Accessing Linearly polarized gluon TMD in back to back J/Ψ and jet

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Plan of Talk

- Gluon TMDs
- Kinematics
- Azimuthal Asymmetry
- TMD parameterization
- Results and Discussion

Gluon Correlator

$$\Phi^{\mu\nu}(x, q_T) = \int \frac{d\xi^- d^2\xi_T}{M_p(2\pi)^3} e^{iq\cdot\xi} \left\langle P \middle| Tr[F^{+\mu}(0) U^{[C]} F^{+\nu}(\xi) U^{[C]}] \middle| P \right\rangle \Big|_{\xi^+=0}$$

Field Strengths **Gauge Links**

		gluon pol.		
nucleon pol.		U	L	linear
	U	f_1^g		$h_1^{\perp g}$
	L		g_{1L}^g	$h_{1L}^{\perp g}$
	T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

D. Boer et al arXiv 1507.05267

- A TMD describes 3D structure of proton
- A Gluon - gluon correlator ⇒ 2-point correlation function with Gauge links
- Gauge links are path ordered exponential connecting the field strength tensors along a definite path that depends on the actual partonic sub-process. Possible configurations are ++ or - - and + - or - +.
- In the literature related to small-x physics, these are known as Weizsacker-Williams (WW) and Dipole distributions respectively.

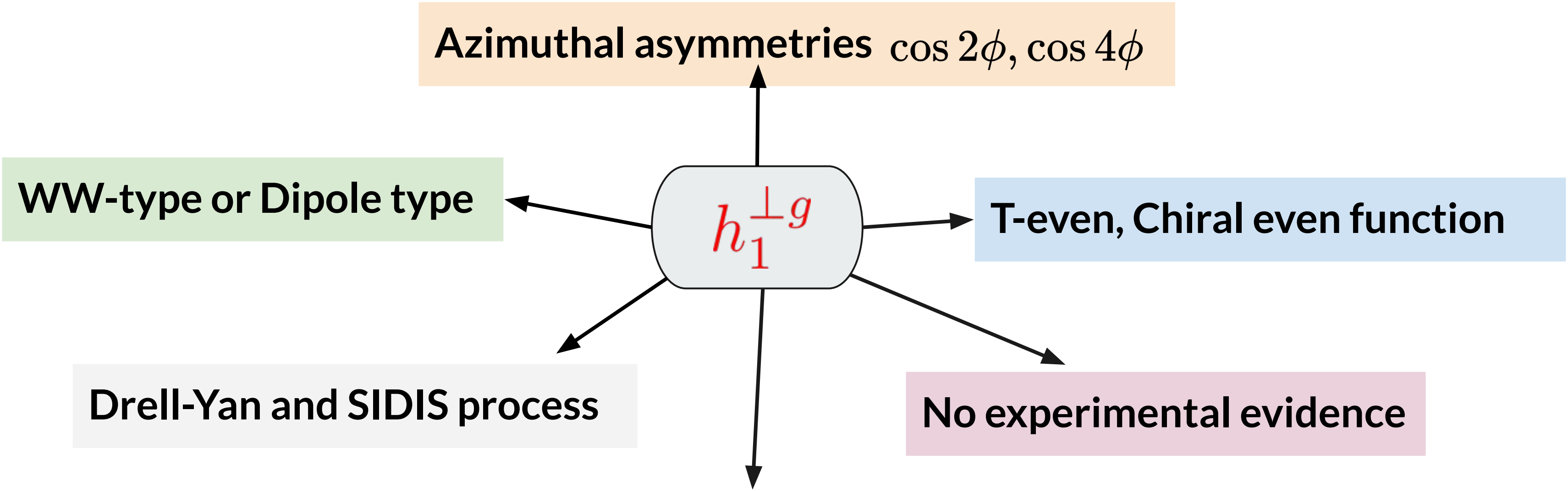
P. J. Mulders and J. Rodrigues PRD 63 094021(2001)

Linearly polarized gluon TMD

$$\Phi_g^{\nu\nu'}(x,\mathbf{p}_T^2) = -\frac{1}{2x}\left\{g_\perp^{\nu\nu'} f_1^g(x,\mathbf{p}_T^2) - \left(\frac{p_T^\nu p_T^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'}\frac{\mathbf{p}_T^2}{2M_p^2}\right) h_1^{\perp g}(x,\mathbf{p}_T^2)\right\}$$

Unpolarized Gluon TMD

Linearly polarized Gluon TMD



$$\frac{\mathbf{k}_\perp^2}{2M_h^2}|h_1^{\perp g}(x,\mathbf{k}_\perp^2)| \leq f_1^g(x,\mathbf{k}_\perp^2)$$

