL}, q_2 = m^*_{uL}, q_3 = m_{uR}, q_4 = m^*_{uR}, q_5 = m_{dL}, q_6 = m^*_{dL}, q_7 = m_{dR}, q_8 = m^*_{dR}, q_9 = M_U, q_{10} = M_D$$

Weak-basis invariants

Using Molien-Weyl formula see e.g. [Y. Wang, B. Yu, S. Zhou (2021)]

$$\mathcal{H}(q_1,\ldots,q_N) = \int [d\mu]_G \prod_{i=1}^{N} \frac{1}{\det(1-q_i R_i(g))}$$

$\mathcal{H}(q_1,\ldots,q_N)$: multigraded Hilbert-series

q_i : arbitrary complex number with $|q_i|<1$

$R_i(g)$: representation of group G where $g \in G$

Haar measure:

$$\int [d\mu]_G = \frac{1}{(2\pi i)^4} \oint_{|z_1|=1} \frac{dz_1}{z_1} \oint_{|z_2|=1} \frac{dz_2}{z_2} \oint_{|z_3|=1} \frac{dz_3}{z_3} \oint_{|z_4|=1} \frac{dz_4}{z_4}$$

ex: $[\det(1-q_1 z_1 z_3^{-1})]^{-1}$

We obtain the multigraded Hilbert series:

$$\mathcal{H}(q_1,\ldots,q_{10}) = \frac{1-q_1q_2q_3q_4q_5q_6q_7q_8}{(1-\underbrace{q_1q_2}_{I_1})(1-\underbrace{q_3q_4}_{I_2})(1-\underbrace{q_5q_6}_{I_3})(1-\underbrace{q_7q_8}_{I_4})(1-\underbrace{q_1q_4q_6q_7}_{I_5})(1-\underbrace{q_2q_3q_5q_8}_{I_6})(1-\underbrace{q_9}_{I_7})(1-\underbrace{q_{10}}_{I_8})}$$

Basic WBIs:

$$I_1 = m_{uL}^2, \quad I_2 = m_{uR}^2, \quad I_3 = m_{dL}^2, \quad I_4 = m_{dR}^2, \quad I_5 = m_{uL} m_{uR}^* m_{dL}^* m_{dR}, \quad I_6 = m_{uL}^* m_{uR} m_{dL} m_{dR}^*$$

$$I_7 = M_U, \quad I_8 = M_D$$

There is one relation among the basic WBIs $I_5 I_6 = I_1 I_2 I_3 I_4$

Weak-basis invariants

By changing $q_1, \ldots, q_{10} \to q$, we obtain the ungraded Hilbert series:

$$H(q) = \frac{1+q^4}{(1-q)^2(1-q^2)^4(1-q^4)}$$

7 independent WBIs

- CP even $\longrightarrow$ $I_1, I_2, I_3, I_4, I_7, I_8$
- CP odd $\longrightarrow$ $J_9 \equiv I_5 - I_6$

CP violating WBI:

$$W = \mathrm{Im}(m_{uL} m^*_{uR} m^*_{dL} m_{dR}) = \frac{J_9}{2i}$$

Next: Finding the source of this CP violating WBI

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WL-WR Mixing, mass eigenvalue and mixing angle

WB freedom:

$$\theta_{V_L} - \theta_U = -\arg(m_{uL})$$
$$\theta_{V_R} - \theta_U = -\arg(m_{uR})$$
$$\theta_{V_L} - \theta_D = -\arg(m_{dL})$$
$$\theta_{V_R} - \theta_D = -\arg(m_{uR}) + \arg(m_{uL}) - \arg(m_{dL})$$

$$m'_{uL} = e^{i(\theta_{V_L}-\theta_U)} m_{uL}$$
$$m'_{uR} = e^{i(\theta_{V_R}-\theta_U)} m_{uR}$$
$$m'_{dL} = e^{i(\theta_{V_L}-\theta_D)} m_{dL}$$
$$m'_{dR} = e^{i(\theta_{V_R}-\theta_D)} m_{dR}$$

$\Rightarrow$

$$m'_{uL} = |m_{uL}|$$
$$m'_{uR} = |m_{uR}|$$
$$m'_{dL} = |m_{dL}|$$
$$m'_{dR} = |m_{dR}| e^{i\theta_{\rm WI}}$$

$$e^{i\theta_{\rm WI}} = e^{i(\arg(m_{dR}) - \arg(m_{uR}) - \arg(m_{dL}) + \arg(m_{uL}))}$$

Lagrangian with the new weak basis:

$$\mathcal{L} = -\overline{\psi_L} m_{uL} U - \overline{\psi_R} m_{uR} U - \overline{\psi_L} m_{dL} D - \overline{\psi_R} m_{dR} e^{i\theta_{\rm WI}} D - \overline{U_L} M_U U_R - \overline{D_L} M_D D_R - h.c.$$

Separate the up-type and down-type:

$$\mathcal{L}_u = -\begin{pmatrix} \overline{u_L} & \overline{U_L} \end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_{uL} \\ m_{uR} & M_U \end{pmatrix}}_{\mathbb{M}_u} \begin{pmatrix} u_R \\ U_R \end{pmatrix} - \begin{pmatrix} \overline{u_R} & \overline{U_R} \end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_{uR} \\ m_{uL} & M_U \end{pmatrix}}_{\mathbb{M}_u^T} \begin{pmatrix} u_L \\ U_L \end{pmatrix}$$

$$\mathcal{L}_d = -\begin{pmatrix} \overline{d_L} & \overline{D_L} \end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_{dL} \\ m_{dR} e^{-i\theta_{\rm WI}} & M_D \end{pmatrix}}_{\mathbb{M}_d} \begin{pmatrix} d_R \\ D_R \end{pmatrix} - \begin{pmatrix} \overline{d_R} & \overline{D_R} \end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_{dR} e^{i\theta_{\rm WI}} \\ m_{dL} & M_D \end{pmatrix}}_{\mathbb{M}_d^\dagger} \begin{pmatrix} d_L \\ D_L \end{pmatrix}$$

