The W boson mass anomaly in the extension of the minimal SU(5) GUT model

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The standard model

Elementary particles are well described by the standard model(SM).

The SM is based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetries.

Strong force Weak force Electromagnetic force

- The origin of neutrino mass
- Dark matter
- Inflation etc.

But there are phenomena that cannot be explained by the SM.

Physics beyond the standard model (BSM) is needed

Grand Unified Theory (GUT)

The theory of embedding $SU(3)_C \times SU(2)_L \times U(1)_Y$ into a large group.

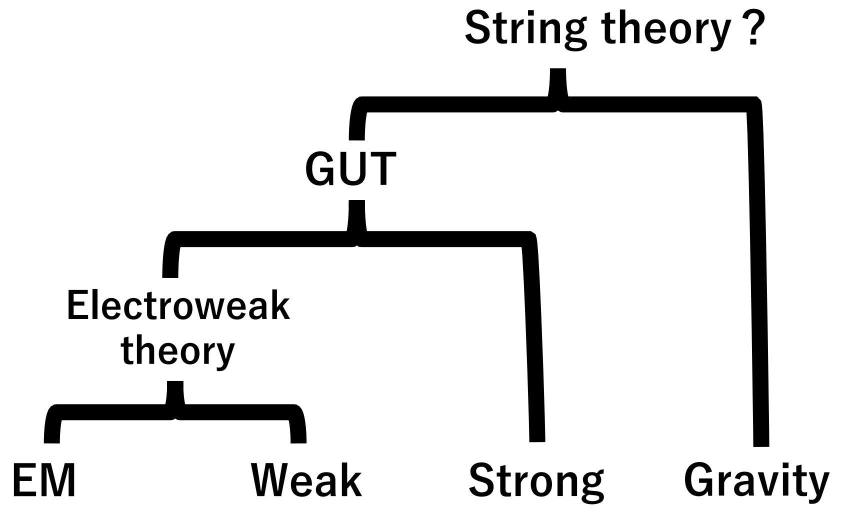
Strong force — Weak force — Electromagnetic force

Ex). SU(5), SO(10)

A simple group

$SU(3)_C \times SU(2)_L \times U(1)_Y$

Strong force — Weak force — Electromagnetic force



The GUT unify the strong, weak, and electromagnetic force.

Minimal SU(5) GUT

H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438

Matter contents : $\overline{5}^i_L$, 10^i_L, A_μ, 5_H, and 24_H $(i = 1\sim 3)$

- Unify the SM gauge interactions into A_μ.

$$A_\mu = \begin{pmatrix} G_\mu - \frac{1}{\sqrt{15}} B_\mu & V^\dagger_\mu \\ V_\mu & W_\mu + \frac{3}{2\sqrt{15}} B_\mu \end{pmatrix}$$

- Unification of quarks and leptons into $\overline{5}$ and 10.

$$\overline{5} = \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e \\ -\nu \end{pmatrix}_L , \qquad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^c_3 & -u^c_2 & -u^1 & -d^1 \\ -u^c_3 & 0 & u^c_1 & -u^2 & -d^2 \\ u^c_2 & -u^c_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & e^c \\ d^1 & d^2 & d^3 & -e^c & 0 \end{pmatrix}_L$$

Symmetry breaking

The 24_H and 5_H break the symmetry by taking the VEVs.

$$SU(5) \xrightarrow{24_H} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{5_H} SU(3)_C \times U(1)_{em}$$

$$<24_H> = \frac{V}{2\sqrt{15}} \text{ Diag } (-2, -2, -2, 3, 3),$$

$$<5_H> = \left(0, 0, 0, 0, \frac{v_h}{\sqrt{2}}\right)$$

Proton decay

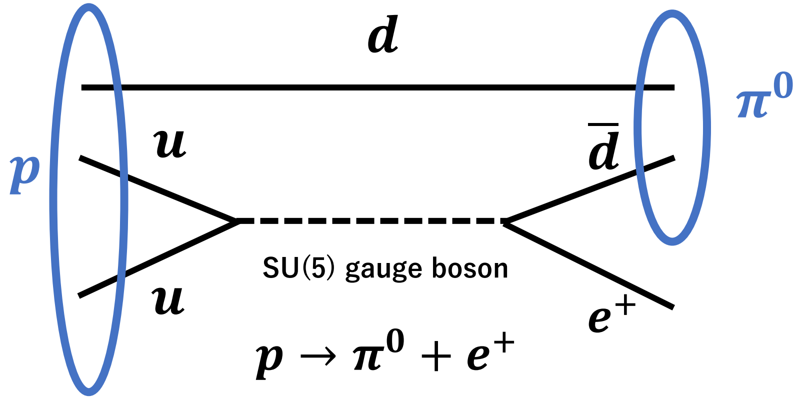
Q. Is it possible to test GUT experimentally? **A. YES**

In GUT, quarks and leptons are embedded into same representations.

→ The GUT predicts the existence of proton decay.

The GUT can be tested by proton decay search!!!

Current experimental results
Super-Kamiokande : $\tau_p(p \to \pi^0 e^+) \gtrsim 2.4 \times 10^{34}$ years
A. Takenaka et al. Phys. Rev. D 102, 112011 (2020)



The problems of GUT

- Inconsistency with experimental results

Minimal SU(5) model : $\tau_p(p \to \pi^0 e^+) \approx 10^{30}\sim 10^{31}$ years