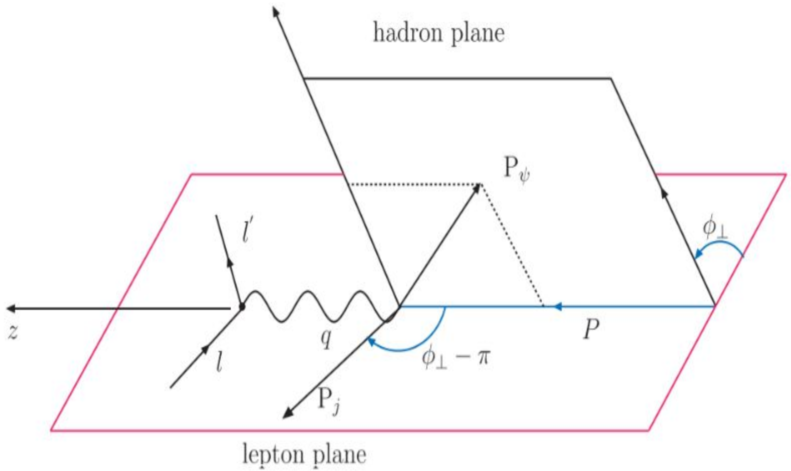
D. Boer et al. PRL 106 132001 (2011)

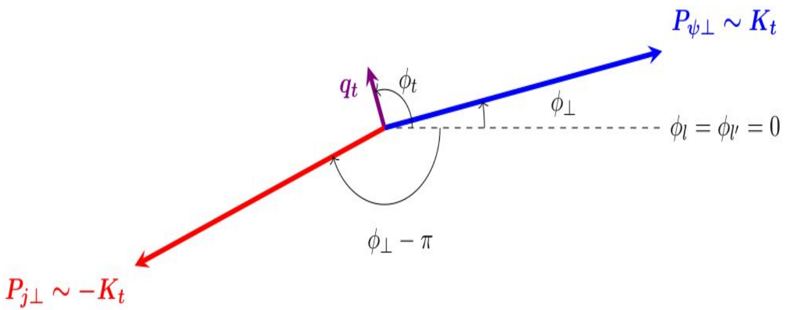
Mulders and Rodrigues, PRD 63, 094021 (2001)

$$e^-(l) + p(\mathrm{P}) \to e^-(l') + J/\psi(\mathrm{P}_\psi) + Jet(\mathrm{P}_j) + X$$

$$\gamma^* + g \to c\bar{c}(^{2S+1}L_J^{(1,8)}) + g$$

D'Alesio, *et al* (2019)





$$Q^2 = -q^2, \quad s = (P+l)^2, \quad x_B = \frac{Q^2}{2P{\cdot}q},$$

$$y = \frac{P{\cdot}q}{P{\cdot}l}, \qquad z = \frac{P{\cdot}\mathrm{P}_\psi}{P{\cdot}q}.$$

$$\mathbf{q}_t \equiv \mathbf{P}_{\psi\perp} + \mathbf{P}_{j\perp}, \quad \mathbf{K}_t \equiv \frac{\mathbf{P}_{\psi\perp} - \mathbf{P}_{j\perp}}{2}$$

$$\Rightarrow |\mathbf{q}_t| \ll |\mathbf{K}_t| \simeq M_\psi$$

TMD Factorization and NRQCD Factorization

$$d\sigma = \frac{1}{2s}\frac{d^3 l'}{(2\pi)^3 2E_{l'}}\frac{d^3 \mathrm{P}_\psi}{2E_\psi (2\pi)^3}\frac{d^3 \mathrm{P}_j}{2E_j (2\pi)^3}\int dx\, d^2\mathbf{p}_T\, (2\pi)^4 \delta^4(q + p_g - \mathrm{P}_j - \mathrm{P}_\psi)$$

$$\frac{1}{Q^4} L^{\mu\mu'}(l,q)\; \Phi_g^{\nu\nu'}(x, \mathbf{p}_T^2)\; \mathcal{M}_{\mu\nu}^{g\gamma^* \to J/\psi\; g}\, \mathcal{M}_{\mu'\nu'}^{*g\gamma^* \to J/\psi\; g}.$$

Leptonic Tensor — Gluon Correlator — Partonic matrix elements

- ★ $k^2 \ll M_c^2 \longrightarrow$ Non-relativistic approx to QCD
- ★ matrix element in NRQCD

Perturbative part — Non-perturbative part (LDMEs)

$$\mathcal{M}^{ab\to J/\psi} = \sum_n \mathcal{M}[ab \to c\bar{c}\left({}^{2S+1}L_J^{(1,8)}\right)] \langle 0|\mathcal{O}^{J/\psi}\left({}^{2S+1}L_J^{(1,8)}\right)|0\rangle$$

Bodwin, Braaten, Lepege (1994)

Radial Angular momentum wave function

$$\mathcal{M}\left(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}]\, g\right) = \sum_{L_z S_z} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Psi_{LL_z}(\mathbf{k}) \langle LL_z; SS_z | JJ_z \rangle \times Tr[O(q, p_g, \mathrm{P}_\psi, k) \mathcal{P}_{SS_z}(\mathrm{P}_\psi, k)]$$

Feynman Diagram contribution **Spin projection operator for J/Ψ**

$$\mathcal{P}_{SS_z}(\mathrm{P}_\psi, k) = \sum_{s_1 s_2} \left\langle \frac{1}{2} s_1; \frac{1}{2} s_2 \middle| SS_z \right\rangle v\left(\frac{\mathrm{P}_\psi}{2} - k, s_1\right) \bar{u}\left(\frac{\mathrm{P}_\psi}{2} + k, s_2\right)$$
$$= \frac{1}{4M_\psi^{3/2}} (-\not{\mathrm{P}}_\psi + 2\not{k} + M_\psi) \Pi_{SS_z} (\not{\mathrm{P}}_\psi + 2\not{k} + M_\psi) + \mathcal{O}(k^2)$$

