Hadronic Vacuum Polarization contribution to muon $g-2$

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Abstract

I show the Hadronic Vacuum Polarization (HVP) contribution to muon $g-2$ in the framework of chiral perturbation theory with vector meson.

I focus on the two pion contribution to HVP.

In the framework, pion form factor and anomalous magnetic moment of muon are calculated.

Outline

- Background
- Purpose
- χPT with vector mesons
- Pion vector form factor
- Numerical calculation of AMM
- Discussion
- Summary

Background

Landé g-factor of muon → quantum correction → Theoretical value from Standard Model

Theoretical value from Standard Model ↕ 4.2σ ↕ Experimental value

anomalous magnetic moment $a_\mu = \frac{g-2}{2}$

$a_\mu^{\mathrm{SM}} = 116591810(43) \times 10^{-11}$ [1]

$a_\mu^{\mathrm{exp}} = 116592061(41) \times 10^{-11}$ [2]

[1] T. Aoyama et al., Phys. Rept. 887, 1 (2020)

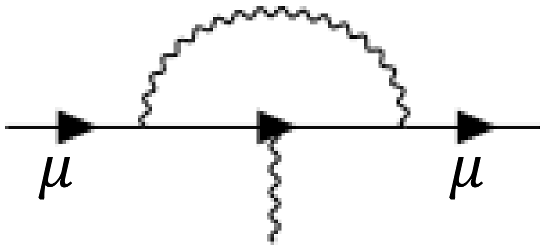
[2] B. Abi et al. (Muon g − 2 Collaboration), Phys. Rev. Lett. 126, 141801 (2021).

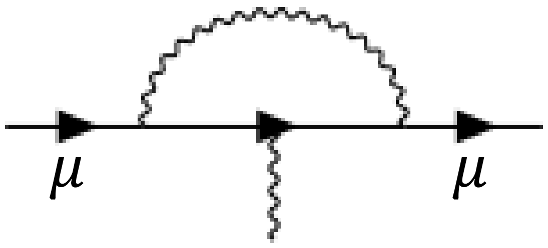
Background

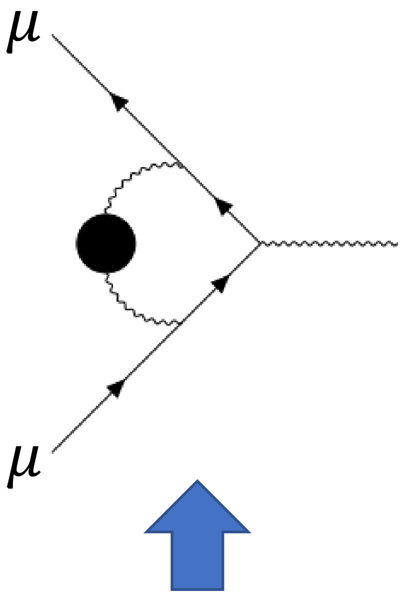
quantum correction

- Quantum Electrodynamics
- Electroweak
- Quantum Chromodynamics
  →Hadronic Vacuum Polarization, Hadronic Light by Light
- Beyond the Standard Model

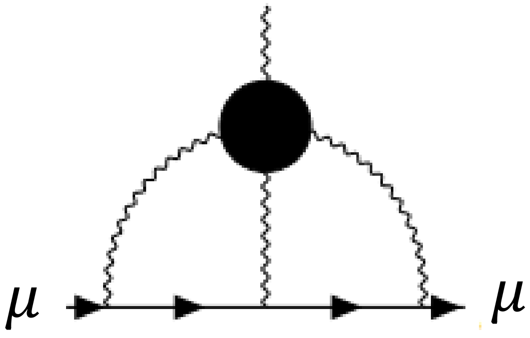
Standard Model







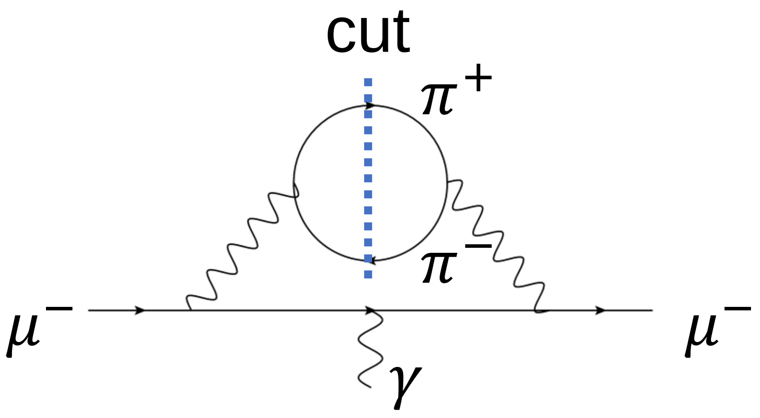
Investigate HVP



Background

We investigate HVP contribution to muon $g-2$ up to LO via $\pi^+\pi^-$ mode.