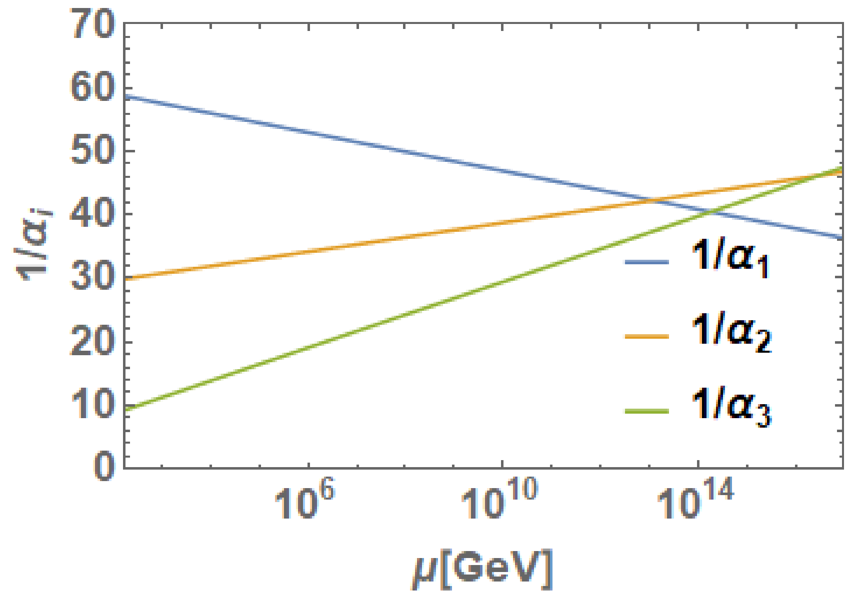
H. Georgi, H.R. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974)

Current experimental results

Super-Kamiokande : $\tau_p(p \to \pi^0 e^+) \gtrsim 2.4 \times 10^{34}$ years

A. Takenaka et al. Phys. Rev. D 102, 112011 (2020)

- No unification of the SM gauge couplings successfully



Blue line : U(1) gauge coupling
Orange line : SU(2) gauge coupling
Green line : SU(3) gauge coupling

The GUT is needed some extensions.

The proton decay search

Current experimental results A. Takenaka et al. Phys. Rev. D 102, 112011 (2020)

Super-Kamiokande : $\tau_p(p \to \pi^0 e^+) \gtrsim 2.4 \times 10^{34}$ years

↓

Future experimental expected limit (2027)

Hyper-Kamiokande : $\tau_p(p \to \pi^0 e^+) \lesssim 1.0 \times 10^{35}$ years

HYPER-KAMIOKANDE collaboration (2019)

The GUT can be testable near future.

Today's talk

We propose a new SU(5) GUT model.

Minimal SU(5) GUT + single vector-like family

- The W boson mass anomaly
- Proton lifetime
- Gauge Unification

Target

The heavy Higgs boson search

The future proton decay search

Contents

✓ Introduction

2. The W boson mass anomaly

3. Our model

4. Summary

The W boson mass anomaly

The W boson mass anomaly

The CDF collaboration has reported an updated result of the W boson mass.

J. de Blas, M. Pierini, L. Reina and L. Silvestrini, Phys.Rev.Lett. 129 (2022) 27, 271801

The SM prediction : $M_W^{\mathrm{SM}} = 80.3500 \pm 0.0056$ GeV

6.5 σ

The CDF collaboration : $M_W^{\mathrm{CDF}} = 80.4335 \pm 0.0094$ GeV

[CDF Collaboration], Science 376, no.6589, 170-176 (2022)

The new physics contribute to the W boson mass.

The W boson mass anomaly

We focus on a real $SU(2)_L$ triplet coming from 24_H to explain the W boson mass anomaly.

$$24_H = \begin{pmatrix} \Sigma_8 - \frac{2}{\sqrt{30}}\Sigma_0 & \Sigma_{(\bar{3},2)} \\ \Sigma_{(3,2)} & \Sigma_3 + \frac{3}{\sqrt{30}}\Sigma_0 \end{pmatrix}$$

$$\Sigma_8 \sim (8,1,0), \quad \Sigma_3 \sim (1,3,0), \quad \Sigma_0 \sim (1,1,0),$$
$$\Sigma_{(3,2)} \sim (3,2,-5/6), \quad \Sigma_{(\bar{3},2)} \sim (\bar{3},2,5/6).$$

R. S. Chivukula, N. D. Christensen, and E. H. Simmons,
Phys. Rev. D 77, 035001 (2008)
P. Fileviez Perez, H. H. Patel and A. D. Plascencia,
Phys. Lett. B 833 (2022), 137371

Triplet Higgs

The SM Higgs + a real triplet with $Y = 0$ model

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, 1/2) \qquad T = \frac{1}{2}\begin{pmatrix} T^0 & \sqrt{2}T^+ \\ \sqrt{2}T^- & -T^0 \end{pmatrix} \sim (1, 3, 0)$$

The Lagrangian for scalar sector :

$$\mathcal{L}_{\mathrm{scalar}} = (D_\mu H)^\dagger (D^\mu H) + \mathrm{Tr}(D_\mu T)^\dagger (D^\mu T) - V(H, T)$$

Covariant derivative :

$$D_\mu H = \partial_\mu H + \mathrm{i} g_1 \frac{B_\mu}{2} + \mathrm{i} g_2 W_\mu, \quad D_\mu T = \partial_\mu T + \mathrm{i}\, g_2 \left[W_\mu , T\right]$$

Scalar potential

Scalar potential :

$$V(H,T)$$
$$= -m_h^2 H^\dagger H + \lambda_0 (H^\dagger H)^2 + M_T^2 \mathrm{Tr}[T^2] + \lambda_1 \mathrm{Tr}[T^4] + \lambda_2 (\mathrm{Tr}[T^2])^2$$
$$+ \alpha (H^\dagger H) \mathrm{Tr}[T^2] + \beta H^\dagger T^2 H + \mu H^\dagger T H$$

The scalar fields can get a vacuum expectation value, v_h and v_T.

$$H = \begin{pmatrix} \phi^+ \\ (v_h + h^0 + \mathrm{i}G^0)/\sqrt{2} \end{pmatrix}, \qquad T = \frac{1}{2}\begin{pmatrix} v_T + t^0 & \sqrt{2}t^+ \\ \sqrt{2}t^- & -v_T - t^0 \end{pmatrix}.$$

$(t^- = (t^+)^*)$

The W boson mass

The real triplet contribute to the W boson mass by getting the vacuum expectation value.

$$M_W^2 = \left(M_W^{\mathrm{SM}}\right)^2 + g_2^2 v_T^2$$

The SM value

New contribution at tree level

The CDF collaboration result can be explained.

