Lepton flavor model and analysis with 3HDM

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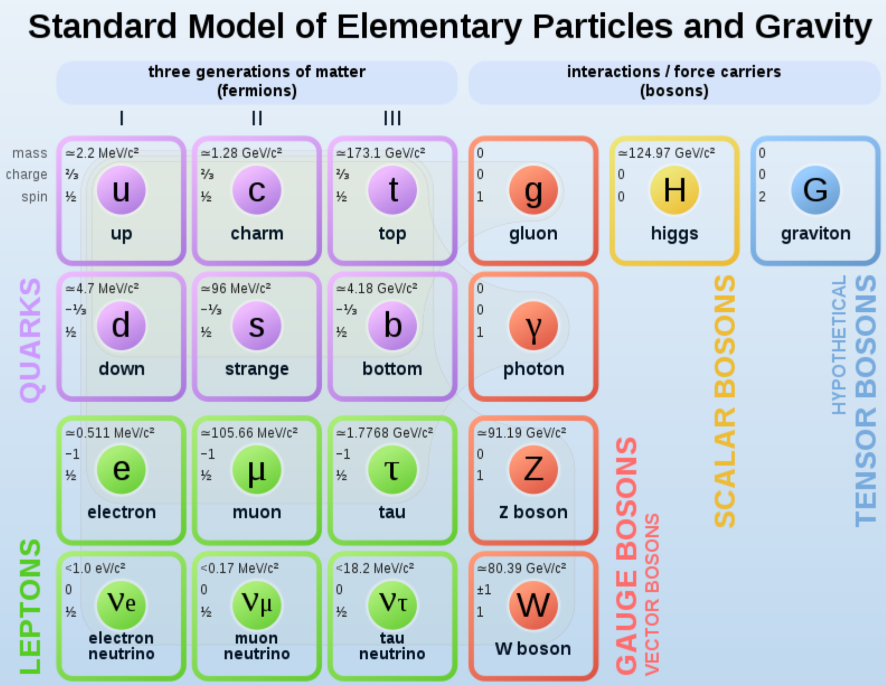
1. Motivation

Quarks and leptons (SM particles) have the generation structure.

Mass differences
(Quantum numbers are same.)



https://www.wikiwand.com/

Especially, leptons have a larger flavor mixing than quarks.

PMNS matrix (notation of PDG) https://pdg.lbl.gov

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

mixing angle(leptons) $\theta_{12} \approx 33°, \theta_{23} \approx 42°, \theta_{13} \approx 8.5°$ (Normal Order)

mixing angle(quarks) $\theta_{12} \approx 0.22°, \theta_{23} \approx 0.041°, \theta_{13} \approx 0.0037°$

NuFIT 5.2.

PDG 2022

The flavor structure can't be explained in the SM.
The Yukawa coupling is free parameter.
→Then we want to extend the SM and discuss flavor structure.

Motivation

We suppose a symmetry among generations. → flavor symmetry

In previous work, Altareli and Feruglio made a flavor model(AF model).

G. Altarelli and F. Feruglio, Nucl. Phys. B741 (2006), 215-235.

AF model

- A_4 symmetry (flavor symmetry) (A_4 symmetry is one of the non-Abelian discrete symmetry.)
- the new particle called 'flavon' (flavon is assigned to gauge singlet and A_4 triplet.)
- SUSY to decide vacuum alignment → high energy theory

In our study, we make new flavor model by using three Higgs doublets model(3HDM) instead of flavon.
→ low energy theory

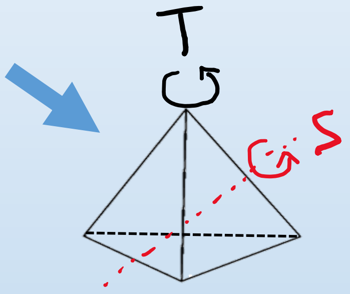
We do the numerical calculations for mixing angles, CP phases and effective mass of the neutrinoless double beta($0\nu\beta\beta$) decay experiment.

2. A_4symmetry

A_4 symmetry：Fourth order alternating group,

Smallest group containing triplet

Algebraic relation：$S^2 = (ST)^3 = T^3 = 1$ (S, T：generators)



regular tetrahedron

Irreducible Representation：

$$1 : \quad S = 1, \quad T = 1$$

$$1' : \quad S = 1, \quad T = e^{\frac{i4\pi}{3}} = \omega^2$$

$$1'' : \quad S = 1, \quad T = e^{\frac{i2\pi}{3}} = \omega$$

$$3: \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

Multiplication rule： $1' \otimes 1' = 1''$, $1'' \otimes 1'' = 1'$, $1' \otimes 1'' = 1$, $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_S \oplus 3_A$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}_3 \otimes \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}_3 = (\alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2)_1 \oplus (\alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1)_{1'} \oplus (\alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1)_{1''}$$

$$\oplus \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix}_{3_S} \oplus \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix}_{3_A}$$

3. 3HDM

Extend SM Higgs doublet to 3 (12 real scalar fields)

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \phi_3 = \begin{pmatrix} \phi_3^+ \\ \phi_3^0 \end{pmatrix}$$

Higgs Potential in 3HDM under $SU(2)_L \otimes U(1)_Y$

$$V = -\sum_{i,j=1}^{3} m_{ij}^2 (\phi_i^\dagger \phi_j) + \frac{1}{2} \sum_{i,j,k,l=1}^{3} \lambda_{ijkl} (\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_l)$$

Potential minimum conditions

$$\left(\frac{\partial V}{\partial \phi_1}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

$$\left(\frac{\partial V}{\partial \phi_2}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

$$\left(\frac{\partial V}{\partial \phi_3}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0$$

⬇ Spontaneous symmetry breaking

3 degrees of freedom are eaten by W and Z bosons.