Flavor basis → mass basis

WL-WR Mixing, mass eigenvalue and mixing angle

$$\mathcal{L}_u = -\left(\overline{u'_L} \quad \overline{U'_L} \right) V_{U_L}^\dagger \begin{pmatrix} 0 & m_{uL} \\ m_{uR} & M_U \end{pmatrix} V_{U_R} \begin{pmatrix} u'_R \\ U'_R \end{pmatrix} - h.c.$$

diagonalize

$$\begin{pmatrix} m_{u_1} & 0 \\ 0 & m_{u_2} \end{pmatrix}$$

$$\mathcal{L}_d = -\left(\overline{d'_L} \quad \overline{D'_L} \right) V_{D_L}^\dagger \begin{pmatrix} 0 & m_{dL} \\ m_{dR} e^{-i\theta_{\mathrm{WI}}} & M_D \end{pmatrix} V_{D_R} \begin{pmatrix} d'_R \\ D'_R \end{pmatrix} - h.c.$$

diagonalize

$$\begin{pmatrix} m_{d_1} & 0 \\ 0 & m_{d_2} \end{pmatrix}$$

$$\begin{pmatrix} u_L \\ U_L \end{pmatrix} = V_{U_L} \begin{pmatrix} u'_L \\ U'_L \end{pmatrix}$$

$$\begin{pmatrix} u_R \\ U_R \end{pmatrix} = V_{U_R} \begin{pmatrix} u'_R \\ U'_R \end{pmatrix}$$

$$\begin{pmatrix} d_L \\ D_L \end{pmatrix} = V_{D_L} \begin{pmatrix} d'_L \\ D'_L \end{pmatrix}$$

$$\begin{pmatrix} d_R \\ D_R \end{pmatrix} = V_{D_R} \begin{pmatrix} d'_R \\ D'_R \end{pmatrix}$$

$V_{U_L}, V_{U_R}, V_{D_L}, V_{D_R}$ are 2 x 2 unitary matrices

$$d_L = \sum_{j=1}^{2} V_{D_{L1j}} d'_{Lj}$$

$$\bar{u}_L = \sum_{i=1}^{2} V^*_{U_{L1i}} u'_{Li}$$

Charge current Lagrangian:

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g_L}{\sqrt{2}} (\bar{u}_L \gamma^\mu d_L W^+_{\mu L} + \bar{d}_L \gamma^\mu u_L W^-_{\mu L}) - \frac{g_R}{\sqrt{2}} (\bar{u}_R \gamma^\mu d_R W^+_{\mu R} + \bar{d}_R \gamma^\mu u_R W^-_{\mu R})$$

$$= -\frac{g_L}{2\sqrt{2}} \left(\bar{u}'_i V^*_{U_{L1i}} V_{D_{L1j}} \gamma^\mu (1-\gamma^5) d'_j W^+_{\mu L} + \bar{d}'_i V^*_{D_{L1i}} V_{U_{L1j}} \gamma^\mu (1-\gamma^5) u'_j W^-_{\mu L} \right)$$

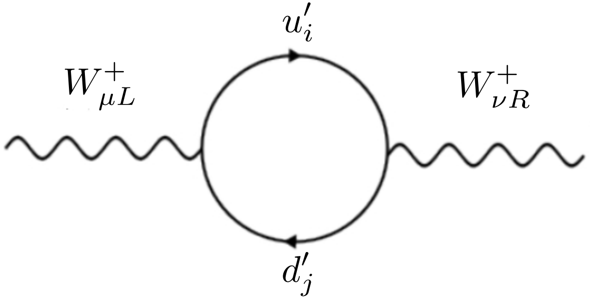
$$-\frac{g_R}{2\sqrt{2}} \left(\bar{u}'_i V^*_{U_{R1i}} V_{D_{R1j}} \gamma^\mu (1+\gamma^5) d'_j W^+_{\mu R} + \bar{d}'_i V^*_{D_{R1i}} V_{U_{R1j}} \gamma^\mu (1+\gamma^5) u'_j W^-_{\mu R} \right)$$

$$V^{\mathrm{CKM}}_{L_{ij}} = V^*_{U_{L1i}} V_{D_{L1j}}$$

$$V^{\mathrm{CKM}}_{R_{ij}} = V^*_{U_{R1i}} V_{D_{R1j}}$$

WL-WR Mixing, mass eigenvalue and mixing angle

$$\mathcal{L}_{\text{CC+mass}} = -\frac{g_L}{2\sqrt{2}}\left(\bar{u}_i' V^*_{U_{L1i}} V_{D_{L1j}} \gamma^\mu (1-\gamma^5) d_j' W^+_{\mu L}\right) - \frac{g_R}{2\sqrt{2}}\left(\bar{d}_i' V^*_{D_{R1i}} V_{U_{R1j}} \gamma^\nu (1+\gamma^5) u_j' W^-_{\nu R}\right)$$
$$-\bar{u}_i' m_{u_i} u_i' - \bar{d}_i' m_{d_i} d_i'$$



$$\mathcal{L}_{\text{eff}} = M^2 W^+_{\mu L} W_R^{-\mu} + h.c.$$

$$\Pi^{\mu\nu}_{W_L W_R}(0) = \frac{g_L g_R}{16\pi^2}(V^*_{U_{L1i}} m_{u_i} V_{U_{R1i}})(V_{D_{L1j}} m_{d_j} V^*_{D_{R1j}}) g_{\mu\nu}[-C_{UV} + \log 2 + 1 + f(m_{u_i}, m_{d_j})]$$

seesaw condition:

$$V^*_{U_{L11}} m_{u_1} V_{U_{R11}} + V^*_{U_{L12}} m_{u_2} V_{U_{R12}} = 0$$
$$V^*_{D_{L11}} m_{d_1} V_{D_{R11}} + V^*_{D_{L12}} m_{d_2} V_{D_{R12}} = 0$$
$$V_{U_{L11}} m_{u_1} V^*_{U_{R11}} + V_{U_{L12}} m_{u_2} V^*_{U_{R12}} = 0$$
$$V_{D_{L11}} m_{d_1} V^*_{D_{R11}} + V_{D_{L12}} m_{d_2} V^*_{D_{R12}} = 0$$

where

$$f(m_{u_i}, m_{d_j}) = \frac{m^2_{u_i}\log(m^2_{u_i}) - m^2_{d_j}\log(m^2_{d_j})}{m^2_{u_i} - m^2_{d_j}}$$

$$\Pi^{\mu\nu}_{W_L W_R}(0) = \frac{g_L g_R}{16\pi^2}(V^*_{U_{L11}} m_{u_1} V_{U_{R11}})(V_{D_{L11}} m_{d_1} V^*_{D_{R11}}) g_{\mu\nu}[f(m_{u_1}, m_{d_1}) + f(m_{u_2}, m_{d_2}) - f(m_{u_1}, m_{d_2}) - f(m_{u_2}, m_{d_1})]$$

