Relativistic Hydrodynamic Model in High-Energy Heavy-Ion Collisions

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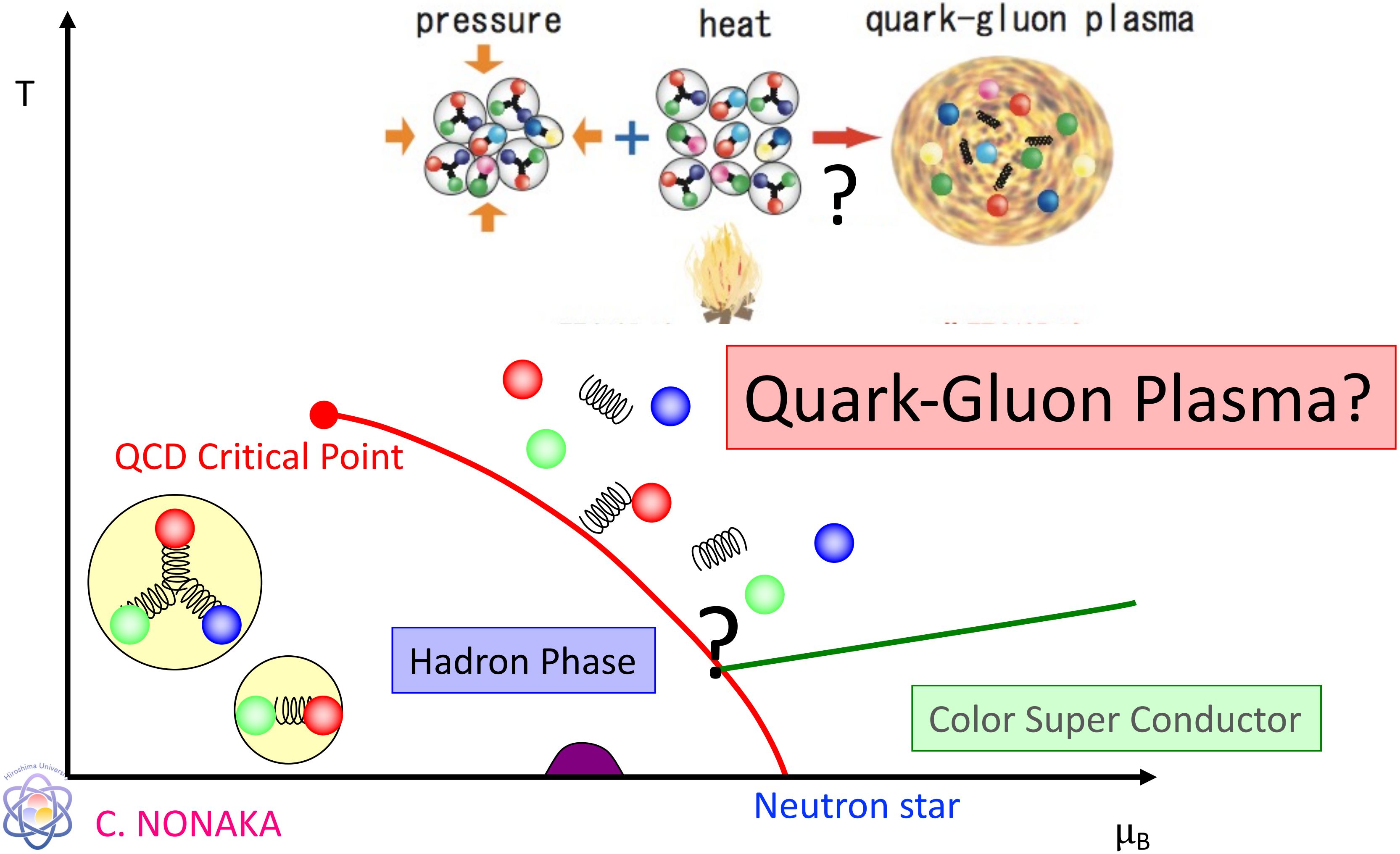
February 21, 2023@3rd IITB-Hiroshima workshop in HEP

What is the QGP?

Quark-Gluon Plasma

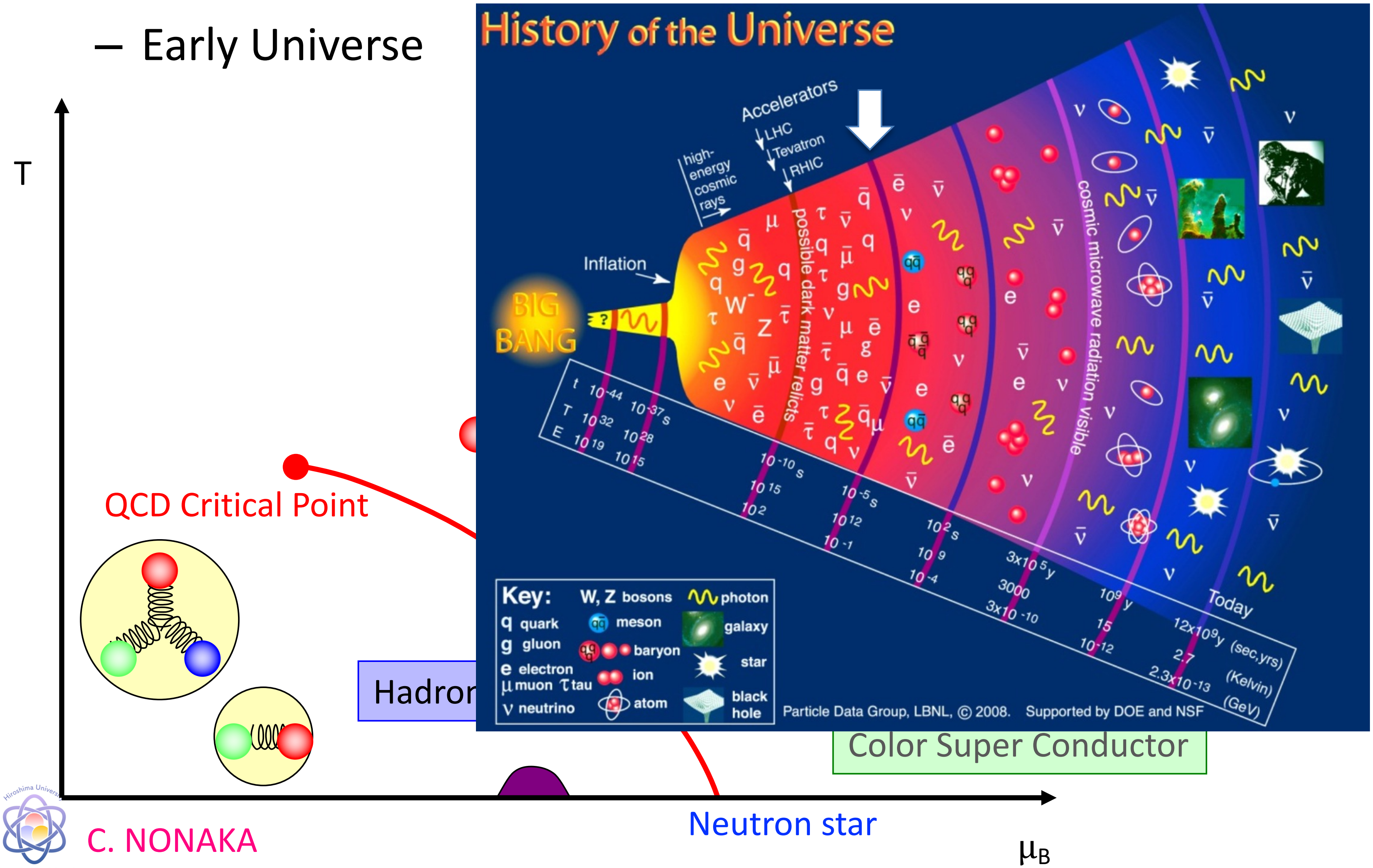
- Quarks and gluons at extreme conditions
  - High temperature and/or high density



What is the QGP?

Quark-Gluon Plasma

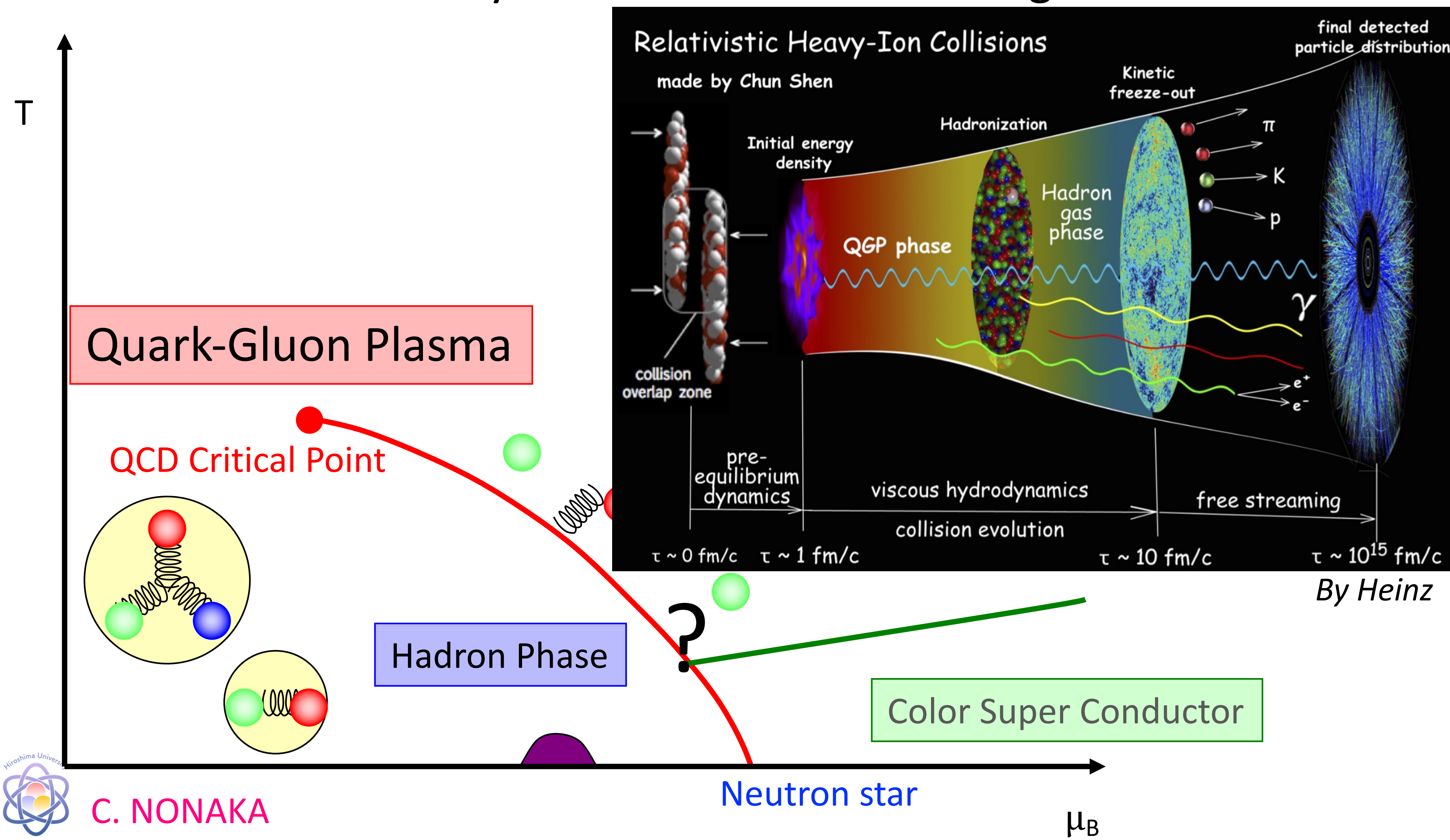
- Quarks and gluons at extreme conditions
  - Early Universe



What is the QGP?

Quark-Gluon Plasma

- Quarks and gluons at extreme conditions
  - Relativistic Heavy Ion Collisions : Little Bang

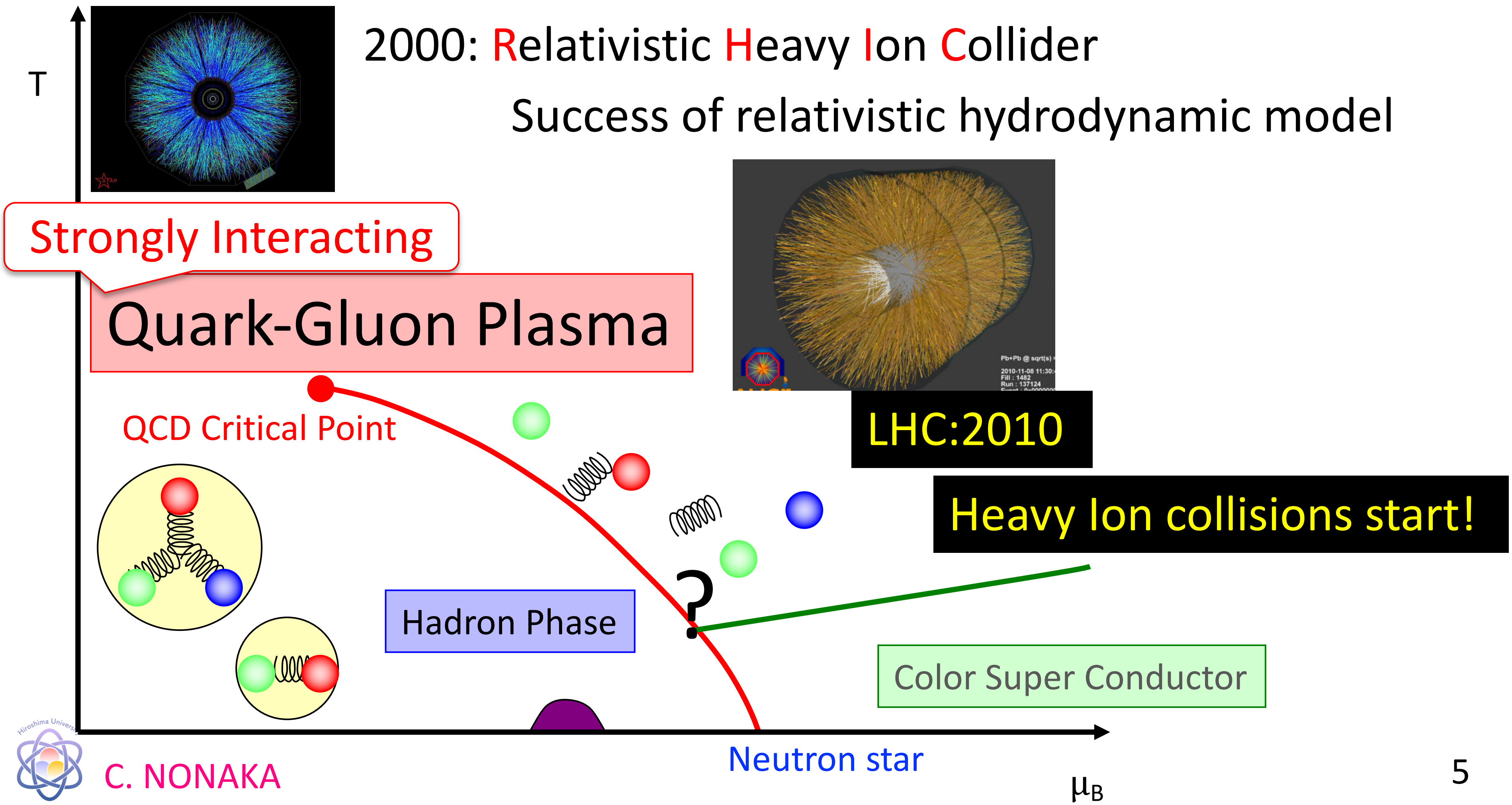


By Heinz

What is the sQGP?