$k^2 \ll M_c^2$ limit, Matrix element ⇛ Taylor series$|_{k=0}$

Non-perturbative part (LDMEs)

D. Boer and C. Pisano (2012)

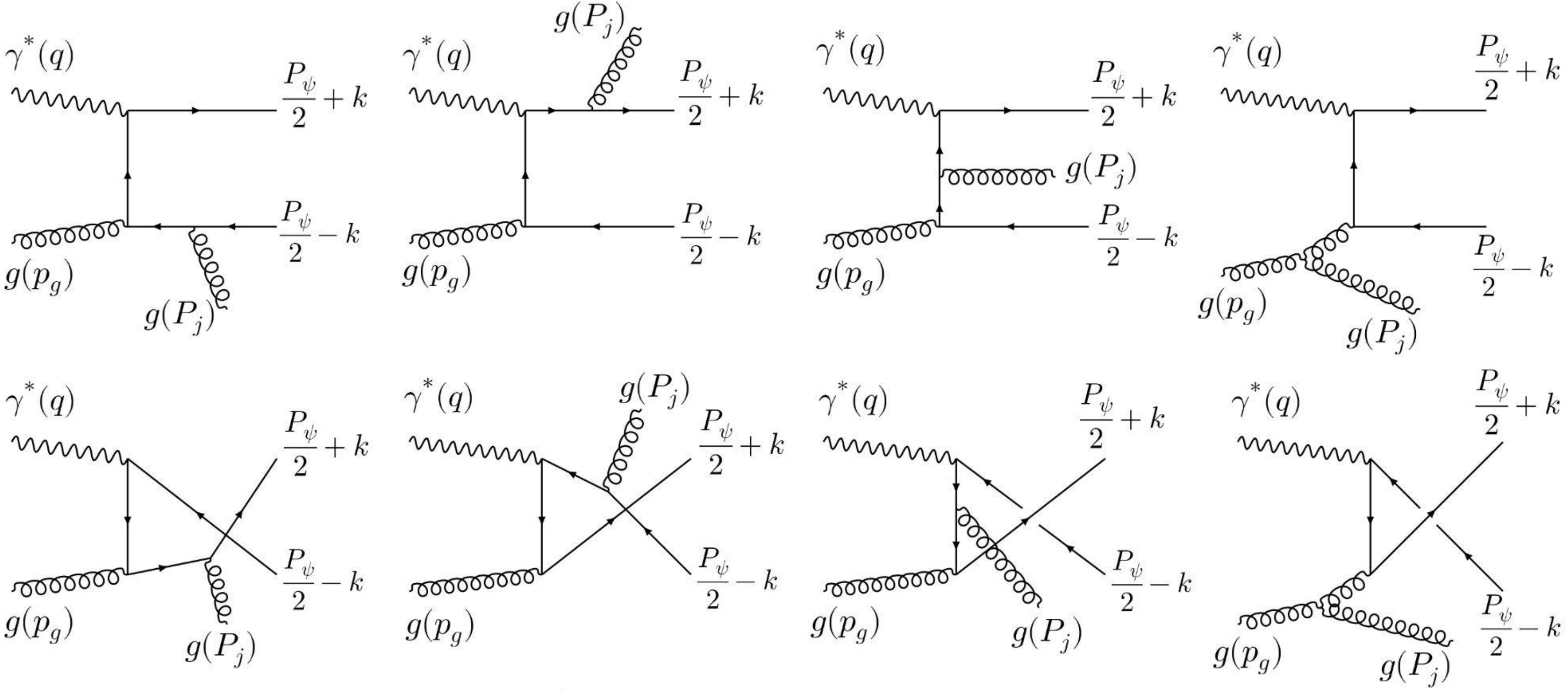
$$\mathcal{M}[^{2S+1}P_J^{(8)}](\mathrm{P}_\psi, k) = -i\sqrt{\frac{3}{4\pi}} R_1'(0) \sum_{L_z S_z} \varepsilon^\alpha_{L_z}(\mathrm{P}_\psi) \langle LL_z; SS_z | JJ_z \rangle \times \frac{\partial}{\partial k^\alpha} Tr[O(q, p_g, \mathrm{P}_\psi, k) \mathcal{P}_{SS_z}(\mathrm{P}_\psi, k)]\Big|_{k=0}$$

$$\mathcal{M}[^{2S+1}S_J^{(1,8)}](\mathrm{P}_\psi, k) = \frac{1}{\sqrt{4\pi}} R_0(0) Tr[O(q, p_g, \mathrm{P}_\psi, k) \mathcal{P}_{SS_z}(\mathrm{P}_\psi, k)]\Big|_{k=0}$$

