Hiroshima – IITB workshop

2023/02/20

Comprehensive Analysis of Equivalence Classes in 5D SU(N) gauge theory with S^1/Z_2 compact space

Kota Takeuchi

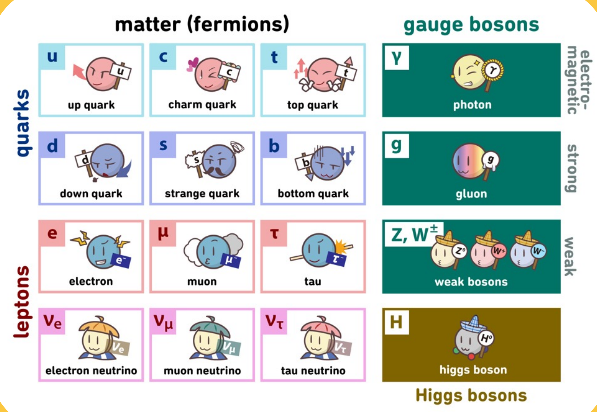
KT, T. Inagaki, arXiv:2301.12938 [hep-th]

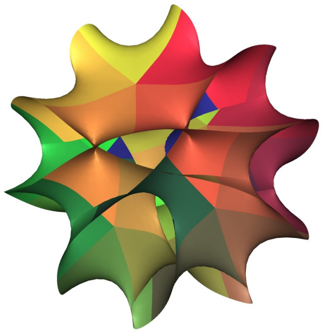
01

Introduction

Extra dimensions

higgstan.com





SM: 4-dimension

String Theory: 10-dimension

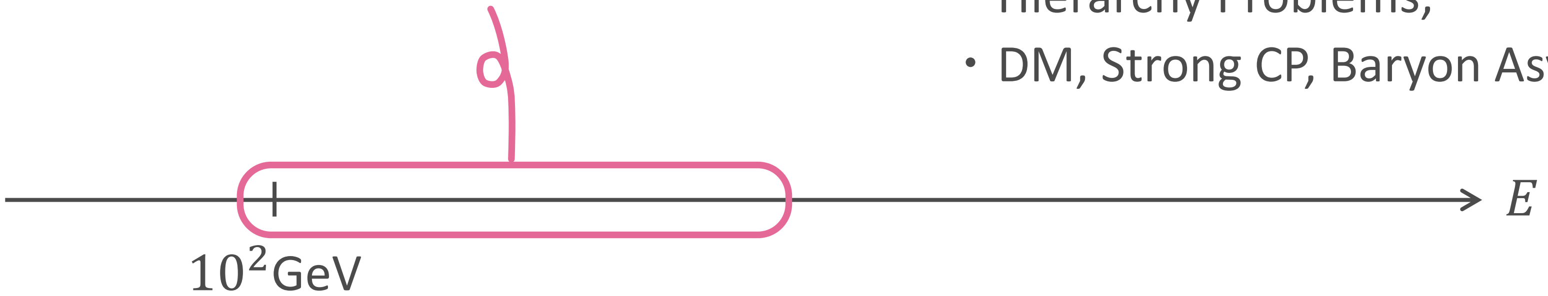
E

10^2GeV

10^{19}GeV

Extra dimensions

Scenario beyond the SM:
4-dimension + Extra-Dimension?



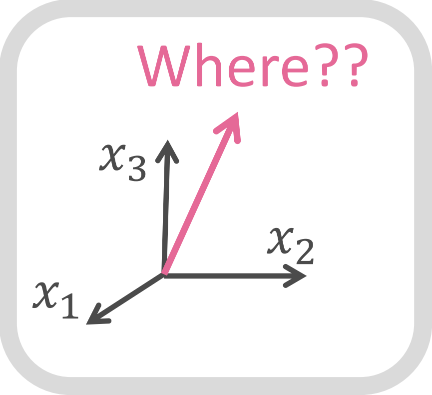
models

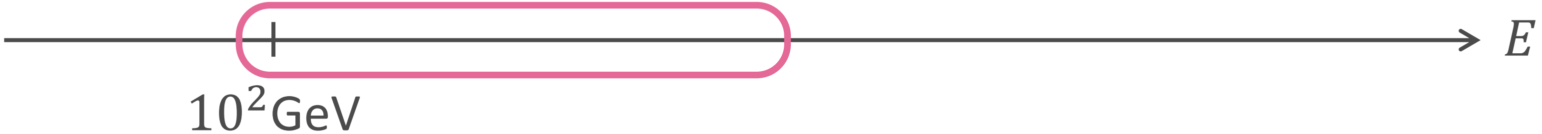
- Gauge Higgs Unification (GHU),
- Large Extra Dimension (LED),
- Randall Sundrum (RS), etc.

approach

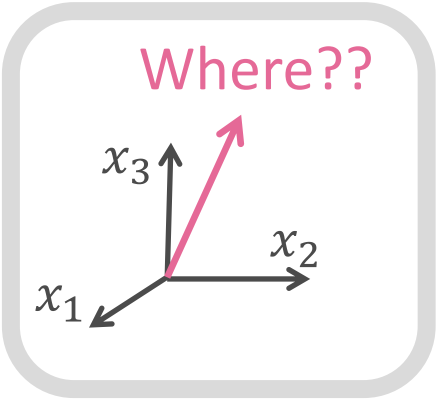
- Hierarchy Problems,
- DM, Strong CP, Baryon Asym, etc.

Compact space



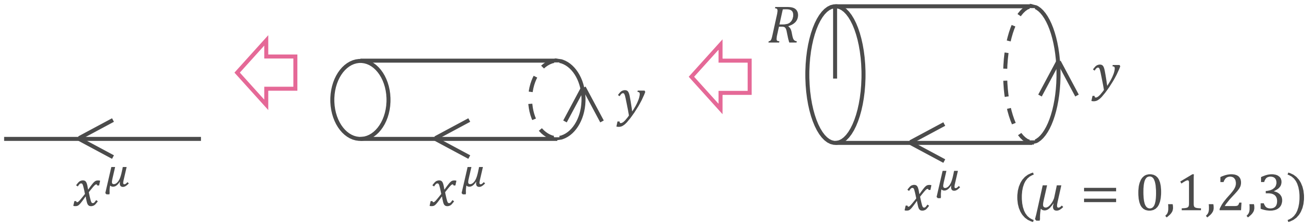


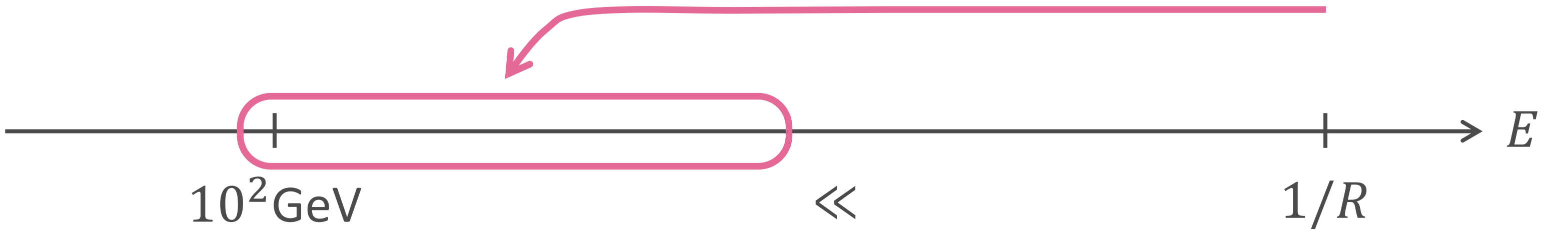
Compact space



4D effective theory:

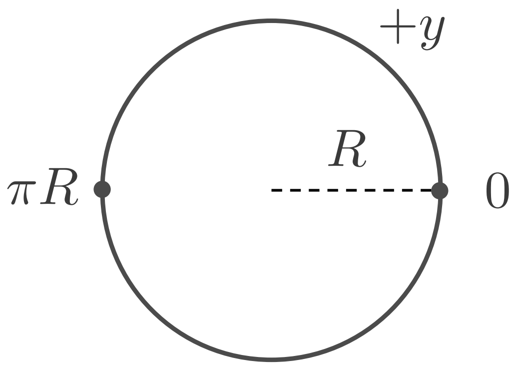
$$\iint d^4x\, dy\, \mathcal{L}^{5D}(x^\mu, y) = \int d^4x\ \mathcal{L}^{4D}_{eff}(x^\mu)$$





Boundary Conditions (BCs)

e.g.) S^1 compactification



$$y \sim y + 2\pi R$$

infinite interval: $y \in (-\infty, +\infty)$

$$\phi(y = \pm\infty) = \partial\phi(y = \pm\infty) = 0$$

finite interval: $y \in (0, 2\pi R)$

$$\phi(y = 0) = +\phi(y = 2\pi R)?$$

$$\phi(y = 0) = -\phi(y = 2\pi R)?$$

$$\phi(y = 0) = e^{i\theta}\phi(y = 2\pi R)?$$

Interesting!