Quark-Gluon Plasma

- Quarks and gluons at extreme conditions
  - Relativistic Heavy Ion Collisions



Heavy Ion Collisions

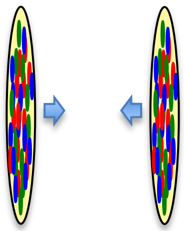
RHIC@BNL

Large Hadron Collider@CERN

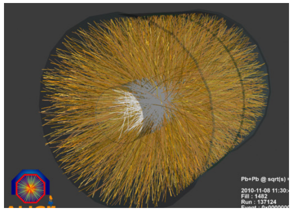
Hydrodynamic Model

Experimental data

collisions



QGP
Production?



Hadrons
Leptons
Photons

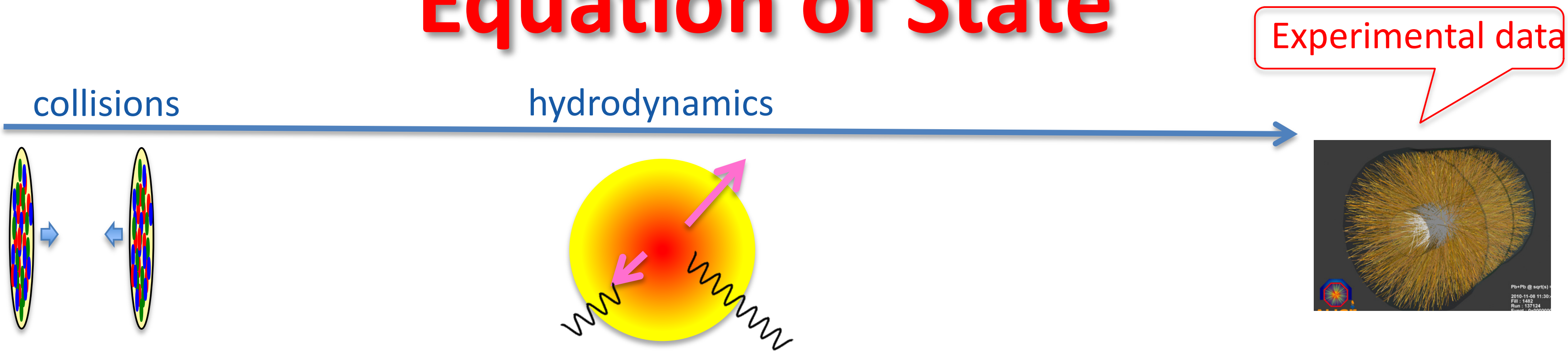
Multiple particle production

➡ Hydrodynamic picture

Landau
Bjorken

Success of hydrodynamic model at RHIC
Relativistic viscous hydrodynamic model
One of important phenomenological models

Equation of State



Hydrodynamics

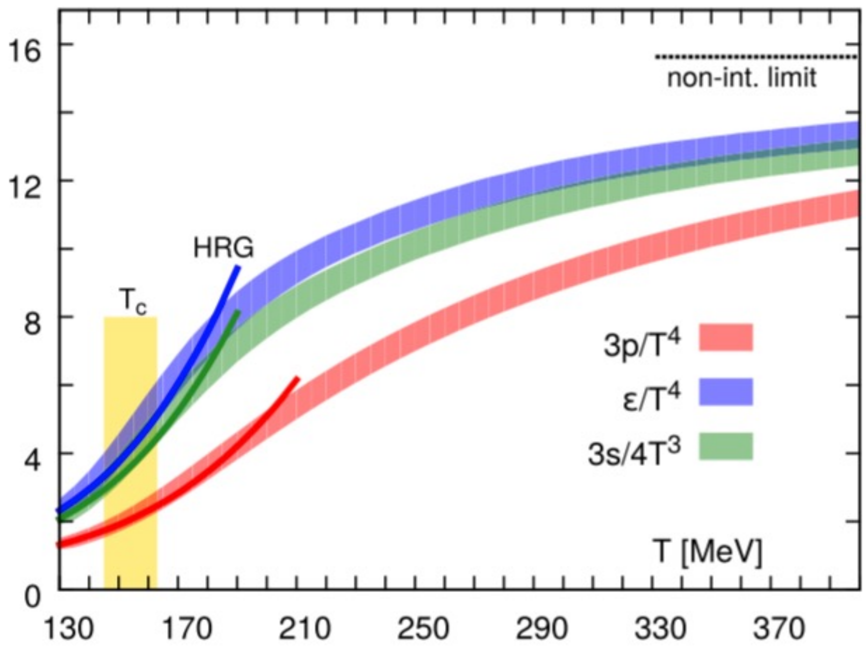
QCD phase diagram

EoS: lattice QCD

Equation of State

- Equation of State
  - Lattice QCD

HotQCD,PRD90,094503(2014)

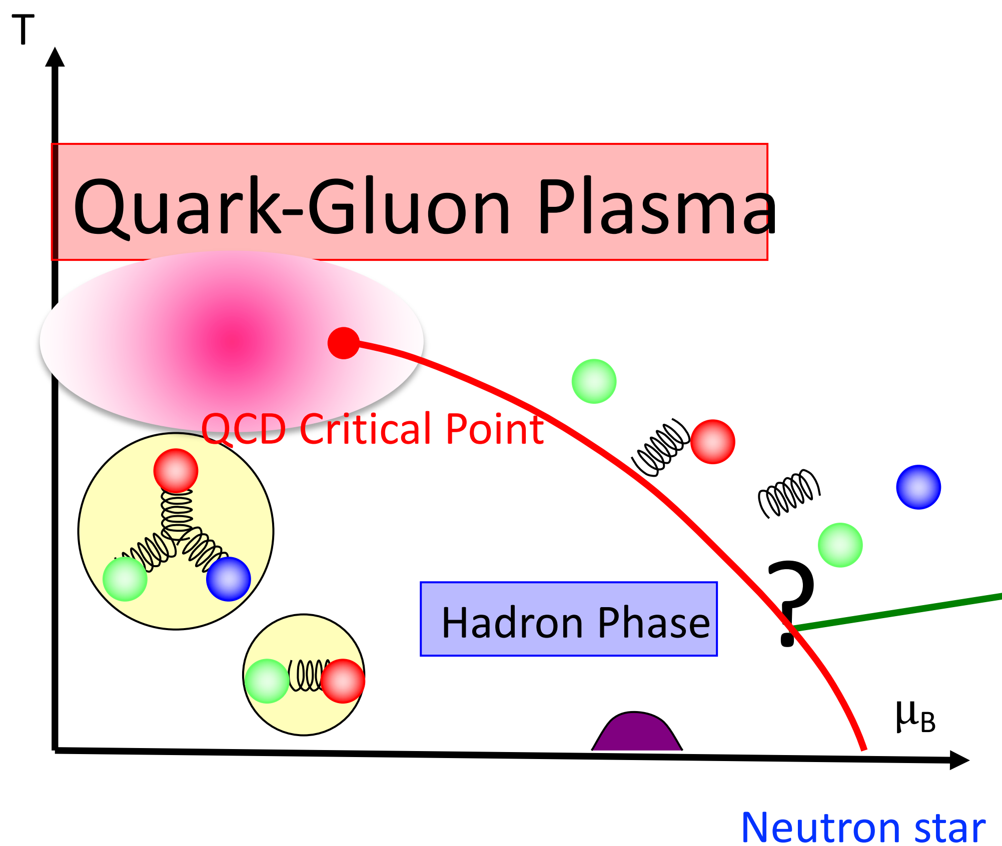


(2+1) flavor, Highly improved staggered quark action

Nt=6,8,10,12,Ns=4Nt → continuum limit

Parametrization of EoS

$T_c \sim 155$ MeV



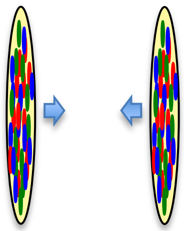
finite μ: sign problem

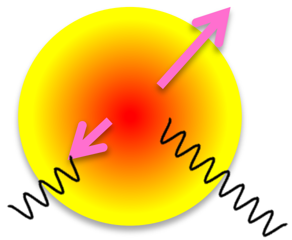
QGP Bulk Properties

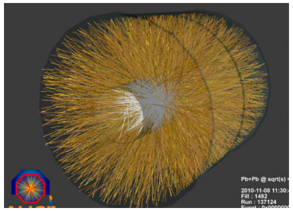
Experimental data

collisions

hydrodynamics







Hydrodynamics

QCD phase diagram

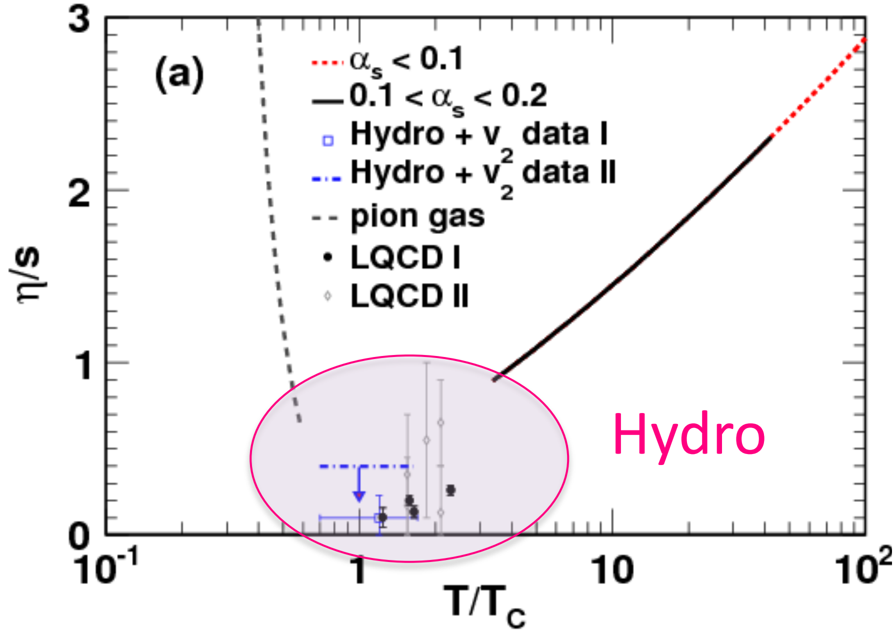
EoS: lattice QCD

Shear and bulk viscosities

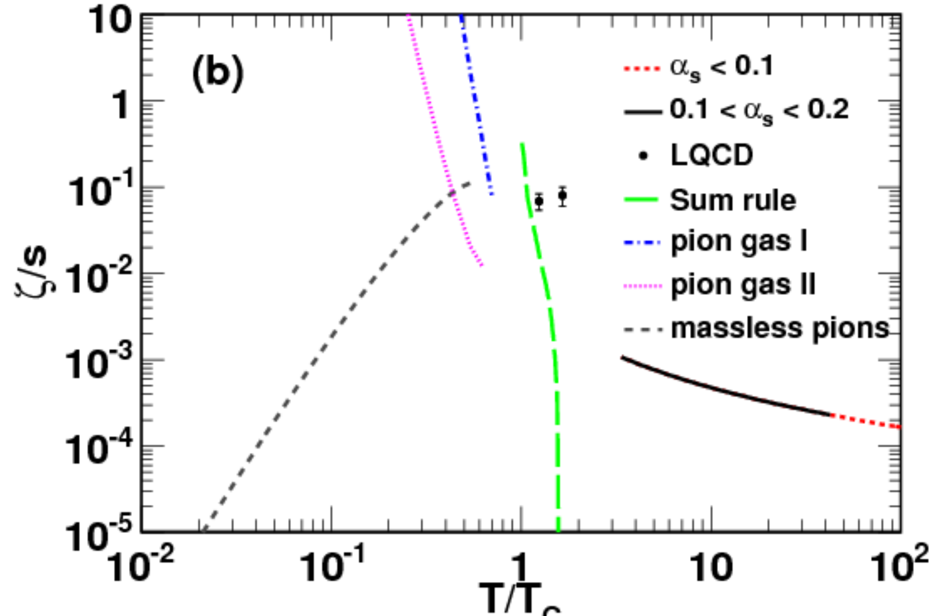
Property of QGP

- Current Status for transport coefficients

shear viscosity



bulk viscosity



Chen,Deng,Dong,Wang,PRC87,024910(2013)

- Shear viscosity takes the minimum around T_c. Cf. $\eta/s=1/4\pi$ AdS/CFT
- Hydrodynamic model constant η/s

- Bulk viscosity Temperature dependence is unclear.
- Hydrodynamic model vanishing

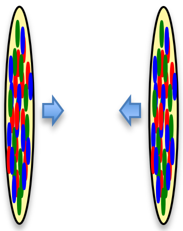
Detailed feature of shear and bulk viscosities