P Wave Scattering amplitude

S Wave Scattering amplitude

Feynman Diagrams



Azimuthal Asymmetry

$$\frac{d\sigma}{dz\, dy\, dx_B\, d^2\mathbf{q}_t\, d^2\mathbf{K}_t} = \frac{1}{(2\pi)^4} \frac{1}{16sz(1-z)Q^4} \Big\{ \big(\mathbb{A}_0 + \mathbb{A}_1 \cos\phi_\perp + \mathbb{A}_2 \cos 2\phi_\perp\big) f_1^g(x, \mathbf{q}_t^2) + \frac{\mathbf{q}_t^2}{M_P^2} h_1^{\perp g}(x, \mathbf{q}_t^2) \big(\mathbb{B}_0 \cos 2\phi_t + \mathbb{B}_1 \cos(2\phi_t - \phi_\perp) + \mathbb{B}_2 \cos 2(\phi_t - \phi_\perp) + \mathbb{B}_3 \cos(2\phi_t - 3\phi_\perp) + \mathbb{B}_4 \cos(2\phi_t - 4\phi_\perp)\big) \Big\}.$$

D'Alesio, *et al* (2019)

$$\langle \cos 2\phi_t \rangle \equiv A^{\cos 2\phi_t} = 2 \frac{\int d\phi_t d\phi_\perp \cos 2\phi_t d\sigma(\phi_t, \phi_\perp)}{\int d\phi_t d\phi_\perp d\sigma(\phi_t, \phi_\perp)}$$

$$\frac{q_t^2}{2M_p^2} |h_1^{\perp g}(x, q_t^2)| = f_1^g(x, q_t^2)$$

$$\langle \cos 2\phi_t \rangle \equiv A^{\cos 2\phi_t} = \frac{\int q_t\, dq_t\, \frac{q_t^2}{M_p^2}\, \mathbb{B}_0\, h_1^{\perp g}(x, \mathbf{q}_t^2)}{\int q_t\, dq_t\, \mathbb{A}_0\, f_1^g(x, \mathbf{q}_t^2)}$$

$$A^{\cos 2\phi_t} \rightarrow U = \frac{2 * |\mathbb{B}_0|}{\mathbb{A}_0}$$

Gaussian Parameterization

- Drell-Yan and SIDIS ⇒ transverse momentum spectra → roughly Gaussian in nature. Stefano Melis,et al. (2014)

TMDs

$$f_1^g(x, \mathbf{q}_t^2) = f_1^g(x, Q) \frac{1}{\pi \langle \mathrm{q}_t^2 \rangle} e^{-\mathrm{q}_t^2 / \langle \mathrm{q}_t^2 \rangle}$$

$$h_1^{\perp g}(x, \mathrm{q}_t^2) = \frac{M_p^2 f_1^g(x, Q)}{\pi \langle \mathrm{q}_t^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \frac{\mathrm{q}_t^2}{r \langle \mathrm{q}_t^2 \rangle}}$$

$$\frac{\mathrm{q}_t^2}{2M_p^2} |h_1^{\perp g}(x, \mathrm{q}_t^2)| \leq f_1^g(x, \mathbf{q}_t^2).$$

Transverse momentum dependent part

$r (0 < r < 1)$ and $\langle \mathrm{q}_t^2 \rangle$ are parameters

$r = 1/3$ $\langle \mathrm{q}_t^2 \rangle = 0.25 \text{ GeV}^2$

D. Boer and C. Pisano (2012)

TMD Evolution

$$\hat{f}(x, \mathbf{b}_t^2; Q_f^2) = \frac{1}{2\pi} \int d^2\mathbf{q}_t \; e^{i\mathbf{q}_t \cdot \mathbf{b}_t} \; f(x, \mathbf{q}_t^2, Q_f^2)$$

$$\hat{f}(x, \mathbf{b}_t^2, Q_f^2) = \frac{1}{2\pi} \sum_{p=q,\bar{q},g} (C_{g/p} \otimes f_1^p)(x, Q_i^2) e^{-\frac{1}{2} S_A(\mathbf{b}_t^2, Q_f^2, Q_i^2)} e^{-S_{np}(\mathbf{b}_t^2, Q_f^2)}$$

$\mathrm{b}_t \ll \frac{1}{\Lambda_{\mathrm{QCD}}}$

$$C_{g/p}(x, Q_i) = \delta_{gp}\delta(1-x) + \sum_{k=1}^{\infty} \sum_{p=g,q,\bar{q}} C_{g/p}^k(x) \left(\frac{\alpha_s(Q_i)}{\pi} \right)^k$$

$$e^{-\frac{1}{2} S_A(\mathrm{b}_t^2, Q_f^2, Q_i^2)} = 1 - \frac{S_A(\mathrm{b}_t^2, Q_f^2, Q_i^2)}{2}$$

Aybat and Rogers (2011)

$$S_A(\mathrm{b}_t^2, Q_f^2, Q_i^2) = \frac{C_A}{\pi} \alpha_s \left(\frac{1}{2} \log^2 \frac{Q_f^2}{Q_i^2} - \frac{11 - 2n_f/C_A}{6} \log \frac{Q_f^2}{Q_i^2} \right)$$

$$f_1^g(x, Q_i^2) = f_1^g(x, Q_f^2) - \frac{\alpha_s}{2\pi} (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, Q_f^2) \log \frac{Q_f^2}{Q_i^2} + \mathcal{O}(\alpha_s^2).$$