➡ ϕ is represented by the expansion of 9 (=12−3) real scalar fields

$$\phi_i = \begin{pmatrix} \rho_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\chi_i) \end{pmatrix}, i = 1,2,3$$

mass eigenstates ➡

(i) Three CP-even scalar fields
(ii) Two CP-odd scalar fields
(iii) Four charged scalar fields

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Higgs potential analysis

3HDM+A_4symmetry

Consider ϕ as A_4 triplet：$\phi=(\phi_1,\phi_2,\phi_3)$，$\left(\phi^\dagger=(\phi_1^\dagger,\phi_3^\dagger,\phi_2^\dagger)\right)$

Calculate Higgs potential $\quad V=-\mu^2\phi^\dagger\phi+\lambda\left(\phi^\dagger\phi\right)^2$

$$T^\dagger=\begin{pmatrix}1&0&0\\0&\omega^2&0\\0&0&\omega\end{pmatrix}$$

Multiplication rule of A_4

$$\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix}_3\otimes\begin{pmatrix}\beta_1\\\beta_2\\\beta_3\end{pmatrix}_3$$
$$=(\alpha_1\beta_1+\alpha_2\beta_3+\alpha_3\beta_2)_1$$
$$\oplus(\alpha_3\beta_3+\alpha_1\beta_2+\alpha_2\beta_1)_{1'}$$
$$\oplus(\alpha_2\beta_2+\alpha_1\beta_3+\alpha_3\beta_1)_{1''}$$
$$\oplus\begin{pmatrix}2\alpha_1\beta_1-\alpha_2\beta_3-\alpha_3\beta_2\\2\alpha_3\beta_3-\alpha_1\beta_2-\alpha_2\beta_1\\2\alpha_2\beta_2-\alpha_1\beta_3-\alpha_3\beta_1\end{pmatrix}_{3_S}$$
$$\oplus\begin{pmatrix}\alpha_2\beta_3-\alpha_3\beta_2\\\alpha_1\beta_2-\alpha_2\beta_1\\\alpha_3\beta_1-\alpha_1\beta_3\end{pmatrix}_{3_A}$$

$$\boxed{\phi^\dagger\phi}=\begin{pmatrix}\phi_1^\dagger\\\phi_3^\dagger\\\phi_2^\dagger\end{pmatrix}_3\otimes\begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix}_3=\left(\phi_1^\dagger\phi_1+\phi_3^\dagger\phi_3+\phi_2^\dagger\phi_2\right)_1=\boxed{|\phi_1|^2+|\phi_2|^2+|\phi_3|^2}$$

$$\boxed{\left(\phi^\dagger\phi\right)^2}=\begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix}_3\otimes\begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix}_3\otimes\begin{pmatrix}\phi_1^\dagger\\\phi_3^\dagger\\\phi_2^\dagger\end{pmatrix}_3\otimes\begin{pmatrix}\phi_1^\dagger\\\phi_3^\dagger\\\phi_2^\dagger\end{pmatrix}_3=(\phi_1\phi_1+2\phi_2\phi_3)_1\otimes\left(\phi_1^\dagger\phi_1^\dagger+2\phi_2^\dagger\phi_3^\dagger\right)_1+(\phi_2\phi_2+2\phi_3\phi_1)_{1''}\otimes\left(\phi_2^\dagger\phi_2^\dagger+2\phi_3^\dagger\phi_1^\dagger\right)_{1'}$$
$$+(\phi_3\phi_3+2\phi_1\phi_2)_{1'}\otimes\left(\phi_3^\dagger\phi_3^\dagger+2\phi_1^\dagger\phi_2^\dagger\right)_{1''}+\frac{2}{3}\begin{pmatrix}\phi_1\phi_1-\phi_2\phi_3\\\phi_3\phi_3-\phi_1\phi_2\\\phi_2\phi_2-\phi_3\phi_1\end{pmatrix}_3\otimes\frac{2}{3}\begin{pmatrix}\phi_1^\dagger\phi_1^\dagger-\phi_2^\dagger\phi_3^\dagger\\\phi_2^\dagger\phi_2^\dagger-\phi_3^\dagger\phi_1^\dagger\\\phi_3^\dagger\phi_3^\dagger-\phi_1^\dagger\phi_2^\dagger\end{pmatrix}_3$$
$$=\boxed{\begin{aligned}&\left|\phi_1^2+2\phi_2\phi_3\right|^2+\left|\phi_2^2+2\phi_3\phi_1\right|^2+\left|\phi_3^2+2\phi_1\phi_2\right|^2\\&+\frac{4}{9}\left[\left|\phi_1^2-\phi_2\phi_3\right|^2+\left|\phi_2^2-\phi_3\phi_1\right|^2+\left|\phi_3^2-\phi_1\phi_2\right|^2\right]\end{aligned}}$$

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Vacuum structure

Potential

$$V = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$$
$$= -\mu^2(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2) + \lambda_1\left|\phi_1^2 + 2\phi_2\phi_3\right|^2 + \lambda_2\left|\phi_2^2 + 2\phi_3\phi_1\right|^2 + \lambda_3\left|\phi_3^2 + 2\phi_1\phi_2\right|^2$$
$$+\lambda_4\left[\left|\phi_1^2 - \phi_2\phi_3\right|^2 + \left|\phi_2^2 - \phi_3\phi_1\right|^2 + \left|\phi_3^2 - \phi_1\phi_2\right|^2\right]$$

Potential minimum conditions

$$\left(\frac{\partial V}{\partial \phi_i}\right)_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle,\phi_3=\langle\phi_3\rangle} = 0, \quad i = 1,2,3$$