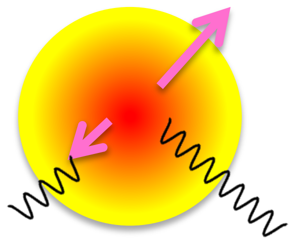
QGP Bulk Properties

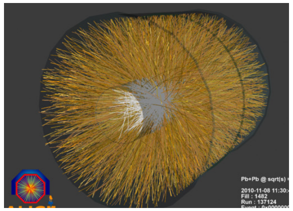
Experimental data

collisions

hydrodynamics



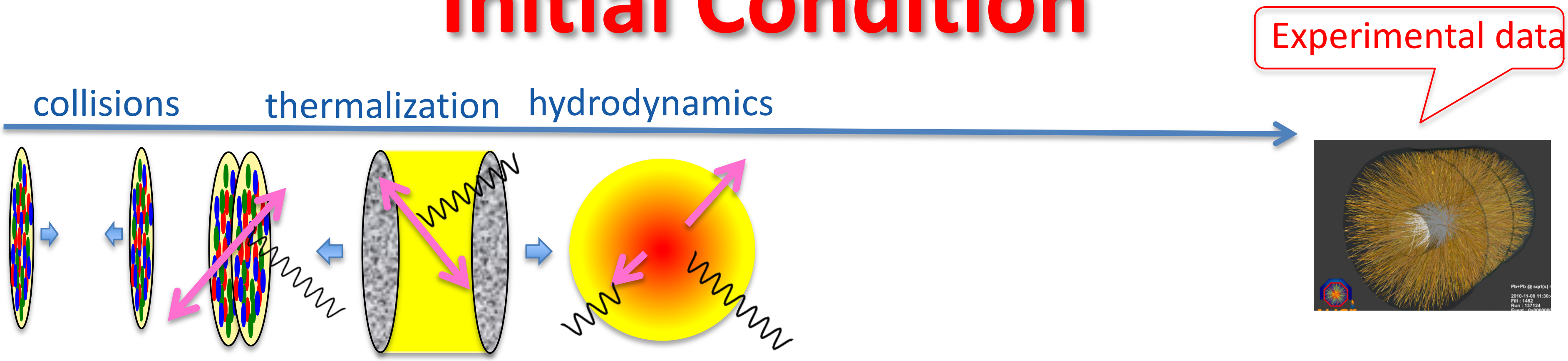




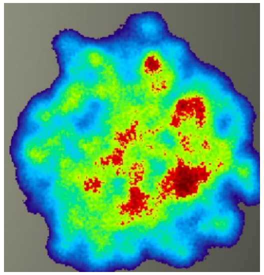
Hydrodynamics

QCD phase diagram
EoS: lattice QCD
Shear and bulk viscosities

Initial Condition



Initial conditions

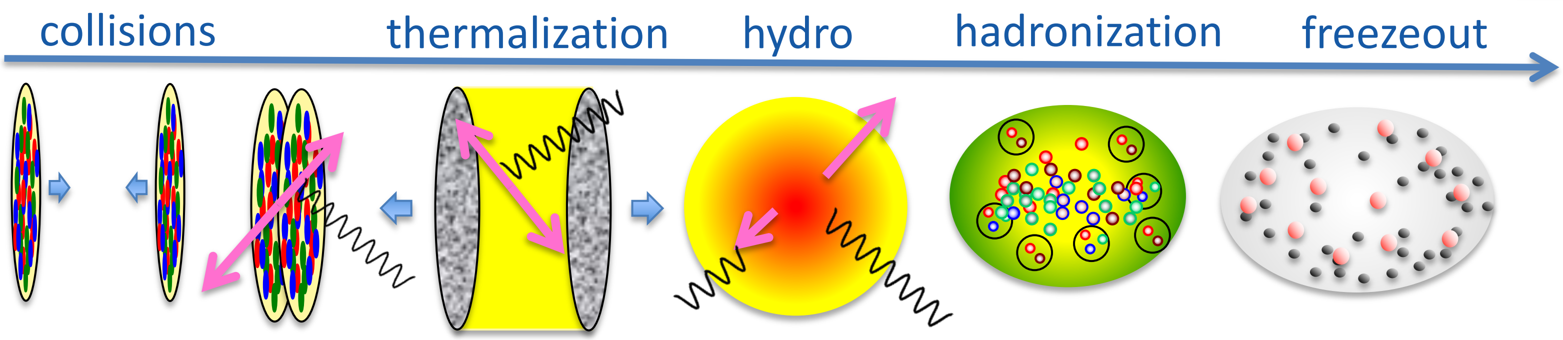


Energy (entropy) density distributions

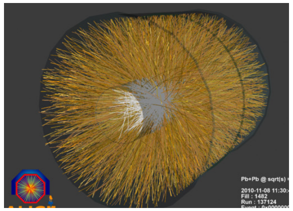
Hydrodynamics

QCD phase diagram
EoS: lattice QCD
Shear and bulk viscosities

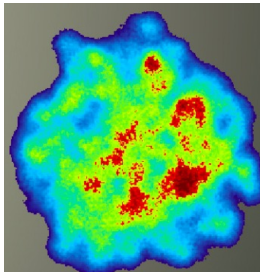
From Fluid to Particle



Experimental data



Initial conditions



Energy (entropy) density distributions

Hydrodynamics

QCD phase diagram
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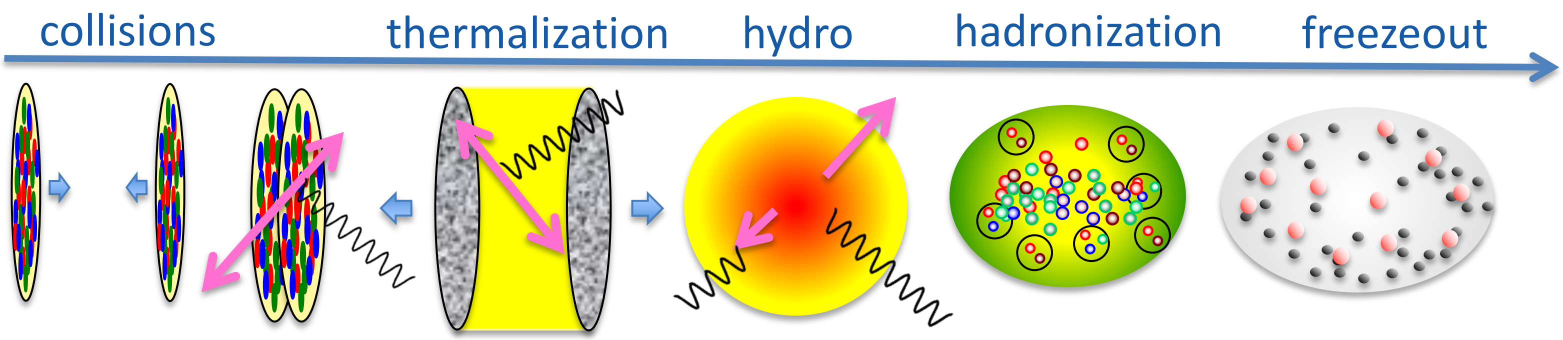
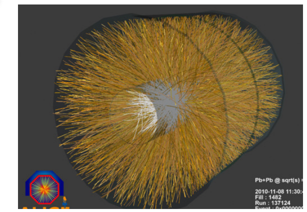
Final state interactions

Hadron based event generator

Quantitative Analyses



Experimental data



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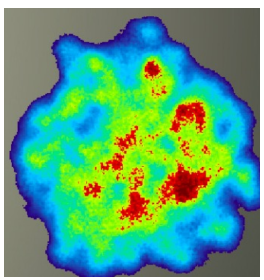


Energy (entropy) density distributions

Hydrodynamics

QCD phase diagram
EoS: lattice QCD
Shear and bulk viscosities

New hydrodynamics code

$$\partial_\mu T^{\mu\nu} = 0$$

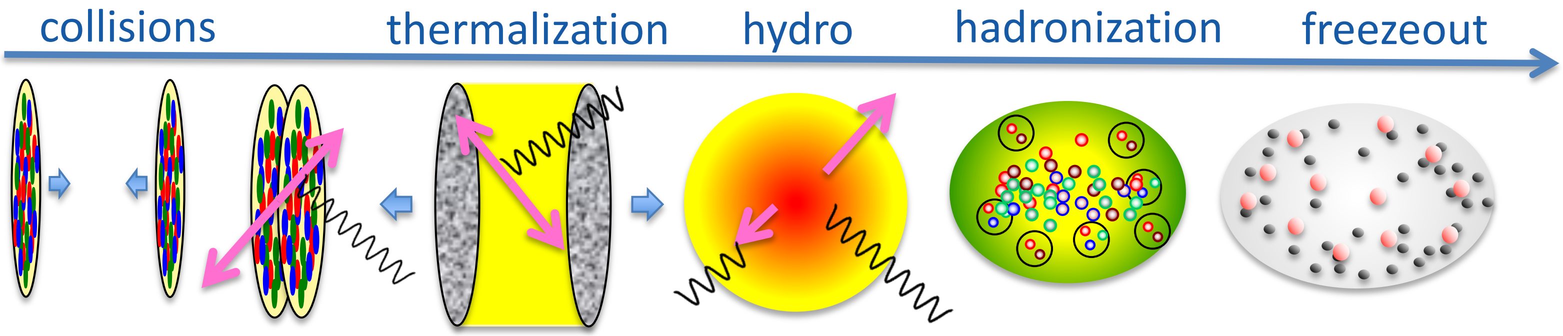
Final state interactions

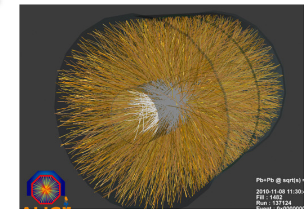
Hadron based event generator

Akamatsu et al, JCP256,34(2014)
Okamoto, Akamatsu, Nonaka, EPJC76,579(2016)
Okamoto and Nonaka, EPJC77,383(2017)

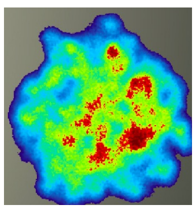
Quantitative Analyses

Experimental data





Initial conditions



TRENTO

Phenomenological model
Parametrization

Moreland et al.,PRC92,011901(2015)
Ke et al.,PRC96,044192(2017)

Hydrodynamics

QGP bulk property
EoS: lattice QCD
Shear and bulk viscosities

New hydrodynamics code

$$\partial_\mu T^{\mu\nu} = 0$$

Final state interactions

Hadron based event generator

UrQMD

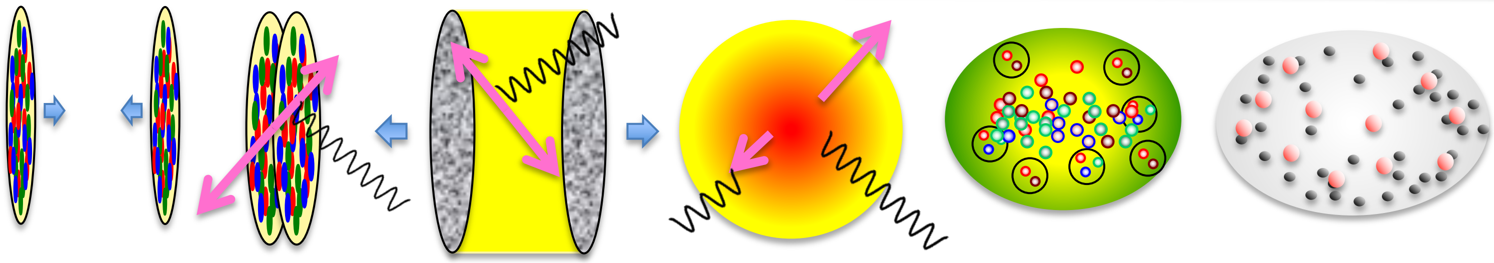
Bass et al., Prog.Part.Nucl.Phys.(1998)
Bleicher et al., J.Phys.G25,1859(1999)

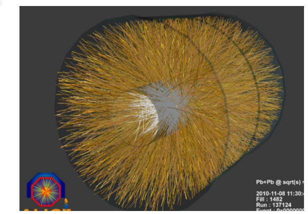
Application to analyses of RHIC and LHC data

Bulk Property of QGP

Okamoto and Nonaka, Phys.Rev. C98 (2018) 054906

collisions thermalization hydro hadronization freezeout

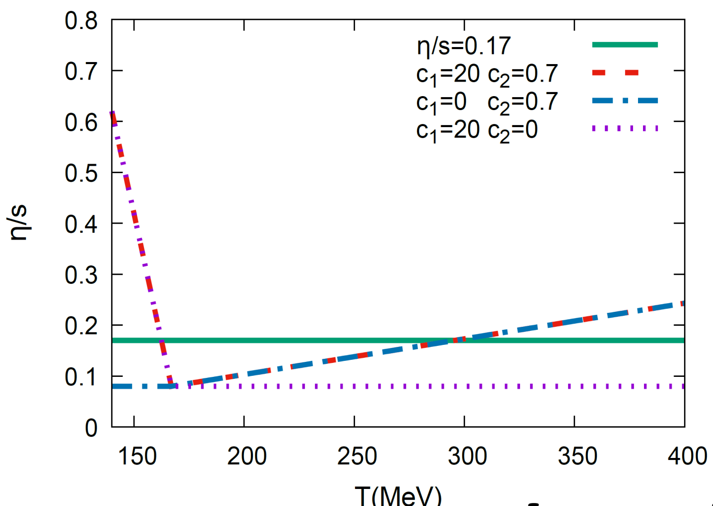




Experimental data

Our Model ⟷ *Experimental data*

Shear viscosity



ALICE Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV, LHC

- ✓ Rapidity distributions
- ✓ P_T distributions
- ✓ Mean P_T
- ✓ Collective flows v_2 and v_3

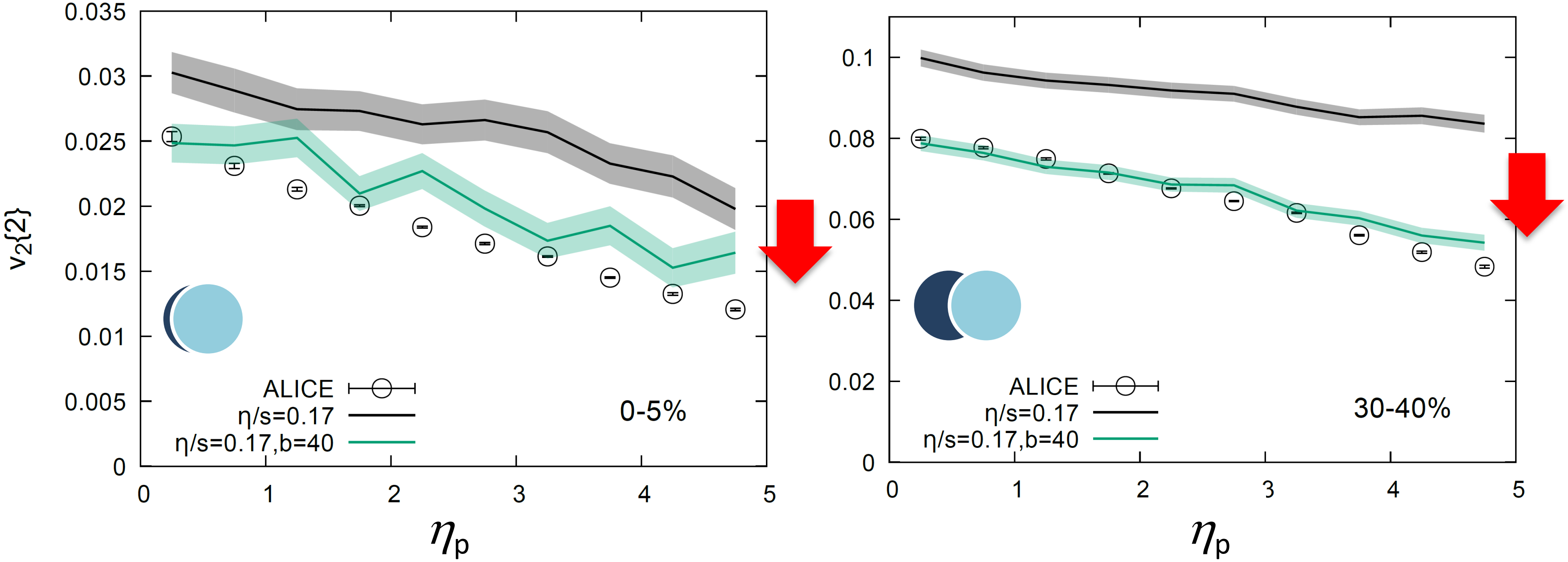
Bulk viscosity

$$\zeta = b\eta\left(\frac{1}{3} - c_s^2\right)^2 \quad b = 40$$

What physical observable is interesting?