$$a_{\mu(\pi^+\pi^-)}^{\mathrm{HVP-LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R_{\pi^+\pi^-}(s)$$



$K(s)$: analytical function (Kernel function)

$$R_{\pi^+\pi^-}(s) = \frac{\sigma(e^+e^- \to \pi^+\pi^-)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$\mathrm{Im}\Pi(s) \propto \sigma(e^+e^- \to \pi^+\pi^-)$

Pion vector Form Factor (PFF): $F_{\pi^+\pi^-}^V(s)$

$$a_{\mu(\pi^+\pi^-)}^{\mathrm{HVP-LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} \frac{1}{4} \sigma_\pi^3(s) \left|F_{\pi^+\pi^-}^V(s)\right|^2$$

Background

$$a_{\mu(\pi^+\pi^-)}^{\mathrm{HVP-LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s^2} \frac{1}{4} \sigma_\pi^3(s) \left| F_{\pi^+\pi^-}^V(s) \right|^2$$

HVP contribution to muon $g-2$ includes errors due to Data-driven method.

Purpose

We investigate the vector mesons contribution to muon $g-2$.

- Lattice QCD or data-driven method

  → It's difficult to extract the contribution of vector mesons
- Chiral Perturbation theory (χPT) with vector mesons

  → We can evaluate its contribution respectively.

Need to evaluate the Applicability of Chiral perturbation theory

- Calculate PFF and compare with experimental value
- Evaluate anomalous magnetic moment

Chiral perturbation theory with vector mesons

Effective field theory to describe low energy QCD

Lagrangian [3,4]

[3] D. Kimura, T. Morozumi, H. Umeeda, Prog. Theor. Exp. Phys. 2018, 123B02 (2018).

[4] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985)

$\mathcal{L}_\chi = \mathcal{L}_P + \mathcal{L}_V + \mathcal{L}_c$ P: Pseudoscalar, V: Vector meson, C: Counter term

$$\mathcal{L}_P = \underbrace{\frac{f^2}{4}\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right)}_{\text{kinetic term}} + \underbrace{B\mathrm{Tr}\left[M\left(U+U^\dagger\right)\right]}_{\text{chiral breaking term}}$$

f: pion decay constant, B: constant

$U = \exp\left(\frac{2i}{f}\pi\right)$: chiral field

$M = diag(m_u, m_d, m_s)$

$D_\mu U = \partial_\mu U + iA_{L\mu}U - iUA_{R\mu}$

$$\mathcal{L}_V = \underbrace{-\frac{1}{2}\mathrm{Tr}F_V^{\mu\nu}F_{V\mu\nu}}_{\text{kinetic term}} + \underbrace{M_V^2\mathrm{Tr}\left(V_\mu - \frac{\alpha_\mu}{g}\right)^2}_{\text{mass term}}$$

M_V: mass term of vector mesons

$F_V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$

Pion vector Form Factor (PFF)

How to find pion PFF

$$\langle \pi^+(p_1)\pi^-(p_2)|j^\mu_{had}(0)|0\rangle = F^V_{\pi^+\pi^-}(q^2)\,(p_1^\mu - p_2^\mu)$$

QCD of 3 flavor quark $\longrightarrow j_q^\mu = \frac{\delta S_{QCD}}{\delta A_\mu^{em}} = N_c Q_q^2 \bar{q}^\alpha \gamma^\mu q^\alpha$

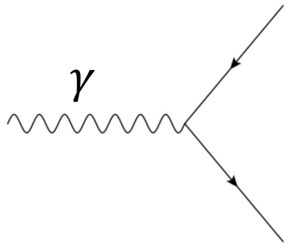
analogy

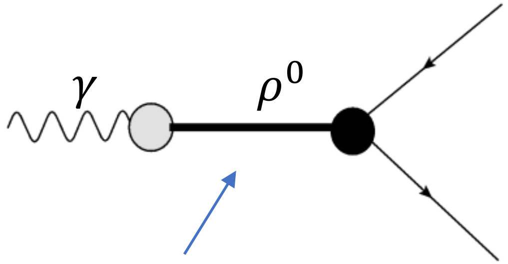
χPT with vector mesons $\longrightarrow j_{had}^\mu = \frac{\delta S_\chi}{\delta A_\mu^{em}} = 2i\left(1 - \frac{M_V^2}{2g^2f^2}\right)\mathrm{Tr}(Q[\pi, \partial^\mu \pi]) - \frac{M_V^2}{g}\mathrm{Tr}(QV^\mu)$