Local vacuum expectation values ($\lambda_1 \neq \lambda_4, 2\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \neq 0$)

$$\langle\phi_1\rangle = v_1$$

$$\langle\phi_2\rangle = -\frac{\lambda_2+\lambda_3-\lambda_4}{2\lambda_1+\lambda_2+\lambda_3+\lambda_4}v_1 \pm \frac{\sqrt{\{-2\lambda_1^2+\lambda_2^2+\lambda_3^2-(\lambda_1-3\lambda_4)(\lambda_2+\lambda_3)-3\lambda_1\lambda_4\}v_1^2+\frac{1}{2}(2\lambda_1+\lambda_2+\lambda_3+\lambda_4)\mu^2}}{2\lambda_1+\lambda_2+\lambda_3+\lambda_4}$$

$$\langle\phi_3\rangle = -\frac{\lambda_2+\lambda_3-\lambda_4}{2\lambda_1+\lambda_2+\lambda_3+\lambda_4}v_1 \pm \frac{\sqrt{\{-2\lambda_1^2+\lambda_2^2+\lambda_3^2-(\lambda_1-3\lambda_4)(\lambda_2+\lambda_3)-3\lambda_1\lambda_4\}v_1^2+\frac{1}{2}(2\lambda_1+\lambda_2+\lambda_3+\lambda_4)\mu^2}}{2\lambda_1+\lambda_2+\lambda_3+\lambda_4}$$

Rewrite VEV with v and β

$$\langle\phi\rangle = \begin{pmatrix} v\cos\beta \\ \frac{1}{\sqrt{2}}v\sin\beta \\ \frac{1}{\sqrt{2}}v\sin\beta \end{pmatrix}$$

v : Higgs VEV
β : free parameter

4. Flavor model

	$\bar{l} = (\bar{l_e}, \bar{l_\mu}, \bar{l_\tau})$	e_R	μ_R	τ_R	$\nu_R = (\nu_1, \nu_2, \nu_3)$	$\phi = (\phi_1, \phi_2, \phi_3)$
$SU(2)_L$	2	1	1	1	1	2
A_4	3	1	$1''$	$1'$	3	3

SM gauge and A_4invariant Lagrangian mass terms： $L_Y = L_l + L_D + L_M + h.c.$

(1) Mass terms of charged leptons： $L_l = y_e \bar{l}\phi e_R + y_\mu \bar{l}\phi\, \mu_R + y_\tau \bar{l}\phi\tau_R$

(2) Mass term of Dirac neutrino： $L_D = y_D \bar{l}\tilde{\phi}\nu_R$

(3) Mass term of right-handed Majorana neutrino： $L_M = M\bar{\nu}_R^c \nu_R$

Calculate mass matrices of charged leptons and left-handed Majorana neutrino

Calculation of mass matrices

Multiplication rule of A_4

$$\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix}_3 \otimes \begin{pmatrix}\beta_1\\\beta_2\\\beta_3\end{pmatrix}_3 = (\alpha_1\beta_1+\alpha_2\beta_3+\alpha_3\beta_2)_1 \oplus (\alpha_3\beta_3+\alpha_1\beta_2+\alpha_2\beta_1)_{1'} \oplus (\alpha_2\beta_2+\alpha_1\beta_3+\alpha_3\beta_1)_{1''} \oplus \begin{pmatrix}2\alpha_1\beta_1-\alpha_2\beta_3-\alpha_3\beta_2\\2\alpha_3\beta_3-\alpha_1\beta_2-\alpha_2\beta_1\\2\alpha_2\beta_2-\alpha_1\beta_3-\alpha_3\beta_1\end{pmatrix}_{3_S} \oplus \begin{pmatrix}\alpha_2\beta_3-\alpha_3\beta_2\\\alpha_1\beta_2-\alpha_2\beta_1\\\alpha_3\beta_1-\alpha_1\beta_3\end{pmatrix}_{3_A}$$

(1) Mass terms of charged leptons

$$L_l = \boxed{y_e\bar{l}\phi e_R} + y_\mu\bar{l}\phi\,\mu_R + y_\tau\bar{l}\phi\tau_R$$

$$\boxed{\begin{aligned}&\underbrace{y_e\begin{pmatrix}\bar{l}_e\\\bar{l}_\mu\\\bar{l}_\tau\end{pmatrix}_3 \otimes \begin{pmatrix}\phi_1\\\phi_2\\\phi_3\end{pmatrix}_3}_{1} \otimes (e_R)_1 = y_e\underline{(\bar{l}_e\phi_1+\bar{l}_\mu\phi_2+\bar{l}_\tau\phi_3)}e_R\\ &\Downarrow\ <\phi>=(v_1,v_2,v_3)\\ &y_e(\bar{e}_Lv_1+\bar{\mu}_Lv_2+\bar{\tau}_Lv_3)e_R\end{aligned}}$$

$$\to y_e(\bar{e}_Lv_1+\bar{\mu}_Lv_3+\bar{\tau}_Lv_2)e_R + y_\mu(\bar{\tau}_Lv_3+\bar{e}_Lv_2+\bar{\mu}_Lv_1)\mu_R + y_\tau(\bar{\mu}_Lv_2+\bar{\tau}_Lv_1+\bar{e}_Lv_3)\tau_R$$

$$\begin{aligned}= &(y_ev_1)\bar{e}_Le_R + (y_\mu v_2)\bar{e}_L\mu_R + (y_\tau v_3)\bar{e}_L\tau_R\\ &+(y_ev_3)\bar{\mu}_Le_R + (y_\mu v_1)\bar{\mu}_L\mu_R + (y_\tau v_2)\bar{\mu}_L\tau_R\\ &+(y_ev_2)\bar{\tau}_Le_R + (y_\mu v_3)\bar{\tau}_L\mu_R + (y_\tau v_1)\bar{\tau}_L\tau_R\end{aligned}$$