WL-WR Mixing, mass eigenvalue and mixing angle

Diagonalization of up-type quark and VLQ mass matrix

$$\begin{pmatrix} u_L \\ U_L \end{pmatrix} = \begin{pmatrix} \cos\theta_{U_l} & -\sin\theta_{U_l} \\ \sin\theta_{U_l} & \cos\theta_{U_l} \end{pmatrix} \begin{pmatrix} \cos\theta_{U_R} & \sin\theta_{U_R} \\ -\sin\theta_{U_R} & \cos\theta_{U_R} \end{pmatrix} \begin{pmatrix} u'_L \\ U'_L \end{pmatrix}$$

$$\begin{pmatrix} u_R \\ U_R \end{pmatrix} = \begin{pmatrix} \cos\theta_{U_R} & \sin\theta_{U_R} \\ -\sin\theta_{U_R} & \cos\theta_{U_R} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u'_R \\ U'_R \end{pmatrix}$$

Lagrangian of up-type mixing become:

$$\begin{aligned}
\mathcal{L}_u &= -\begin{pmatrix} \overline{u'_L} & \overline{U'_L} \end{pmatrix} \begin{pmatrix} \cos\theta_{U_R} & -\sin\theta_{U_R} \\ \sin\theta_{U_R} & \cos\theta_{U_R} \end{pmatrix} \begin{pmatrix} \cos\theta_{U_l} & \sin\theta_{U_l} \\ -\sin\theta_{U_l} & \cos\theta_{U_l} \end{pmatrix} \begin{pmatrix} 0 & m_{u_L} \\ m_{u_R} & M_U \end{pmatrix} \begin{pmatrix} \cos\theta_{U_R} & \sin\theta_{U_R} \\ -\sin\theta_{U_R} & \cos\theta_{U_R} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u'_R \\ U'_R \end{pmatrix} \\
&= -\begin{pmatrix} \overline{u'_L} & \overline{U'_L} \end{pmatrix} \begin{pmatrix} \cos\theta_{U_L} & -\sin\theta_{U_L} \\ \sin\theta_{U_L} & \cos\theta_{U_L} \end{pmatrix} \begin{pmatrix} 0 & m_{u_L} \\ m_{u_R} & M_U \end{pmatrix} \begin{pmatrix} -\cos\theta_{U_R} & \sin\theta_{U_R} \\ \sin\theta_{U_R} & \cos\theta_{U_R} \end{pmatrix} \begin{pmatrix} u'_R \\ U'_R \end{pmatrix} \\
&= -\begin{pmatrix} \overline{u'_L} & \overline{U'_L} \end{pmatrix} \begin{pmatrix} m_{u_1} & 0 \\ 0 & m_{u_2} \end{pmatrix} \begin{pmatrix} u'_R \\ U'_R \end{pmatrix}
\end{aligned}$$

Mass eigenvalue:

$$m_{u_1} = -\frac{\sqrt{M_U^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_U^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

$$m_{u_2} = \frac{\sqrt{M_U^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_U^2 + (m_{u_R} + m_{u_L})^2}}{2}$$

Mixing angle:

$$\theta_{U_L} = \theta_{U_R} - \theta_{U_l}$$

$$\tan\theta_{U_l} = \frac{m_{u_R} - m_{u_L}}{M_U}$$

$$\tan 2\theta_{U_R} = \frac{2M_U m_{u_R}}{M_U^2 + m_{u_L}^2 - m_{u_R}^2}$$

WL-WR Mixing, mass eigenvalue and mixing angle

Diagonalization of down-type quark and VLQ mass matrix

$$\begin{pmatrix} d_L \\ D_L \end{pmatrix} = \begin{pmatrix} \cos\theta_{D_l} & -\sin\theta_{D_l} \\ \sin\theta_{D_l} & \cos\theta_{D_l} \end{pmatrix} \begin{pmatrix} \cos\theta_{D_R} & \sin\theta_{D_R} \\ -\sin\theta_{D_R} & \cos\theta_{D_R} \end{pmatrix} \begin{pmatrix} d'_L \\ D'_L \end{pmatrix}$$

$$\begin{pmatrix} d_R \\ D_R \end{pmatrix} = \begin{pmatrix} e^{i\theta_{\rm WI}} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_{D_R} & \sin\theta_{D_R} \\ -\sin\theta_{D_R} & \cos\theta_{D_R} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} d'_R \\ D'_R \end{pmatrix}$$

Lagrangian of down-type mixing become:

$$\mathcal{L}_d = -\begin{pmatrix} \overline{d'_L} & \overline{D'_L} \end{pmatrix} \begin{pmatrix} \cos\theta_{D_L} & -\sin\theta_{D_L} \\ \sin\theta_{D_L} & \cos\theta_{D_L} \end{pmatrix} \begin{pmatrix} 0 & m_{d_L} \\ m_{d_R} e^{-i\theta_{\rm WI}} & M_D \end{pmatrix} \begin{pmatrix} -e^{i\theta_{\rm WI}} \cos\theta_{D_R} & e^{i\theta_{\rm WI}} \sin\theta_{D_R} \\ \sin\theta_{D_R} & \cos\theta_{D_R} \end{pmatrix} \begin{pmatrix} d'_R \\ D'_R \end{pmatrix}$$

$$= -\begin{pmatrix} \overline{d'_L} & \overline{D'_L} \end{pmatrix} \begin{pmatrix} m_{d_1} & 0 \\ 0 & m_{d_2} \end{pmatrix} \begin{pmatrix} d'_R \\ D'_R \end{pmatrix}$$

Mass eigenvalue:

$$m_{d_1} = -\frac{\sqrt{M_D^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_D^2 + (m_{d_R} + m_{d_L})^2}}{2}$$

$$m_{d_2} = \frac{\sqrt{M_D^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_D^2 + (m_{d_R} + m_{d_L})^2}}{2}$$