Effect on Collective Flow

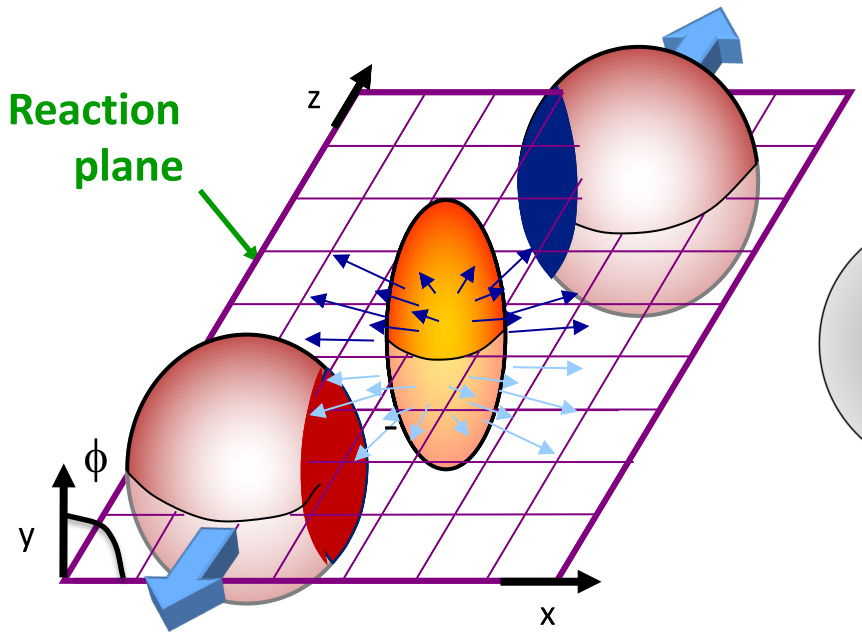
- Collective flow as a function of η_p

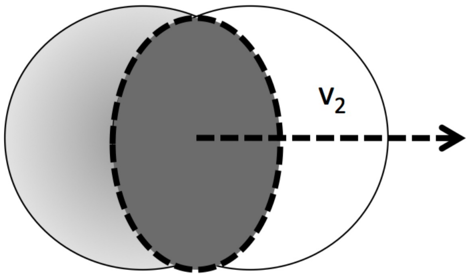


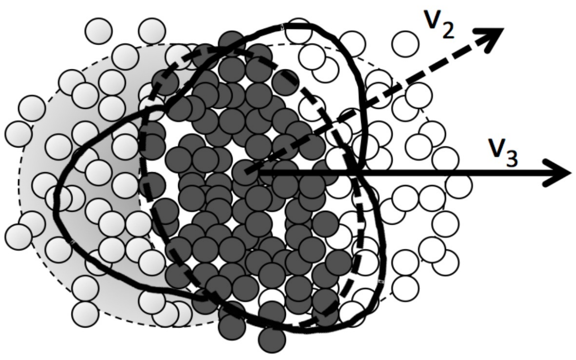
- (3+1)-d calculation
- v_n with bulk viscosity is much closer to the ALICE data: amplitude and slope
- Effect of bulk viscosity at forward rapidity is large.

Finite bulk viscosity

Collective Flow



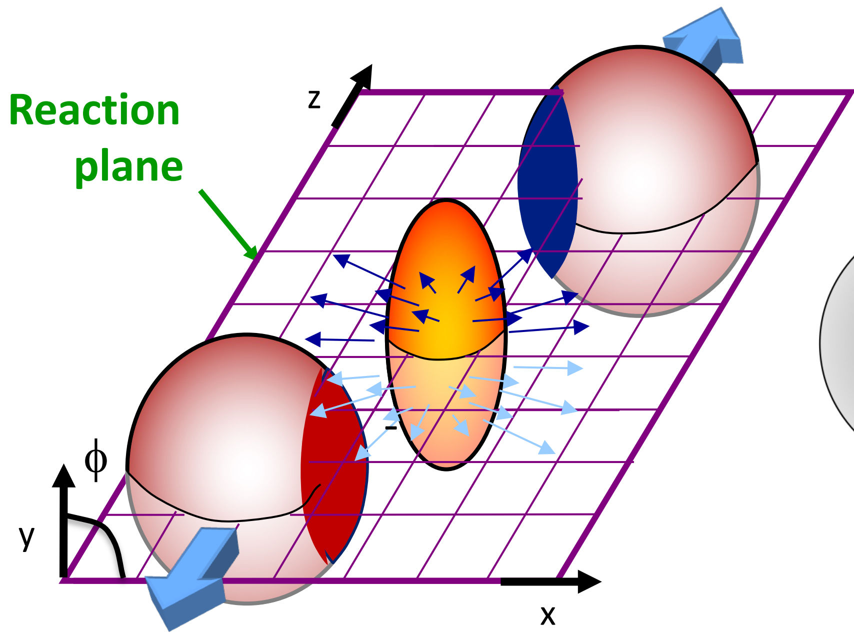




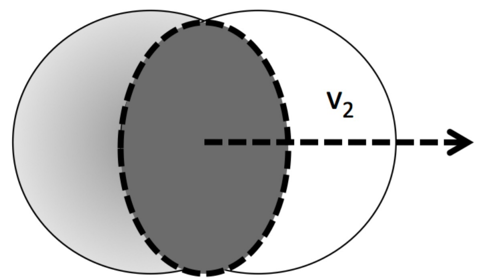
$$\frac{dN}{d\phi} \sim N_0(1 + \underline{2v_1} \cos\phi + \underline{2v_2} \cos 2\phi)$$

Directed flow　　Elliptic flow

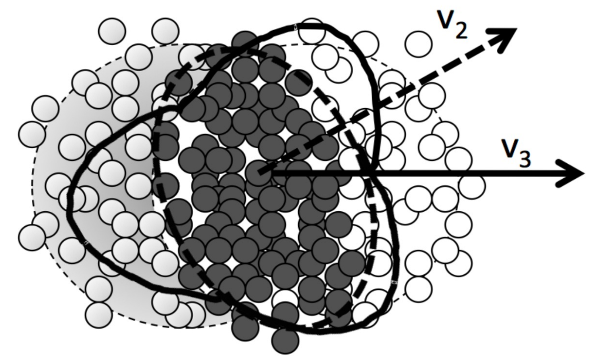
Collective Flow



almond shape



more realistic….

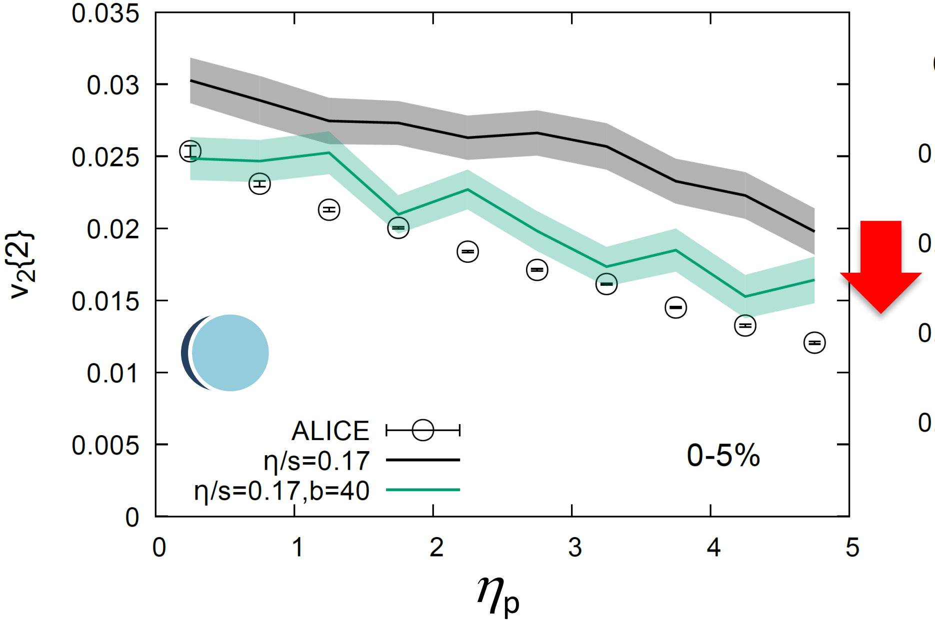


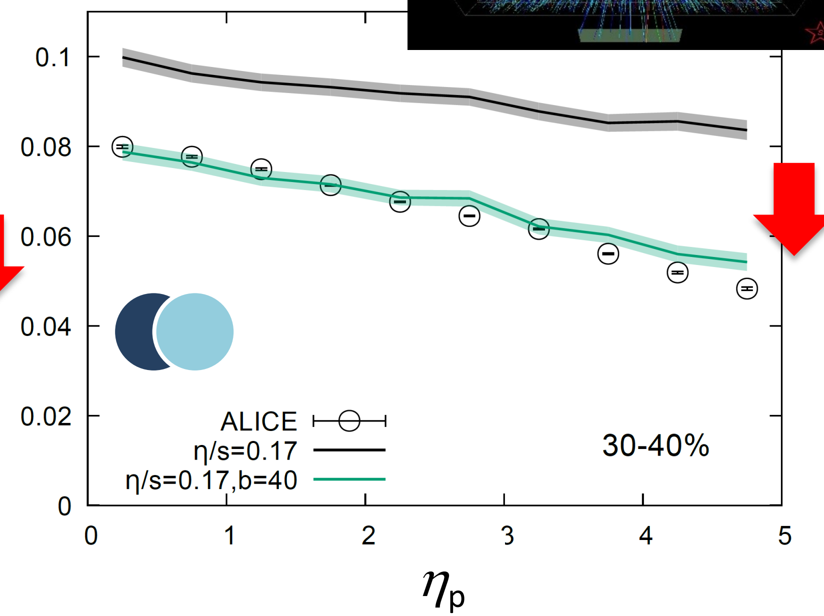
$$\frac{dN}{d\phi} \sim N_0(1 + \underline{2v_1} \cos\phi + \underline{2v_2} \cos 2\phi + 2v_3 \cos 3\phi + 2v_4 \cos 4\phi + \cdots)$$

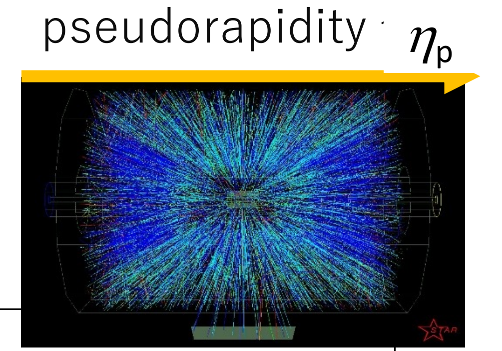
Directed flow   Elliptic flow

Effect on Collective Flow

- Collective flow as a function of η_p





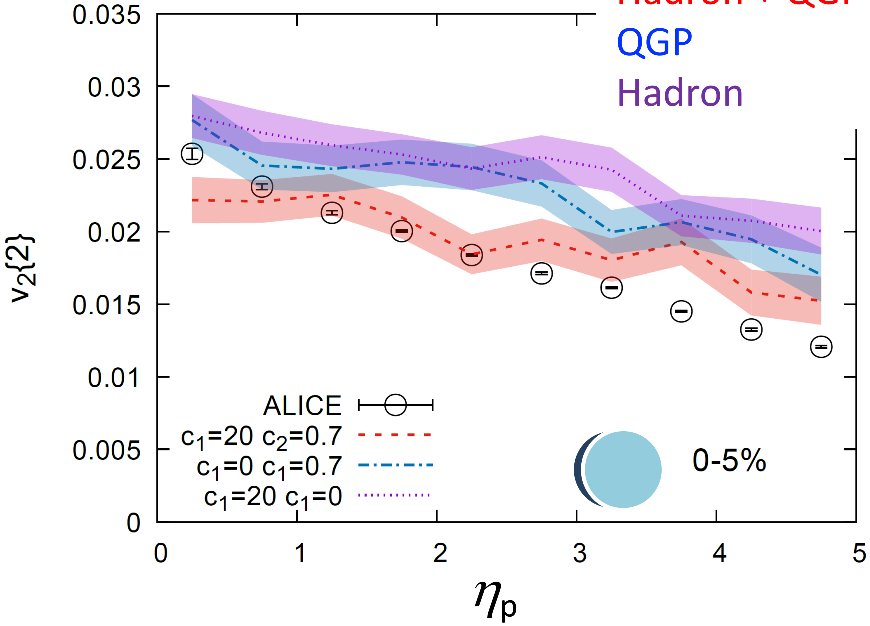


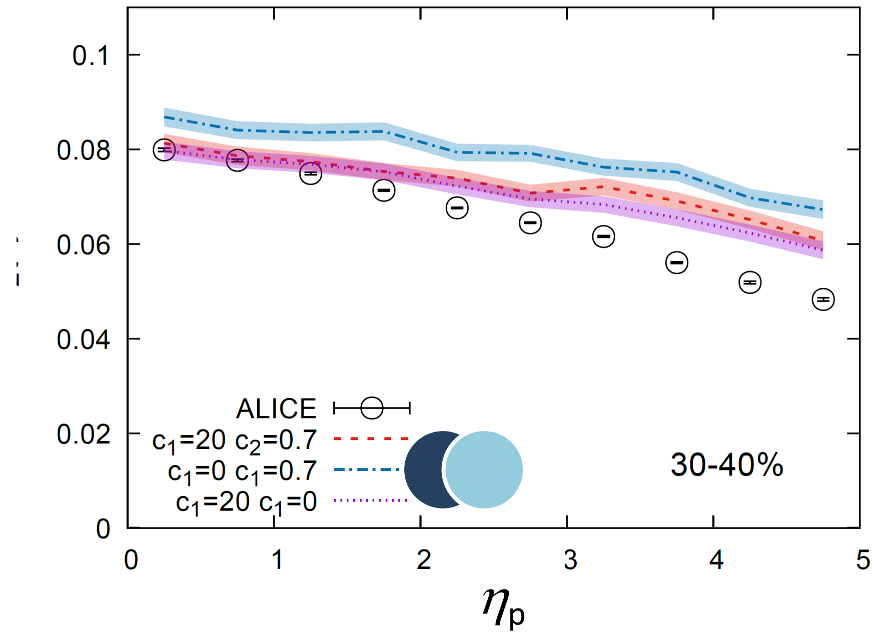
- (3+1)-d calculation
- v_n with bulk viscosity is much closer to the ALICE data: amplitude and slope
- Effect of bulk viscosity at forward rapidity is large.