Mass matrix of charged leptons

$$M_l = \begin{pmatrix}y_ev_1 & y_\mu v_2 & y_\tau v_3\\ y_ev_3 & y_\mu v_1 & y_\tau v_2\\ y_ev_2 & y_\mu v_3 & y_\tau v_1\end{pmatrix}_{LR}$$

Calculation of mass matrices

(2) Mass term of Dirac neutrino

$$L_D = y_D \bar{l} \tilde{\phi} \nu_R$$

$$= \boxed{y_D \underbrace{\begin{pmatrix} \bar{l}_e \\ \bar{l}_\mu \\ \bar{l}_\tau \end{pmatrix}_3 \otimes \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_3 \\ \tilde{\phi}_2 \end{pmatrix}_3}_{3}} \otimes \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix}_3 = \boxed{\left(\frac{y_{DS}}{3} \begin{pmatrix} 2\bar{l}_1\tilde{\phi}_1 - \bar{l}_2\tilde{\phi}_2 - \bar{l}_3\tilde{\phi}_3 \\ 2\bar{l}_3\tilde{\phi}_2 - \bar{l}_1\tilde{\phi}_3 - \bar{l}_2\tilde{\phi}_1 \\ 2\bar{l}_2\tilde{\phi}_3 - \bar{l}_3\tilde{\phi}_1 - \bar{l}_1\tilde{\phi}_2 \end{pmatrix}_{3S} + \frac{y_{DA}}{2} \begin{pmatrix} \bar{l}_2\tilde{\phi}_2 - \bar{l}_3\tilde{\phi}_3 \\ \bar{l}_1\tilde{\phi}_3 - \bar{l}_2\tilde{\phi}_1 \\ \bar{l}_3\tilde{\phi}_1 - \bar{l}_1\tilde{\phi}_2 \end{pmatrix}_{3A} \right)} \otimes \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix}_3$$

$$\rightarrow \frac{y_{DS}}{3}\left[(2\bar{\nu}_e v_1 - \bar{\nu}_\mu v_2 - \bar{\nu}_\tau v_3)\nu_{R1} + (2\bar{\nu}_\tau v_2 - \bar{\nu}_e v_3 - \bar{\nu}_\mu v_1)\nu_{R3} + (2\bar{\nu}_\mu v_3 - \bar{\nu}_\tau v_1 - \bar{\nu}_e v_2)\nu_{R2}\right]$$

$$+ \frac{y_{DA}}{2}\left[(\bar{\nu}_\mu v_2 - \bar{\nu}_\tau v_3)\nu_{R1} + (\bar{\nu}_e v_3 - \bar{\nu}_\mu v_1)\nu_{R3} + (\bar{\nu}_\tau v_1 - \bar{\nu}_e v_2)\nu_{R2}\right]$$

Mass matrix of Dirac neutrino

$$M_D = y_{DS} \begin{pmatrix} \frac{2}{3}v_1 & -\frac{1}{3}v_2 & -\frac{1}{3}v_3 \\ -\frac{1}{3}v_2 & \frac{2}{3}v_3 & -\frac{1}{3}v_1 \\ -\frac{1}{3}v_3 & -\frac{1}{3}v_1 & \frac{2}{3}v_2 \end{pmatrix} + y_{DA} \begin{pmatrix} 0 & -\frac{1}{2}v_2 & \frac{1}{2}v_3 \\ \frac{1}{2}v_2 & 0 & -\frac{1}{2}v_1 \\ -\frac{1}{2}v_3 & \frac{1}{2}v_1 & 0 \end{pmatrix}$$

Multiplication rule of A_4

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}_3 \otimes \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}_3$$

$$= (\alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2)_1$$

$$\oplus (\alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1)_{1'}$$

$$\oplus (\alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1)_{1''}$$

$$\oplus \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix}_{3S}$$

$$\oplus \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix}_{3A}$$

Calculation of mass matrices

(3) Mass term of right-handed Majorana neutrino

$$L_M = \frac{1}{2} M \bar{\nu}_R^c \nu_R$$

$$= \frac{1}{2} M \underline{\begin{pmatrix} \bar{\nu}_{R1}^c \\ \bar{\nu}_{R2}^c \\ \bar{\nu}_{R3}^c \end{pmatrix}_3} \otimes \underline{\begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix}_3} = \frac{1}{2} M \left(\overline{\nu_1^c} \nu_1 + \overline{\nu_2^c} \nu_3 + \overline{\nu_3^c} \nu_2 \right)$$

Mass matrix of right-handed Majorana neutrino

$$M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}$$

Calculate mass matrix of left-handed Majorana neutrino by using type-I seesaw mechanism

Minkowski '77; Gell-Mann, Ramond, Slansky; Yanagida; Glashow; Mohapatra, Senjanovic '79

$$m_\nu = -M_D M_R^{-1} M_D^\dagger$$

$$= \begin{pmatrix} \frac{-4y_{DS}^2(2v_1^2 + v_2 v_3) + 9y_{DA}^2 v_2 v_3}{18M} & \frac{4y_{DS}(2y_{DS} - 3y_{DA})v_3^2 + (4y_{DS}^2 - 24y_{DS}y_{DA} - 9y_{DA}^2)v_1 v_2}{36M} & \frac{4y_{DS}(2y_{DS} + 3y_{DA})v_2^2 + (4y_{DS}^2 + 24y_{DS}y_{DA} - 9y_{DA}^2)v_1 v_3}{36M} \\ \frac{4y_{DS}(2y_{DS} - 3y_{DA})v_3^2 + (4y_{DS}^2 - 24y_{DS}y_{DA} - 9y_{DA}^2)v_1 v_2}{36M} & \frac{-(2y_{DS} - 3y_{DA})^2 v_2^2 + 8y_{DS}(2y_{DS} + 3y_{DA})v_1 v_3}{36M} & \frac{-4y_{DS}^2(v_1^2 + 5v_2 v_3) + 9y_{DA}^2(v_1^2 + v_2 v_3)}{36M} \\ \frac{4y_{DS}(2y_{DS} + 3y_{DA})v_2^2 + (4y_{DS}^2 + 24y_{DS}y_{DA} - 9y_{DA}^2)v_1 v_3}{36M} & \frac{-4y_{DS}^2(v_1^2 + 5v_2 v_3) + 9y_{DA}^2(v_1^2 + v_2 v_3)}{36M} & \frac{8y_{DS}(2y_{DS} - 3y_{DA})v_1 v_2 - 9(2y_{DS} - 3y_{DA})^2 v_3^2}{36M} \end{pmatrix}$$