TMD Evolution

Non perturbative region

functional form Scarpa, et al (2020)

bt range (0.0–1.5 GeV^{-1}) (C.P. Yuan, et al (2003), Scarpa, et al (2020))

$$S_{np} = \frac{A}{2}\log\left(\frac{Q_f}{Q_{np}}\right)\mathrm{b}_c^2,$$

$$Q_{np} = 1\ \mathrm{GeV}$$

$$\mathrm{b}_c = \sqrt{\mathrm{b}_t^2 + \left(\frac{2e^{-\gamma_E}}{Q_f}\right)^2}$$

Aybat and Rogers (2011)

$$f_1^g(x, \mathrm{q}_t^2) = \frac{1}{2\pi}\int_0^\infty \mathrm{b}_t d\mathrm{b}_t J_0(\mathrm{b}_t \mathrm{q}_t)\Bigg\{ f_1^g(x, Q_f^2)$$
$$- \frac{\alpha_s}{2\pi}\Bigg[\left(\frac{C_A}{2}\log^2\frac{Q_f^2}{Q_i^2} - \frac{11C_A - 2n_f}{6}\log\frac{Q_f^2}{Q_i^2}\right) f_1^g(x, Q_f^2)$$
$$+ (P_{gg}\otimes f_1^g + P_{gi}\otimes f_1^i)(x, Q_f^2)\log\frac{Q_f^2}{Q_i^2} - 2f_1^g(x, Q_f^2)\Bigg]\Bigg\} \times e^{-S_{np}(\mathrm{b}_t^2)}.$$

$$\frac{\mathrm{q}_t^2}{M_p^2} h_1^{\perp g(2)}(x, \mathrm{q}_t^2) = \frac{\alpha_s}{\pi^2}\int_0^\infty d\mathrm{b}_t \mathrm{b}_t\ J_2(\mathrm{q}_t \mathrm{b}_t)\left[C_A \int_x^1 \frac{d\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_1^g(\hat{x}, Q_f^2)\right.$$
$$\left. + C_F \sum_{p=q,\bar{q}} \int_x^1 \frac{d\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_1^p(\hat{x}, Q_f^2)\right] \times e^{-S_{np}(\mathrm{b}_t{}^2)}.$$

Spectator Model

- In this model proton ⇒ g+X
- spectator particle ⇒ On-Shell ⇒ M_x ⇒ continuous values
- P-g-X coupling is given by vertex ⇒ 2 form factors
- Model parameters fixed by integrating the transverse momentum and fitting with gluon pdfs (NNPDF3.1)

$$\hat{f}_1^g(x, \mathbf{q}_t^2; M_X) = -\frac{1}{2} g^{ij} \left[\Phi^{ij}(x, \mathbf{q}_t, S) + \Phi^{ij}(x, \mathbf{q}_t, -S)\right]$$
$$= \Big[\left(2Mxg_1 - x(M+M_X)g_2\right)^2 \left[(M_X - M(1-x))^2 + \mathbf{q}_t^2\right]$$
$$+ 2\mathbf{q}_t^2\left(\mathbf{q}_t^2 + xM_X^2\right) g_2^2 + 2\mathbf{q}_t^2 M^2 (1-x)\left(4g_1^2 - xg_2^2\right)\Big]$$
$$\times \left[(2\pi)^3\, 4xM^2 \left(L_X^2(0) + \mathbf{q}_t^2\right)^2\right]^{-1},$$

$$\hat{h}_1^{\perp g}(x, \mathbf{q}_t^2; M_X) = \frac{M^2}{\varepsilon_t^{ij}\delta^{jm}(p_t^j p_t^m + g^{jm}\mathbf{q}_t^2)}\, \varepsilon_t^{ln}\delta^{nr}\left[\Phi^{nr}(x, \mathbf{q}_t, S) + \Phi^{nr}(x, \mathbf{q}_t, -S)\right]$$
$$= \left[4M^2(1-x)\, g_1^2 + \left(L_X^2(0) + \mathbf{q}_t^2\right) g_2^2\right] \times \left[(2\pi)^3\, x \left(L_X^2(0) + \mathbf{q}_t^2\right)^2\right]^{-1}$$

$$F^g(x, q_t^2) = \int_M^\infty dM_X\, \rho_X(M_X)\, \hat{F}^g(x, q_t^2; M_X)$$

$$\rho_X(M_X) = \mu^{2a}\left[\frac{A}{B+\mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X-D)^2}{\sigma^2}}\right]$$