Octet $\pi = \begin{pmatrix} \frac{1}{2}\pi^0 + \frac{1}{2\sqrt{3}}\eta^8 & \frac{1}{\sqrt{2}}\pi^+ & \frac{1}{\sqrt{2}}K^+ \\ \frac{1}{\sqrt{2}}\pi^- & -\frac{1}{2}\pi^0 + \frac{1}{2\sqrt{3}}\eta^8 & \frac{1}{\sqrt{2}}K^0 \\ \frac{1}{\sqrt{2}}K^- & \frac{1}{\sqrt{2}}K^0 & -\frac{1}{\sqrt{3}}\eta^8 \end{pmatrix}$ $V_\mu = \begin{pmatrix} \frac{1}{2}\rho_\mu^0 + \frac{1}{2\sqrt{3}}\phi_\mu^8 & \frac{1}{\sqrt{2}}\rho_\mu^+ & \frac{1}{\sqrt{2}}K_\mu^{*+} \\ \frac{1}{\sqrt{2}}\rho_\mu^- & -\frac{1}{2}\rho_\mu^0 + \frac{1}{2\sqrt{3}}\phi_\mu^8 & \frac{1}{\sqrt{2}}K_\mu^{*0} \\ \frac{1}{\sqrt{2}}K_\mu^{*-} & \frac{1}{\sqrt{2}}K_\mu^{*0} & -\frac{1}{\sqrt{3}}\phi_\mu^8 \end{pmatrix}$

Pion vector Form Factor (PFF)

$$\left\langle \pi^+(p_1)\pi^-(p_2)|j^\mu_{\text{had}}(0)|0\right\rangle = \left\{F^V_{\pi^+\pi^-}(q^2)\Big|_{\text{direct}} + F^V_{\pi^+\pi^-}(q^2)\Big|_{\text{vector meson}}\right\}(p_1^\mu - p_2^\mu)$$





propagator of ρ meson $$D^{\mu\nu}_\rho = \frac{g^{\mu\nu} - \frac{q^\mu q^\nu}{M_V^2}(1 - i\mathrm{Im}\Pi(q^2))}{M_V^2 - q^2(1 - i\mathrm{Im}\Pi(q^2))}$$

The whole contribution (left + right diagram)

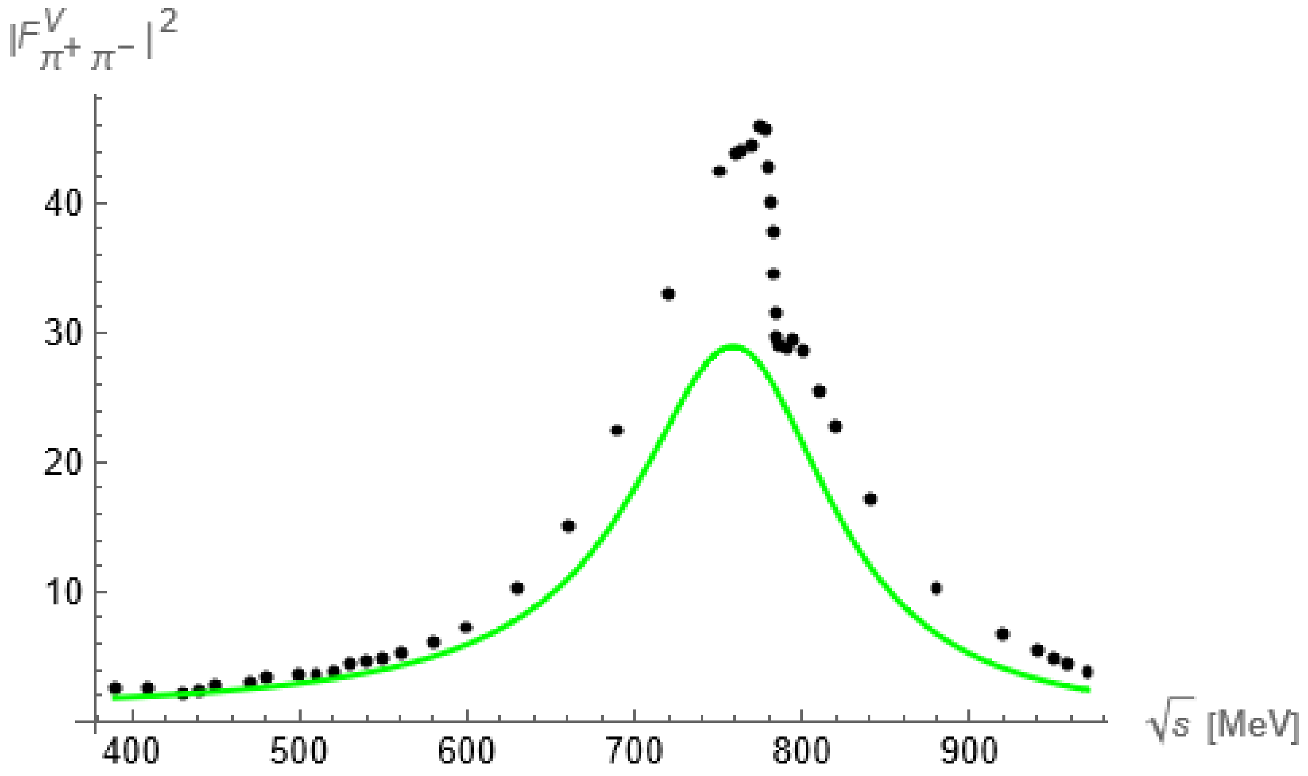
$$F^V_{\pi^+\pi^-}(q^2) = -\left(1 + a\frac{\left(1 - i\mathrm{Im}\Pi(q^2)\right)q^2}{M_V^2 - \left(1 - i\mathrm{Im}\Pi(q^2)\right)q^2}\right)$$