Which BCs should we choose?

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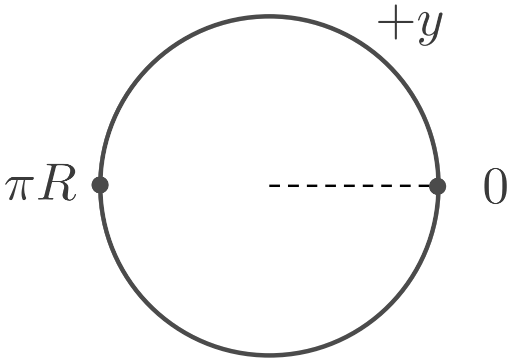
BCs on S^1/Z_2

S^1 circle

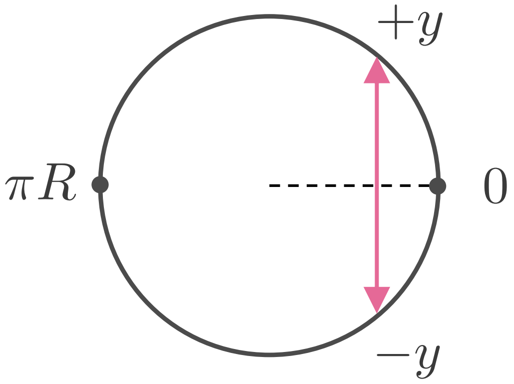


$$y \sim y + 2\pi R$$

S^1 circle

cannot realize
Chiral Asymmetry...

S^1/Z_2 Orbifold



$$\begin{cases} y \sim y + 2\pi R \\ y \sim -y \end{cases}$$

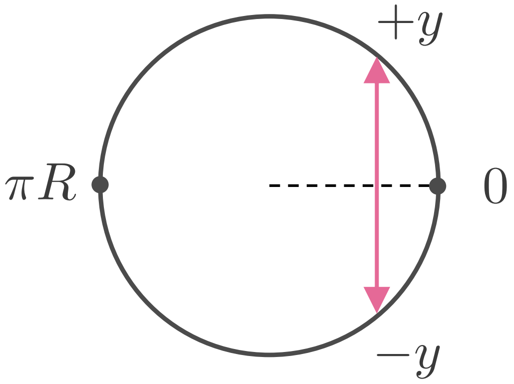
S^1/Z_2 orbifold

realize

Chiral Asymmetry!

realistic minimum model

S^1/Z_2 Orbifold



$$\Rightarrow$$

Two fixed points

$y_0 = 0,\ y_1 = \pi R$

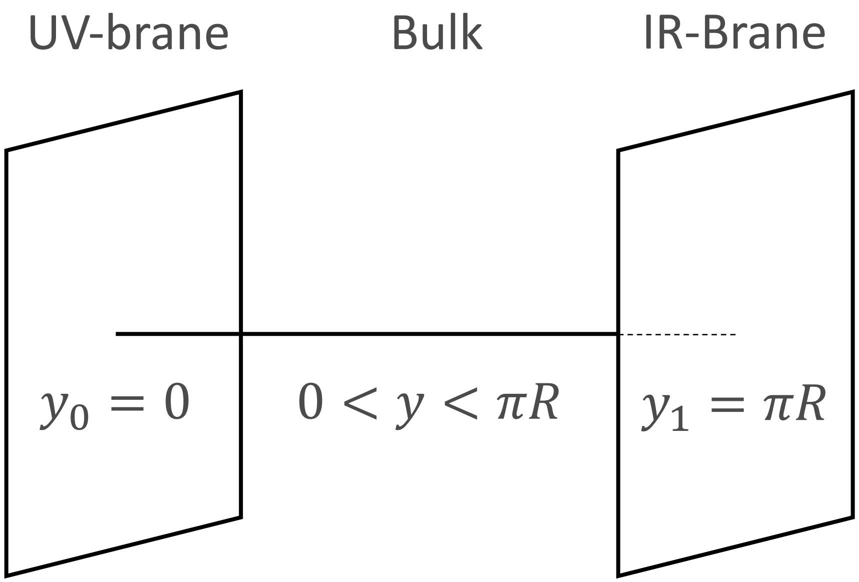
$$\begin{cases} y \sim y + 2\pi R \\ y \sim -y \end{cases}$$

$$\begin{cases} \hat{\mathcal{T}} & : y \to y + 2\pi R \\ \hat{\mathcal{P}}_0 & : -y \to y \\ \hat{\mathcal{P}}_1 & : \pi R - y \to \pi R + y \end{cases}$$

$$\hat{\mathcal{P}}_1 \hat{\mathcal{P}}_0 = \hat{\mathcal{T}}, \quad \hat{\mathcal{P}}_0^2 = \hat{\mathcal{P}}_1^2 = 1$$

Brane World Scenario

There are two branes at the fixed points in the bulk.



$SU(N)$ model on S^1/Z_2 orbifold

$$
\begin{cases}
\mathcal{L} = -\frac{1}{4}F_{MN}F^{MN}(x,y) + \bar{\Psi} i \Gamma^M D_M \Psi(x,y) \\
\Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \\
A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \qquad (i = 0, 1) \\
A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i
\end{cases}
$$

$$\Gamma^M = (\gamma^\mu, i\gamma^5)$$

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN} I_{4\times 4}$$

$$D_M = \partial_M - igA_M$$

$$F_{MN} = \frac{i}{g}[D_M, D_N]$$

$$y_0 = 0,\ y_1 = \pi R$$

Unitary + Parity → Hermite

$P_i^{-1} = P_i^\dagger$ $\qquad P_i^2 = 1$ $\qquad P_i^\dagger = P_i = P_i^{-1}$

P_i : Hermitian N×N matrices with ± 1 eigenvalues

Symmetry Breaking by BCs

Different BCs generally produce different symmetry.

e.g.) SU(3)model

P_0, P_1	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
A_μ^0	$\begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \star \\ 0 & \star & 0 \end{pmatrix}$
generators	$\{T^1, T^2, T^3, T^8\}$	$\{T^3, T^8\}$	$\{\frac{\sqrt{3}}{2}T^3 + \frac{1}{2}T^8\}$
symmetry	SU(2)×U(1)	U(1)×U(1)	U(1)

03

ECs on S^1/Z_2

Equivalence classes

Different BCs always lead to different theories?

P_0, P_1	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
A_μ^0	$\begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \star \\ 0 & \star & 0 \end{pmatrix}$
generators	$\{T^1, T^2, T^3, T^8\}$	$\{T^3, T^8\}$	$\{\frac{\sqrt{3}}{2}T^3 + \frac{1}{2}T^8\}$
symmetry	SU(2)×U(1)	U(1)×U(1)	U(1)

Equivalence classes

Different BCs always lead to different theories? → Not necessarily!

P_0, P_1	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
A_μ^0	$\begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \star \\ 0 & \star & 0 \end{pmatrix}$
generators	$\{T^1, T^2, T^3, T^8\}$	$\{T^3, T^8\}$	$\{\frac{\sqrt{3}}{2}T^3 + \frac{1}{2}T^8\}$
symmetry	SU(2)×U(1)	U(1)×U(1)	U(1)

Gauge transformation for BCs

$$\begin{cases} \Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \quad (P_i^\dagger = P_i = P_i^{-1}) \\ A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \\ A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i \end{cases}$$

Gauge invariant?

> Gauge Transformation
>
> $$\Psi'(x, y) = \Omega(x, y)\Psi(x, y)$$
>
> $$A'_M(x, y) = \Omega(x, y) A_M(x, y) \Omega^{-1}(x, y) - \frac{i}{g}\Omega(x, y)\partial_M \Omega^{-1}(x, y)$$

Gauge transformation for BCs

$$\begin{cases} \Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \quad (P_i^\dagger = P_i = P_i^{-1}) \\ A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \\ A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i \end{cases}$$

$$\downarrow$$

$$\begin{cases} \Psi'(x, y_i - y) = P_i' \gamma^5 \Psi'(x, y_i + y) \\ A_\mu'(x, y_i - y) = P_i' A_\mu'(x, y_i + y) P_i'^\dagger - P_i' \partial_\mu P_i'^\dagger \\ A_y'(x, y_i - y) = -P_i' A_y'(x, y_i + y) P_i'^\dagger - P_i' (-\partial_y) P_i'^\dagger \end{cases}$$

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y)$$

Not invariant!