Mixing angle:

$$\theta_{D_L} = \theta_{D_R} - \theta_{D_l}$$

$$\tan\theta_{D_l} = \frac{m_{d_R} - m_{d_L}}{M_D}$$

$$\tan 2\theta_{D_R} = \frac{2 M_D m_{d_R}}{M_D^2 + m_{d_L}^2 - m_{d_R}^2}$$

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Hierarchy limit

$$M_U, M_D \gg v_R \gg v_L$$

$$\begin{pmatrix} \cos\theta_{U_L} & -\sin\theta_{U_L} \\ \sin\theta_{U_L} & \cos\theta_{U_L} \end{pmatrix} \begin{pmatrix} 0 & m_{u_L} \\ m_{u_R} & M_U \end{pmatrix} \begin{pmatrix} -\cos\theta_{U_R} & \sin\theta_{U_R} \\ \sin\theta_{U_R} & \cos\theta_{U_R} \end{pmatrix} = \begin{pmatrix} m_{u_1} & 0 \\ 0 & m_{u_2} \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta_{D_L} & -\sin\theta_{D_L} \\ \sin\theta_{D_L} & \cos\theta_{D_L} \end{pmatrix} \begin{pmatrix} 0 & m_{d_L} \\ m_{d_R} e^{-i\theta_{\mathrm{WI}}} & M_D \end{pmatrix} \begin{pmatrix} -e^{i\theta_{\mathrm{WI}}}\cos\theta_{D_R} & e^{i\theta_{\mathrm{WI}}}\sin\theta_{D_R} \\ \sin\theta_{D_R} & \cos\theta_{D_R} \end{pmatrix} = \begin{pmatrix} m_{d_1} & 0 \\ 0 & m_{d_2} \end{pmatrix}$$

Mass eigenvalues:

$$m_{u_1} \simeq \frac{m_{u_R} m_{u_L}}{M_U} = \frac{v_R y_{uR} y_{uL} v_L}{2M_U}$$

$$m_{u_2} \simeq M_U$$

$$m_{d_1} \simeq \frac{m_{d_R} m_{d_L}}{M_D} = \frac{v_R y_{dR} y_{dL} v_L}{2M_D}$$

$$m_{d_2} \simeq M_D$$

Mixing angle:

$$\sin\theta_{U_L} \simeq \frac{m_{uL}}{M_U} \qquad \sin\theta_{U_R} \simeq \frac{m_{uR}}{M_U}$$

$$\sin\theta_{D_L} \simeq \frac{m_{dL}}{M_D} \qquad \sin\theta_{D_R} \simeq \frac{m_{dR}}{M_D}$$

$$\cos\theta_{D_L} = \cos\theta_{D_R} = \cos\theta_{U_L} = \cos\theta_{U_R} \simeq 1$$

Recall:

$$W = \mathrm{Im}(m_{uL} m_{uR}^* m_{dL}^* m_{dR}) = m_{uL} m_{uR} m_{dL} m_{dR} \mathrm{Im}(e^{i\theta_{\mathrm{WI}}})$$

$$\Pi^{\mu\nu}_{W_L W_R}(0) = \frac{g_L g_R}{16\pi^2} (V^*_{U_{L11}} m_{u_1} V_{U_{R11}})(V_{D_{L11}} m_{d_1} V_{D_{R11}}) g_{\mu\nu} [f(m_{u_1}, m_{d_1}) + f(m_{u_2}, m_{d_2}) - f(m_{u_1}, m_{d_2}) - f(m_{u_2}, m_{d_1})]$$

$$\mathrm{Im}\left(\Pi^{\mu\nu}_{W_L W_R}(0)\right) = \mathrm{Im}\left(\frac{m_{uL} m_{uR} m_{dL} m_{dR}}{M_U M_D} e^{-i\theta_{\mathrm{WI}}}\right) = -\frac{W}{M_U M_D}$$

Hierarchy limit

$$M_U, M_D \gg v_R \gg v_L$$

This hierarchy limit is applied for:

1st generation quark

$$m_u \simeq \frac{m_{u_R} m_{u_L}}{M_U} = \frac{v_R y_{uR} y_{uL} v_L}{2M_U}$$

$$m_{u'} \simeq M_U$$

$$m_d \simeq \frac{m_{d_R} m_{d_L}}{M_D} = \frac{v_R y_{dR} y_{dL} v_L}{2M_D}$$

$$m_{d'} \simeq M_D$$

2nd generation quark

$$m_c \simeq \frac{m_{c_R} m_{c_L}}{M_C} = \frac{v_R y_{cR} y_{cL} v_L}{2M_C}$$

$$m_{c'} \simeq M_C$$

$$m_s \simeq \frac{m_{s_R} m_{s_L}}{M_S} = \frac{v_R y_{sR} y_{sL} v_L}{2M_S}$$

$$m_{s'} \simeq M_S$$

Hierarchy limit

$$M_U, M_D \gg v_R \gg v_L$$

$$V_L^{\rm CKM} = \begin{pmatrix} \cos\theta_{U_L}\cos\theta_{D_L} & \cos\theta_{U_L}\sin\theta_{D_L} \\ \sin\theta_{U_L}\cos\theta_{D_L} & \sin\theta_{U_L}\sin\theta_{D_L} \end{pmatrix} \simeq \begin{pmatrix} \simeq 1 & \frac{m_{dL}}{M_D} \\ \frac{m_{uL}}{M_U} & \frac{m_{uL}}{M_U}\frac{m_{dL}}{M_D} \end{pmatrix}$$

$$V_R^{\rm CKM} = \begin{pmatrix} \cos\theta_{U_R}\cos\theta_{D_R} & \cos\theta_{U_R}\sin\theta_{D_R} \\ \sin\theta_{U_R}\cos\theta_{D_R} & \sin\theta_{U_R}\sin\theta_{D_R} \end{pmatrix} \begin{pmatrix} -e^{i\theta_{\rm WI}} & 0 \\ 0 & e^{i\theta_{\rm WI}} \end{pmatrix} \simeq \begin{pmatrix} \simeq 1 & \frac{m_{dR}}{M_D} \\ \frac{m_{uR}}{M_U} & \frac{m_{uR}}{M_U}\frac{m_{dR}}{M_D} \end{pmatrix} \begin{pmatrix} -e^{i\theta_{\rm WI}} & 0 \\ 0 & e^{i\theta_{\rm WI}} \end{pmatrix}$$

$$\mathcal{L}_{\rm CC} = -\frac{g_L}{\sqrt{2}}\left(\bar{u}'_{Li} V^{\rm CKM}_{L_{ij}} \gamma^\mu d'_{Lj} W^+_{\mu L} + h.c\right) - \frac{g_R}{\sqrt{2}}\left(\bar{u}'_{Ri} V^{\rm CKM}_{R_{ij}} \gamma^\mu d'_{Rj} W^+_{\mu R} + h.c.\right)$$