Calculation of Yukawa couplings

①Calculate $|y_e|^2, |y_\mu|^2, |y_\tau|^2$

Denote $h_e \equiv |y_e|^2, h_\mu \equiv |y_\mu|^2, h_\tau \equiv |y_\tau|^2$

Mass matrix of charged leptons

$$M_l = \begin{pmatrix} y_e v_1 & y_\mu v_2 & y_\tau v_3 \\ y_e v_3 & y_\mu v_1 & y_\tau v_2 \\ y_e v_2 & y_\mu v_3 & y_\tau v_1 \end{pmatrix}_{LR},$$

VEV

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v\cos\beta \\ \frac{1}{\sqrt{2}} v\sin\beta \\ \frac{1}{\sqrt{2}} v\sin\beta \end{pmatrix}$$

$$M_l M_l^\dagger = \begin{pmatrix} h_e v^2\cos^2(\beta) + \frac{1}{2}(h_\mu + h_\tau)v^2\sin^2(\beta) & -\frac{1}{\sqrt{2}}(h_e + h_\mu)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_\tau v^2\sin^2(\beta) & -\frac{1}{\sqrt{2}}(h_e + h_\tau)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_\mu v^2\sin^2(\beta) \\ -\frac{1}{\sqrt{2}}(h_e + h_\mu)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_\tau v^2\sin^2(\beta) & h_\mu v^2\cos^2(\beta) + \frac{1}{2}(h_e + h_\tau)v^2\sin^2(\beta) & -\frac{1}{\sqrt{2}}(h_\mu + h_\tau)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_e v^2\sin^2(\beta) \\ -\frac{1}{\sqrt{2}}(h_e + h_\tau)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_\mu v^2\sin^2(\beta) & -\frac{1}{\sqrt{2}}(h_\mu + h_\tau)v^2\cos(\beta)\sin(\beta) + \frac{1}{2}h_e v^2\sin^2(\beta) & h_\tau v^2\cos^2(\beta) + \frac{1}{2}(h_e + h_\mu)v^2\sin^2(\beta) \end{pmatrix}$$

Diagonalize $M_l M_l^\dagger$ with unitary matrix V_l

$$V_l^\dagger M_l M_l^\dagger V_l = \begin{pmatrix} m_e^2 & & \\ & m_\mu^2 & \\ & & m_\tau^2 \end{pmatrix}$$

Solve the eigenvalues equation

$$\begin{cases} \mathrm{Tr}(M_l M_l^\dagger) = m_e^2 + m_\mu^2 + m_\tau^2 \\ \det(M_l M_l^\dagger) = m_e^2 m_\mu^2 m_\tau^2 \\ \left[\mathrm{Tr}(M_l M_l^\dagger)\right]^2 - \mathrm{Tr}(M_l M_l^\dagger M_l M_l^\dagger) = 2(m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_\tau^2 m_e^2) \end{cases}$$

We get $h_e = |y_e|^2, h_\mu = |y_\mu|^2, h_\tau = |y_\tau|^2$

Calculation of physical quantity

②Calculate unitary matrix V_l

Substitute the obtained $|y_e|^2, |y_\mu|^2, |y_\tau|^2$ into $M_l M_l^\dagger$ ➡ Calculate unitary matrix V_l

③Consider the same for neutrinos and find the unitary matrix V_ν that diagonalizes $m_\nu m_\nu^\dagger$

④Calculate $U_{PMNS}^{\mathrm{model}} = V_l^\dagger V_\nu \equiv U$ and mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}$$

$$\boxed{\begin{array}{l} c_{12} = \cos\theta_{12}\,,\ s_{12} = \sin\theta_{12} \\ c_{23} = \cos\theta_{23}\,,\ s_{23} = \sin\theta_{23} \\ c_{13} = \cos\theta_{13}\,,\ s_{13} = \sin\theta_{13} \end{array}}$$

$$\theta_{12} = \tan^{-1}\left(\left|\frac{U_{e2}}{U_{e1}}\right|\right), \theta_{23} = \tan^{-1}\left(\left|\frac{U_{\mu3}}{U_{\tau3}}\right|\right), \theta_{13} = \sin^{-1}|U_{e3}|$$

Calculation of physical quantity

⑤Calculate δ_{CP}

Jarlskog invariant： $J_{CP} = \mathrm{Im}\left[U_{\alpha i}U_{\beta i}^{*}U_{\alpha j}^{*}U_{\beta j}\right] \Rightarrow \mathrm{Im}\left[U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2}\right] = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^{2}\sin\delta_{CP}$

$$\therefore \sin\delta_{CP} = \frac{\mathrm{Im}\left[U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2}\right]}{s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^{2}}$$

and $|U_{\tau 1}|^{2} = s_{12}^{2}s_{23}^{2} + c_{12}^{2}s_{13}^{2}c_{23}^{2} - 2s_{12}s_{23}c_{12}s_{13}c_{23}\cos\delta_{CP}$ ➡ $\cos\delta_{CP} = \dfrac{s_{12}^{2}s_{23}^{2} + c_{12}^{2}s_{13}^{2}c_{23}^{2} - |U_{\tau 1}|^{2}}{2s_{12}s_{23}c_{12}s_{13}c_{23}}$