A. Bachetta, *et al* (2020)

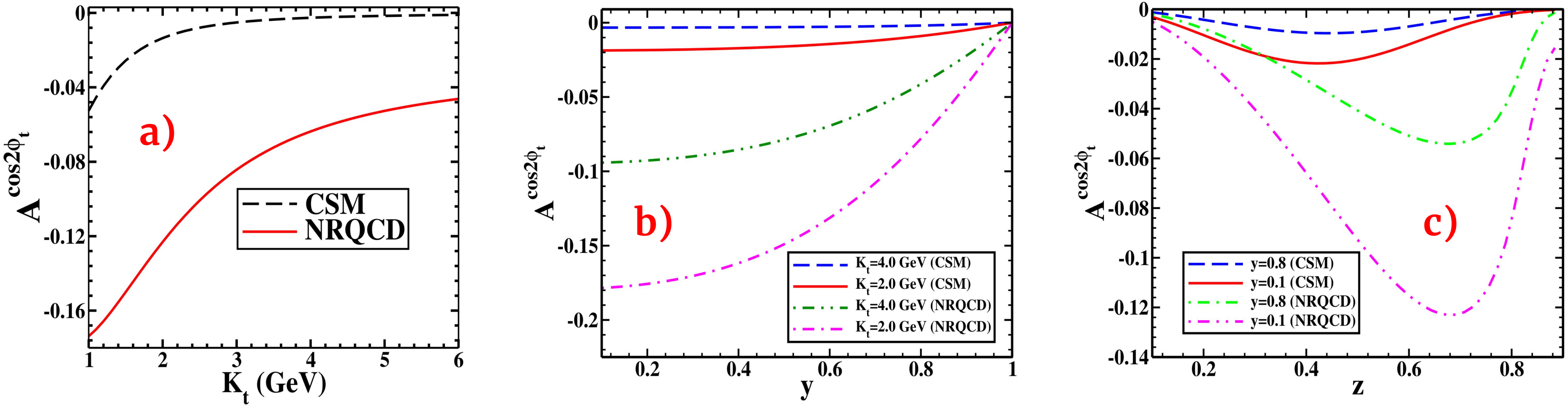
Spectral function

Model dependent form factors

Results

Gaussian Parameterization

√s = 140 GeV



y∈(0,1)