This hierarchy limit is applied for:

1st generation

$$j_L^{\mu-} = \bar{u}_L\gamma^\mu d_L + \frac{m_{dL}}{M_D}\bar{u}_L\gamma^\mu d'_L + \frac{m_{uL}}{M_U}\bar{u}'_L\gamma^\mu d_L + \frac{m_{uL}}{M_U}\frac{m_{dL}}{M_D}\bar{u}'_L\gamma^\mu d'_L$$

$$j_R^{\mu-} = \bar{u}_R\gamma^\mu d_R + \frac{m_{dR}}{M_D}\bar{u}_R\gamma^\mu d'_R + \frac{m_{uR}}{M_U}\bar{u}'_R\gamma^\mu d_R + \frac{m_{uR}}{M_U}\frac{m_{dR}}{M_D}\bar{u}'_R\gamma^\mu d'_R$$

2nd generation

$$j_L^{\mu-} = \bar{c}_L\gamma^\mu s_L + \frac{m_{sL}}{M_S}\bar{c}_L\gamma^\mu s'_L + \frac{m_{cL}}{M_C}\bar{c}'_L\gamma^\mu s_L + \frac{m_{cL}}{M_C}\frac{m_{sL}}{M_S}\bar{c}'_L\gamma^\mu s'_L$$

$$j_R^{\mu-} = \bar{c}_R\gamma^\mu s_R + \frac{m_{sR}}{M_S}\bar{c}_R\gamma^\mu s'_R + \frac{m_{cR}}{M_C}\bar{c}'_R\gamma^\mu s_R + \frac{m_{cR}}{M_C}\frac{m_{sR}}{M_S}\bar{c}'_R\gamma^\mu s'_R$$

Hierarchy limit

$$M_D \gg v_R \gg v_L$$

$$v_R > M_U \gg v_L$$

$$\begin{pmatrix} \cos\theta_{U_L} & -\sin\theta_{U_L} \\ \sin\theta_{U_L} & \cos\theta_{U_L} \end{pmatrix} \begin{pmatrix} 0 & m_{u_L} \\ m_{u_R} & M_U \end{pmatrix} \begin{pmatrix} -\cos\theta_{U_R} & \sin\theta_{U_R} \\ \sin\theta_{U_R} & \cos\theta_{U_R} \end{pmatrix} = \begin{pmatrix} m_{u_1} & 0 \\ 0 & m_{u_2} \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta_{D_L} & -\sin\theta_{D_L} \\ \sin\theta_{D_L} & \cos\theta_{D_L} \end{pmatrix} \begin{pmatrix} 0 & m_{d_L} \\ m_{d_R} e^{-i\theta_{\mathrm{WI}}} & M_D \end{pmatrix} \begin{pmatrix} -e^{i\theta_{\mathrm{WI}}}\cos\theta_{D_R} & e^{i\theta_{\mathrm{WI}}}\sin\theta_{D_R} \\ \sin\theta_{D_R} & \cos\theta_{D_R} \end{pmatrix} = \begin{pmatrix} m_{d_1} & 0 \\ 0 & m_{d_2} \end{pmatrix}$$

Mass eigenvalues:

$$m_{u_1} \simeq m_{uL}$$

$$m_{u_2} \simeq m_{uR}$$

$$m_{d_1} \simeq \frac{m_{d_R} m_{d_L}}{M_D} = \frac{v_R y_{dR} y_{dL} v_L}{2M_D}$$

$$m_{d_2} \simeq M_D$$

Mixing angle:

$$\sin\theta_{U_L} \simeq \frac{M_U m_{uL}}{m_{uR}^2}$$

$$\sin\theta_{U_R} \simeq 1$$

$$\cos\theta_{U_L} \simeq 1$$

$$\cos\theta_{U_R} \simeq \frac{M_U}{m_{uR}}$$

$$\sin\theta_{D_L} \simeq \frac{m_{dL}}{M_D}$$

$$\sin\theta_{D_R} \simeq \frac{m_{dR}}{M_D}$$

$$\cos\theta_{D_L} = \cos\theta_{D_R} \simeq 1$$

$$\mathrm{Im}\left(\Pi^{\mu\nu}_{W_L W_R}(0)\right) = \mathrm{Im}\left(\frac{m_{uL} M_U m_{dL} m_{dR}}{m_{uR} M_D} e^{-i\theta_{\mathrm{WI}}}\right) = -\frac{M_U W}{m_{uR}^2 M_D}$$

Hierarchy limit

$$\boldsymbol{M_D \gg v_R \gg v_L}$$

$$\boldsymbol{v_R > M_U \gg v_L}$$

This hierarchy limit applied for:

3rd generation quark

$$m_t \simeq m_{t_L} = \frac{y_{tL} v_L}{\sqrt{2}}$$

$$m_{t'} \simeq m_{t_R} = \frac{y_{tR} v_R}{\sqrt{2}}$$

$$m_b \simeq \frac{m_{b_R} m_{b_L}}{M_B} = \frac{v_R y_{bR} y_{bL} v_L}{2 M_B}$$

$$m_{b'} \simeq M_B$$

Hierarchy limit

$$M_D \gg v_R \gg v_L$$

$$v_R > M_U \gg v_L$$

$$V_L^{\mathrm{CKM}} = \begin{pmatrix} \cos\theta_{U_L}\cos\theta_{D_L} & \cos\theta_{U_L}\sin\theta_{D_L} \\ \sin\theta_{U_L}\cos\theta_{D_L} & \sin\theta_{U_L}\sin\theta_{D_L} \end{pmatrix} \simeq \begin{pmatrix} \simeq 1 & \frac{m_{dL}}{M_D} \\ \frac{M_U m_{uL}}{m_{uR}^2} & \frac{M_U m_{uL}}{m_{uR}^2}\frac{m_{dL}}{M_D} \end{pmatrix}$$

$$V_R^{\mathrm{CKM}} = \begin{pmatrix} \cos\theta_{U_R}\cos\theta_{D_R} & \cos\theta_{U_R}\sin\theta_{D_R} \\ \sin\theta_{U_R}\cos\theta_{D_R} & \sin\theta_{U_R}\sin\theta_{D_R} \end{pmatrix} \begin{pmatrix} -e^{i\theta_{\mathrm{WI}}} & 0 \\ 0 & e^{i\theta_{\mathrm{WI}}} \end{pmatrix} \simeq \begin{pmatrix} \frac{M_U}{m_{uR}} & \frac{M_U}{m_{uR}}\frac{m_{dR}}{M_D} \\ \simeq 1 & \frac{m_{dR}}{M_D} \end{pmatrix} \begin{pmatrix} -e^{i\theta_{\mathrm{WI}}} & 0 \\ 0 & e^{i\theta_{\mathrm{WI}}} \end{pmatrix}$$