$$a = \frac{M_V^2}{2g^2f^2} = \frac{96\pi f_\pi^2\Gamma_\rho}{M_V^3\sigma_\pi^3(M_V^2)}$$

$\Rightarrow$ $a = 1$

Direct contribution (left diagram) vanishes.
(Vector Meson Dominance: VMD)

Pion vector Form Factor (PFF)



Green line: The squared PFF using ρ propagator (only decay width)
→ It's not consistent with the height of peak of data around 800 MeV.

Pion vector Form Factor (PFF)

Propagator of ρ^0 meson with 1-loop self energy

[3] D. Kimura, T. Morozumi, H. Umeeda, Prog. Theor. Exp. Phys. 2018, 123B02 (2018).

only imaginary part (decay width)

$$D_\rho^{\mu\nu} = \frac{g^{\mu\nu} - \frac{q^\mu q^\nu}{M_V^2}(1 - i\mathrm{Im}\Pi(q^2))}{M_V^2 - q^2(1 - i\mathrm{Im}\Pi(q^2))}$$

with real part [3]

$$D_\rho^{\mu\nu} = \frac{g^{\mu\nu} - \frac{q^\mu q^\nu}{M_\rho^2}\delta B_\rho}{M_\rho^2 - q^2 \delta B_\rho}$$

$$F_{\pi^+\pi^-}^V(q^2)\Big|_{\text{whole}} = -\left(1 + a\frac{\delta B_\rho q^2}{M_\rho^2 - \delta B_\rho q^2}\right)$$

- $Z_V^r(\mu)$, $g_{\rho\pi\pi}^2$, C_1^r, C_2^r: calculated analytically
- M_V: fixed by χ^2 fitting