→ $v_T = 5.57\ \mathrm{GeV}$

The minimization conditions

The minimization conditions :

$$m_h^2 - \lambda_0 v_h^2 - \frac{A}{2} v_h^2 + \frac{\mu}{2} v_T = 0,$$

$$M_T^2 - \frac{\mu v_h^2}{4 v_T} + \frac{A}{2} v_h^2 + \frac{B}{2} v_T^2 = 0. \qquad \left(A = \alpha + \frac{\beta}{2},\ B = \lambda_1 + 2\lambda_2\right)$$

$\xrightarrow{v_h \gg v_T}$

$$M_T^2 \approx \frac{\mu v_h^2}{4 v_T} \left(v_T \approx \frac{\mu v_h^2}{4 M_T^2} \right)$$

The scalar mass matrix

We can obtain the real scalar and charged scalar mass matrix.

$$M_0^2 = \begin{pmatrix} 2\lambda_0 v_h^2 & -\frac{\mu v_h}{2} + A v_h v_T \\ -\frac{\mu v_h}{2} + A v_h v_T & B v_T^2 + \frac{\mu v_h^2}{4 v_T} \end{pmatrix}, \qquad M_\pm^2 = \begin{pmatrix} \mu v_T & \frac{\mu v_h^2}{2} \\ \frac{\mu v_h^2}{2} & \frac{\mu v_h^2}{4 v_T} \end{pmatrix}.$$

$$A = \alpha + \frac{\beta}{2}, \qquad B = \lambda_1 + 2\lambda_2$$

The mass eigenstates :

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} h^0 \\ t^0 \end{pmatrix}, \qquad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \begin{pmatrix} -\sin\theta_+ & \cos\theta_+ \\ \cos\theta_+ & \sin\theta_+ \end{pmatrix} \begin{pmatrix} \phi^\pm \\ t^\pm \end{pmatrix}.$$

The mass eigenvalues

In the case $v_h \gg v_T$, the mixing angle is too small, $\theta_0 \ll 1$.

$$\tan 2\theta_0 = \frac{4v_h v_T(-\mu + 2Av_T)}{8\lambda_0 v_h^2 v_T - 4Bv_T^3 - \mu v_h^2},$$

$$\tan 2\theta_+ = \frac{4v_h v_T}{4v_T^2 - v_h^2}.$$

$$M_h^2 = 2\lambda_0 v_h^2,$$ ← the SM-like Higgs

$$M_H^2 = Bv_T^2 + \frac{\mu v_h^2}{4v_T},$$

$$M_{H^\pm}^2 = \mu v_T + \frac{\mu v_h^2}{4v_T}.$$

$v_h \gg v_T$ →

$$M_H^2 = M_{H^\pm}^2 \approx \frac{\mu v_h^2}{4v_T} \left(= M_T^2\right)$$

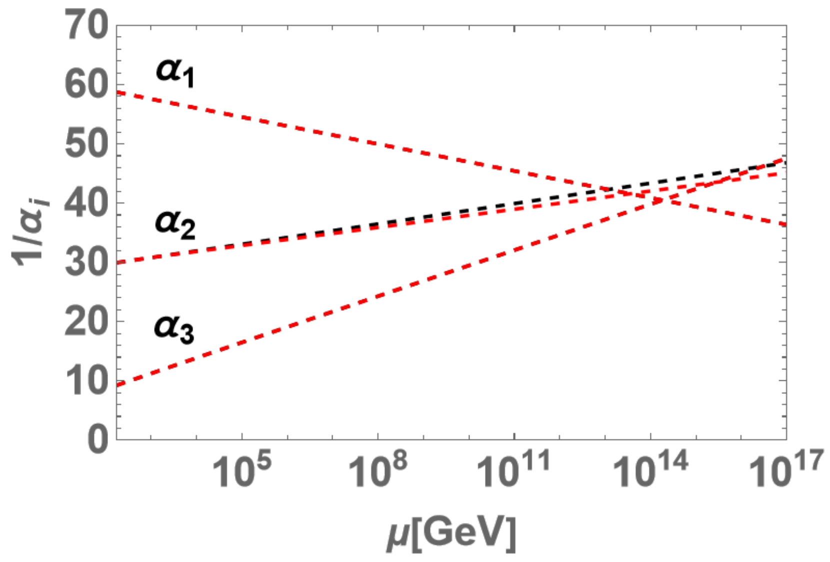
The gauge unification

J. L. Evans, T. T. Yanagida and N. Yokozaki, Phys. Lett. B 833 (2022), 137306

A real triplet contributes only to the running of the $SU(2)_L$ gauge coupling.

M_T=1.5 TeV

- No unification of the SM gauge couplings
- Inconsistency with experimental results



Black : the SM
Red : including a real triplet

Today's talk

We propose a new SU(5) GUT model.

Minimal SU(5) GUT＋single vector-like family

✓ The W boson mass anomaly

● Proton lifetime

● Gauge Unification

Target

The heavy Higgs boson search

The future proton decay search

Our model

Our model

Minimal SU(5) GUT + single vector-like family

$\overline{\mathbf{5}}_L^i$, $\mathbf{10}_L^i$, A_μ, 5_{H}, 24_{H} $\overline{\mathbf{5}}_{L,R}^4$, $\mathbf{10}_{L,R}^4$

New

Vector-like representation of SU(5) :

$$\overline{\mathbf{5}}_{L,R}^4 = D^c\left(\overline{3}, 1, \frac{1}{3}\right) \oplus L\left(1, 2, -\frac{1}{2}\right),$$

$$\mathbf{10}_{L,R}^4 = Q\left(3, 2, \frac{1}{6}\right) \oplus U^c\left(\overline{3}, 1, -\frac{2}{3}\right) \oplus E^c(1, 1, 1).$$

Yukawa interaction

We introduce a Z_2 symmetry to forbid the mixing between the SM and vector-like fermions.