Gauge transformation for BCs

$$\begin{cases} \Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \quad (P_i^\dagger = P_i = P_i^{-1}) \\ A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \\ A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i \end{cases}$$

$$\downarrow$$

$$\begin{cases} \Psi'(x, y_i - y) = P_i' \gamma^5 \Psi'(x, y_i + y) \\ A'_\mu(x, y_i - y) = P_i' A'_\mu(x, y_i + y) P_i'^\dagger - \cancel{P_i' \partial_\mu P_i'^\dagger} \\ A'_y(x, y_i - y) = -P_i' A'_y(x, y_i + y) \underbrace{P_i'^\dagger}_{= P_i} - \cancel{P_i' (-\partial_y) P_i'^\dagger} \end{cases}$$

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y)$$

$$P_i' = P_i \ \ (\text{i=0,1})$$

$\Rightarrow$ gauge invariant

Gauge transformation for BCs

$$\begin{cases} \Psi(x, y_i - y) = P_i \gamma^5 \Psi(x, y_i + y) \quad (P_i^\dagger = P_i = P_i^{-1}) \\ A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i \\ A_y(x, y_i - y) = -P_i A_y(x, y_i + y) P_i \end{cases}$$

$$\downarrow$$

$$\begin{cases} \Psi'(x, y_i - y) = P_i' \gamma^5 \Psi'(x, y_i + y) \\ A_\mu'(x, y_i - y) = P_i' A_\mu'(x, y_i + y) P_i'^\dagger - \cancel{P_i' \partial_\mu P_i'^\dagger} \\ A_y'(x, y_i - y) = -P_i' A_y'(x, y_i + y) P_i'^\dagger - \cancel{P_i' (-\partial_y) P_i'^\dagger} \end{cases}$$

$$P_i' = \Omega(x, y_i - y) P_i \Omega^\dagger(x, y_i + y)$$

$$\begin{cases} \partial_M P_i' = 0 \\ P_i'^\dagger = P_i' \end{cases} \quad \text{(i=0,1)}$$

$\Rightarrow$ not invariant, but P_i' become other BCs !

Equivalence Classes (1/2)

- Equivalence Classes (ECs)

$$(P_0, P_1) \sim (P_0', P_1')$$

- ECs gauge transformations

ECs gauge conditions

$$\Omega(y) = \exp\left[i f^a(y) T^a\right] \quad \text{,with} \quad \begin{cases} \partial_M P_i' = 0 \\ P_i'^{\dagger} = P_i' \end{cases}$$

f^a: parameters
T^a: generators

Equivalence Classes (2/2)

e.g.) SU(3)model

P_0, P_1	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
symmetry	U(1)×U(1)	SU(2)×U(1)	U(1)

$$\Omega(y): \quad \exp\left[i\,\pi\,\frac{y}{2\pi R}\begin{pmatrix} 0 & 0 \\ 0 & \sigma_1 \end{pmatrix}\right] \qquad \exp\left[i\left(-\frac{\pi}{2}\right)\frac{y}{2\pi R}\begin{pmatrix} 0 & 0 \\ 0 & \sigma_1 \end{pmatrix}\right]$$

ECs in $SU(N)$ on S^1/Z_2

- Well-known ECs gauge transformations on S^1/Z_2

$$\Omega(y) = \exp\left[i\frac{y}{2\pi R}\theta^a T^a\right] \longrightarrow \boxed{\begin{aligned}[p,q,r,s] &\sim [p-1, q+1, r+1, s-1] \\ &\sim [p+1, q-1, r-1, s+1]\end{aligned}}$$

- Rearrangement

$$P_0 = \mathrm{diag}\overbrace{(+1,\ldots,+1,+1,\ldots,+1,-1,\ldots,-1,-1,\ldots,-1)}^{N},$$
$$P_1 = \mathrm{diag}(\underbrace{+1,\ldots,+1}_{p},\underbrace{-1,\ldots,-1}_{q},\underbrace{+1,\ldots,+1}_{r},\underbrace{-1,\ldots,-1}_{s=N-p-q-r}),$$

$$(P_0, P_1) = [p,q,r,s]$$

ECs in $SU(N)$ on S^1/Z_2

- Well-known ECs gauge transformations on S^1/Z_2

$$\Omega(y) = \exp\left[i\frac{y}{2\pi R}\theta^a T^a\right] \longrightarrow \boxed{\begin{aligned}[p,q,r,s] &\sim [p-1,q+1,r+1,s-1]\\ &\sim [p+1,q-1,r-1,s+1]\end{aligned}}$$

Only?

- No other ECs gauge transformations?
- Different ECs are unrelated to each other?

04

Analysis of ECs transformations

arXiv:2301.12938 [hep-th]

Questions

Is there a set of parameters $f^a(\hat{y})$ satisfying ECs conditions?

$$(P_0, P_1) \overset{?}{\sim} (P'_0, P'_1)$$

$$\partial_M P'_i = 0, \quad P'^{\dagger}_i = P'_i$$

$$P'_i = \Omega(x, \hat{y}_i - \hat{y}) P_i \Omega^\dagger(x, \hat{y}_i + \hat{y})$$

$$\Omega(\hat{y}) = \exp\left[i f^a(\hat{y}) T^a\right] \quad \left(\hat{y} = \frac{y}{2\pi R}\right)$$

Diagonalizablity

Haba(2004)
Kawamura(2020)

- On $S^1/\mathbb{Z}_2$, the BCs (P_o, P_1) are always diagonalizable.

$$(P_0, P_1) \sim (P_0^{diag}, P_1^{diag})$$

- Rearrangement

$$\begin{aligned} P_0 &= \mathrm{diag}\,\overbrace{(+1,\ldots,+1,+1,\ldots,+1,-1,\ldots,-1,-1,\ldots,-1)}^{N}, \\ P_1 &= \mathrm{diag}(\underbrace{+1,\ldots,+1}_{p},\underbrace{-1,\ldots,-1}_{q},\underbrace{+1,\ldots,+1}_{r},\underbrace{-1,\ldots,-1}_{s=N-p-q-r}), \end{aligned}$$

$$(P_0, P_1) = [p, q, r, s]$$

Classification of Generators

Kawamura(2020)

$$T^{a++} = \begin{pmatrix} \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 \\ 0 & 0 & \star & 0 \\ 0 & 0 & 0 & \star \end{pmatrix}, \quad T^{a+-} = \begin{pmatrix} 0 & \star & 0 & 0 \\ \star & 0 & 0 & 0 \\ 0 & 0 & 0 & \star \\ 0 & 0 & \star & 0 \end{pmatrix},$$

$$T^{a-+} = \begin{pmatrix} 0 & 0 & \star & 0 \\ 0 & 0 & 0 & \star \\ \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 \end{pmatrix}, \quad T^{a--} = \begin{array}{c} \begin{pmatrix} 0 & 0 & 0 & \star \\ 0 & 0 & \star & 0 \\ 0 & \star & 0 & 0 \\ \star & 0 & 0 & 0 \end{pmatrix} \\ \begin{matrix} p & q & r & s \end{matrix} \end{array} \begin{matrix} p \\ q \\ r \\ s \end{matrix},$$

★ : a sub-matrix

0 : zero sub-matrix

$$\boxed{\begin{aligned} [T^{a++}, P_0] &= [T^{a++}, P_1] = 0, \\ [T^{a+-}, P_0] &= \{T^{a+-}, P_1\} = 0, \\ \{T^{a-+}, P_0\} &= [T^{a-+}, P_1] = 0, \\ \{T^{a--}, P_0\} &= \{T^{a--}, P_1\} = 0, \end{aligned}}$$

Classification of Transformations

(i) Commutative type: $\{T^{a_{++}}\}$

$$[\Omega_{++}, P_0] = [\Omega_{++}, P_1] = 0$$

(ii) Mixed type: $\{T^{a_{+-}}\}$ or $\{T^{a_{-+}}\}$

$$[\Omega_{+-}, P_0] = \{\Omega_{+-}, P_1\} = 0$$

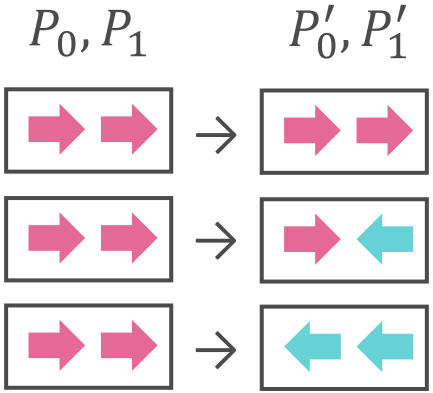
(iii) Anti-commutative type: $\{T^{a_{--}}\}$

$$\{\Omega_{--}, P_0\} = \{\Omega_{--}, P_1\} = 0$$

Global ECs transformations

$$\Omega = \exp\left[i\,\frac{c^a}{2}T^a\right] \qquad c^a\text{: constant}$$

$\{T^a\}$	(P_0', P_1')
commutative	(P_0, P_1)
mixed	$(P_0, e^{iT^{+-}}P_1),\ (e^{iT^{-+}}P_0, P_1)$
anti-commutative	$(e^{iT^{--}}P_0, e^{iT^{--}}P_1)$



Is there a set of parameters such that the rotation factor e^{iT^-} becomes a non-trivial real diagonal matrix?