Calculate δ_{CP} from $\sin\delta_{CP}$ and $\cos\delta_{CP}$

⑥Calculate effective mass $m_{\beta\beta}$ in neutrinoless double beta ($0\nu\beta\beta$) decay experiment and Majorana phases η_1, η_2

$$m_{\beta\beta} = \left|m_1U_{e1}^{2} + m_2U_{e2}^{2} + m_3U_{e3}^{2}\right|$$

$$\eta_1 = \arg\left[\frac{U_{e1}U_{e3}^{*}}{\cos\theta_{12}\cos\theta_{13}\sin\theta_{13}e^{i\delta_{CP}}}\right], \quad \eta_2 = \arg\left[\frac{U_{e2}U_{e3}^{*}}{\sin\theta_{12}\cos\theta_{13}\sin\theta_{13}e^{i\delta_{CP}}}\right]$$

Numeric calculation

⑦Use the data from PDG(2021) and NuFIT 5.1.

e

$J = \frac{1}{2}$

Mass $m = (548.579909070 \pm 0.000000016) \times 10^{-6}$ u
Mass $m = 0.5109989461 \pm 0.0000000031$ MeV
$\left|m_{e^+} - m_{e^-}\right|/m < 8 \times 10^{-9}$, CL = 90%
$\left|q_{e^+} + q_{e^-}\right|/e < 4 \times 10^{-8}$
Magnetic moment anomaly
$(g-2)/2 = (1159.65218091 \pm 0.00000026) \times 10^{-6}$
$(g_{e^+} - g_{e^-}) \,/\, g_{\text{average}} = (-0.5 \pm 2.1) \times 10^{-12}$
Electric dipole moment $d < 0.11 \times 10^{-28}$ $e\,$cm, CL = 90%
Mean life $\tau > 6.6 \times 10^{28}$ yr, CL = 90% [a]

μ

$J = \frac{1}{2}$

Mass $m = 0.1134289257 \pm 0.0000000025$ u
Mass $m = 105.6583745 \pm 0.0000024$ MeV

τ

$J = \frac{1}{2}$

Mass $m = 1776.86 \pm 0.12$ MeV

https://pdg.lbl.gov

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
$\delta_{\rm CP}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ {\rm eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

http://www.nu-fit.org/

⑧Take β at random

VEV of Higgs

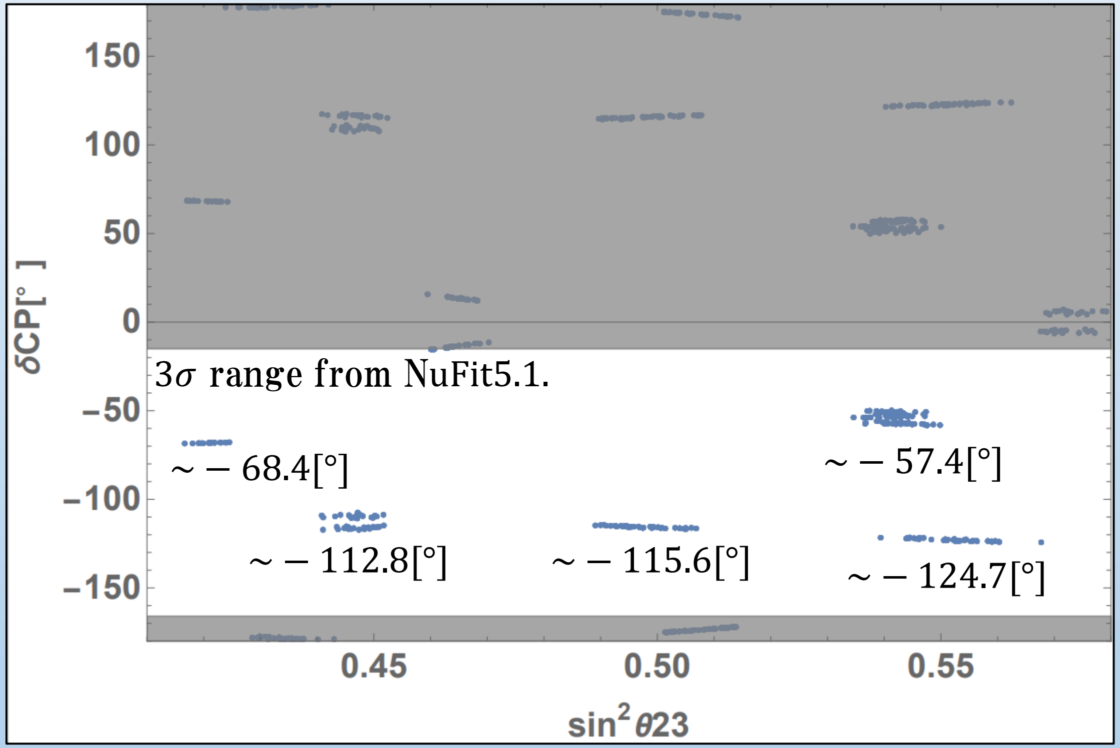
$$\langle\phi\rangle = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v\cos\beta \\ \dfrac{1}{\sqrt{2}} v\sin\beta \\ \dfrac{1}{\sqrt{2}} v\sin\beta \end{pmatrix}$$

$$\beta: -\frac{\pi}{2} \le \beta \le \frac{\pi}{2}$$
$$v = 173\ \text{GeV}$$

5. Result

Numerical result (1)