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g_L}{\sqrt{2}}\left(\bar{u}'_{Li} V_{L_{ij}}^{\mathrm{CKM}} \gamma^\mu d'_{Lj} W_{\mu L}^+ + h.c\right) - \frac{g_R}{\sqrt{2}}\left(\bar{u}'_{Ri} V_{R_{ij}}^{\mathrm{CKM}} \gamma^\mu d'_{Rj} W_{\mu R}^+ + h.c.\right)$$

This hierarchy limit is applied for:

3rd generation quark

$$j_L^{\mu -} = \bar{t}_L \gamma^\mu b_L + \frac{m_{dL}}{M_D}\bar{t}_L \gamma^\mu b'_L + \frac{M_U m_{uL}}{m_{uR}^2}\bar{t}'_L \gamma^\mu b_L + \frac{M_U m_{uL}}{m_{uR}^2}\frac{m_{dL}}{M_D}\bar{t}'_L \gamma^\mu b'_L$$

$$j_R^{\mu -} = \frac{M_U}{m_{uR}}\bar{t}_R \gamma^\mu b_R + \frac{M_U}{m_{uR}}\frac{m_{dR}}{M_D}\bar{t}_R \gamma^\mu b'_R + \bar{t}'_R \gamma^\mu b_R + \frac{m_{dR}}{M_D}\bar{t}'_R \gamma^\mu b'_R$$

Outline

- Introduction
- The model
- Weak-basis invariants
- WL-WR mixing, mass eigenvalue, and mixing angle
- Hierarchy limit
- Summary

Summary

- We study quark sector of universal seesaw model, in particular in one generation case
- There is one CP violating weak-basis invariant

$$W = \mathrm{Im}(m_{uL} m^*_{uR} m^*_{dL} m_{dR}) = \frac{J_9}{2i}$$

  which appears in the WL-WR mixing

- We have applied the hierarchy limit to the appropriate generation
- On going work:
  Effective theory →
  Integrated out down-type and up-type VLQ and obtained the dim 5 and 6 effective lagrangian

THANK YOU

BACKUP

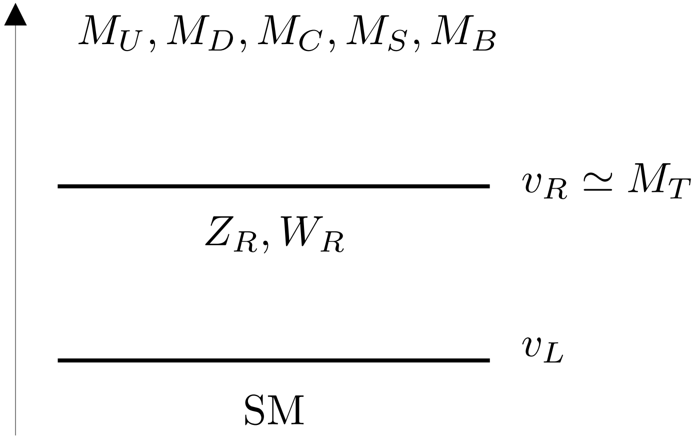
Effective theory: a preliminary study

Most BSM model, reduction to SM at low energies proceeds via decoupling of heavy particles with masses of order Λ

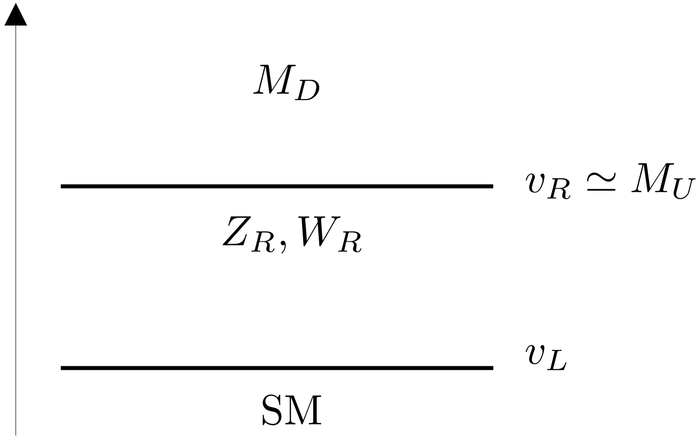
[B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek (2010)]

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda}\sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)} Q_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

Mass hierarchy (three generation case) :



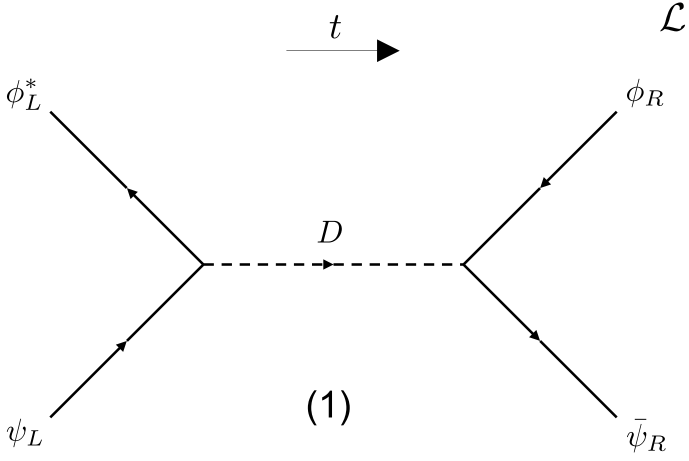
Mass hierarchy (one generation case) :