- The Yukawa interaction of the SM

$$\mathcal{L}_{\mathrm{SM}} \supset \sum_{i,j=1}^{3}\left[Y_1^{ij} 5_{\mathrm{H}} 10_L^i 10_L^j\right] + \sum_{i,j=1}^{3}\left[Y_2^{ij} 5_{\mathrm{H}}^* \overline{5}_L^i 10_L^j\right] + \mathrm{h.c.}$$

- The Yukawa interaction of the vector-like fermion

$$\mathcal{L}_{\mathrm{VL}} \supset \overline{5}_L^4 (M_5 + Y_5 24_{\mathrm{H}}) 5_R^4 + \overline{10}_L^4 (M_{10} + Y_{10} 24_{\mathrm{H}}) 10_R^4 + \mathrm{h.c.}$$

Yukawa interaction

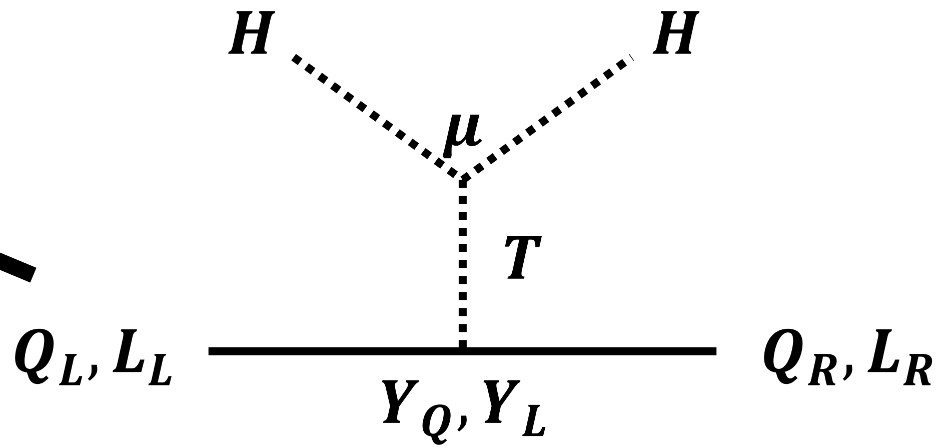
$T \sim (1, 3, 0)$

In our model, a real triplet from 24_H also get the VEVs.

→ Vector-like quark doublet and lepton doublet acquire the mass through the Yukawa interaction of a real triplet.

$$\mathcal{L}_{\mathrm{VL}} \supset Y_Q T \bar{Q}_L Q_R + Y_L T \bar{L}_L L_R + \mathrm{h.c.}$$

$$T = \frac{1}{2}\begin{pmatrix} v_T + t^0 & \sqrt{2} t^+ \\ \sqrt{2} t^- & -v_T - t^0 \end{pmatrix}, \quad v_T \approx \frac{\mu v_h^2}{4 M_T^2}$$



Vector-like fermion masses

After spontaneous symmetry breaking, vector-like fermion acquire the masses.

$$M_Q = M_{10} - \frac{Y_{10}}{4\sqrt{15}}V + Y_Q \frac{\mu v_h^2}{8M_T^2}, \quad M_L = M_5 - \frac{3Y_5}{2\sqrt{15}}V + Y_L \frac{\mu v_h^2}{8M_T^2},$$

$$M_U = M_{10} + \frac{Y_{10}}{\sqrt{15}}V, \quad M_D = M_5 + \frac{Y_5}{\sqrt{15}}V.$$

$$M_E = M_{10} - \frac{3Y_{10}}{2\sqrt{15}}V.$$

Vector-like fermion masses

$$M_Q = M_{10} - \frac{Y_{10}}{4\sqrt{15}} V + Y_Q \frac{\mu v_h^2}{8M_T^2},$$

$$M_U = M_{10} + \frac{Y_{10}}{\sqrt{15}} V,$$

$$M_E = M_{10} - \frac{3Y_{10}}{2\sqrt{15}} V.$$

$$M_{10} \approx \frac{Y_{10}}{4\sqrt{15}} V$$

$$M_Q = Y_Q \frac{\mu v_h^2}{8M_T^2},$$

$$M_U = M_E = \frac{5Y_{10}}{4\sqrt{15}} V.$$

Theoretical bound

- Perturbative unitarity of the WW scattering cross section

R. S. Chivukula, N. D. Christensen and E. H. Simmons, Phys. Rev. D 77 (2008), 035001

$$M_H, M_{H^\pm} \leq \frac{2\sqrt{\pi} v_h^2}{v_T} \longrightarrow \mu < \frac{16\pi v_h^2}{v_T}$$

$$M_H^2 = M_{H^\pm}^2 \approx \frac{\mu v_h^2}{4 v_T}$$

$$M_Q = Y_Q \frac{\mu v_h^2}{8 M_T^2}$$

$v_h = \left(\sqrt{2} G_F\right)^{-1/2} \approx 246.22$ GeV
$v_T \approx 5.57$ GeV

$$M_Q \times M_T^2 < 4.14 \times Y_Q (\mathrm{TeV})^3$$

Experimental constraints

- Vector-like quark mass

A. M. Sirunyan et al. [CMS], Eur. Phys. J. C 79 (2019), 90

$$M_Q > 1660\ \mathrm{GeV}$$

- Charged Higgs boson mass

G. Aad et al. [ATLAS], JHEP 06 (2021), 145

$$M_{H^\pm} > 1000\ \mathrm{GeV}$$

- Heavy neutral Higgs boson mass

G. Aad et al. [ATLAS], Phys. Rev. D 102 (2020) no.3, 032004

$$M_H > 1400\ \mathrm{GeV}$$

The allowed mass range

Theoretical bound : $M_Q \times M_T^2 < 4.14\ (\mathrm{TeV})^3\ (Y_Q = 1)$

$M_Q > 1660$ GeV

$M_H > 1400$ GeV
($M_{H^\pm} > 1000$ GeV)

$M_T = M_H = M_{H^\pm} < 1580$ GeV

$M_Q < 2114$GeV
($M_Q < 4144$ GeV)

- 1660 GeV< M_Q <2114(4144) GeV
- 1400(1000) GeV< M_T <1580 GeV

The mass setup

We assume M_Q and M_D is the same for simplicity.