Finite bulk viscosity

Temperature Dependent η/s

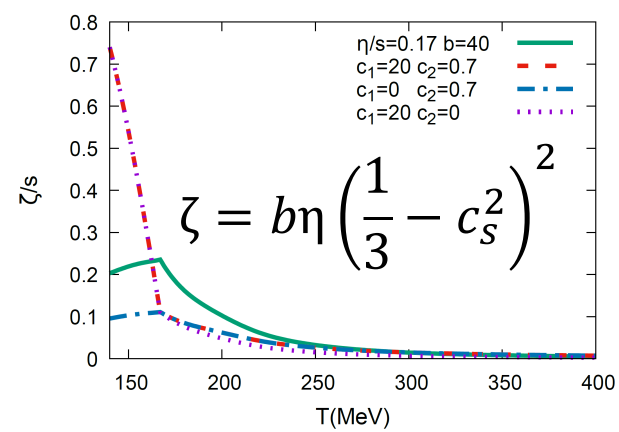
Hadron + QGP
QGP
Hadron

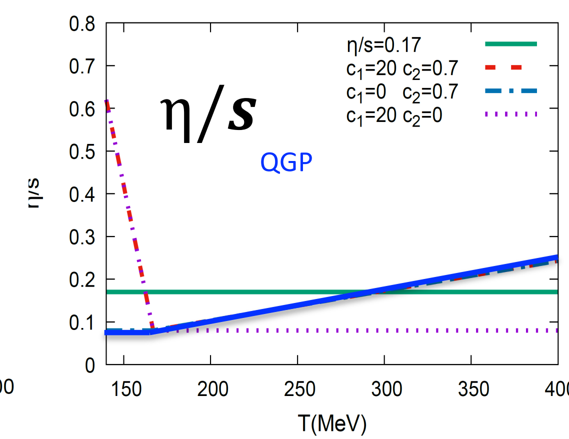




QGP phase: $\eta/s(T)$
Hadron phase: $\eta/s=0.08$

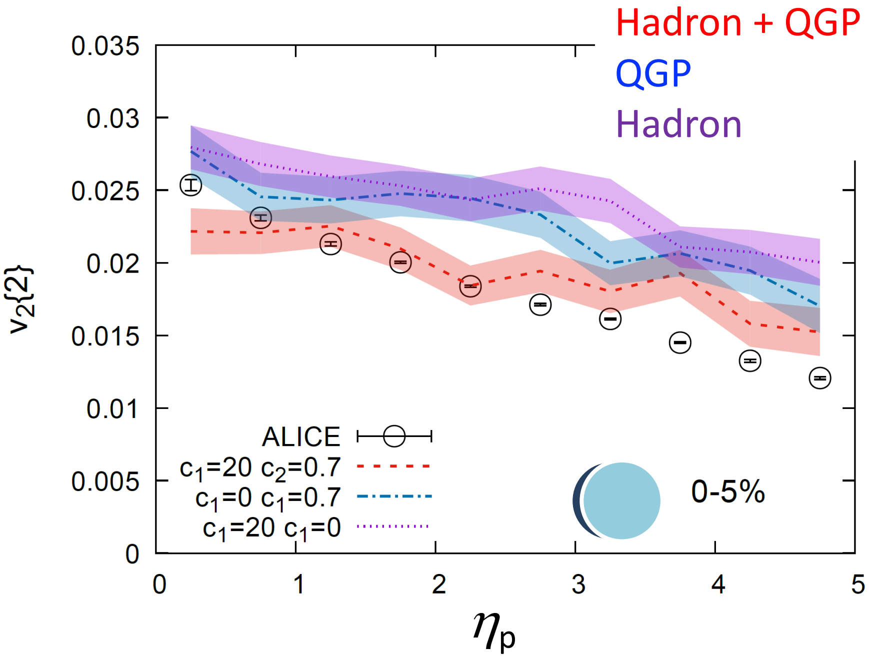
In both centrality classes $v_2(\eta_p)$ is larger.

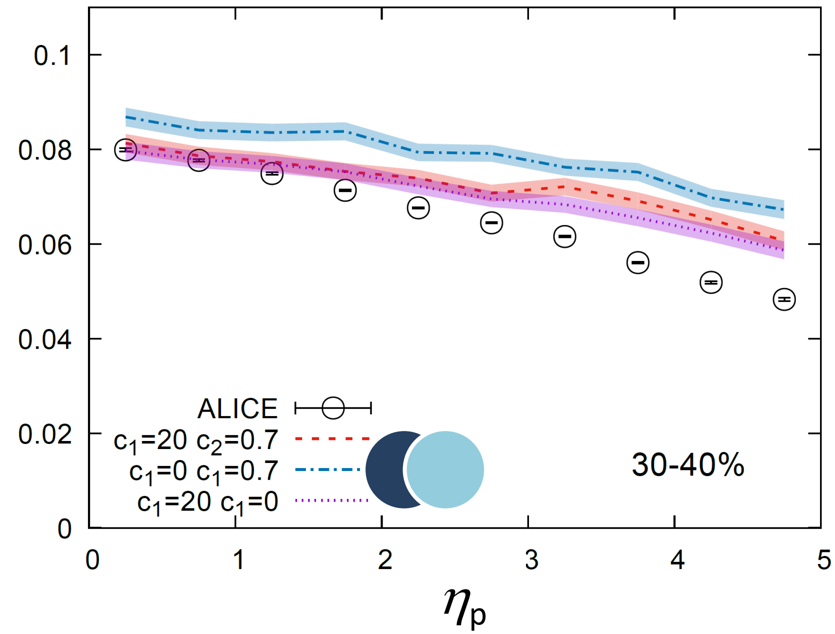




$T_{\rm SW} = 150$ MeV

Temperature Dependent η/s



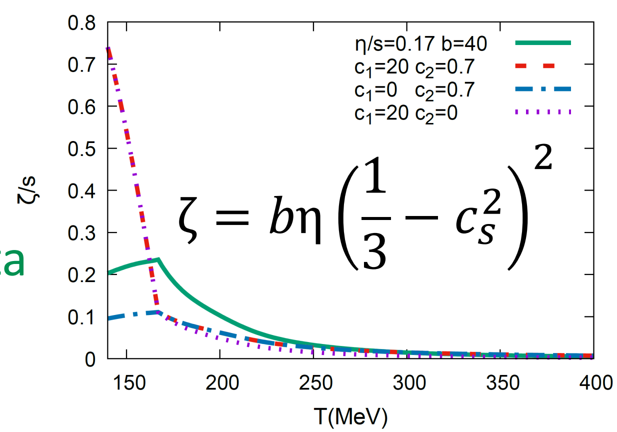


QGP phase: η/s=0.08

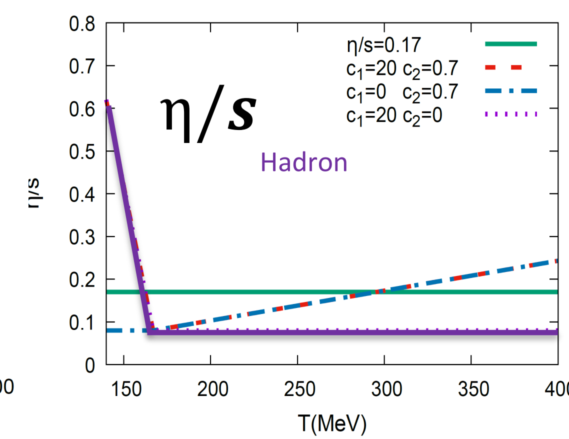
Hadron phase: $\eta/s(T)$

0-5%: larger than ALICE data

30-40%: close to ALICE data



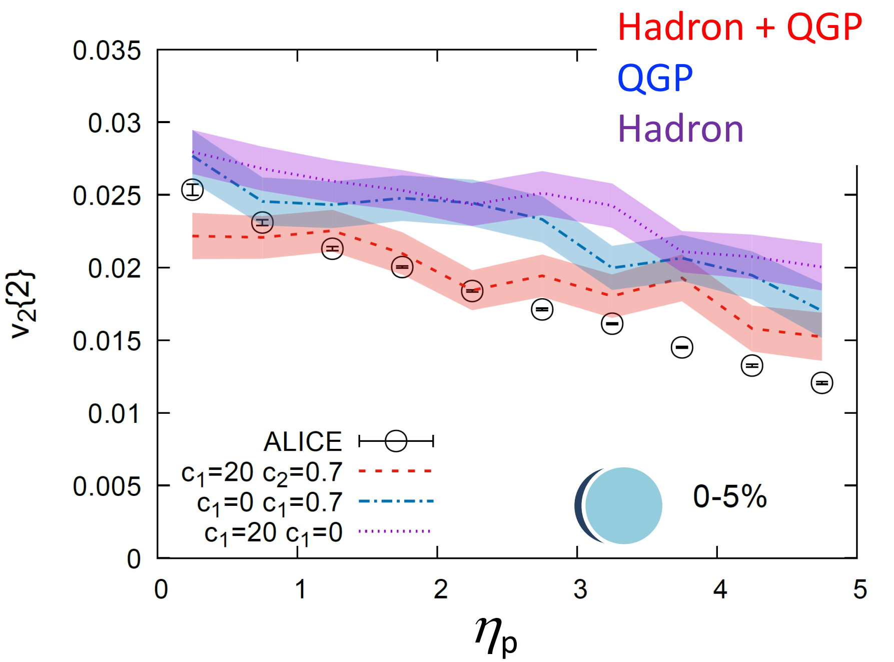
$$\zeta = b\eta\left(\frac{1}{3} - c_s^2\right)^2$$

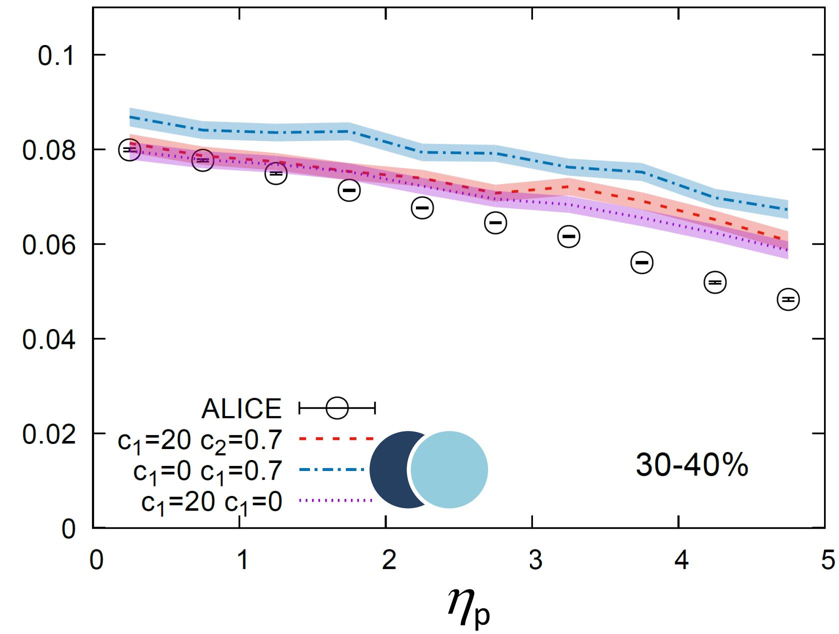


$T_{\rm SW} = 150$ MeV

C. NONAKA

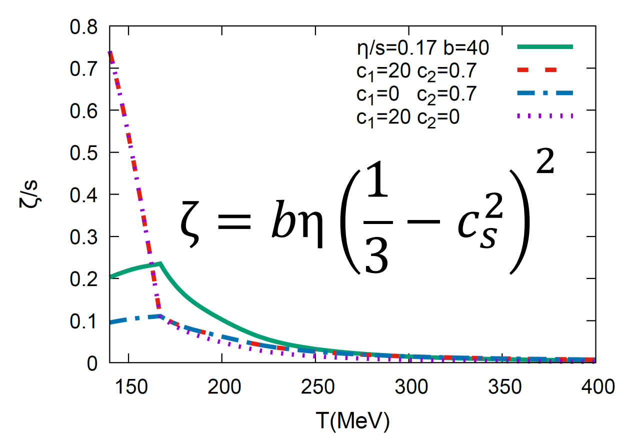
Temperature Dependent η/s

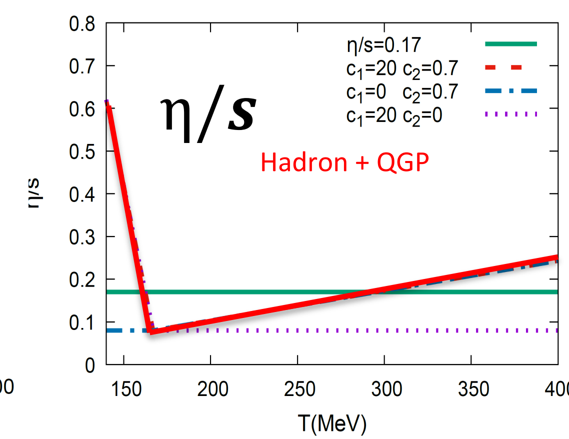




QGP phase: *η/s(T)*
Hadron phase: *η*/s(T)

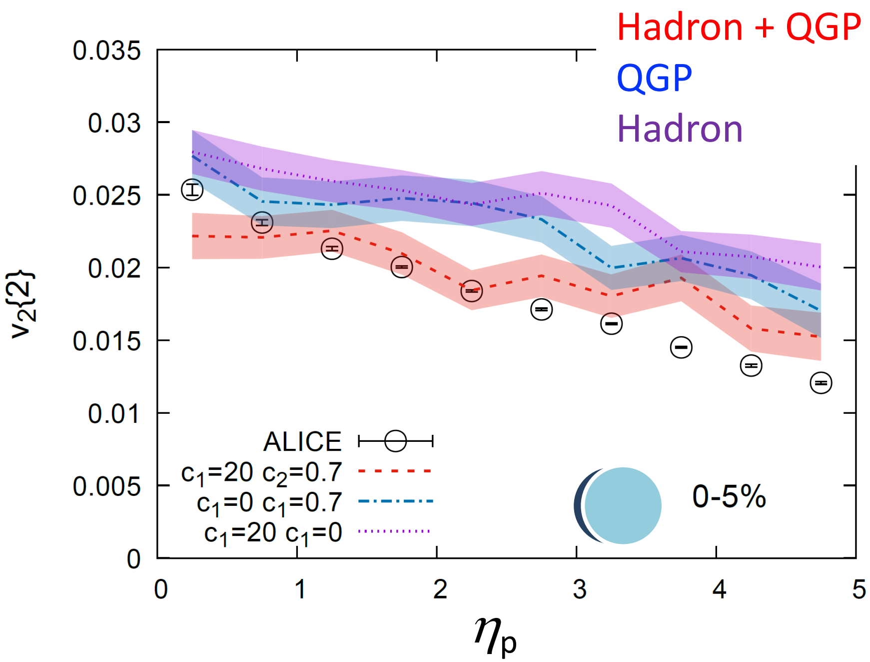
In both central classes,
close to ALICE data

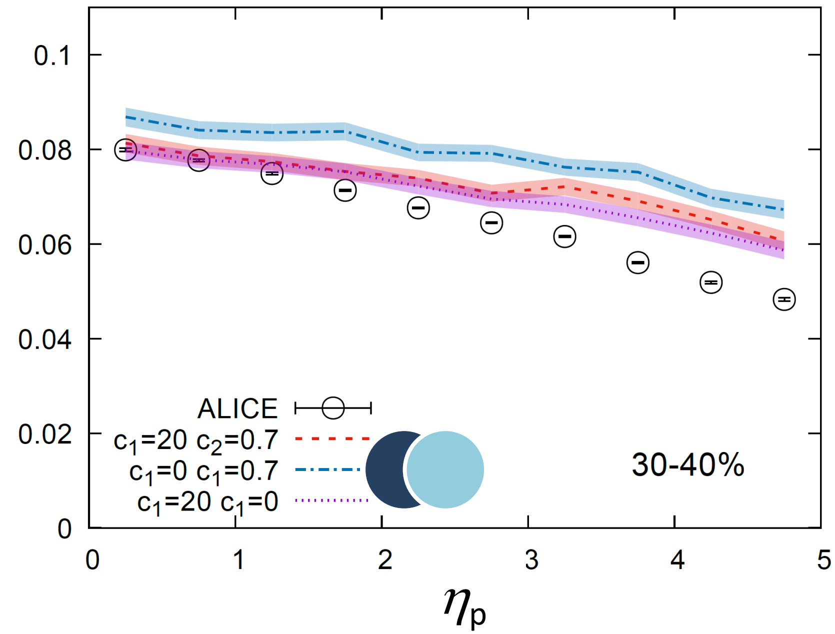




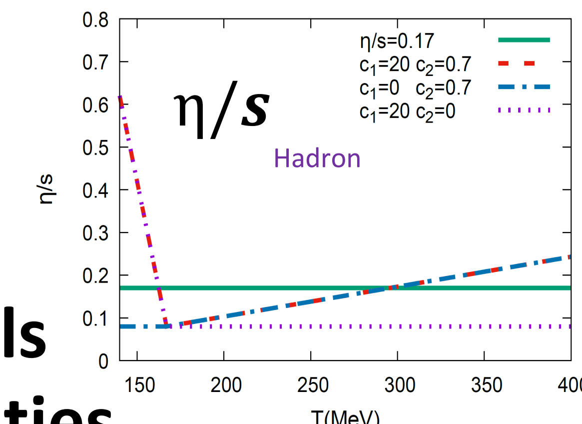
$T_{\mathrm{SW}} = 150$ MeV

Temperature Dependent η/s





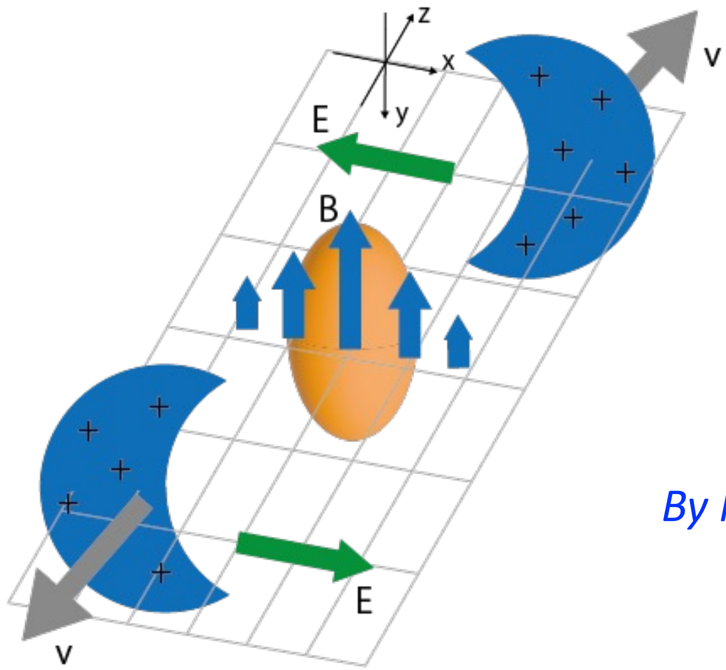
- 0-5 % centrality
  η/s of QGP and hadron phases is important.
- 30-40 % centrality
  η/s of hadron phase is important.