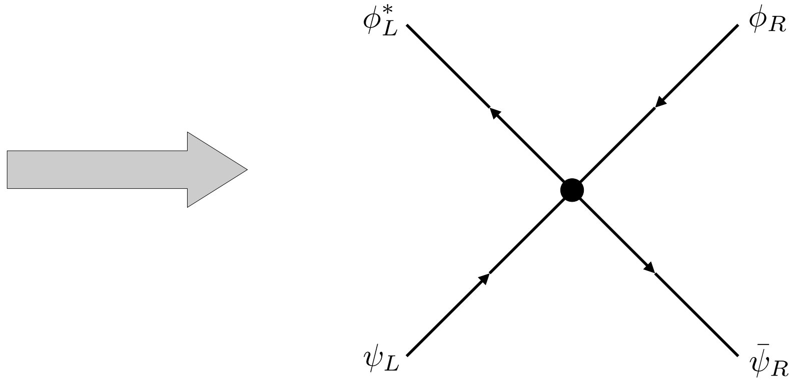


Effective theory: a preliminary study

Integrate out down-type VLQ at tree level

$$\mathcal{L}_1 = -y_{dR}\bar{\psi}_R\phi_R D - y_{dL}^*\bar{D}\phi_L^\dagger\psi_L$$





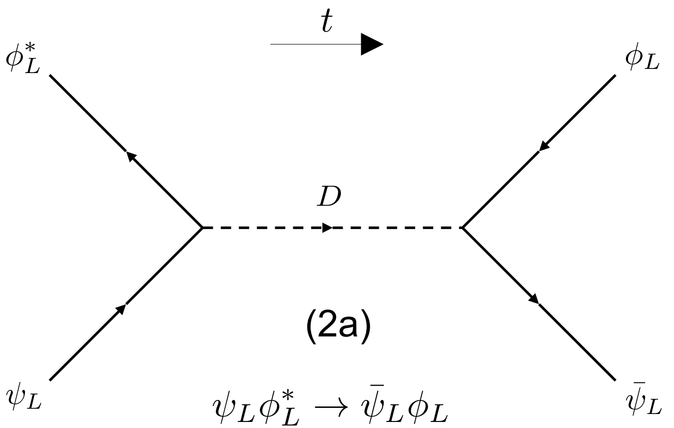
$\psi_L\phi_L^* \to \bar{\psi}_R\phi_R$

$$\mathcal{A}_1 = \frac{y_{dR}y_{dL}^*}{M_D}\left(\bar{\psi}_R(x)\phi_R(x)\right)\left(\phi_L^\dagger(x)\psi_L(x)\right)$$

$$\mathcal{L}_{\text{eff}}^{(5)} = C_D^{(5)}(\mu)\left(\bar{\psi}_R(x)\phi_R(x)\right)\left(\phi_L^\dagger(x)\psi_L(x)\right) + h.c.$$

$$C_D^{(5)}(M_D) = \frac{y_{dR}y_{dL}^*}{M_D}$$

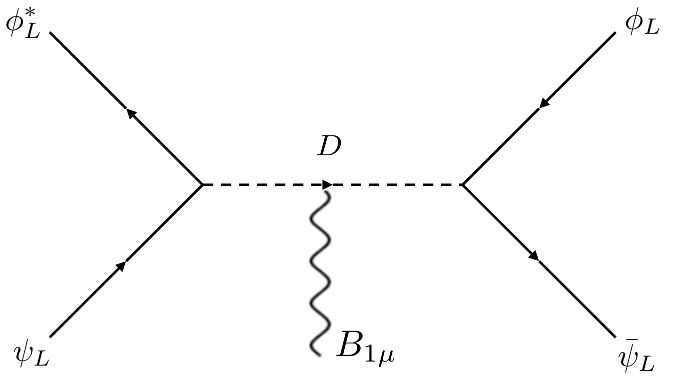
Effective theory: a preliminary study



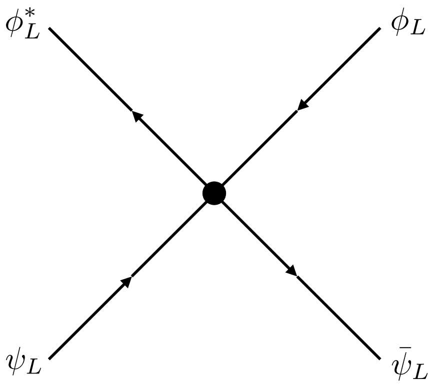
(2a)

$\psi_L \phi_L^* \to \bar{\psi}_L \phi_L$

$$\mathcal{L}_2 = -y_{dL} \bar{\psi}_L \phi_L D - y_{dL}^* \bar{D} \phi_L^\dagger \psi_L$$



(2b)



$$\mathcal{L}_{\text{eff}}^{(6)} = C_D^{(6)}(\mu) \left(\bar{\psi}_L(x)\phi_L(x)\right) i\not{D}_D \left(\phi_L^\dagger(x)\psi_L(x)\right)$$

$$i\not{D}_D = i\not{\partial}_\mu + \frac{1}{3} g_1 \not{B}_1$$

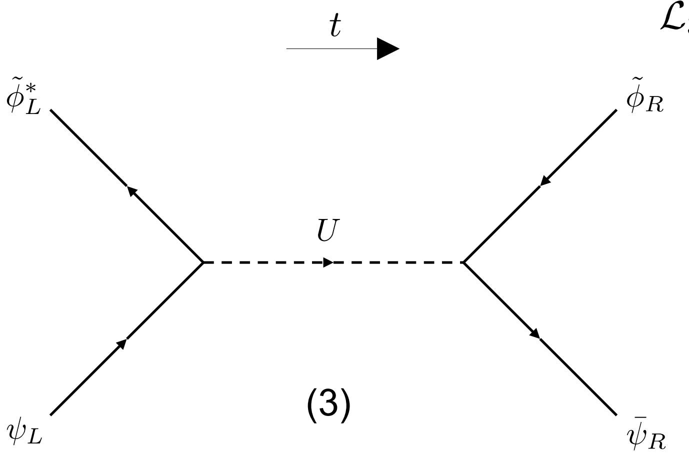
$$\mathcal{A}_{2a} + \mathcal{A}_{2b} = \frac{y_{dL} y_{dL}^*}{M_D^2} \left(\bar{\psi}_L(x)\phi_L(x)\right) i\not{D}_D \left(\phi_L^\dagger(x)\psi_L(x)\right)$$