Simple Example

e.g.) mixed type for $U(2)$ matrix

$$T^{+-} = \begin{pmatrix} 0 & c \\ c^* & 0 \end{pmatrix}$$

$$e^{iT^{+-}} = \begin{pmatrix} \cos|c| & \frac{ic}{|c|}\sin|c| \\ \frac{ic^*}{|c|}\sin|c| & \cos|c| \end{pmatrix}$$

$$\mathrm{diag}(e^{iT^{+-}}) = I_2,\ -I_2$$

$$P_0 = \begin{pmatrix} + & \\ & + \end{pmatrix}, \quad P_1 = \begin{pmatrix} + & \\ & - \end{pmatrix}$$

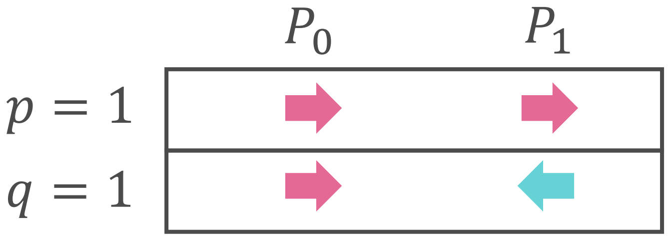
$$\downarrow$$

$$P_0' = P_0 = \begin{pmatrix} + & \\ & + \end{pmatrix}, \quad P_1' = e^{iT^{+-}} P_1 = \begin{pmatrix} - & \\ & + \end{pmatrix}$$

Simple Example

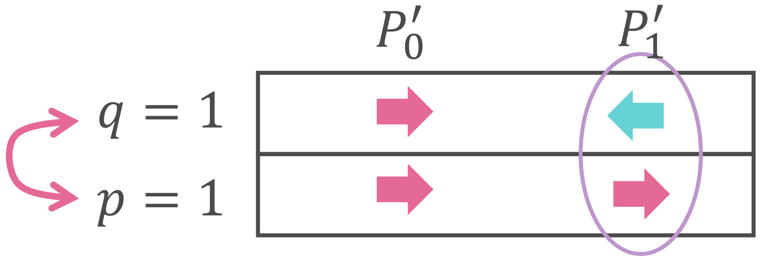
e.g.) mixed type for $U(2)$ matrix

$$P_0 = \begin{pmatrix} + & \\ & + \end{pmatrix}, \quad P_1 = \begin{pmatrix} + & \\ & - \end{pmatrix} \quad \to \quad P'_0 = \begin{pmatrix} + & \\ & + \end{pmatrix}, \quad P'_1 = \begin{pmatrix} - & \\ & + \end{pmatrix}$$



[1,1,0,0]

$\to$



[1,1,0,0]

A set of BCs $[p, q, r, s]$ are invariant

General Example

e.g.) anti-commutative type for $U(N)$ matrix

$$T^{-}=\begin{pmatrix} 0 & 0 & 0 & A \\ 0 & 0 & B & 0 \\ 0 & B^{\dagger} & 0 & 0 \\ A^{\dagger} & 0 & 0 & 0 \end{pmatrix} \qquad \mathrm{diag}(e^{iT^{-}})=\begin{pmatrix} \tilde{I}_{p,X} & 0 & 0 & 0 \\ 0 & \tilde{I}_{q,Y} & 0 & 0 \\ 0 & 0 & \tilde{I}_{r,Y} & 0 \\ 0 & 0 & 0 & \tilde{I}_{s,X} \end{pmatrix}$$

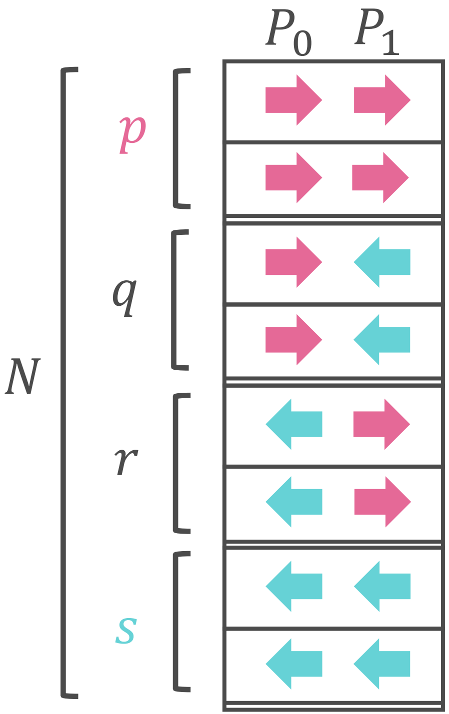
A, B: sub-matrices

$\tilde{I}_{p,X}$: diagonal $(p\times p)$ sub-matrix with ± 1

X : the number of -1 components

General Example

e.g.) anti-commutative type for $U(N)$ matrix

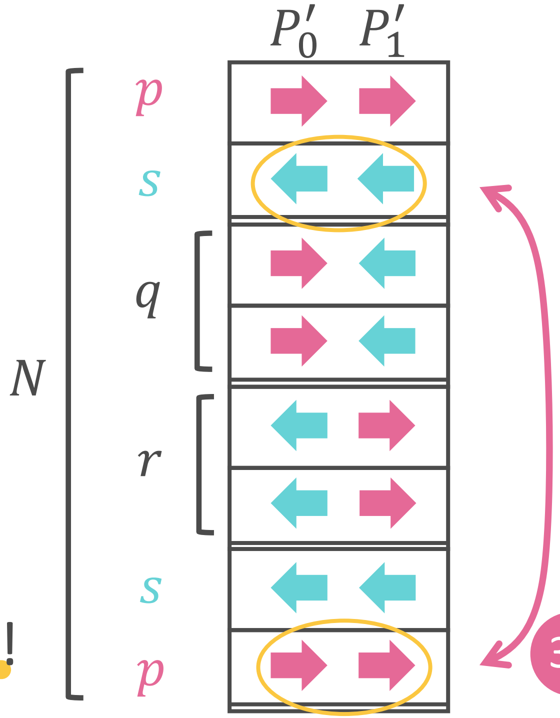


e.g.)

$X = 1$

$Y = 0$

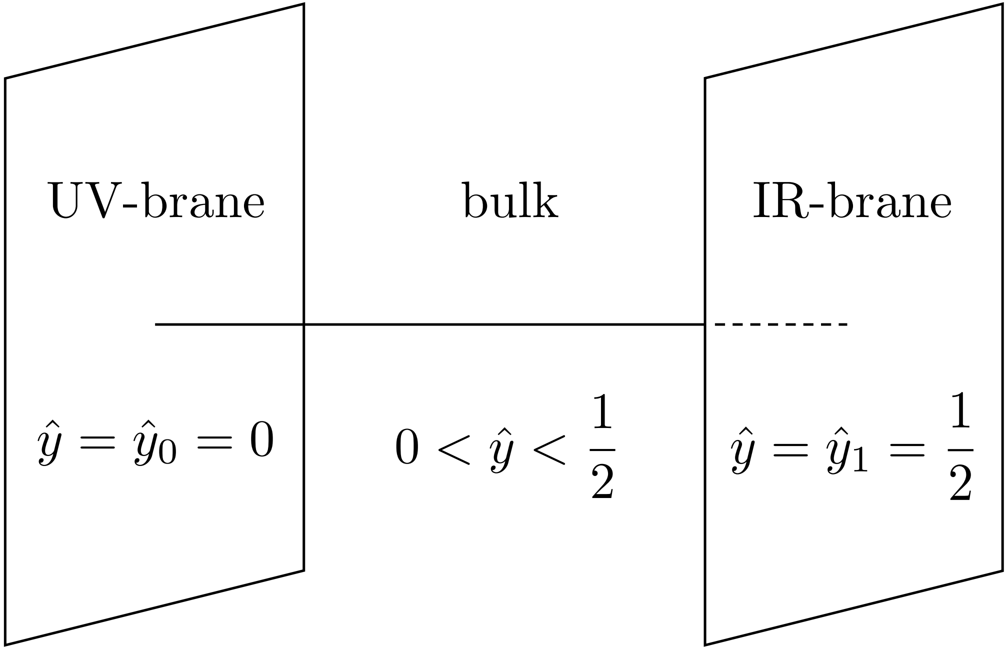
$\rightarrow$



$[p, q, r, s]$ is invariant!

