CP violation due to a Majorana phase in two flavor neutrino oscillations with decays

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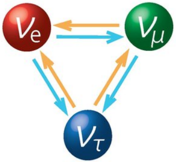
Plan of this talk

1. Motivation
2. Neutrino oscillations
3. Majorana phase in neutrino oscillations
4. *CP* properties of oscillation probabilities with neutrino decay

Motivation

- Neutrino oscillation probabilities depend on the elements of the mixing matrix and the mass-squared differences, in general, but no dependence on the Majorana phase.
- Benatti et al. (2001) showed that the neutrino oscillation probabilities depend on Majorana phases in case of a new form of neutrino decoherence with an off-diagonal term in the decoherence matrix.
- Are there other possibilities under which the Majorana phases appear in neutrino oscillation probabilities and lead to *CP* violation?

- About 65 billion (6.5×10^{10}) neutrinos coming from Sun's interior pass through 1 square centimeter per second. Homestake experiment's observed value was 1/3 of the predicted flux. This lead Pontecorvo to suggest neutrino oscillations.
- Up-down asymmetry of atmospheric muon neutrino flux by IMB and KamioKande experiments gave additional hint of neutrino oscillations. (*T. Kajita* (SK), *A. McDonald* (SNO), 2015)



- Experiments : Solar (e.g. Homestake, Gallex/SAGE, SNO), Atmospheric (e.g. Super Kamiokande), Reactor (e.g. CHOOZ, KamLAND, Daya-Bay, RENO), Accelerator (e.g. T2K, MINOS, NOνA, DUNE (upcoming))

- We restrict ourselves to two flavor oscillations. The flavor states ν_e and ν_μ mix to form the mass eigenstates ν_1 and ν_2 with masses m_1 and m_2, respectively.
- In two-flavor neutrino oscillations
  Propagation states $\rightarrow \{|\nu_1\rangle\,,\ \ |\nu_2\rangle\}$;
  Flavor states $\rightarrow \{|\nu_e\rangle\,,\ \ |\nu_\mu\rangle\}$
- General state of a neutrino in flavor basis:

$$|\Psi(t)\rangle = \nu_e(t)\,|\nu_e\rangle + \nu_\mu(t)\,|\nu_\mu\rangle$$

- Same state in propagation basis looks like:

$$|\Psi(t)\rangle = \nu_1(t)\,|\nu_1\rangle + \nu_2(t)\,|\nu_2\rangle$$

- The coefficients in two representations are connected via a *unitary* matrix

$$\nu_\alpha(t) = U\nu_i(t). \tag{1}$$

where $\nu_\alpha(t) = \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix}$; $\qquad \nu_i(t) = \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix}$

- Time evolution of mass eigenstates

$$\nu_i(t) = E\nu_i(0); \qquad E = diag(e^{-iE_1 t}, e^{-iE_2 t}) \tag{2}$$

Hence, $\nu_f(t) = UEU^{-1}\ \nu_{\mathbf{f}}(0) = U_f\ \nu_f(0).$

- For antineutrinos

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = U^* \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix}.$$

- If $P(\nu_\alpha \to \nu_\beta) \neq P(\nu_{\bar{\alpha}} \to \nu_{\bar{\beta}})$
  $\Rightarrow$ CP is violated.
- CP violation in neutrino oscillations requires complex values of neutrino mixing matrix.

Basically, a $N \times N$ unitary matrix can be parameterized via N^2 independent real parameters

$$^{N}C_2 = \frac{N(N-1)}{2}, \rightarrow \text{mixing angles,}$$

$$\frac{N(N+1)}{2} \rightarrow \text{phases.}$$

$$\textit{i.e.}, \; U_{N\times N} \rightarrow O_{N\times N} \text{ modified by } \frac{N(N+1)}{2} \text{ phases.}$$

Hence, a 2×2 unitary matrix (with 1 mixing angle and 3 phases) can be represented as

$$U = \begin{pmatrix} \cos\theta e^{i\omega_1} & \sin\theta e^{i(\omega_2+\eta)} \\ -\sin\theta e^{i(\omega_1-\eta)} & \cos\theta e^{i\omega_2} \end{pmatrix}$$

$$U = \begin{pmatrix} e^{i\omega_1} & 0 \\ 0 & e^{i(\omega_1-\eta)} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\omega_2-\omega_1+\eta)} \end{pmatrix}$$

or,

$$U = U_{ph}^{left} O \; U_{ph}^{right}$$

In case of Fermion mixing, not all of these phases are physical observables, some of them can be absorbed by rephasing fermion fields.