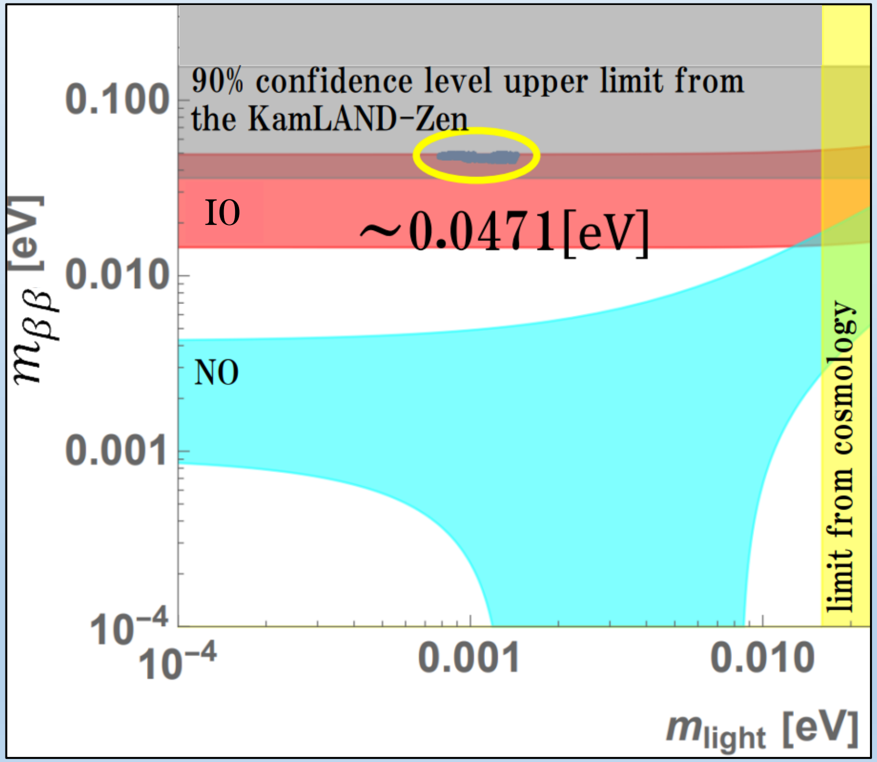
Prediction of δ_{CP} and $\sin^2\theta_{23}$



Strong prediction of δ_{CP}

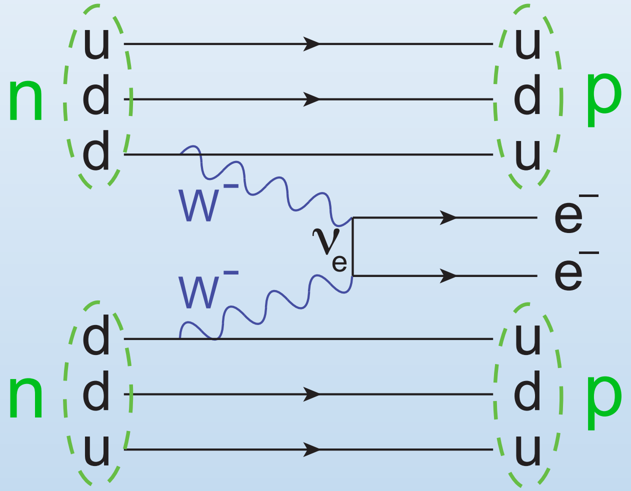
Numerical result (2)

Prediction of the effective neutrino mass $m_{\beta\beta}$ in the $0\nu\beta\beta$ decay experiment and the lightest neutrino mass m_{light}



Our model can be confirmed in the near future.

$0\nu\beta\beta$ decay



https://en.wikipedia.org/wiki/Double_beta_decay

Decay rate

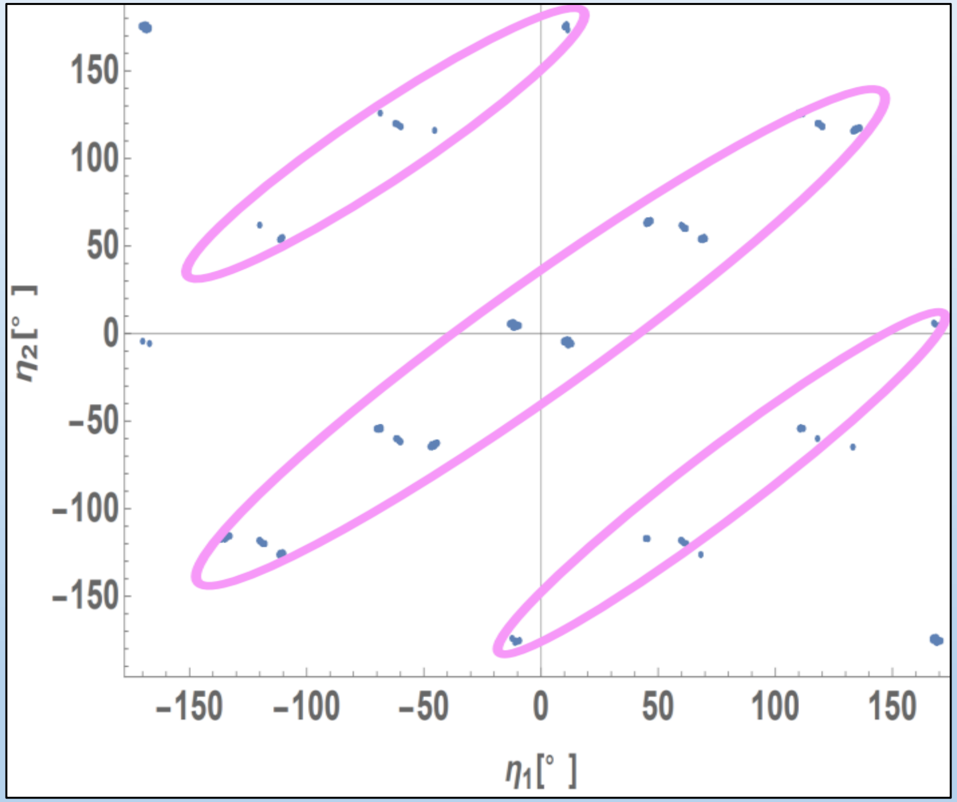
$$\Gamma \propto m_{\beta\beta}^2$$

Effective mass of electron neutrino

$$m_{\beta\beta} = \left| m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 \right|$$