Central dependence of $v_2(\eta_p)$ reveals temperature dependence of viscosities.

Electromagnetic Field in HIC

- Strong Electromagnetic field ?
  - Au + Au ($\sqrt{s_{NN}} = 200$ GeV) : 10^{14} T ~10 m_π^2
  - Pb + Pb ($\sqrt{s_{NN}} = 2.76$ TeV) : 10^{15} T

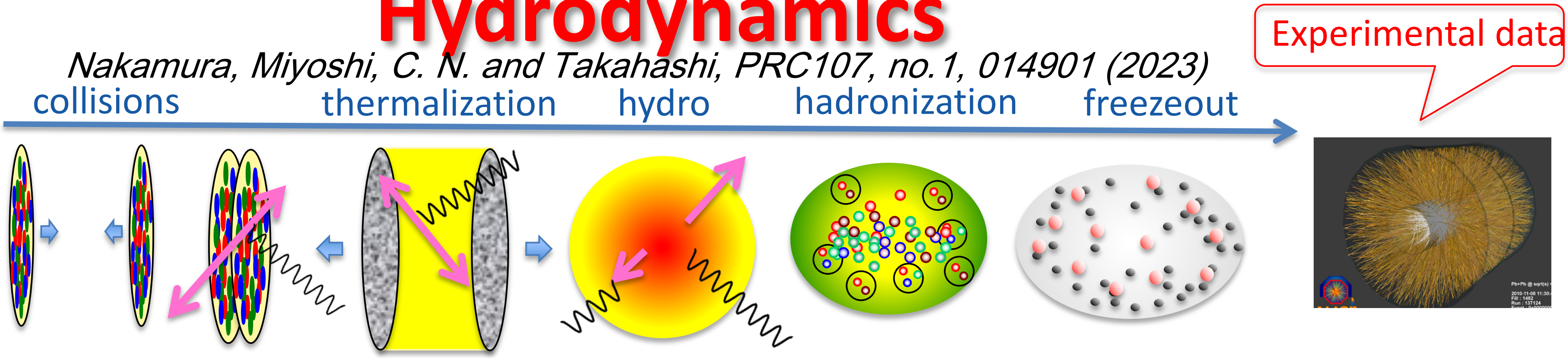


By Nakamura

Nakamura, Miyoshi, C. N. and Takahashi, Phys. Rev. C 107, no.1, 014901 (2023)
Nakamura, Miyoshi, C. N. and Takahashi, arXiv:2211.02310 [nucl-th]
Nakamura, Miyoshi, C. N. and Takahashi, arXiv:2212.02124 [nucl-th]

Relativistic Resistive Magneto-Hydrodynamics

Nakamura, Miyoshi, C. N. and Takahashi, PRC107, no.1, 014901 (2023)



Initial conditions

Hydrodynamics

Glauber model
+approximate solutions of Maxwell eq.

Hydrodynamic eq. + Maxwell eq. + Ohm's law

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad J^\mu = \sigma e^\mu$$

Relativistic Resistive Magneto-Hydrodynamics (RRMHD)

Nakamura, Miyoshi, C.N. and Takahashi, arXiv:2211.02310 [nucl-th]

■ RRMHD equation

➢ Conservation law＋Maxwell eq. ＋Ohm's law

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$

$$J^\mu = J_c^\mu + q u^\mu$$

e: energy density
p: pressure
$p_{em} = (E^2 + B^2)/2$

$\varepsilon = (e+p)\gamma^2 - p + p_{em}$
$m^i = (e+p)\gamma^2 v^i + \epsilon^{ijk} B_j E_k$
$\Pi^{ij} = (e+p)\gamma^2 v^i v^j + (p + p_{em}) g^{ij} - E^i E^j - B^i B^j$

Energy Conservation

$$\partial_t \varepsilon + \nabla \cdot m = 0$$

Momentum conservation

$$\partial_t m^i + \nabla \cdot \Pi^i = 0$$

Faraday's law

$$\partial_t \vec{B} + \nabla \times \vec{E} = 0$$

Ohm's law

$$\vec{J} = q\vec{v} + \underbrace{\sigma\gamma[\vec{E} + \vec{v} \times \vec{B} - (\vec{v} \cdot \vec{E})\vec{v}]}_{=: \vec{J}_c}$$

Ampere's law

$$\partial_t \vec{E} - \nabla \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{E} - \nabla \times \vec{B} = q\vec{v}$$

$$\partial_t \vec{E} = \vec{J}_c \quad \text{演算子分離}$$

- Integration with quasi-analytic solutions

$$\vec{E}_\perp = -\vec{v} \times \vec{B} + \left(E_\perp^0 + \vec{v} \times \vec{B}\right) \exp(-\sigma\gamma t)$$

$$\vec{E}_\parallel = E_\parallel^0 \exp(-\sigma t/\gamma)$$

Komissarov, Mon. Not. R. Astron. Soc. 382, 995-1004 (2007)

RRMHD eq. in the Milne Coordinates

Nakamura, Miyoshi, C. N, and Takahashi, arXiv:2211.02310 [nucl-th]

- Milne Coordinates
  - **Expansion system in the longitudinal coordinates** $(\boldsymbol{\tau}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta_s})$
    - Strong velocity in the longitudinal direction -> 0
    - Saves the number of cells in the collision axis direction

$$\tau = \sqrt{t^2 - z^2}$$
$$\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z}$$

RRMHD eq.

$$\partial_\tau(\tau U) + \partial_i(\tau F^i) = \tau S$$

$$U = \begin{pmatrix} D \\ m_j \\ \varepsilon \\ B^j \\ E^j \\ q \end{pmatrix}, \mathrm{F}^{\mathrm{i}} = \begin{pmatrix} Dv^i \\ \Pi^{ji} \\ m^i \\ \varepsilon^{jik}E_k \\ \varepsilon^{jik}B_k \\ J^i \end{pmatrix}, S = \begin{pmatrix} 0 \\ \frac{1}{2}T^{ik}\partial_j g_{ik} \\ -\frac{1}{2}T^{ik}\partial_0 g_{ik} \\ 0 \\ J_c^i \\ 0 \end{pmatrix}$$

Newly developed RRMHD code in Milne coordinates

Validation of the Code

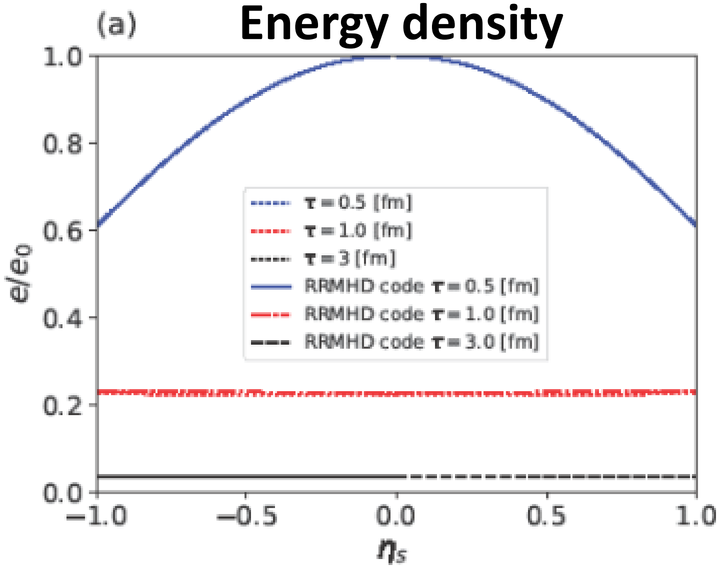
Nakamura, Miyoshi, C. N.and Takahashi, arXiv:2211.02310 [nucl-th]

- RRMHD in the Milne coordinates

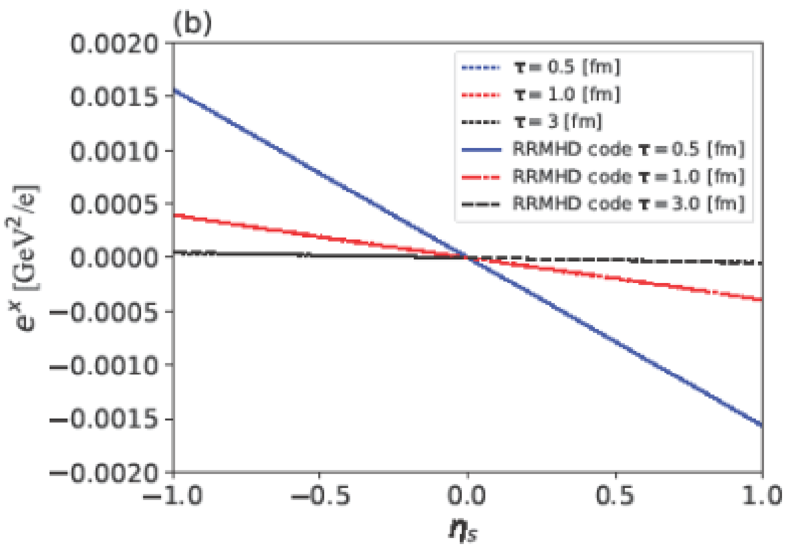
New Test Problem

- **(1+1) dimensional expansion system** $u^\mu = (\cosh Y, 0, 0, \sinh Y)$
  - **Comparison between quasi-analytical solution and RRMHD simulation**

Energy density



Electric field



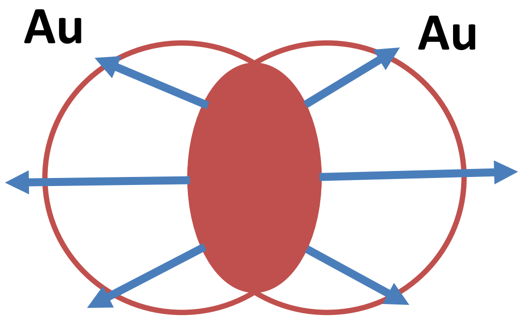
Solid line : RRMHD code
Dashed line: quasi-analytical solution

➡ **Application to Heavy Ion Collisions**

Symmetric and Asymmetric Systems

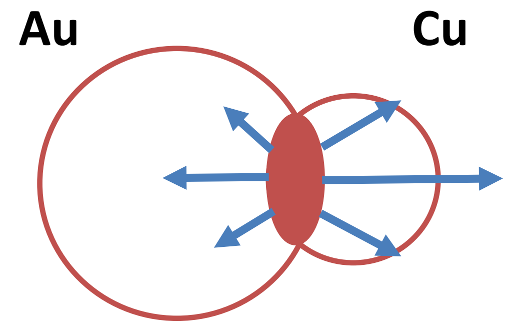
RHIC $\sqrt{s_{NN}} = 200\ \mathrm{GeV}$

■Au-Au collisions



- Symmetric pressure gradient
- Almond-shaped medium

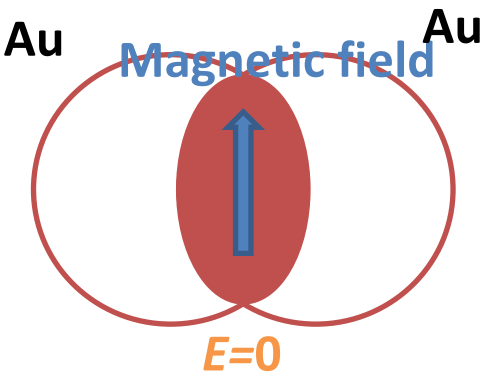
■Cu-Au collisions



- Asymmetric pressure gradient
- Distorted Almond-shaped medium

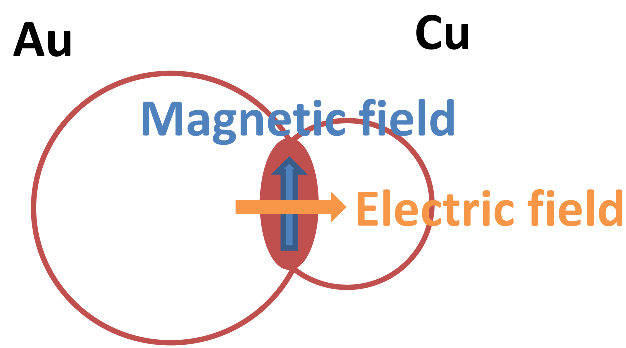
Electromagnetic Field in Symmetric and Asymmetric Systems

■Au-Au collisions



- Magnetic field
  - Strong magnetic field
- Electric field
  - No electric field

■Cu-Au collisions



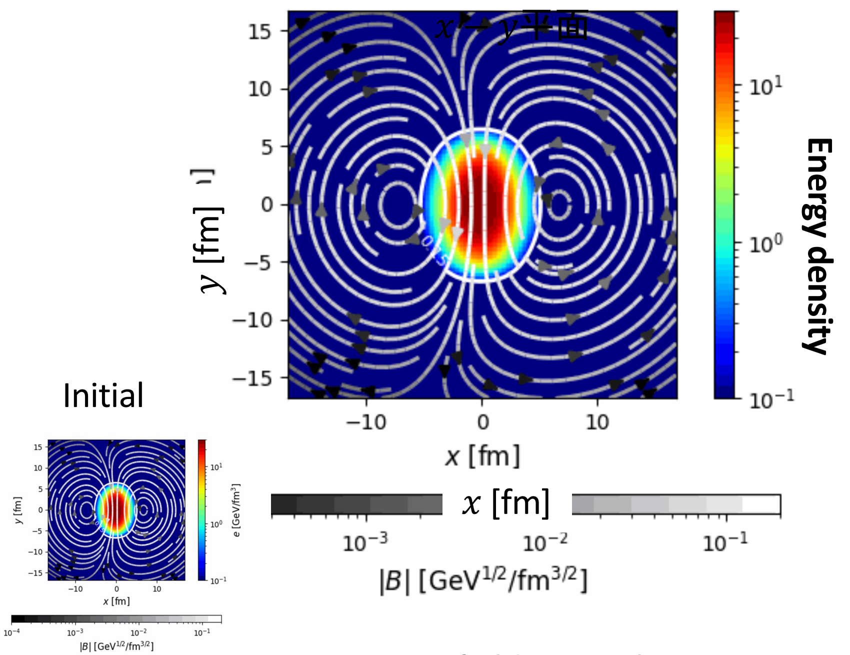
- Magnetic field
  - Strong magnetic field
- Electric field
  - $E \neq 0$ **due to the asymmetry of the charge distribution**