Majorana Mixing matrix

Hence, in neutrino mixing $\quad |\nu_{\mathbf{f}}\rangle = U_{ph}^{left} O\ U_{ph}^{right} |\nu_{\mathbf{i}}\rangle$

$$|\tilde{\nu}_{\mathbf{f}}\rangle = U_{ph}^{left^\dagger} |\nu_{\mathbf{f}}\rangle = O\ [U_{ph}^{right} |\nu_{\mathbf{i}}\rangle]$$

we have the form of mixing matrix for Majorana neutrinos as

$$U = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\mathbf{O}} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}}_{U_{ph}} \tag{3}$$

- The phase ϕ is unphysical if neutrinos are Dirac particles but is physical if they are Majorana.
- Moreover, both vacuum and matter modified oscillation probabilities are independent of ϕ whether neutrinos are Dirac or Majorana.

- Benatti and Floreanini (PRD **64** (2001) 085015) considered a novel form of neutrino decoherence with an off-diagonal term in the decoherence matrix.
- For such decoherence, the oscillation probabilities depend on ϕ. Also, these probabilities are CP violating.

We ask "what are the other possibilities under which the Majorana phase ϕ appears in neutrino oscillation probabilities and causes CP violation?"

The Hamiltonian in mass eigenbasis

$$\mathcal{H} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = \frac{(a_1 + a_2)}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -(a_2 - a_1) & 0 \\ 0 & a_2 - a_1 \end{pmatrix},$$

where $a_1 = m_1^2/2E$ and $a_2 = m_2^2/2E$, $m_i \rightarrow$ mass eigenvalues, $E \rightarrow$ energy.

Evolution equations in mass eigenbasis

$$i\frac{d}{dt}\nu_i(t) = \left[\frac{(a_1 + a_2)}{2}\sigma_0 - \frac{a_2 - a_1}{2}\sigma_z\right]\nu_i(t), \quad (4)$$

where, $\sigma_0 \equiv \mathbf{I}_{2\times 2}$ and σ_z is the diagonal Pauli matrix.

In flavor basis $(\nu_i(0) = U^\dagger \nu_f(0) = U_{ph}^\dagger O^T \nu_f(0))$

$$i\frac{d}{dt}\nu_\alpha(t) = \left[\frac{(a_1 + a_2)}{2}\sigma_0 - \frac{(a_2 - a_1)}{2} O U_{ph} \sigma_z U_{ph}^\dagger O^T\right]\nu_\alpha(t). \quad (5)$$

Since

$$O U_{ph} \sigma_z U_{ph}^\dagger O^T = O \sigma_z O^T \qquad ([U_{ph}, \sigma_z] = 0)$$

$\Rightarrow$ U_{ph} matrix containing ϕ is gone!

Most general neutrino evolution

If, $\mathcal{H} = ((-\frac{(a_2-a_1)}{2})\sigma_z + b\sigma_x + c\sigma_y)$

$$\boxed{OU_{ph}[-\frac{(a_2-a_1)}{2}\sigma_z + b\sigma_x + c\sigma_y]U_{Ph}^{\dagger}O^T \neq O\sigma_z O^T},$$

where σ_x and σ_y are the off-diagonal Pauli matrices. It is trivial to see

$$[U_{ph}, \sigma_x] \neq 0 \ \text{ and } \ [U_{ph}, \sigma_y] \neq 0$$

- We consider the case of the most general Hamiltonian in neutrino mass basis including the decay terms
$$\mathcal{H} = M - i\Gamma/2,$$
where M and Γ are hemitian matrices.
- Presence of Γ makes $\mathcal{H}$ non-hermitian.
- Since $\mathcal{H}$ is in mass basis, M is diagonal, but Γ need not be in general.
- We assume that the decay eigenstates are not the same as mass eigenstates.
- Hence Γ is non-diagonal and does not commute with M.