Local ECs transformations

Local ECs transformations behave differently on the UV-brane and the others.



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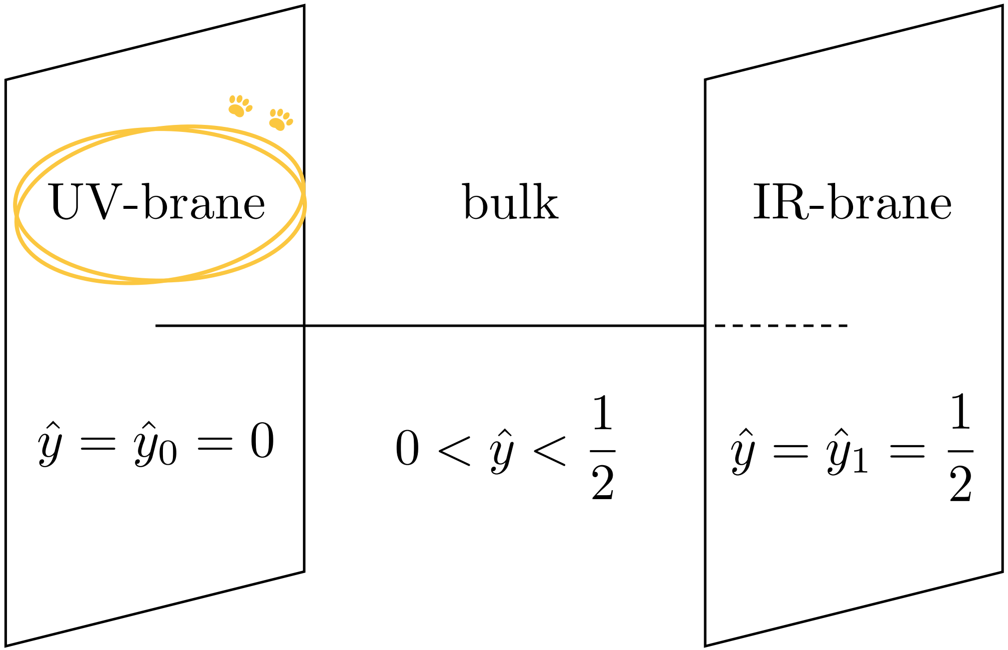


On the UV-brane

On the UV-brane ($\hat{y} \to 0$), for any local transformations,

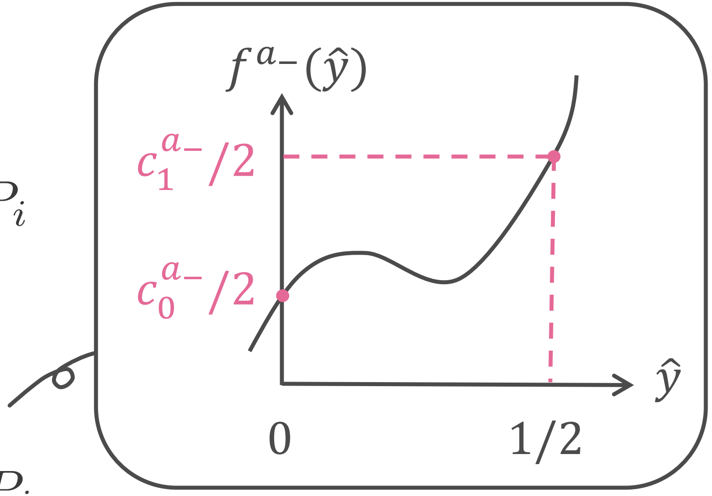
$$\Omega(\hat{y}) = \exp\left[i f^a(\hat{y}) T^a\right]$$

Commutative type:

$$P_i' = \exp\left[-i \left\{f^{a_+}(\hat{y}_i + \hat{y}) - f^{a_+}(\hat{y}_i - \hat{y})\right\} T^{a_+}\right] P_i$$
$$\to P_i \quad (\hat{y} \to 0)$$

Anti-commutative type

$$P_i' = \exp\left[-i \left\{f^{a_-}(\hat{y}_i + \hat{y}) + f^{a_-}(\hat{y}_i - \hat{y})\right\} T^{a_-}\right] P_i$$
$$\to e^{-i c_i^{a_-} T^{a_-}} P_i \quad (\hat{y} \to 0)$$



On the UV-brane

On the UV-brane ($\hat{y} \to 0$)

$\{T^a\}$	(P_0', P_1')
commutative	(P_0, P_1)
mixed	$(P_0, e^{iT^{+-}} P_1),\ (e^{iT^{-+}} P_0, P_1)$
anti-commutative	$(e^{iT_0^{--}} P_0, e^{iT_1^{--}} P_1)$

$$T_0^{--} = c_0^{a--} T_0^{a--}$$
$$T_1^{--} = c_1^{a--} T_1^{a--}$$

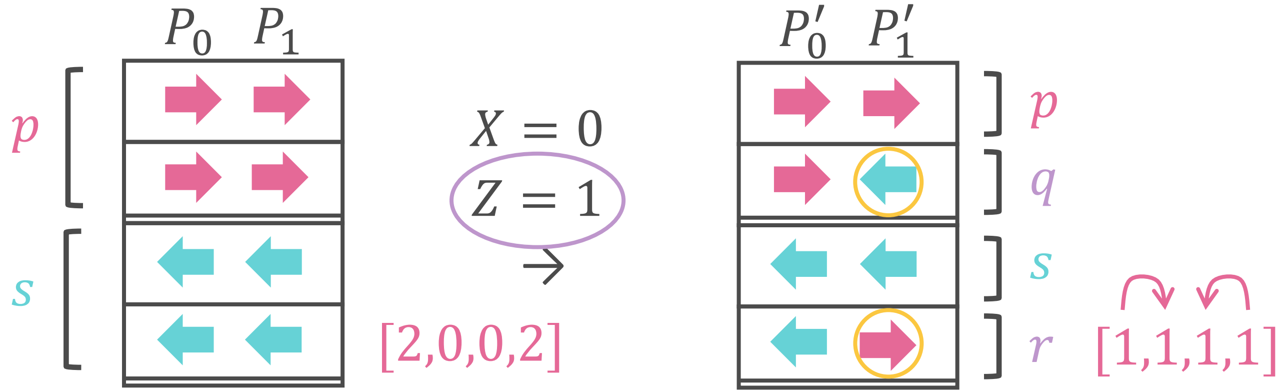
not simultaneously,
but independently

On the UV-brane

e.g.) anti-commutative type: $[p,q,r,s] = [2,0,0,2]$

$$\mathrm{diag}(e^{iT_0^{--}}) = \mathrm{diag}(\tilde{I}_{p,X}, \tilde{I}_{s,X})$$

$$\mathrm{diag}(e^{iT_1^{--}}) = \mathrm{diag}(\tilde{I}_{p,Z}, \tilde{I}_{s,Z})$$



On the UV-brane

On the UV-brane ($\hat{y} \to 0$)

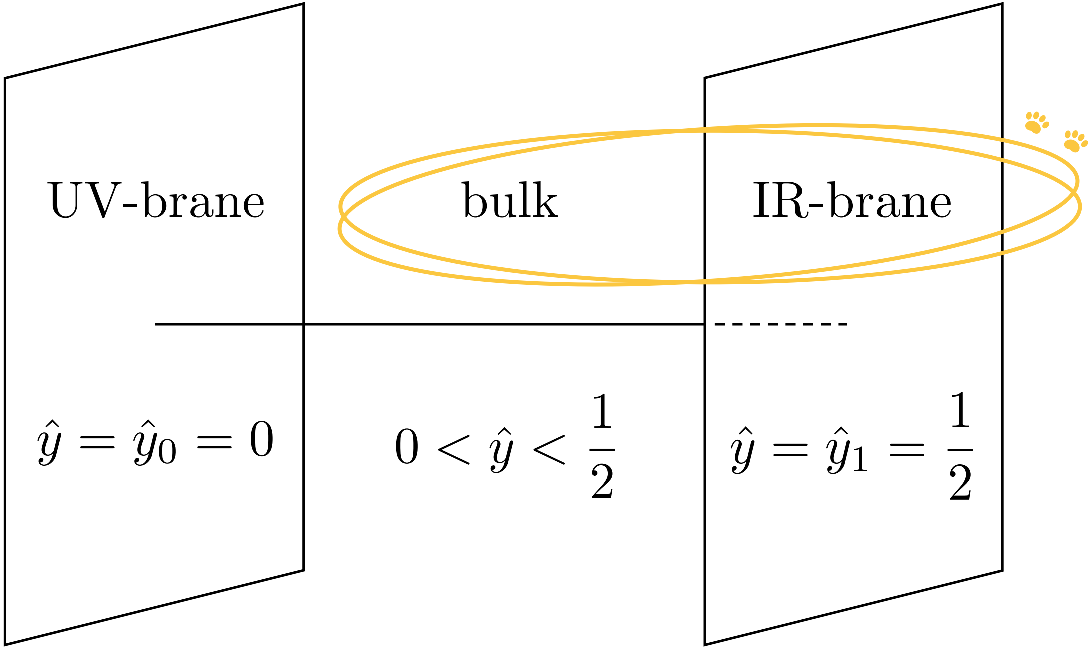
$\{T^a\}$	(P_0', P_1')
commutative	(P_0, P_1)
mixed	$(P_0, e^{iT^{+-}} P_1),\ (e^{iT^{-+}} P_0, P_1)$
anti-commutative	$(e^{iT_0^{--}} P_0, e^{iT_1^{--}} P_1)$