Space-time Evolution

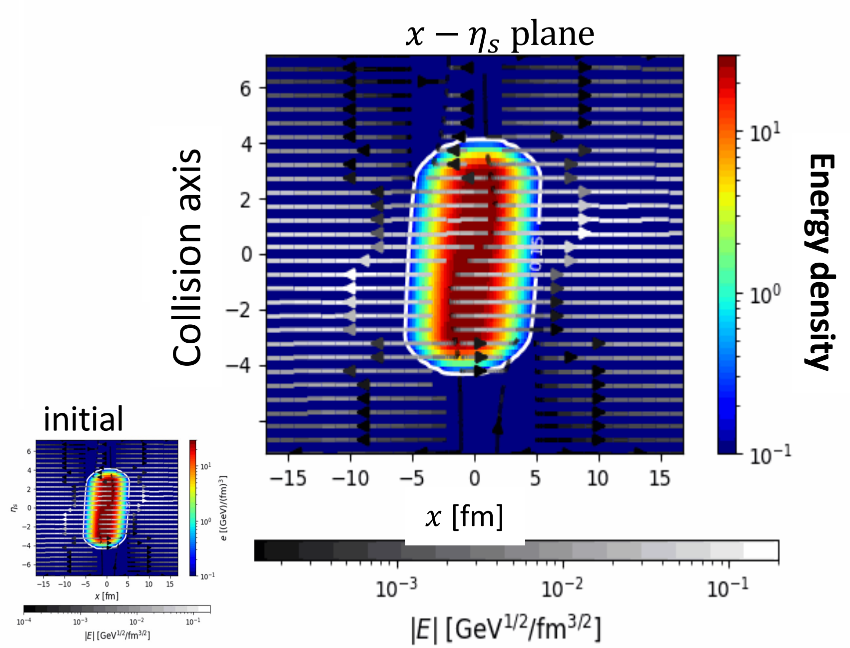
Nakamura, Miyoshi, C. N. and Takahashi, PRC 107, no.1, 014901 (2023)

Au+Au collision system

First calculation in HIC with RRMHD code



Magnetic field strength



Electric field strength

Analysis of Heavy Ion Collisions

Directed Flow

Nakamura, Miyoshi, C. N. and Takahashi, PRC 107, no.1, 014901 (2023)

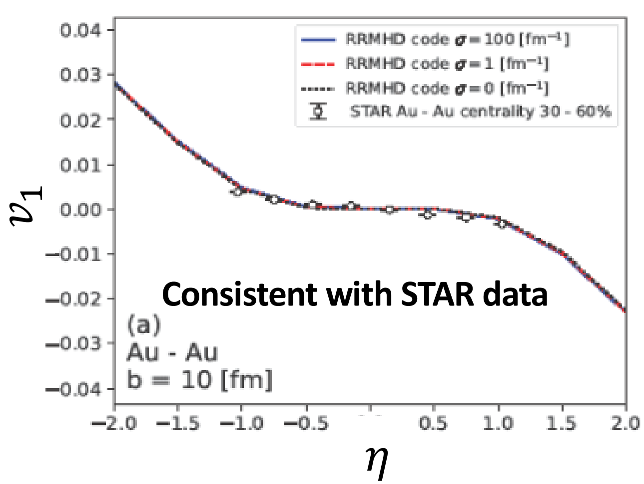
- $v_1 := \langle \cos(\phi - \Psi_1) \rangle \sim \langle \frac{p_x}{p_T} \rangle$

$$\eta = \frac{1}{2}\ln\frac{|p| + p_z}{|p| - p_z}$$

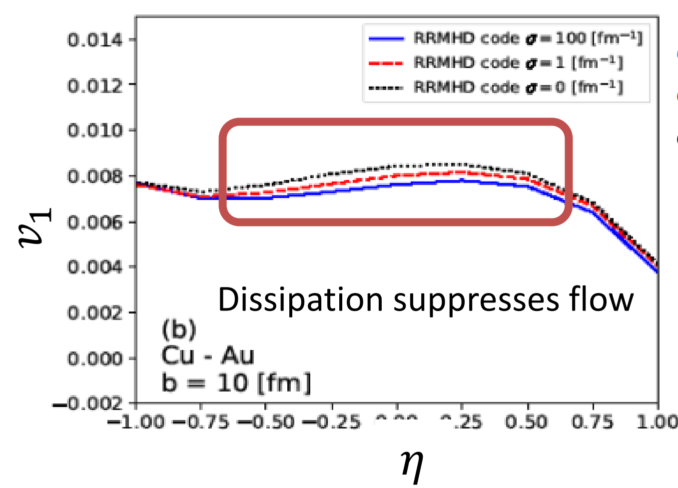
➤ Au-Au collisions ($\sqrt{s_{NN}} = 200\ \mathrm{GeV}$)
- Parameter fixed in initial condition from comparison with STAR data

➤ Cu-Au collisions ($\sqrt{s_{NN}} = 200\ \mathrm{GeV}$)
- Decreases with conductivity
- Dissipation suppresses flow in the Cu direction

STAR Collaboration, Phys. Rev. Lett. **101** (2008), 252301



$\sigma = 100$ fm^{-1}
$\sigma = 1$ fm^{-1}
$\sigma = 0$ fm^{-1}
◦ : STAR



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$\sigma = 0$ fm^{-1}

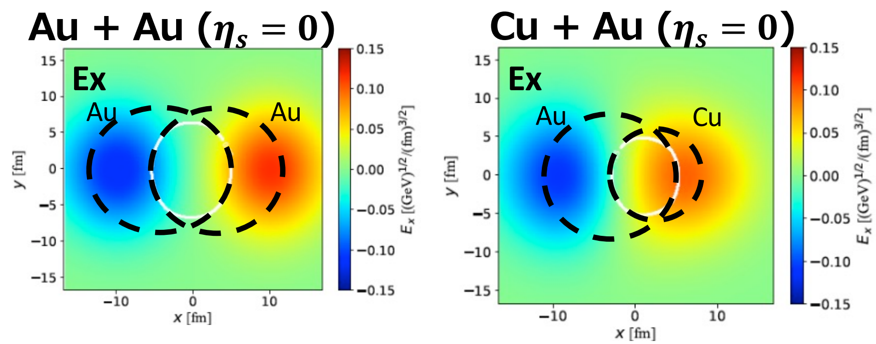
Energy Transfer by Ohm Dissipation

Nakamura, Miyoshi, C. N. and Takahashi, PRC 107, no.1, 014901 (2023)

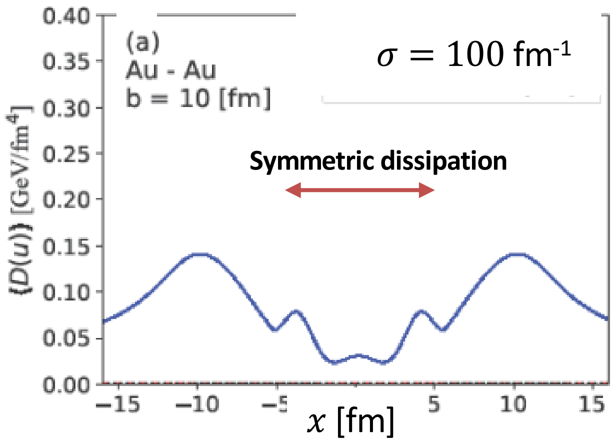
■ Energy Transfer

$$D(u) := j^\mu e_\mu = \gamma[j\cdot(E+v\times B) - q(v\cdot E)]$$

energy of
the electromagnetic field → Thermal energy / Kinetic energy

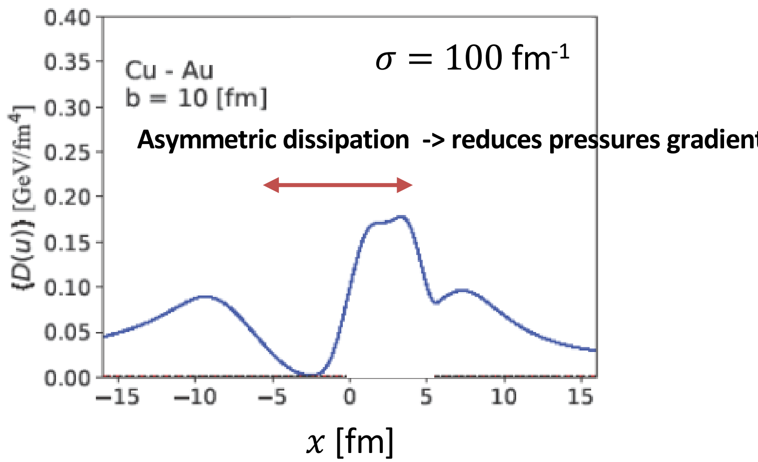


Au+Au collisions



no contribution to $\boldsymbol{v_1}$

Cu+Au collisions



contribution to $\boldsymbol{v_1}$

Directed Flow

Nakamura, Miyoshi, C. N. and Takahashi, PRC 107, no.1, 014901 (2023)

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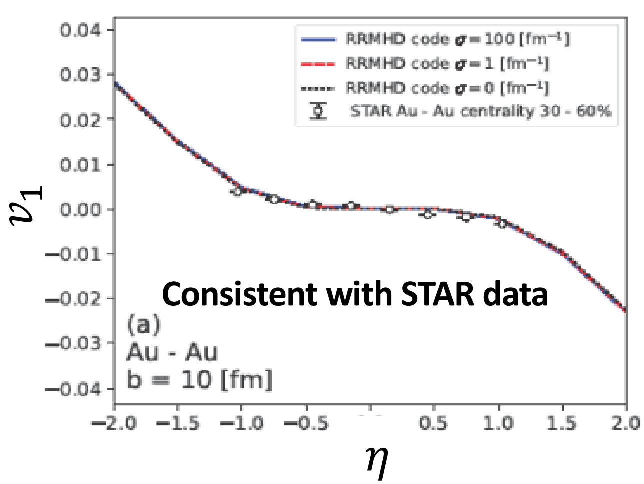

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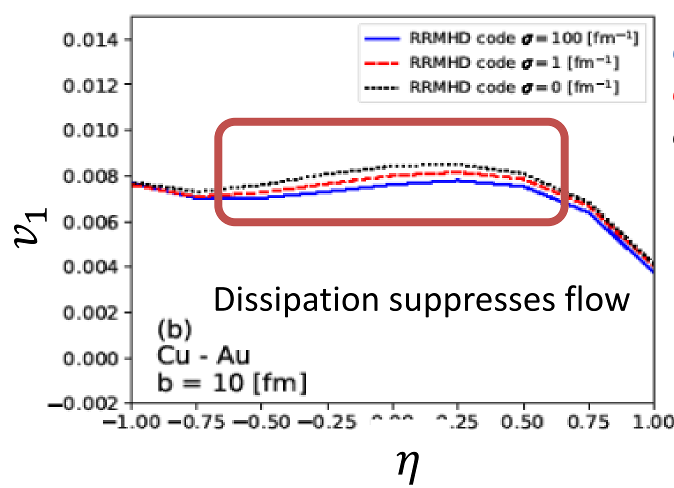
◦ : STAR

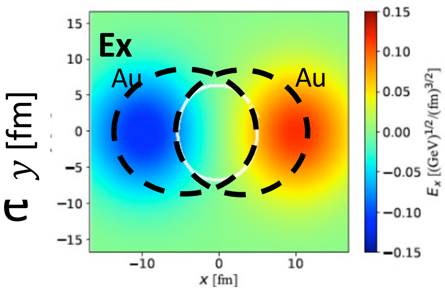


$\sigma = 100$ fm^{-1}
$\sigma = 1$ fm^{-1}
$\sigma = 0$ fm^{-1}

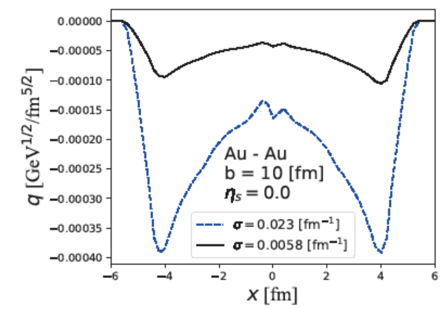
Charge Dependence of Δv_2 : Au + Au

Nakamura, Miyoshi, C. N. and Takahashi, PRC 107, no.1, 014901 (2023)

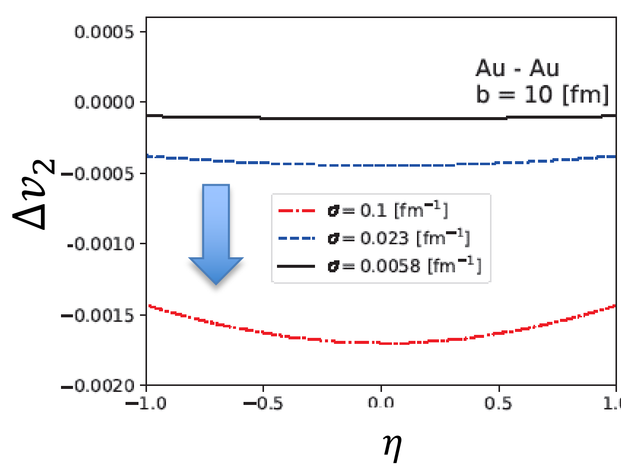
- $\Delta v_2 = v_2^{\pi^+}(\eta) - v_2^{\pi^-}(\eta)$
  - **Negative Elliptic Flow**
    - Contribution of negative charge on freezeout hypersurface
    - Symmetric structure: initial electric field to the collision axis
    - **Electric conductivity dependence is observed even in the symmetry system.**



Charge distribution
on freezeout hypersurface



x [fm] x [fm]



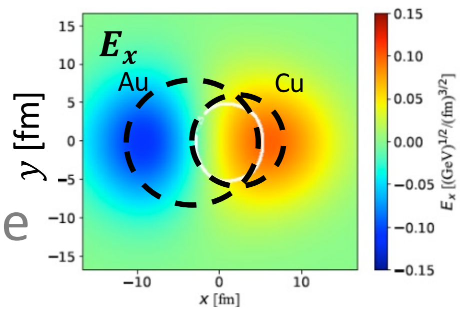
$\sigma = 0.1$ fm^{-1}
$\sigma = 0.023$ fm^{-1}
$\sigma = 0.0058$ fm^{-1}

$$\eta = \frac{1}{2}\ln\frac{|p| + p_z}{|p| - p_z}$$

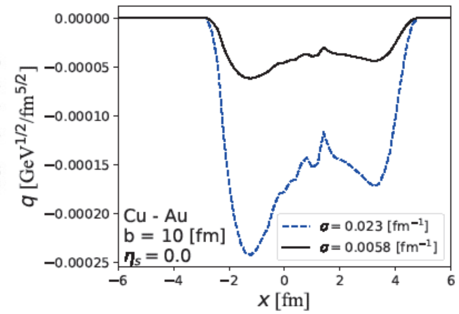
Charge Dependence of Δv_2: Cu＋Au

Nakamura, Miyoshi, C. N. and Takahashi, PRC 107, no.1, 014901 (2023)