Numerical result (2)

Prediction of Majorana phases η_1, η_2



$$\eta_1 = \arg\left[\frac{U_{e1}U_{e3}^*}{\cos\theta_{12}\cos\theta_{13}\sin\theta_{13}e^{i\delta_{CP}}}\right], \quad \eta_2 = \arg\left[\frac{U_{e2}U_{e3}^*}{\sin\theta_{12}\cos\theta_{13}\sin\theta_{13}e^{i\delta_{CP}}}\right]$$

6. Conclusion

We consider A_4 symmetry as flavor symmetry.
We consider Higgs field ϕ as A_4 triplet.
→ We perform Higgs potential analysis and obtain local VEV.

$$\langle\phi\rangle = \begin{pmatrix} v\cos\beta \\ \frac{1}{\sqrt{2}} v\sin\beta \\ \frac{1}{\sqrt{2}} v\sin\beta \end{pmatrix}$$

We build new flavor model by using 3HDM and A_4 symmetry.
We calculate mass matrices of charged leptons and neutrinos under new flavor model.

Mass matrix of charged leptons

$$M_l = \begin{pmatrix} y_e v_1 & y_\mu v_2 & y_\tau v_3 \\ y_e v_3 & y_\mu v_1 & y_\tau v_2 \\ y_e v_2 & y_\mu v_3 & y_\tau v_1 \end{pmatrix}_{LR}$$

Mass matrix of left-handed Majorana neutrinos

$$m_\nu = -M_D M_R^{-1} M_D^\dagger \quad \left(M_D = y_{DS}\begin{pmatrix} \frac{2}{3}v_1 & -\frac{1}{3}v_2 & -\frac{1}{3}v_3 \\ -\frac{1}{3}v_2 & \frac{2}{3}v_3 & -\frac{1}{3}v_1 \\ -\frac{1}{3}v_3 & -\frac{1}{3}v_1 & \frac{2}{3}v_2 \end{pmatrix} + y_{DA}\begin{pmatrix} 0 & -\frac{y_{DA}}{2}v_2 & \frac{y_{DA}}{2}v_3 \\ \frac{y_{DA}}{2}v_2 & 0 & -\frac{y_{DA}}{2}v_1 \\ -\frac{y_{DA}}{2}v_3 & \frac{y_{DA}}{2}v_1 & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix} \right)$$

We perform numerical analysis and calculate δ_{CP} , effective mass $m_{\beta\beta}$ and Majorana phases η_1, η_2.
We obtain strong predictions of δ_{CP} and $m_{\beta\beta}$ ($m_{\beta\beta} \approx 0.0471$[eV]).
→This flavor model can be confirmed by neutrino experiments in the near future.

7. Future work

We will add the soft breaking term to Higgs potential.

Potential

$$V = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \underline{(m_{12}\phi_1^\dagger\phi_2 + h.c.)}$$

$$= -\mu^2(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2) + \lambda_1|\phi_1^2 + 2\phi_2\phi_3|^2 + \lambda_2|\phi_2^2 + 2\phi_3\phi_1|^2 + \lambda_3|\phi_3^2 + 2\phi_1\phi_2|^2$$

$$+\lambda_4\left[|\phi_1^2 - \phi_2\phi_3|^2 + |\phi_2^2 - \phi_3\phi_1|^2 + |\phi_3^2 - \phi_1\phi_2|^2\right]$$

$$+\underline{(m_{12}\phi_1^\dagger\phi_2 + h.c.)}$$

➡ We try to solve domain wall problem and increase the heavy Higgs masses.

Quark sector

quark sector

$$L = y^d_{ij}\bar{q}_{Li}\phi q^d_{Rj} + y^u_{ij}\,\bar{q}_{Li}\tilde{\phi} q^u_{Rj}$$

A_4 3   3   3   3   3   3

$$q_L = \left(\begin{pmatrix} u_L \\ d_L \end{pmatrix} \ \begin{pmatrix} c_L \\ s_L \end{pmatrix} \ \begin{pmatrix} t_L \\ b_L \end{pmatrix}\right)$$

$$q^d_{R1} = d_{R1} \quad q^d_{R2} = s_{R2} \quad q^d_{R3} = b_{R3}$$

$$q^u_{R1} = u_{R1} \quad q^u_{R2} = c_{R2} \quad q^u_{R3} = t_{R3}$$

However, the quark sector has strict restrictions on experimentation.
The mixing angles of quarks are precisely measured.

Alternating group

Alternating group：the set of even permutations of the symmetric group

置換

symmetric group(permutation group)： set of n-dimensional permutation

even permutation：permutation expressed as a product of even number of transposition

互換

parameter

SM　Yukawa coupling→3∗3∗2=18
Flavor structure　Yukawa coupling→$y_e, y_\mu, y_\tau, y_{DS}, y_{DA}, \phi_{DA}$→7

Higgs VEV parameter β

Physical quantity we used

$v, m_e, m_\mu, m_\tau, \Delta m_{21}^2, \Delta m_{32}^2$

Physical quantity(prediction)

$\theta_{12}, \theta_{23}, \theta_{13} \quad \rightarrow \quad \delta_{CP}, m_{ee}, \eta_1, \eta_2$