z=0.7

K_t=3.0 GeV

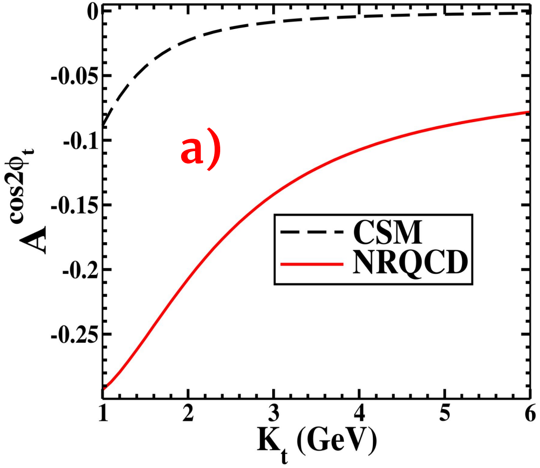
CMSWZ set of LDMEs

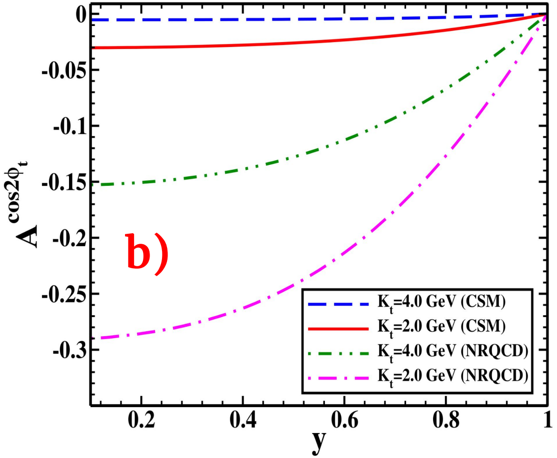
$$Q^2 = M_\psi^2 + K_t^2$$

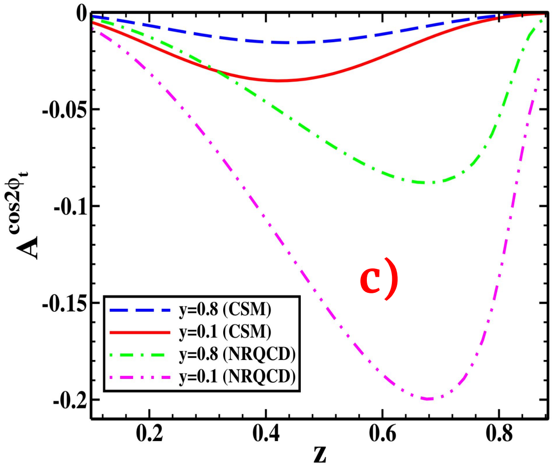
Results

Spectator Model

$\sqrt{s} = 140$ GeV







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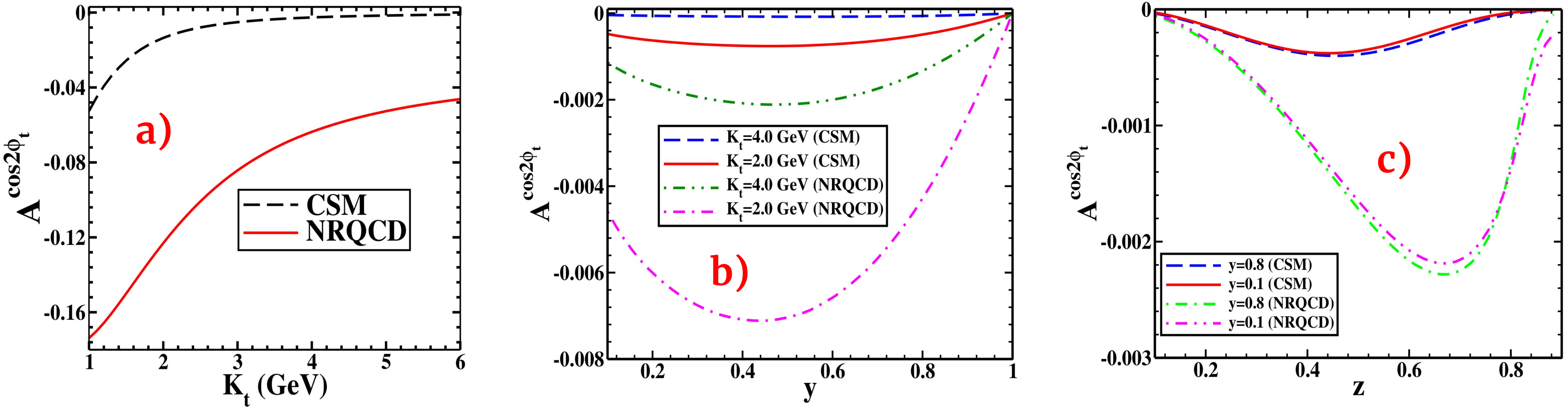
CMSWZ set of LDMEs

$$Q^2 = M_\psi^2 + K_t^2$$

Results

TMD Evolution

$\sqrt{s}$ = 140 GeV



y∈(0,1)

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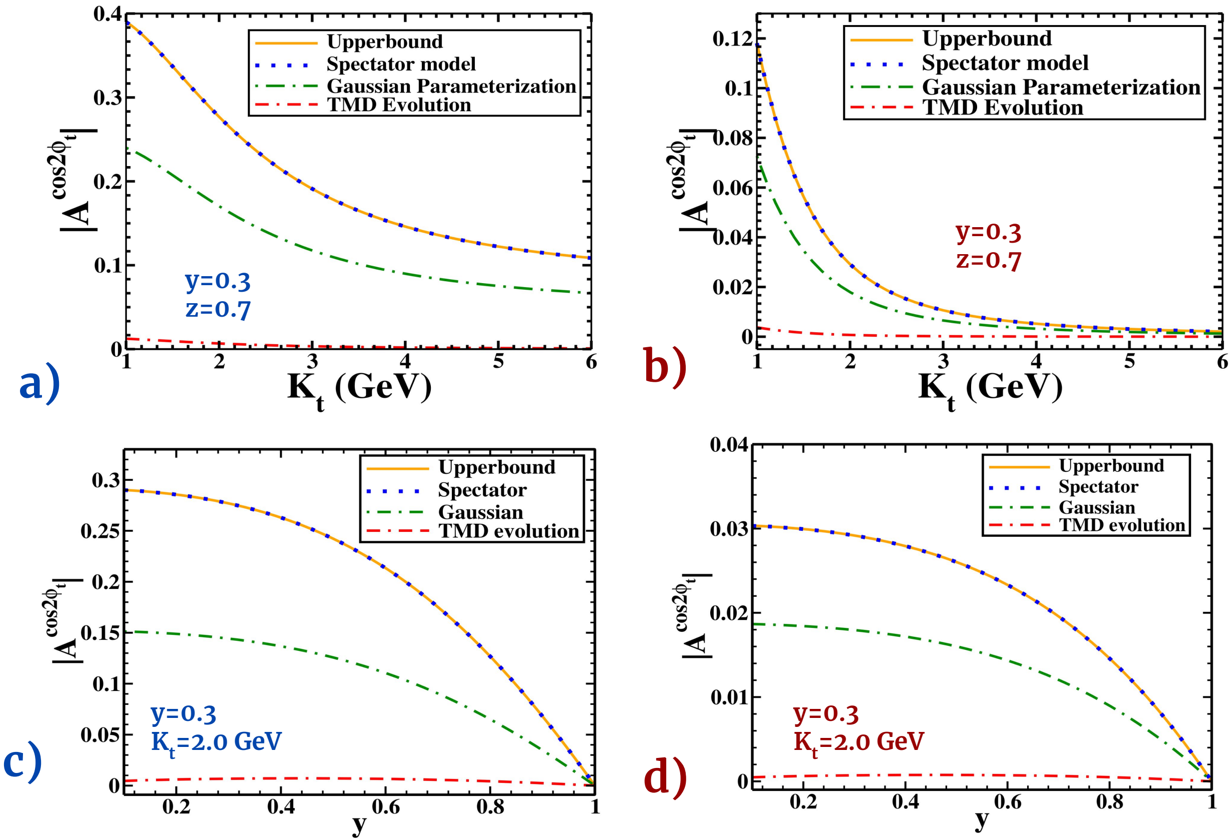
CMSWZ set of LDMEs