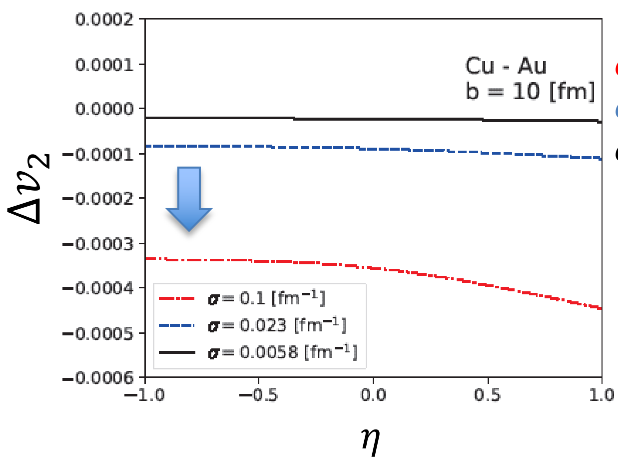
- $\Delta v_2 = v_2^{\pi^+}(\eta) - v_2^{\pi^-}(\eta)$
  - Negative Elliptic Flow
    - Contribution of negative charge on freezeout hypersurface
    - Asymmetric structure: initial electric field to the collision axis
    - **Electric conductivity dependence is observed.**



Charge distribution on freezeout hypersurface



x [fm]



x [fm]

$\sigma = 0.1$ fm^{-1}
$\sigma = 0.023$ fm^{-1}
$\sigma = 0.0058$ fm^{-1}

$$\eta = \frac{1}{2}\ln\frac{|p| + p_z}{|p| - p_z}$$

Δv_2: initial electromagnetic field distribution, electrical conductivity

Charge Dependence of Δv_1 : Au＋Au

Nakamura, Miyoshi, C. N. and Takahashi, arXiv:2212.02124 [nucl-th]

- $\Delta v_1 = v_1^{\pi^+}(\eta) - v_1^{\pi^-}(\eta)$
  - **Clear dependence of charge conductivity**
    - **Proportion to electric conductivity**
    - **Negative charge induced in the opposite direction of fluid flow**

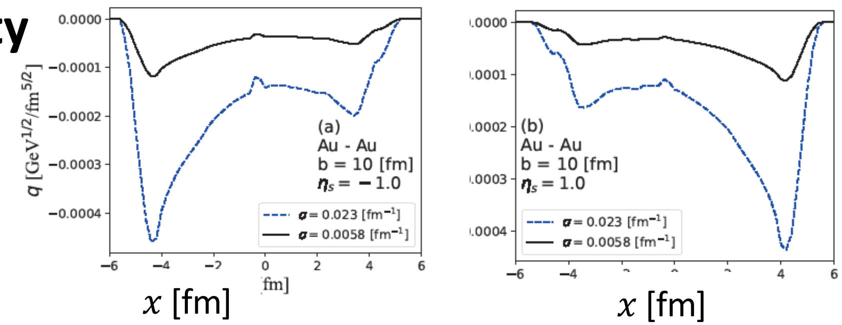
      suppression of v_1 of negative charge
  - **Δv_1 with finite σ is consistent with STAR data**
    - $\sigma = 0.0058\ \mathrm{fm}^{-1}$

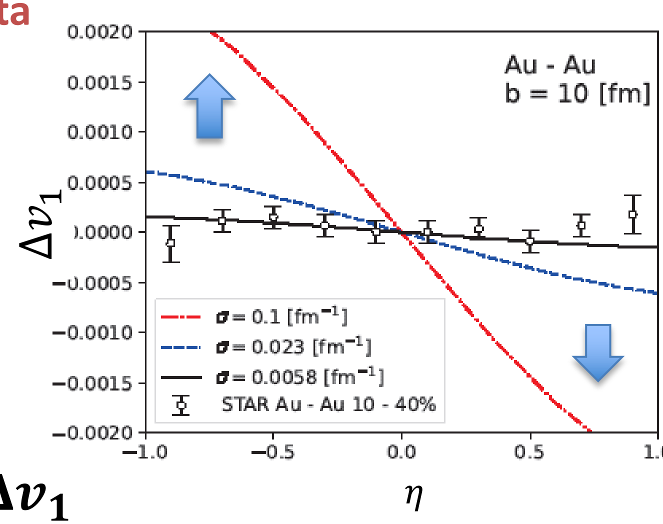
      ex. $\sigma_{LQCD} = 0.023\ \mathrm{fm}^{-1}$ from lattice QCD

      Gert Aarts, et al. Phys. Rev. Lett., 99:022002, 2007.

✓ **QGP electrical conductivity from high-precision measurement of Δv_1**

Charge distribution on freezeout hypersurface





$\sigma = 0.1$ fm⁻¹
$\sigma = 0.023$ fm⁻¹
$\sigma = 0.0058$ fm⁻¹
∘ : **STAR data**

STAR collaboration, Phys. Rev. Lett. **112** (2014) no.16, 162301

$$\eta = \frac{1}{2}\ln\frac{|p| + p_z}{|p| - p_z}$$

Charge Dependence of Δv_1 : Au+Au

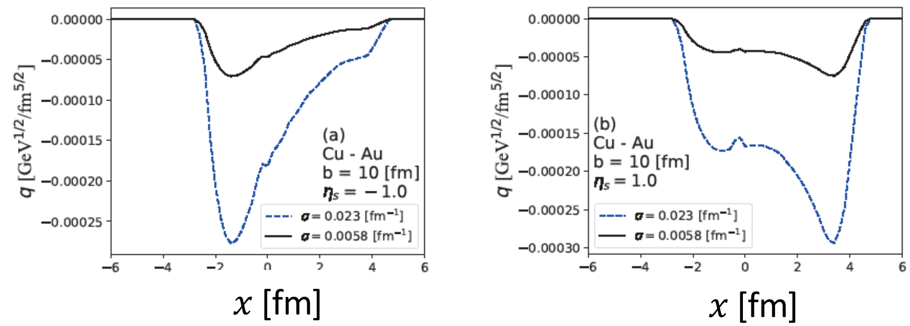
Nakamura, Miyoshi, C. N. and Takahashi, arXiv:2212.02124 [nucl-th]

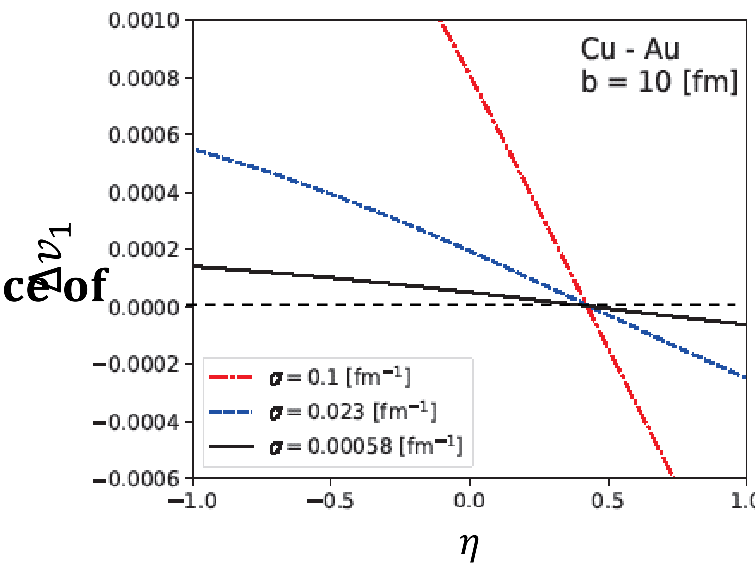
- $\Delta v_1 = v_1^{\pi^+}(\eta) - v_1^{\pi^-}(\eta)$
  - **Electric field created by initial condition**
    - Δv_1 is finite at $\eta = 0$
    - Asymmetry structure to $\eta = 0$
  - **Proportion to electric conductivity**
    - Δv_1 vanishes at $\eta = 0.5$.

✓ **Electrical conductivity <- Δv_1 at $\eta = 0$**

✓ **Initial electrical field from η dependence of Δv_1**

Charge distribution on freezeout hypersurface





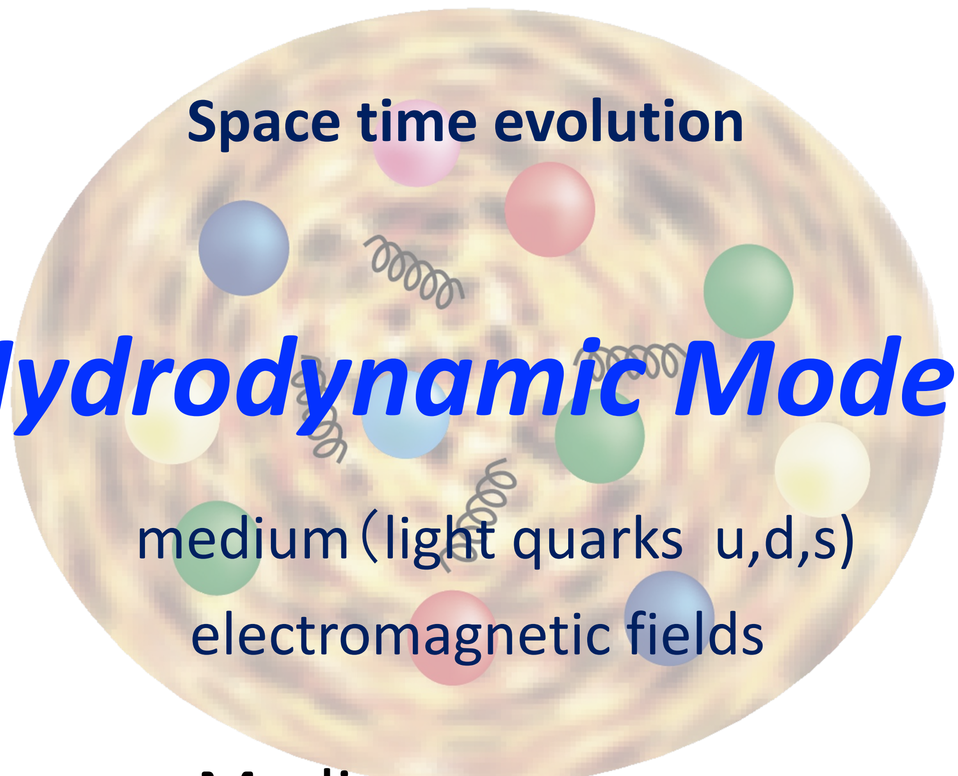
$\sigma = 0.1$ fm^{-1}
$\sigma = 0.023$ fm^{-1}
$\sigma = 0.0058$ fm^{-1}

Asymmetric system had advantage in explore of QGP electrical conductivity.

Summary

- **Tools for analyses of relativistic heavy ion collisions**
  - New relativistic viscous hydrodynamics code
  - New relativistic resistive hydrodynamic model

Quantitative Analysis on QGP bulk property



Medium

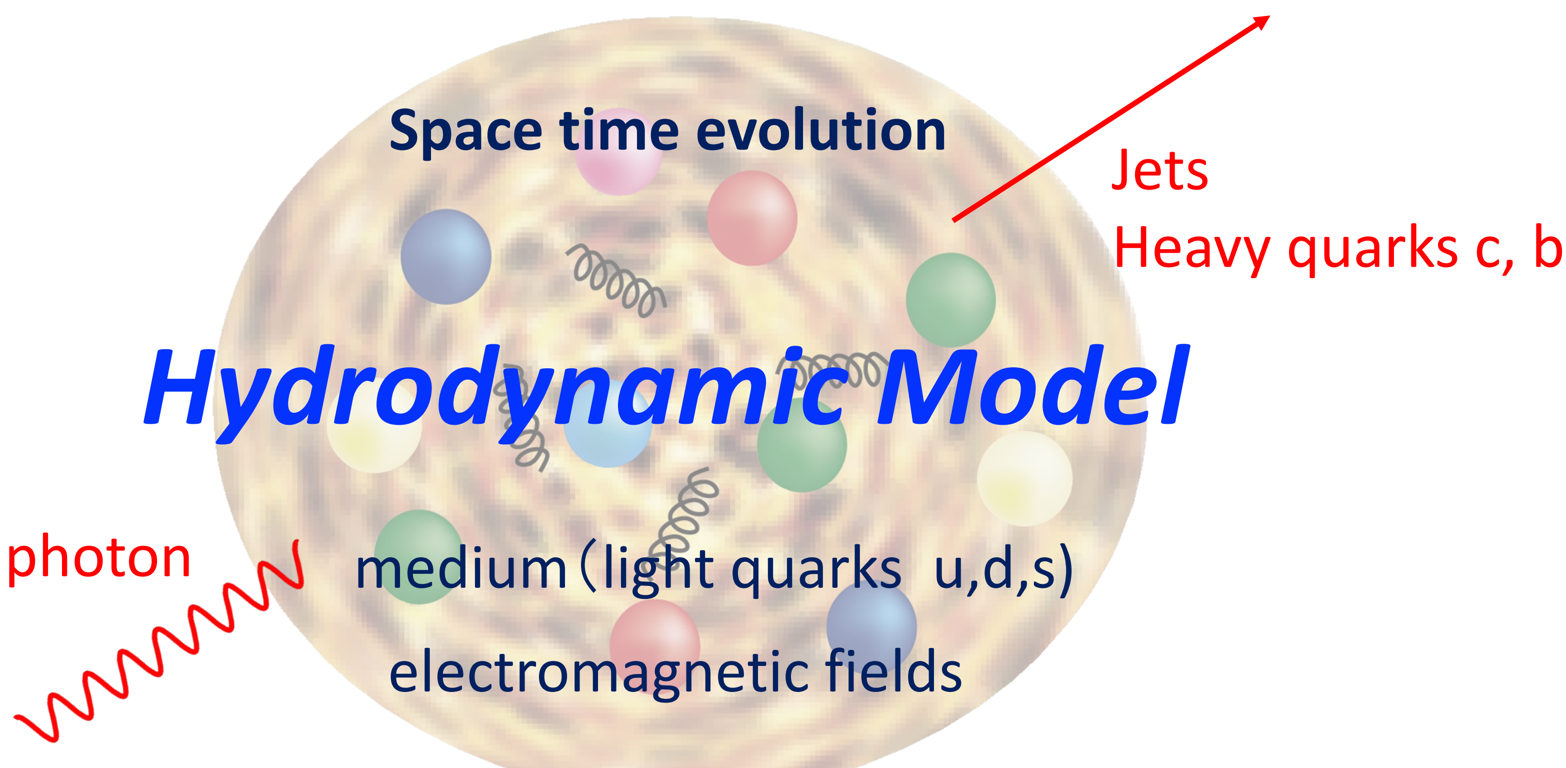
temperature dependence of viscosities

Electric conductivity

Summary

- **Tools for analyses of relativistic heavy ion collisions**
  - Application to other physical observables

    Jets, heavy quarks, photons, electromagnetic probes…



Medium + Physical Observables