$$[p,q,r,s] \sim [p-1, q+1, r+1, s-1]$$
$$\sim [p+1, q-1, r-1, s+1]$$

Only this ECs!

Local ECs transformations

Local ECs transformations behave differently on the UV-brane and the others.

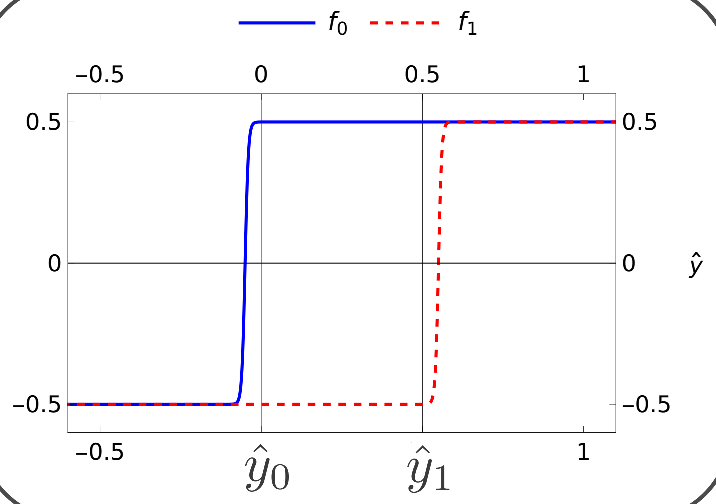


On the Bulk and IR-brane

Commutative type:

$$P_i' = \exp\left[-i\left\{f(\hat{y}_i+\hat{y}) - f(\hat{y}_i-\hat{y})\right\}T^{++}\right]P_i$$

$$f_0(\hat{y}) = \frac{1}{2}\tanh\left[\lambda\left(\hat{y}-(\hat{y}_0-\epsilon)\right)\right],$$
$$f_1(\hat{y}) = \frac{1}{2}\tanh\left[\lambda\left(\hat{y}-(\hat{y}_1+\epsilon)\right)\right]$$



Bulk area: $1/\lambda \ll \epsilon < \hat{y} < 1/2$

$$\begin{cases} \partial_M P_i' = 0 \\ P_i'^{\dagger} = P_i' \end{cases}$$

⇒ ECs gauge conditions are achieved!

On the Bulk and IR-brane

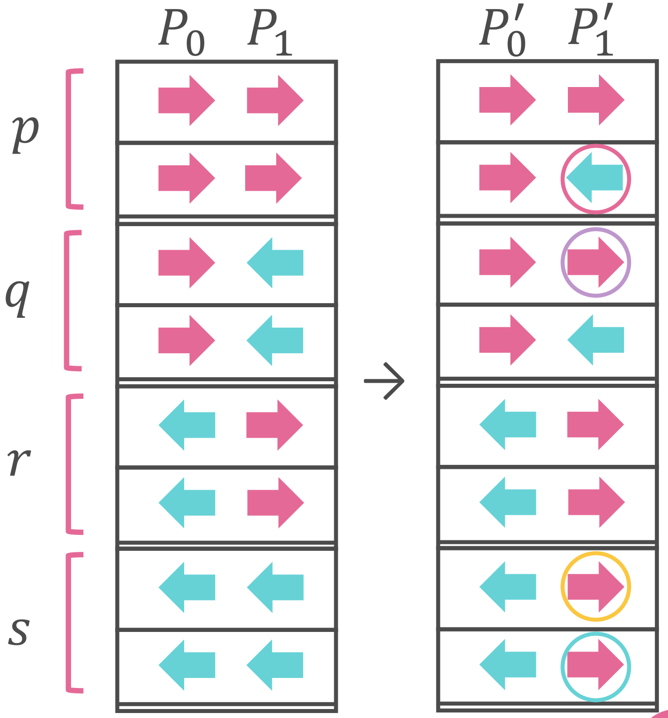
Using parameters $f_1(\hat{y})$,

$$(P_0', P_1') = (P_0,\, e^{-iT_1^{++}} P_1)$$

$$\begin{pmatrix} \star & 0 & 0 & 0 \\ 0 & \star & 0 & 0 \\ 0 & 0 & \star & 0 \\ 0 & 0 & 0 & \star \end{pmatrix}$$

$$\mathrm{diag}\left(e^{iT_1^{++}}\right) = \begin{pmatrix} e^{ia_1} & & 0 \\ & \ddots & \\ 0 & & e^{ia_N} \end{pmatrix}$$

$$= \tilde{I}$$: diagonal matrix with ± 1



Freely flip!

All set of the BCs are connected!

05

Summary

Summary (1/2)

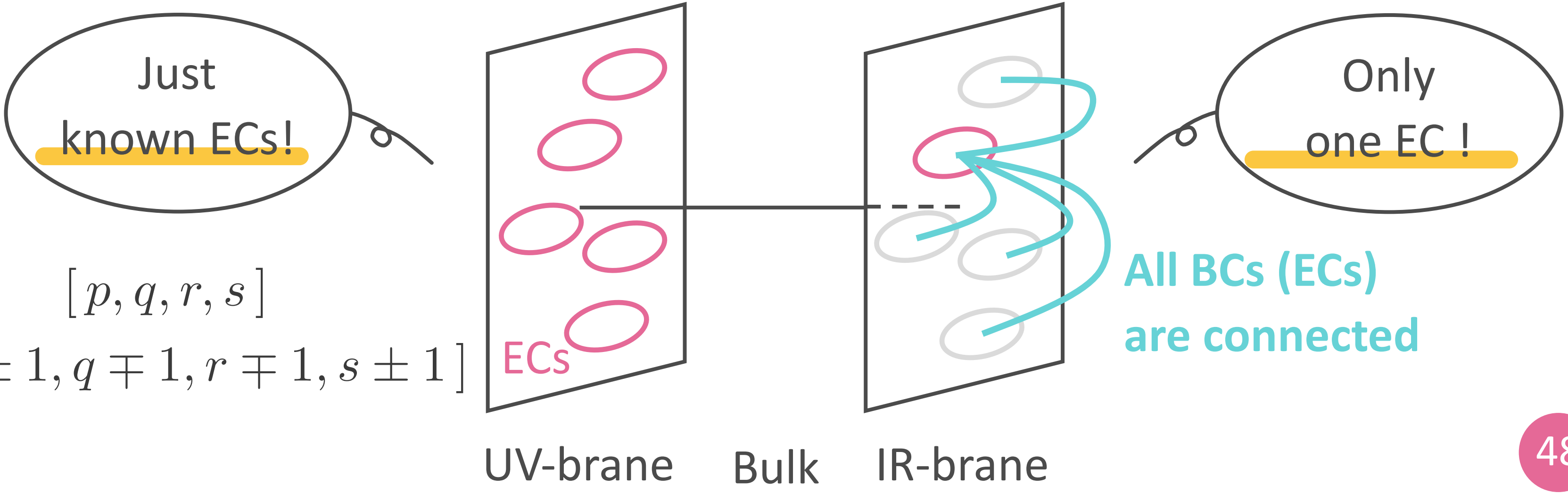
- Symmetry breaking are caused by Boundary Conditions (BCs).
- Gauge-connected BCs belong to Equivalence Class (EC).

P_0, P_1	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
symmetry	U(1)×U(1)	SU(2)×U(1)	U(1)

belong to the same EC

Summary (2/2)

- On the UV-brane, only the known ECs are constructed.
- On the bulk and IR-brane, all sets of the BCs are connected.