$$Q^2 = M_\psi^2 + K_t^2$$

Results

Upper bound and absolute values of asymmetries

NRQCD CSM



CMSWZ set of LDMEs $\sqrt{s}$ = 140 GeV $Q^2 = M_\psi^2 + K_t^2$

Results

$$\hat{f}_1^g(x, \mathbf{q}_t^2; M_X) = -\frac{1}{2}\, g^{ij}\ \left[\Phi^{ij}(x, \mathbf{q}_t, S) + \Phi^{ij}(x, \mathbf{q}_t, -S)\right]$$
$$= \Big[\left(2Mxg_1 - x(M+M_X)g_2\right)^2 \left[(M_X - M(1-x))^2 + \mathbf{q}_t^2\right]$$
$$+2\mathbf{q}_t^2\left(\mathbf{q}_t^2 + xM_X^2\right) g_2^2 + 2\mathbf{q}_t^2 M^2 (1-x)\left(4g_1^2 - xg_2^2\right)\Big]$$
$$\times\left[(2\pi)^3\, 4xM^2\left(L_X^2(0) + \mathbf{q}_t^2\right)^2\right]^{-1},$$

$$\hat{h}_1^{\perp g}(x, \mathbf{q}_t^2; M_X) = \frac{M^2}{\varepsilon_t^{ij}\delta^{jm}(p_t^j p_t^m + g^{jm}\mathbf{q}_t^2)}\,\varepsilon_t^{ln}\delta^{nr}\ \left[\Phi^{nr}(x, \mathbf{q}_t, S) + \Phi^{nr}(x, \mathbf{q}_t, -S)\right]$$
$$= \left[4M^2(1-x)\,g_1^2 + \left(L_X^2(0) + \mathbf{q}_t^2\right) g_2^2\right] \times \left[(2\pi)^3\, x\left(L_X^2(0) + \mathbf{q}_t^2\right)^2\right]^{-1}.$$

$$F^g(x, \mathbf{q}_t^2) = \int_M^\infty dM_X\, \rho_X(M_X)\, \hat{F}^g(x, \mathbf{q}_t^2; M_X)$$

$$\frac{\mathrm{q}_t^2}{2M_p^2}\left|h_1^{\perp g}(x, \mathrm{q}_t^2)\right| = f_1^g(x, \mathrm{q}_t^2)$$

$$\langle\cos 2\phi_t\rangle \equiv A^{\cos 2\phi_t} = \frac{\int \mathrm{q}_t\ \mathrm{dq}_t\ \frac{\mathbf{q}_t^2}{M_p^2}\ \mathbb{B}_0\ h_1^{\perp g}(x, \mathbf{q}_t^2)}{\int \mathrm{q}_t\ \mathrm{dq}_t\ \mathbb{A}_0\ f_1^g(x, \mathbf{q}_t^2)}$$

$$A^{\cos 2\phi_t} \rightarrow U = \frac{2*|\mathbb{B}_0|}{\mathbb{A}_0}$$

Results

$$f_1^g(x,\mathbf{q}_t^2) = \frac{1}{2\pi}\int_0^\infty \mathrm{b}_t d\mathrm{b}_t J_0(\mathrm{b}_t\mathrm{q}_t)\Bigg\{ f_1^g(x,Q_f^2) - \frac{\alpha_s}{2\pi}\Bigg[\bigg(\frac{C_A}{2}\log^2\frac{Q_f^2}{Q_i^2} - \frac{11C_A - 2n_f}{6}\log\frac{Q_f^2}{Q_i^2}\bigg) f_1^g(x,Q_f^2) + (P_{gg}\otimes f_1^g + P_{gi}\otimes f_1^i)(x,Q_f^2)\,\log\frac{Q_f^2}{Q_i^2} - 2f_1^g(x,Q_f^2)\Bigg]\Bigg\} \times e^{-S_{np}(\mathbf{b}_t^2)}$$

α_s^0

$$\frac{\mathbf{q}_t^2}{M_p^2} h_1^{\perp g(2)}(x,\mathbf{q}_t^2) = \frac{\alpha_s}{\pi^2}\int_0^\infty d\mathrm{b}_t\mathrm{b}_t\; J_2(\mathrm{q}_t\mathrm{b}_t)\left[C_A\int_x^1\frac{d\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x}-1\right) f_1^g(\hat{x},Q_f^2) + C_F\sum_{p=q,\bar{q}}\int_x^1\frac{d\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x}-1\right) f_1^p(\hat{x},Q_f^2)\right]\times e^{-S_{np}(\mathbf{b_t}^2)}$$

α_s^1

$$\langle\cos 2\phi_t\rangle \equiv A^{\cos 2\phi_t} = \frac{\int \mathrm{q}_t\; d\mathrm{q}_t\; \frac{\mathbf{q}_t^2}{M_p^2}\,\mathbb{B}_0\; h_1^{\perp g}(x,\mathbf{q}_t^2)}{\int \mathrm{q}_t\; d\mathrm{q}_t\; \mathbb{A}_0\; f_1^g(x,\mathbf{q}_t^2)}$$

Conclusion

- NRQCD → J/Ψ and Jet production rate
- parameterized gluon TMDs → Gaussian, Spectator model and implemented TMD Evolution
- Asymmetry: TMD evolution < Gaussian < Spectator ≤ Upper bound.
- These kinematical regions will be accessible at EIC.
- These results will help us to extract the Linearly polarized gluon TMD

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Thank You