- $1660\ \text{GeV} < M_Q = M_D < 2114(4144)\ \text{GeV}$
- $1400(1000)\ \text{GeV} < M_T < 1580\ \text{GeV}$
- $M_U = M_E = \frac{5Y_{10}}{4\sqrt{15}} V = 3.0 \times 10^{13}\ \text{GeV}$
- $M_L = \mathcal{O}(\text{GUT})$

The gauge unification

Benchmark :

$M_Q = M_D$ =2000 GeV, M_T=1500 GeV

$M_U = M_E = 3.0 \times 10^{13}$ GeV

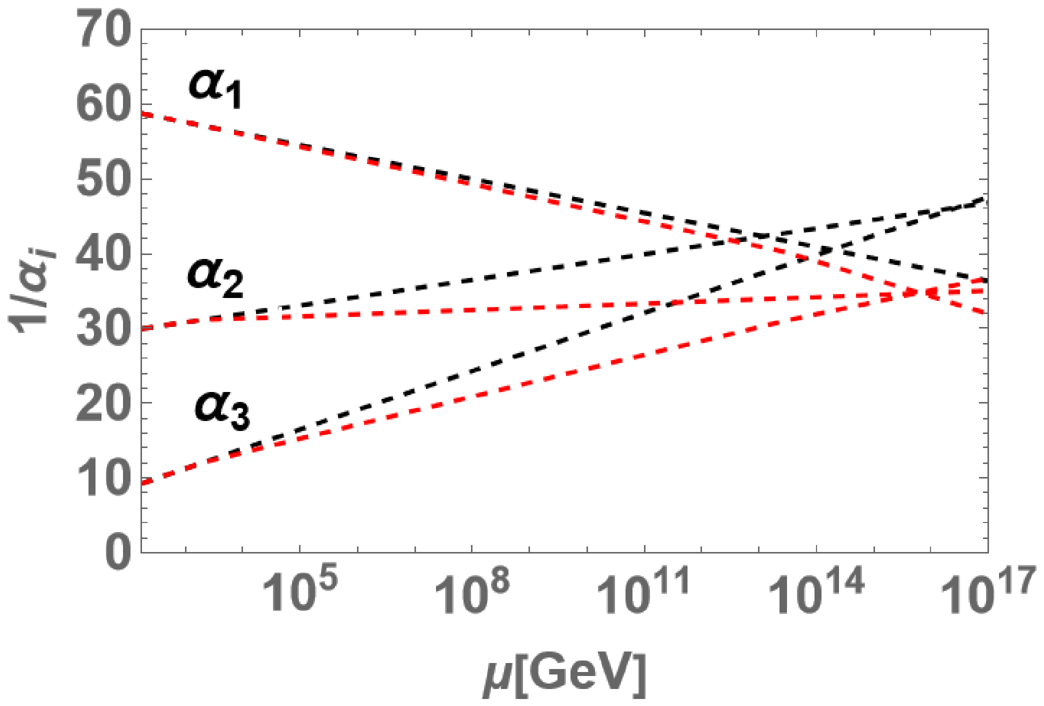
$M_{\rm GUT} \approx 6.49 \times 10^{15}$GeV

$\alpha_{\rm GUT} = \alpha_1 = \alpha_2 = \alpha_3 \approx 1/34.7$

$$\tau_p(p \to \pi^0 e^+) \approx \frac{1}{\alpha^2_{\rm GUT}} \frac{M^4_{\rm GUT}}{m_p^5}$$

$$\approx 6.12 \times 10^{34}\text{years}$$

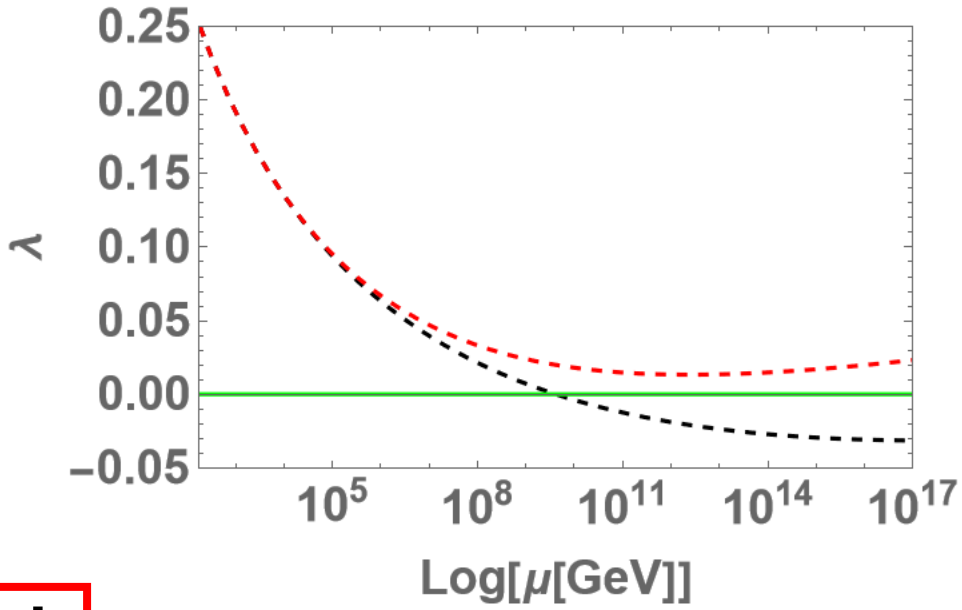
P. Nath and P. Fileviez Perez, Phys. Rept. 441 (2007), 191-317



Black : the SM
Red : Our model

The running of quartic coupling

Benchmark :
$M_Q = M_D$ =2000 GeV, M_T=1500 GeV
$M_U = M_E = 3.0 \times 10^{13}$ GeV