- We consider the following form of the Hamiltonian

$$\mathcal{H} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} - i \begin{pmatrix} b_1 & \frac{1}{2}\eta e^{i\xi} \\ \frac{1}{2}\eta e^{-i\xi} & b_2 \end{pmatrix}.$$

- Γ should be positive semi-definite ($b_i > 0$ and $\eta \leq 4b_1^2 b_2^2$).
- When Γ is non-diagonal ($\eta \neq 0$), *i.e.*, the mass eigenstates are **not** decay eigenstates, the evolution of mass eigenstates

$$i\frac{d}{dt}\nu_i(t) = \left[\frac{(a_1 + a_2)}{2}\sigma_0 - \frac{(a_2 - a_1)}{2}\sigma_z - \frac{i}{2}\left((b_1 + b_2)\sigma_0 + \vec{\sigma}.\vec{\Gamma}\right)\right] \nu_i(t),$$

where $\vec{\Gamma} = [\eta\cos\xi, -\eta\sin\xi, -(b_2 - b_1)]$.

- The evolution equation in terms of flavor states is

$$i\frac{d}{dt}\nu_\alpha(t) = \left[\frac{(a_1+a_2)}{2}\sigma_0 - \frac{(a_2-a_1)}{2}O\sigma_z O^T - \frac{i}{2}(b_1+b_2)\sigma_0 - \frac{i}{2}OU_{ph}(\vec{\sigma}.\vec{\Gamma})U_{ph}^\dagger O^T\right]\nu_\alpha(t).$$

- The matrix $\vec{\sigma}.\vec{\Gamma}$ does not commute with U_{ph} (since σ_x and σ_y do not commute with U_{ph}), the phase ϕ remains in the evolution equation.
- And hence, ϕ also appears in oscillation probabilities.

Neutrino oscillations with most general decay Hamiltonian

- Evolution operator $\mathcal{U} = e^{-i\mathcal{H}t}$ can be expanded in the basis spanned by σ_0 and Pauli matrices σ_i.
- This expansion is parameterized by a complex 4-vector $n_\mu \equiv (n_0, \vec{n})$, with $n_\mu = Tr[(-i\mathcal{H}t).\sigma_\mu]/2$, where

$$n_0 = -\frac{i}{2}(a_1 + a_2)t - \frac{1}{2}(b_1 + b_2)t, \quad n_x = -\frac{1}{2}t\eta\cos\xi,$$
$$n_y = \frac{1}{2}t\eta\sin\xi, \qquad n_z = \frac{i}{2}(a_2 - a_1)t + \frac{1}{2}(b_2 - b_1)t. \tag{6}$$

- The evolution matrix $\mathcal{U}$ is

$$\mathcal{U} = e^{n_0}\left[\cosh n\ \sigma_0 + \frac{\vec{n}.\vec{\sigma}}{n}\sinh n\right], \tag{7}$$

where

$$n = \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{t}{2}\sqrt{\eta^2 - (a_2 - a_1 - i(b_2 - b_1))^2}. \tag{8}$$

evolution matrix in flavor basis $\mathcal{U}_f = U_{mix}\mathcal{U}U_{mix}^{-1}$.

- Neglecting terms of $\mathcal{O}(\eta^2)$ and higher order and assuming $b_1 = b = b_2$, we get the survival probabilities

$$P_{ee} = e^{-2bt}\left(P^{vac}_{ee} - \eta\cos(\xi-\phi)\mathcal{A}\right)$$
$$P_{\mu\mu} = e^{-2bt}\left(P^{vac}_{\mu\mu} + \eta\cos(\xi-\phi)\mathcal{A}\right)$$

and the oscillation probabilities

$$P_{e\mu} = e^{-2bt}\left(P^{vac}_{e\mu} + 2\eta\sin(\xi-\phi)\mathcal{B}\right)$$
$$P_{\mu e} = e^{-2bt}\left(P^{vac}_{\mu e} - 2\eta\sin(\xi-\phi)\mathcal{B}\right)$$

where

$$\begin{aligned}\mathcal{A} &= \frac{\sin(2\theta)\sin[(a_2-a_1)t]}{(a_2-a_1)} \\ \mathcal{B} &= \frac{\sin(2\theta)\sin^2\left[\frac{1}{2}(a_2-a_1)t\right]}{(a_2-a_1)}.\end{aligned} \quad (9)$$

$$P^{vac}_{e\mu} = \sin^2 2\theta \sin^2\left(\frac{(a_2-a_1)t}{2}\right), \qquad a_2 - a_1 = \Delta m^2/2E. \quad (10)$$

The relations between different vacuum oscillation probabilities, $P^{vac}_{ee} = 1 - P^{vac}_{e\mu} = P^{vac}_{\mu\mu}$ and $P^{vac}_{\mu e} = P^{vac}_{e\mu}$.