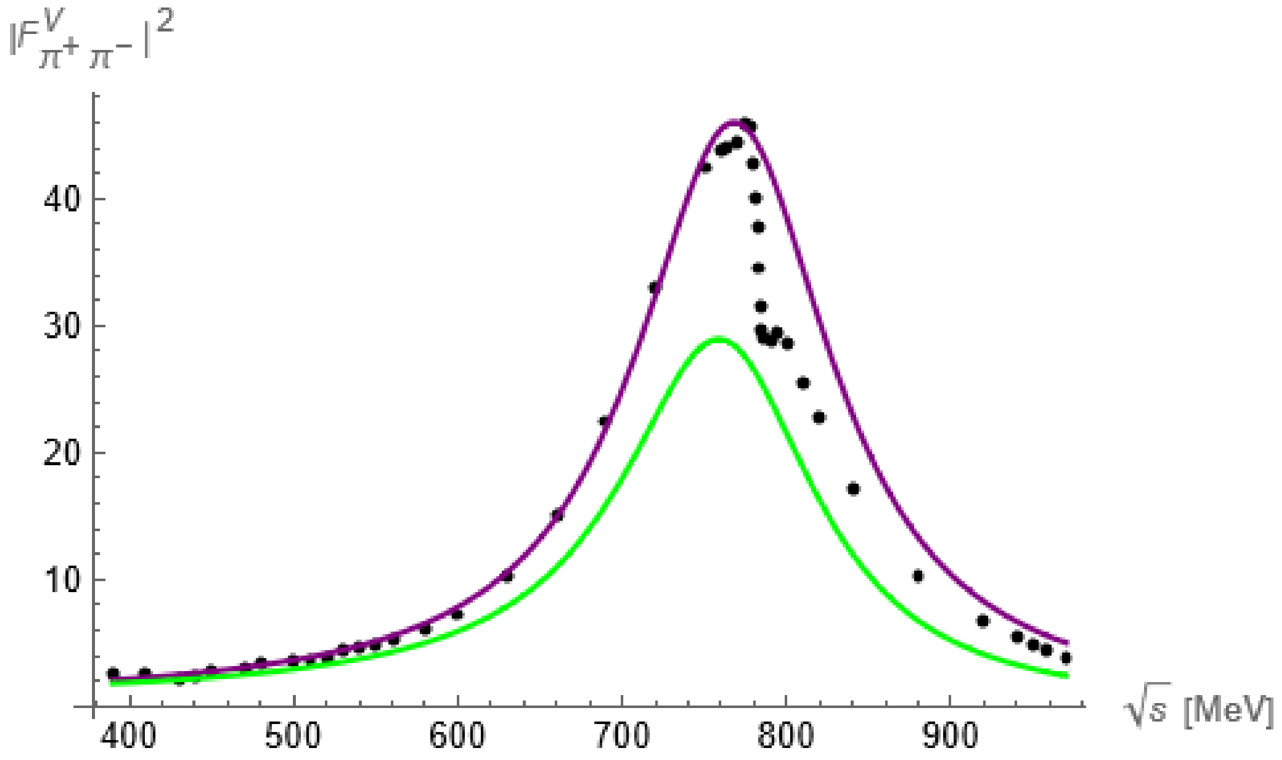
$$a = \frac{M_V^2}{2g^2 f_\pi^2} = \frac{2f_\pi^2}{M_V^2} g_{\rho\pi\pi}^2$$

Loop function

$$\delta B_\rho = Z_V^r(\mu) + g_{\rho\pi\pi}^2\left(4M_\pi^r + M_{K^+}^r + M_{K^0}^r\right)$$

$$M_\rho^2 = M_V^2 + C_1^r M_\pi^2 + C_2^r(M_{K^+}^2 + M_{K^0}^2 + M_\pi^2)$$

Pion vector Form Factor (PFF)



[5] M. N. Achasov et al., J. Exp. Theor. Phys. 101, 1053-1070 (2005).

M_V: 712.5 MeV (best fit value)

- $390\ \mathrm{MeV} \le \sqrt{s} \le 720\ \mathrm{MeV} \rightarrow$ Purple line is consistent with Data points.
- $720\ \mathrm{MeV} \le \sqrt{s} \le 970\ \mathrm{MeV} \rightarrow$ Purple line is not consistent with Data points.

Numerical calculation of AMM

$$a_{\mu(\pi^+\pi^-)}^{\mathrm{HVP-LO}}(390\ \mathrm{MeV} \leq \sqrt{s} \leq 970\ \mathrm{MeV}) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int ds \frac{K(s)}{s^2} \times \frac{1}{4}\sigma_\pi^3(s)\left|F_{\pi^+\pi^-}^V(s)\right|^2$$

AMM	Results
data-driven method	791.89×10^{-10}
χPT with vector mesons (M_V fixed with data)	573.558×10^{-10}
χPT with vector mesons (improved) (M_V fixed with data)	856.512×10^{-10}

- χPT with vector mesons < χPT with vector mesons (improved)

  67%

- Data-driven method $\cong$ χPT with vector mesons (improved)

  92%

Discussion

χPT with vector mesons

ρ meson

Need to consider $\boldsymbol{\omega}$ meson

- Pion vector Form Factor (Exp)
- a_μ (data-driven)

ρ meson, **$\boldsymbol{\omega}$ meson**

Summary

χPT with vector mesons → Pion Form Factor $F^V_{\pi^+\pi^-}(s)$, anomalous magnetic moment a_μ

① PFF: If free parameter M_V is fixed by 実験値とおおむね一致

② 異常磁気モーメント: データドリブンな方法に基づく値は, 模型による計算の約92%

適用性: 約90%以上の精度でベクトル中間子の寄与を調べられる模型

Thank you.

Back up