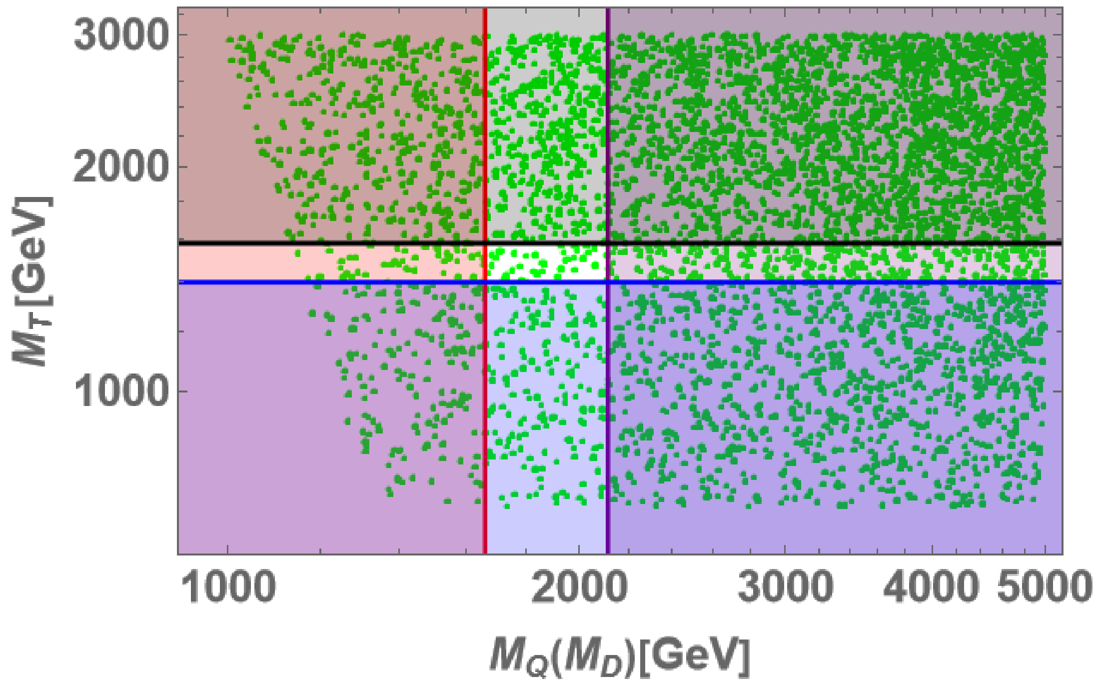


The SM Higgs potential is stabilized.

Black : the SM
Red : Our model

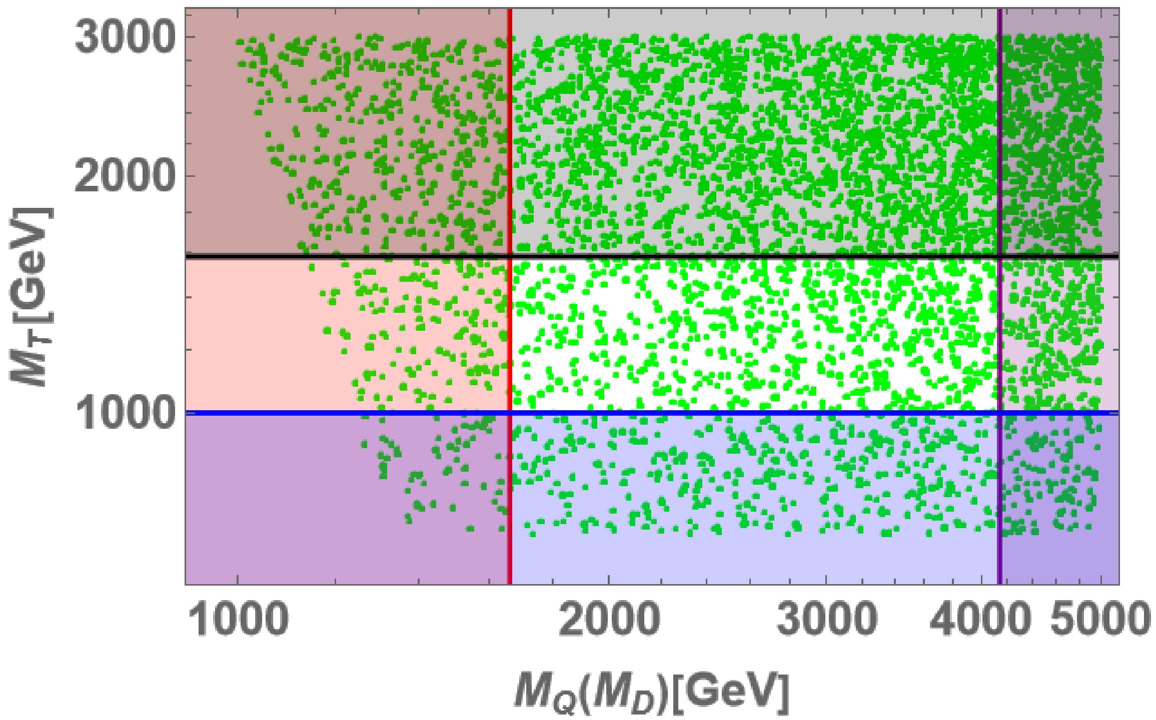
The relation of new particles

- The heavy neutral Higgs



1660 GeV< $M_Q(M_D)$ <2114 GeV
1400 GeV< M_T <1580 GeV

- The charged Higgs



1660 GeV< $M_Q(M_D)$ <4144 GeV
1000 GeV< M_T <1580 GeV

Summary

We propose a new SU(5) GUT model.