- Majorana phase ϕ appears in the probability expressions.
- This appearance is proportional to $\Gamma_{12} \propto \eta$.
- Unlike in vacuum oscillations $P_{ee} \neq P_{\mu\mu}$ and $P_{e\mu} \neq P_{\mu e}$.
- $P_{e\mu}$ and $P_{\mu e}$ are sensitive to the mass ordering.

Antineutrino oscillation probabilities

- For antineutrinos, we have

$$\bar{M} = M \quad \text{and} \quad \bar{\Gamma} = \Gamma^*. \tag{11}$$

- Hence, antineutrino probability expressions can be obtained by making the substitutions $\phi \rightarrow -\phi$ and $\xi \rightarrow -\xi$.

$$P_{\bar{e}\bar{e}} = e^{-2bt} \left(P^{vac}_{\bar{e}\bar{e}} - \eta \cos(\xi - \phi)\mathcal{A}\right) \quad \text{and} \quad P_{\bar{\mu}\bar{\mu}} = e^{-2bt} \left(P^{vac}_{\bar{\mu}\bar{\mu}} + \eta \cos(\xi - \phi)\mathcal{A}\right)$$

$$P_{\bar{e}\bar{\mu}} = e^{-2bt} \left(P^{vac}_{\bar{e}\bar{\mu}} - 2\eta \sin(\xi - \phi)\mathcal{B}\right) \quad \text{and} \quad P_{\bar{\mu}\bar{e}} = e^{-2bt} \left(P^{vac}_{\bar{\mu}\bar{e}} + 2\eta \sin(\xi - \phi)\mathcal{B}\right).$$

- We find $P_{\bar{e}\bar{e}} = P_{ee}$, $P_{\bar{\mu}\bar{\mu}} = P_{\mu\mu}$ and $P_{\bar{\mu}\bar{e}} = P_{e\mu}$, *i.e.*, *CPT* is conserved.
- However, there is *CP*-violation ($P_{\bar{e}\bar{\mu}} \neq P_{e\mu}$) and *T*-violation ($P_{\mu e} \neq P_{e\mu}$).

CP-violation with Majorana phase

- The CP violating term in the oscillation probabilities is proportional to $\eta \sin(\xi - \phi)$.
- There are three possibilities of CP-violation
  1. CP-violation in mass - if $\eta \neq 0$ and $\xi = 0$, but $\phi \neq 0$
  2. CP-violation in decay - if $\phi = 0$, but $\eta \neq 0$ and $\xi \neq 0$
  3. CP-violation in mass and decay - if $\eta \neq 0$, $\xi \neq 0$ and $\phi \neq 0$ but $\xi \neq \phi$
- Non-zero value of η is a necessary condition for CP-violation but is not sufficient.
- For the two special cases,(i) $\phi = 0 = \xi$ and (ii) $\phi = \xi$, there is no CP-violation even when $\eta \neq 0$.
- However, the presence of η is discernible in the survival probabilities.

- A bound on $\tau_\nu \geq 5.7 \times 10^5$ s (m_ν/eV) is obtained from the neutrino data of Supernova 1987A. (Frieman et al,. PLB **200**, 115 (1988))
  i.e., $\Gamma_\nu \equiv b \approx 10^{-21}$ eV for a neutrino of mass 1 eV.
- $\eta = b$, which satisfies the semipositivity constraint $\eta \leq 2b$.
- The effects of non diagonal decay terms considered in this work are of order $\eta/(a_2 - a_1) = \eta E/\Delta m^2$.
- For $\Delta m^2 \approx 10^{-4}\ eV^2$, $E \approx 10^{16}$ eV or 10^7 GeV, these effects are of order 10%.

Ultrahigh energy neutrinos from astrophysical sources provide a platform to study the effects of the off-diagonal decay term.

Summary & Conclusions

- We point out scenarios in which the Majorana phase of two flavor oscillations can appear in neutrino oscillation probabilities and also causes CP violation.
- It was demonstrated earlier that off-diagonal terms in the decoherence matrix is one such scenario, which violates CP.
- We found another scenario which involves neutrino decay, where the decay eigenstates are not the same as the mass eigenstates.
- We pointed out types of possible CP violation: due to the Majorana phase ϕ (CP violation in mass), due to the phase ξ of decay matrix (CP violation in decay) or both.
- In the two special cases, when ϕ and ξ are equal to each other or when both are zero, there is no CP violation even if the decay eigenstates are different from the mass eigenstates.
- The CP-violating terms in this scenario are sensitive to the neutrino mass ordering.

THANK
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