$$C_D^{(6)}(M_D) = \frac{y_{dL} y_{dL}^*}{M_D^2}$$

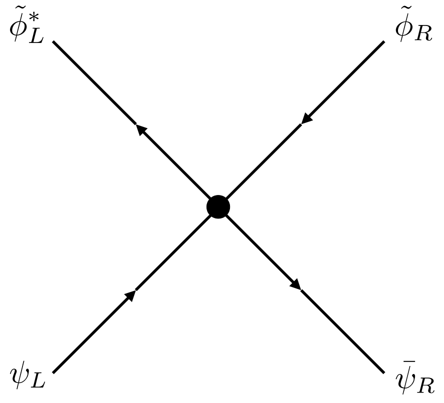
Effective theory: a preliminary study

Integrate out up-type VLQ at tree level

$$\mathcal{L}_3 = -y_{uR}\bar{\psi}_R\tilde{\phi}_R U - y_{uL}^*\bar{D}\tilde{\phi}_L^\dagger\psi_L$$



$$\psi_L\tilde{\phi}_L^* \to \bar{\psi}_R\phi_R$$

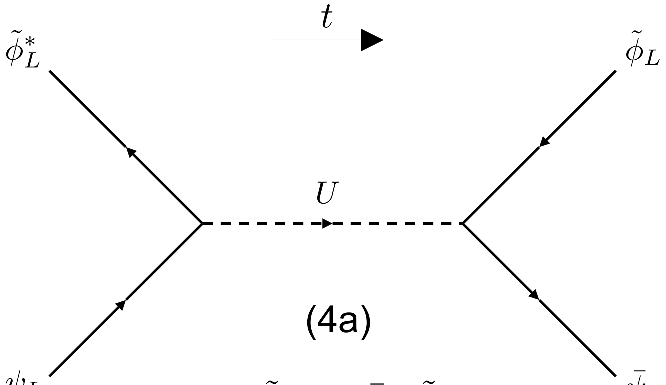


$$\mathcal{A}_3 = \frac{y_{uR}y_{uL}^*}{M_U}\left(\bar{\psi}_R(x)\tilde{\phi}_R(x)\right)\left(\tilde{\phi}_L^\dagger(x)\psi_L(x)\right)$$

$$\mathcal{L}_{\text{eff}}^{(5)} = C_U^{(5)}(\mu)\left(\bar{\psi}_R(x)\tilde{\phi}_R(x)\right)\left(\tilde{\phi}_L^\dagger(x)\psi_L(x)\right) + h.c.$$

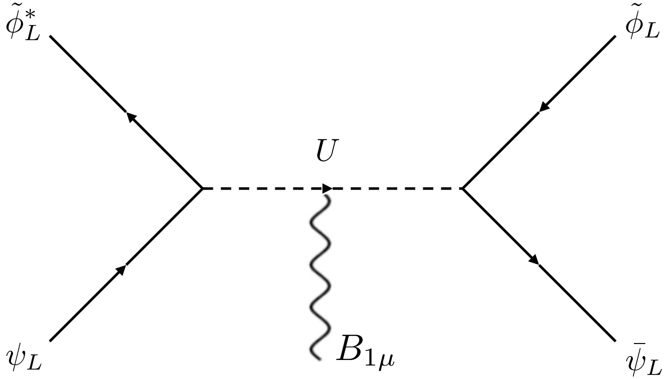
$$\boxed{C_U^{(5)}(M_U) = \frac{y_{uR}y_{uL}^*}{M_U}}$$

Effective theory: a preliminary study



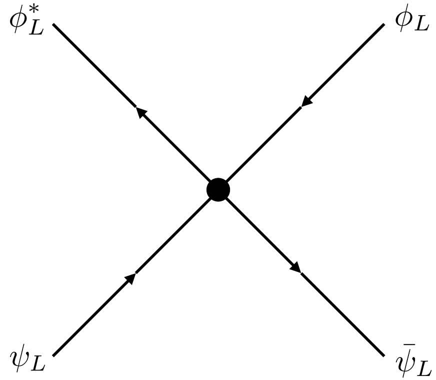
(4a)

$\psi_L \tilde{\phi}_L^* \to \bar{\psi}_L \tilde{\phi}_L$



(4b)

$$\mathcal{L}_4 = -y_{uL}\bar{\psi}_L\tilde{\phi}_L U - y_{uL}^*\bar{U}\tilde{\phi}_L^\dagger\psi_L$$



$$\mathcal{L}_{\text{eff}}^{(6)} = C_U^{(6)}(\mu)\left(\bar{\psi}_L(x)\tilde{\phi}_L(x)\right) i\not{D}_U \left(\tilde{\phi}_L^\dagger(x)\psi_L(x)\right)$$

$$i\not{D}_U = i\not{\partial}_\mu - \frac{2}{3}g_1\not{B}_1$$

$$\mathcal{A}_{4a} + \mathcal{A}_{4b} = \frac{y_{uL}y_{uL}^*}{M_U^2}\left(\bar{\psi}_L(x)\tilde{\phi}_L(x)\right) i\not{D}_U \left(\tilde{\phi}_L^\dagger(x)\psi_L(x)\right)$$

$$C_U^{(6)}(M_U) = \frac{y_{uL}y_{uL}^*}{M_U^2}$$

Effective theory: a preliminary study

Summary:

$$\mathcal{L}_{\text{eff}}^{(5)} = \frac{y_{dR} y_{dL}^*}{M_D} \left(\bar{\psi}_R(x)\phi_R(x)\right) \left(\phi_L^\dagger(x)\psi_L(x)\right) + h.c.$$
$$+ \frac{y_{uR} y_{uL}^*}{M_U} \left(\bar{\psi}_R(x)\tilde{\phi}_R(x)\right) \left(\tilde{\phi}_L^\dagger(x)\psi_L(x)\right) + h.c.$$

$$\mathcal{L}_{\text{eff}}^{(6)} = \frac{y_{dL} y_{dL}^*}{M_D^2} \left(\bar{\psi}_L(x)\phi_L(x)\right) i\not{D}_D \left(\phi_L^\dagger(x)\psi_L(x)\right) + (L \to R)$$
$$+ \frac{y_{uL} y_{uL}^*}{M_U^2} \left(\bar{\psi}_L(x)\tilde{\phi}_L(x)\right) i\not{D}_U \left(\tilde{\phi}_L^\dagger(x)\psi_L(x)\right) + (L \to R)$$