Minimal SU(5) GUT + single vector-like family

$\overline{\mathbf{5}}_L^i$, $\mathbf{10}_L^i$, A_μ, $5_{\rm H}$, $24_{\rm H}$ $\overline{\mathbf{5}}_{L,R}^4$, $\mathbf{10}_{L,R}^4$

New

- ✓ The W boson mass anomaly
- ✓ Proton lifetime
- ✓ Gauge Unification

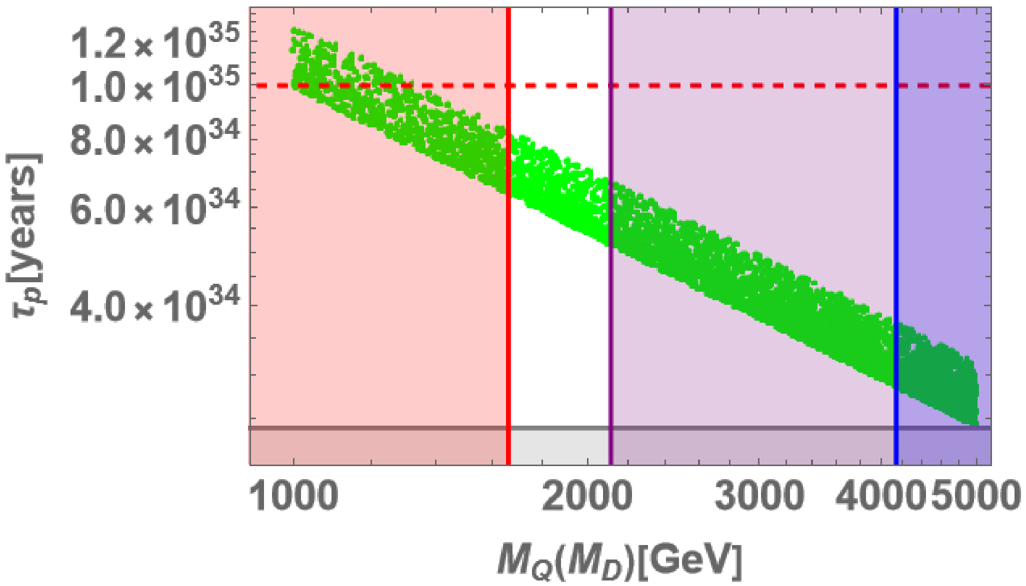
Target

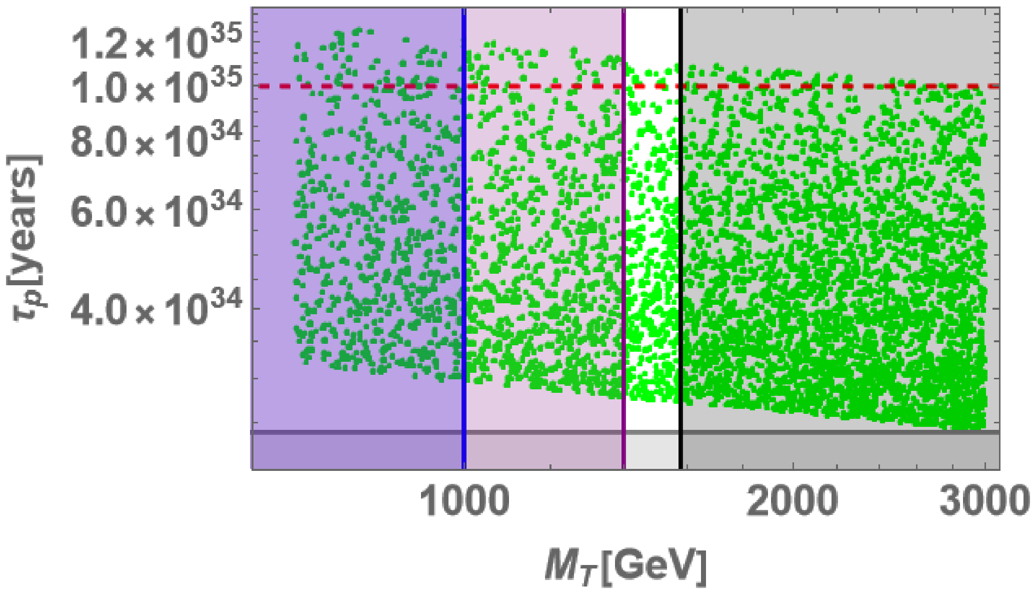
The heavy Higgs boson search

The future proton decay search

Back up

New particles-proton lifetime





1660 GeV< M_Q <2114(4144) GeV

1400(1000) GeV< M_T <1580 GeV

The SM particle masses

- The Yukawa interaction of the SM

$$\mathcal{L}_{\mathrm{SM}} \supset \sum_{i,j=1}^{3} \left[Y_1^{ij} 5_{\mathrm{H}} 10_L^i 10_L^j \right] + \sum_{i,j=1}^{3} \left[Y_2^{ij} 5_{\mathrm{H}}^* \bar{5}_L^i 10_L^j \right] + \mathrm{h.c.}$$

$$\longrightarrow \quad m_{di} = m_{ei} \quad \text{Wrong}$$

We need to add non-renormalizable operators to obtain the correct mass relation.

Ex). $\frac{Y_u^{ij}}{\Lambda} 24_{\mathrm{H}} 5_{\mathrm{H}} 10_L^i 10_L^j, \quad \frac{Y_d^{ij}}{\Lambda} 24_{\mathrm{H}} 5_{\mathrm{H}}^* \bar{5}_L^i 10_L^j$

The mixing angle

$$v_h = \left(\sqrt{2}G_F\right)^{-1/2} \approx 246.22\ \text{GeV},\ v_T \approx 5.57\ \text{GeV}$$

$$\tan 2\theta_0 = \frac{4v_h v_T(-\mu + 2Av_T)}{8\lambda_0 v_h^2 v_T - 4Bv_T^3 - \mu v_h^2} \ll 1$$

$$\tan 2\theta_+ = \frac{4v_h v_T}{4v_T^2 - v_h^2} = -0.0452$$

We can determine the charged Higgs boson couplings.