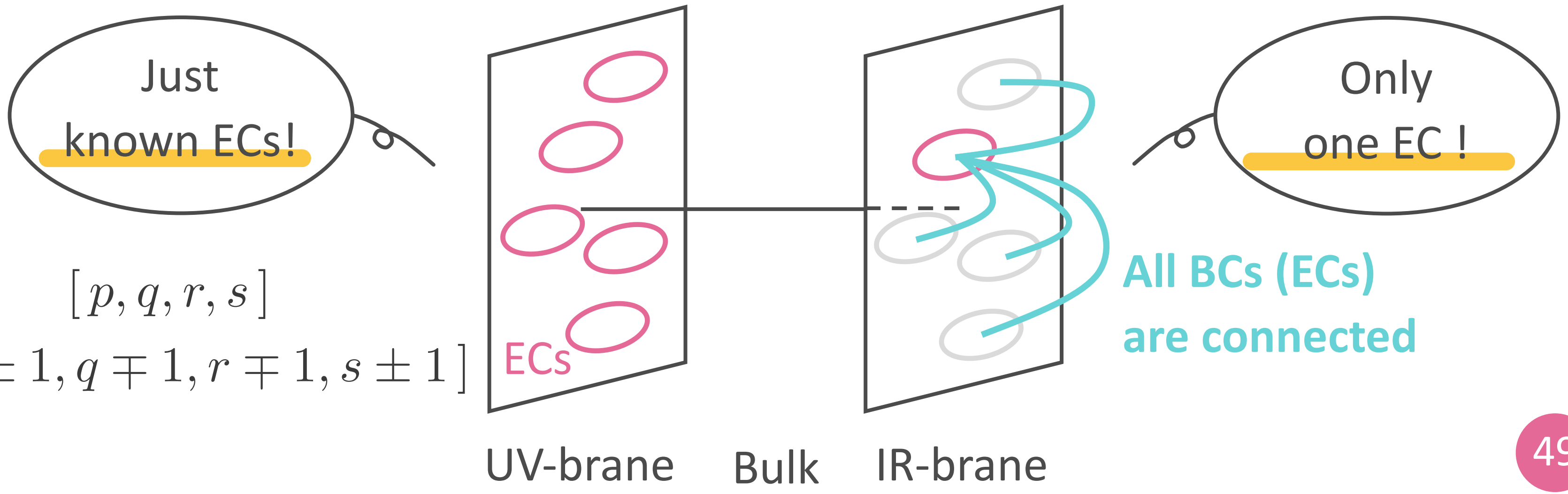


$$[\,p, q, r, s\,]$$
$$\sim [\,p \pm 1, q \mp 1, r \mp 1, s \pm 1\,]$$

Summary (2/2)

Thank you for listening!

- On the UV-brane, only the known ECs are constructed.
- On the bulk and IR-brane, all sets of the BCs are connected.



$$[\,p, q, r, s\,] \sim [\,p \pm 1, q \mp 1, r \mp 1, s \pm 1\,]$$

06

Follow-up

Applications (1/2)

- Apply to phenomenological models

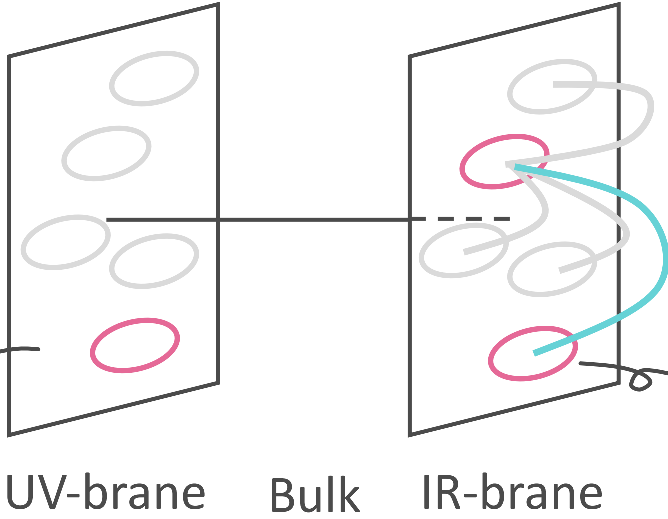
e.g.) $P_0 = P_1 = \mathrm{diag}\,(-1, -1, +1)$

$SU(3)$

$$A_\mu^0 = \begin{pmatrix} \star & \star & \star \\ \star & \star & \star \\ \star & \star & \star \end{pmatrix}$$

$SU(2)\times U(1)$

$$A_\mu^0 = \begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$$



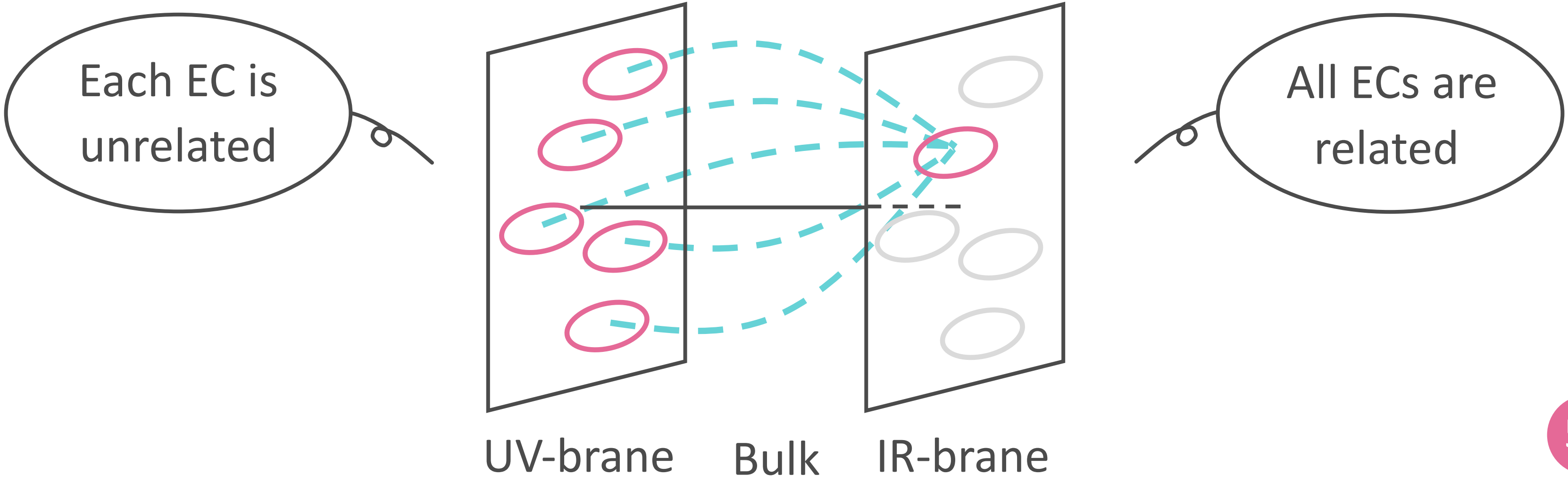
$SU(2)\times U(1)$

$$A_\mu^0 = \begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$$

Applications (2/2)

- Approach to the arbitrariness problem of BCs:

Which type of BCs should be selected
without relying on phenomenological information?



Symmetry Breaking by BCs

e.g.) SU(3)model

$$P_0 = P_1 = diag\,(-1, -1, +1)$$

$$A_\mu(x, y_i - y) = P_i A_\mu(x, y_i + y) P_i$$

$$= \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}$$

(P_0, P_1) parity

$SU(2)\times U(1)$ 4-D effective theory

Symmetry Breaking by BCs

$$\mathcal{G}^{5D} = \{T^a\} \quad \to \quad \mathcal{H}^{BC} = \{T^a \in \mathcal{G} \,|\, [T^a, P_i] = 0 \ (i = 0, 1)\}$$

P_0, P_1	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
A_μ^0	$\begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \star \\ 0 & \star & 0 \end{pmatrix}$
$\mathcal{H}^{BC}$	$\{T^1, T^2, T^3, T^8\}$	$\{T^3, T^8\}$	$\{\frac{\sqrt{3}}{2}T^3 + \frac{1}{2}T^8\}$
symmetry	SU(2)×U(1)	U(1)×U(1)	U(1)

Global ECs transformations (1/2)

$$\Omega_\pm = \exp\left[i\frac{c^{a_\pm}}{2}T^{a_\pm}\right]$$ $c^{a_\pm}$: constant

$$[\Omega_+, P] = \{\Omega_-, P\} = 0$$

$$P' = \Omega_+ P\,\Omega_+^\dagger = \Omega_+\Omega_+^\dagger P = P$$

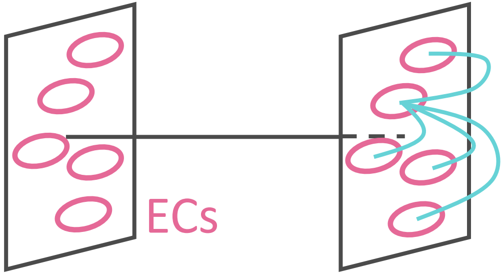
$$P' = \Omega_- P\,\Omega_-^\dagger = \Omega_-\Omega_- P = e^{iT^-}P \quad (T^- = c^{a_-}T^{a_-})$$

rotation factor

ECs in $SU(N)$ on S^1/Z_2 (4/4)

Conclusion:

- At a particular point, there are no further ECs.
- Outside of that point,
  there is a new type of ECs transformations!