Decay mode

P. Fileviez Perez, H. H. Patel and A. D. Plascencia, Phys. Lett. B 833 (2022), 137371

- The heavy neutral Higgs $(\theta_0 \to 0)$

Main decay mode : $H \to WW$

- The charged Higgs $(\theta_0 \to 0)$

Main decay mode : $H^+ \to \tau^+ \nu_\tau$

$H^+ \to t\bar{b}$

$H^+ \to W^+ Z$

$H^+ \to h W^+$

Feynman Rule

P. Fileviez Perez, H. H. Patel and A. D. Plascencia, Phys. Lett. B 833 (2022), 137371

Interaction	Feynman Rule
hff	$i(M_f/v_0)$
$H^+\bar{\nu}_i e_i$	$-i\frac{\sqrt{2}}{v_0}M_e^i \sin\theta_+ P_R$
$H^+\bar{u}d$	$-i\frac{\sqrt{2}}{v_0}\sin\theta_+\,(-M_u V_{\rm CKM}P_L + V_{\rm CKM}M_d P_R)$
ZZh	$(2iM_Z^2/v_0)g^{\mu\nu}$
$ZW^\pm H^\mp$	$ig_2\big(-g_2 x_0 c_+ c_w + \frac{1}{2}g_Y v_0 s_+ s_w\big)g^{\mu\nu}$
W^+W^-h	$ig_2^2\big(\frac{1}{2}v_0\big)g^{\mu\nu}$
W^+W^-H	$ig_2^2\big(2x_0\big)g^{\mu\nu}$
γH^+H^-	$ie\,(p'-p)^\mu$
ZH^+H^-	$i\big(g_2 c_w - \frac{M_Z}{v_0}s_+^2\big)(p'-p)^\mu$
$W^\pm hH^\mp$	$\pm ig_2\big(\frac{1}{2}s_+\big)(p'-p)^\mu$
$W^\pm HH^\mp$	$\pm ig_2 c_+\,(p'-p)^\mu$

TABLE I: Feynman Rules in the limit when h is SM-like ($\theta_0 \to 0$)

The Yukawa couplings

- The heavy neutral Higgs

1660 GeV$< M_Q$
1400 GeV$< M_T$

$$Y_Q > 0.785$$

$$M_Q \times M_T^2 < 4.14 \times Y_Q (\mathrm{TeV})^3$$

- The charged Higgs

1660 GeV$< M_Q$
1000 GeV$< M_T$

$$Y_Q > 0.401$$

Accuracy of unification

We define the unification successfully as an accuracy of unification is 1% or less.

Accuracy :

The SM gauge couplings α_1, α_2, and α_3
We assume $r_{12} = \frac{\alpha_2}{\alpha_1}$、 $r_{23} = \frac{\alpha_3}{\alpha_2}$.
If $0.99 < \frac{r_{23}}{r_{12}} < 1.01$, accuracy of unification is 1% or less.

The calculation of beta function

The contributions of new particles are added from the mass of each particle.

Beta coefficient : $b_i = -\frac{11}{3}N + \frac{2}{3}T(R_f)N_f^c + \frac{1}{3}T(R_s)N_s \quad (i = 1 \sim 3)$

N : N of SU(N) group
N_f^c : the number of chiral fermion
N_s : the number of complex scalar

$$T(R) = \mathrm{Tr}[L^i L^j] = \begin{cases} \frac{1}{2}\delta_{ij} & (R : \text{basic representation}) \\ N\delta_{ij} & (R : \text{adjoint representation}) \end{cases}$$

Ex). One SU(3) triplet chiral fermion case is $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

Beta coefficient

$$b_1 : \mathrm{U}(1),\ b_2 : \mathrm{SU}(2),\ b_3 : \mathrm{SU}(3)$$

$$D : b_1 = \frac{2}{15}, b_2 = 0, b_3 = \frac{1}{3}$$

$$L : b_1 = \frac{1}{5}, b_2 = \frac{1}{3}, b_3 = 0$$

$$Q : b_1 = \frac{1}{15}, b_2 = 1, b_3 = \frac{2}{3}$$

$$U : b_1 = \frac{1}{3}, b_2 = 0, b_3 = \frac{8}{15}$$

$$E : b_1 = \frac{2}{5}, b_2 = 0, b_3 = 0$$

$$T : b_1 = 0\ , b_2 = \frac{1}{3}, b_3 = 0$$

$$\overline{5}^4_{L,R} = D^c\left(\overline{3}, 1, \frac{1}{3}\right) \oplus L\left(1, 2, -\frac{1}{2}\right),$$

$$10^4_{L,R} = Q\left(3, 2, \frac{1}{6}\right) \oplus U^c\left(\overline{3}, 1, -\frac{2}{3}\right) \oplus E^c(1, 1, 1).$$

$$T = \frac{1}{2}\begin{pmatrix} T^0 & \sqrt{2}T^+ \\ \sqrt{2}T^- & -T^0 \end{pmatrix} \sim (1, 3, 0)$$

The product of SU(5) representations

$$\mathbf{5} \times \mathbf{5} = \mathbf{10} + \mathbf{15}$$

$$\overline{\mathbf{5}} \times \mathbf{10} = \mathbf{5} + \overline{\mathbf{45}}$$

$$\mathbf{10} \times \mathbf{10} = \overline{\mathbf{5}} + \mathbf{45} + \mathbf{50}$$

$$\mathbf{10} \times \overline{\mathbf{10}} = \mathbf{1} + \mathbf{24} + \mathbf{75}$$

Minimal SU(5) GUT

$$A_\mu = \begin{pmatrix} G_\mu - \frac{1}{\sqrt{15}} B_\mu & V_\mu^\dagger \\ V_\mu & W_\mu + \frac{3}{2\sqrt{15}} B_\mu \end{pmatrix}, \qquad \bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L, \qquad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & e^c \\ d^1 & d^2 & d^3 & -e^c & 0 \end{pmatrix}_L$$

$$24_H = \begin{pmatrix} \Sigma_8 - \frac{2}{\sqrt{30}} \Sigma_0 & \Sigma_{(\bar{3},2)} \\ \Sigma_{(3,2)} & \Sigma_3 + \frac{3}{\sqrt{30}} \Sigma_0 \end{pmatrix}, \quad 5_H = \begin{pmatrix} H^1 \\ H^2 \\ H^3 \\ \phi^+ \\ \phi^0 \end{pmatrix}.$$