Set-up in this section

- 5次元時空座標

$$x^M = (x^\mu, y) \qquad (\mu = 0, 1, 2, 3)$$

- 複素スカラー場の理論

$$S = \int dx^4 dy \left\{ -\frac{1}{2} \Phi^\dagger \partial_M \partial^M \Phi(x, y) \right\} \qquad g_{MN} = diag(+, -, -, -, -)$$

$$= \int dx^4 dy \left\{ -\frac{1}{2} \Phi^\dagger (\partial_\mu \partial^\mu - (\partial_y)^2) \Phi(x, y) \right\}$$

KK expansion on S^1

- KK展開

$$\Phi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \Phi^{(n)}(x) \exp\left(-i \frac{n + \frac{\beta}{2\pi}}{R} y \right)$$

- 4 次元有効理論 $(1/R \to \infty)$

mass spectrum

$$S = \int dx^4 \mathcal{L}^{4D} = \int dx^4 \left\{ -\frac{1}{2} \sum_{n=-\infty}^{\infty} \Phi^{(n)\dagger} \left(\partial_\mu \partial^\mu + m_n^2(\beta) \right) \Phi^{(n)}(x) \right\}$$

$$m_n^2(\beta) \equiv \left(\frac{n + \frac{\beta}{2\pi}}{R} \right)^2$$

KK expansion on $S^1/\mathbb{Z}_2$

(i) $(P_0, P_1) = (+,+)$

$$\Phi(x,y) = \frac{1}{\sqrt{\pi R}}\Phi^{(0)}(x) + \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}\Phi^{(n)}(x)\cos\left(\frac{n}{R}y\right)$$

(ii) $(P_0, P_1) = (-,-)$

$$\Phi(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}\Phi^{(n)}(x)\sin\left(\frac{n}{R}y\right)$$

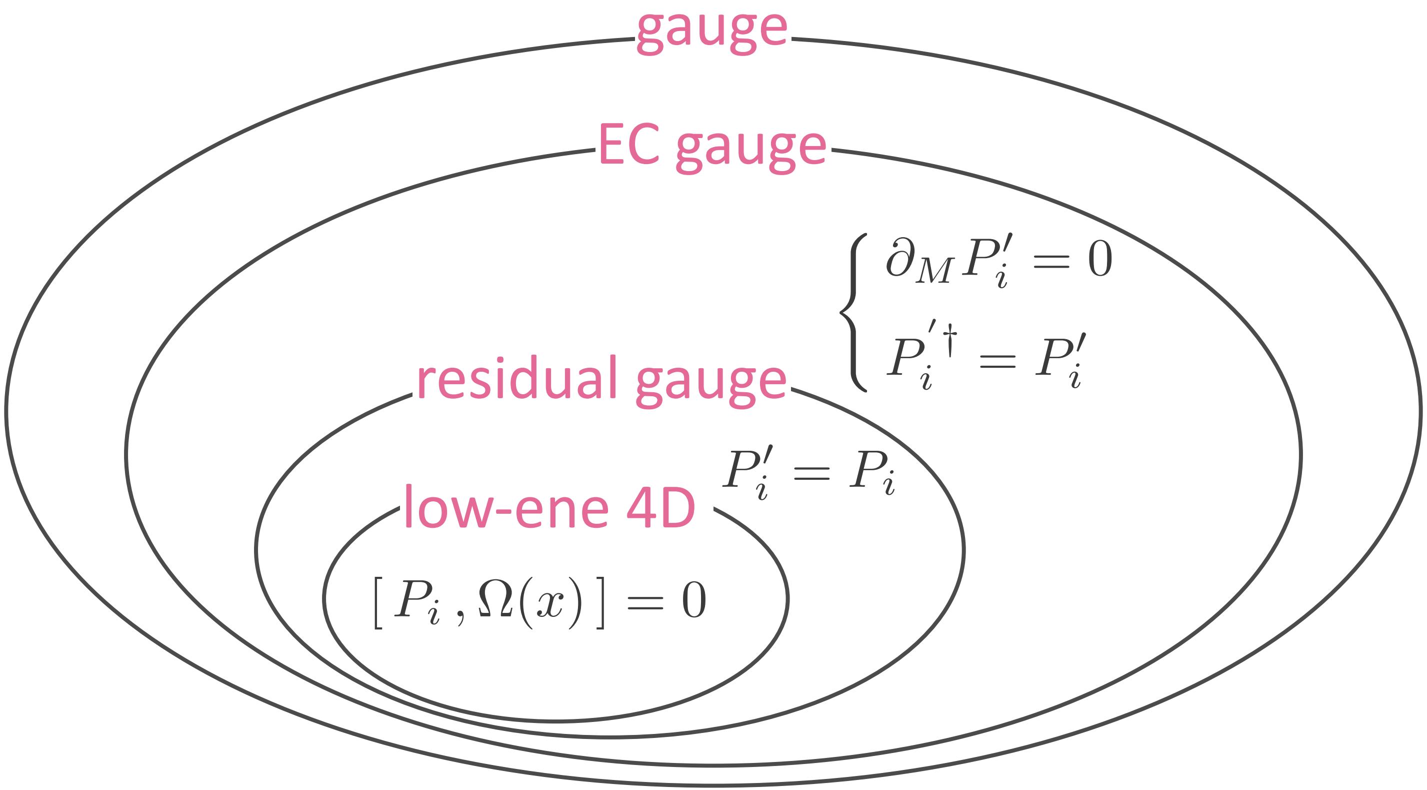
(iii) $(P_0, P_1) = (+,-)$

$$\Phi(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n=0}^{\infty}\Phi^{(n+\frac{1}{2})}(x)\cos\left(\frac{n+\frac{1}{2}}{R}y\right)$$

(iv) $(P_0, P_1) = (-,+)$

$$\Phi(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n=0}^{\infty}\Phi^{(n+\frac{1}{2})}(x)\sin\left(\frac{n+\frac{1}{2}}{R}y\right)$$

Equivalence Classes



EC Gauge Transformation

e.g.) SU(2)model

$$\underline{P_0 = P_1 = \sigma_3} \qquad \Omega(y) = \exp\left[i\pi \frac{y}{2\pi R}\sigma_1\right]$$

$$\begin{cases} P_0' = \Omega(-y)P_0\Omega^\dagger(y) = \Omega(-y)\Omega(y)P_0 = P_0 \\ P_1' = \Omega(\pi R - y)\Omega(\pi R + y)P_1 = \Omega(2\pi R)P_1 = -P_1 \end{cases}$$

$$(\sigma_3, \sigma_3) \sim (\sigma_3, -\sigma_3) \qquad [1,0,0,1] \sim [0,1,1,0]$$

In general,

$$[p,q,r,s] \sim [p-1, q+1, r+1, s-1] \sim [p+1, q-1, r-1, s+1]$$

Physical Symmetry

- Wilson line phase (AB phase)

$$W_T \equiv \mathcal{P}\exp\left[ig\int_0^{2\pi R} dy A_y(x,y)\right] T \qquad (T = P_1 P_0)$$

- １つの同値類で実現される物理は１つ　Hosotani(1989)

$$V_{eff}(A_M^c; P_0, P_1) = V_{eff}(A_M^{\prime c}; P_0^\prime, P_1^\prime)$$

- Physical symmetry

$$(A_y^c\ ; P_o, P_1) \sim (A_y^{\prime c} = 0\ ; P_o^{phys}, P_1^{phys})$$

$$\mathcal{H}^{phys} = \left\{ T^a \in \mathcal{G} \,\middle|\, \left[T^a, P_i^{phys}\right] = 0 \ (i = 0, 1) \right\}$$

Number of ECs

- $SU(N)$ゲージ理論の同値類の個数

$$
{}_{N+3}C_3 - \sum_{k=0}^{N-2} {}_{k+1}C_1 \, {}_{N-k-1}C_1 = (N+1)^2
$$