Lifetimes of B-mesons in the context of the $B \to DM$ puzzle

Hiroyuki Umeeda
(Institute of High Energy Physics, Chinese Academy of Sciences)

Collaborators: Albertus Hariwangsa Panuluh (Hiroshima University)
Satoshi Tanaka (Hiroshima University)

Ongoing work

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Introduction

◉ Exclusive B-meson decays

— Hadronic final states are specified. Examples: $B \to D^{(*)}\ell\nu, B \to K^*\gamma$

— Theory: difficult

— For $\bar{B} \to D^{(*)}M$, a puzzle has been pointed out recently. Bordone *et al.* [2007.10338]

— One of the possiblities to resolve this issue is new physics (NP).

◉ Inclusive B-meson decays

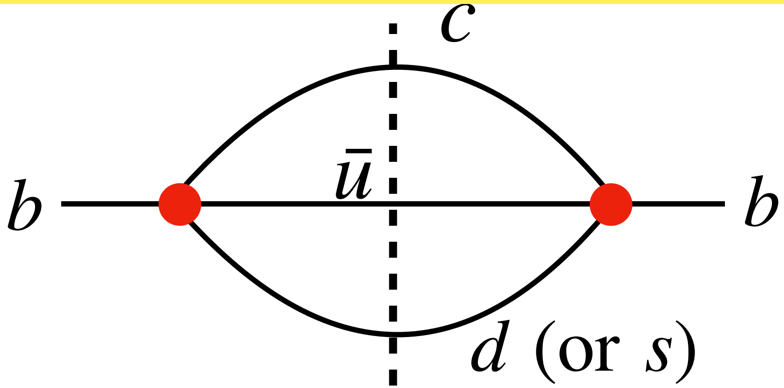
— Inclusive = sum of exclusive (quark-hadron duality).

— Examples: $B \to$ anything, $B \to X_c\ell\nu$

— Theory: easier due to operator product expansion (OPE)

This work: Correlation between inclusive and exclusive decays

$\Gamma_{\text{tot}} \propto$



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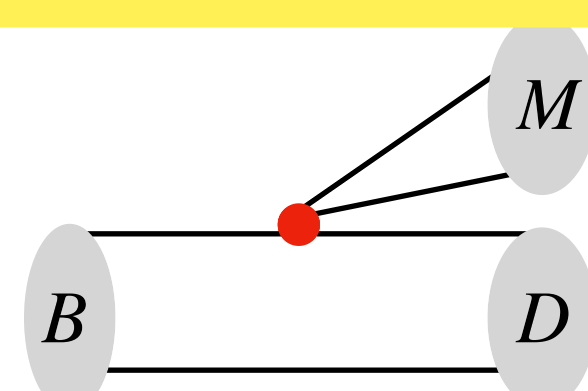

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(1) $\bar{B} \to D^{(*)}M$ decays

— Theoretical analysis based on the QCD factorization

— Final state interactions

(2) Lifetimes of B-mesons

— Heavy quark expansion (HQE)

— Quark-hadron duality

(3) Numerical results

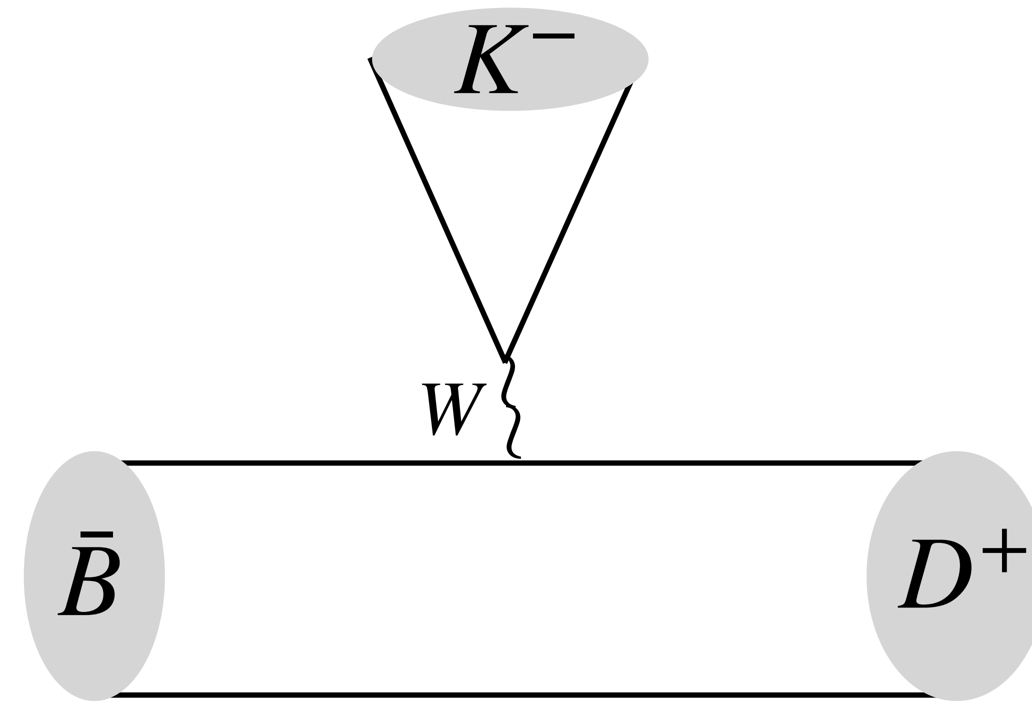
— Lifetimes in the presence of NP

— Constraints on the exclusive parameters

(4) Summary

$\bar{B} \to D^+K^-$ decay

Color-allowed tree diagram



- Non-leptonic decays require non-perturbative analysis in QCD.
- There is a difficulty in evaluating $B \to MM$ in lattice QCD.

QCD factorization approach ($m_b \to \infty$)

Beneke, *et al.* [0006124]

Process = (hard scattering) $\otimes$ (soft objects)

Perturbation theory — Universal

➡ Predictability in $\bar{B} \to D^+K^-$ decay

Comparison between data and QCDF

Branching ratio	PDG	QCDF	(in 10^{-4} units)
$B_s \to D_s^- \pi^+$	29.8 ± 1.4	40.9 ± 2.1	4.4σ
$B_s \to D_s^{*-} \pi^+$	19^{+5}_{-4}	44.6 ± 2.2	4.7σ
$B \to D^- K^+$	2.05 ± 0.08	3.03 ± 0.15	5.8σ
$B \to D^{*-} K^+$	2.12 ± 0.15	3.27 ± 0.16	5.2σ

Endo, Iguro and Mishima [2109.10811]

(1) Higher order corrections of $\mathcal{O}(1/m_b)$?

- The two particle Fock state of twist-three the light meson wave function gives $\alpha_s(\Lambda/m_b)^2$ correction due to the cancellation for $\alpha_s(\Lambda/m_b)$ terms.
- The hard-collinear gluon exchange between b or c and kaon, leading to a twist-four contribution.
- The soft gluon exchange between $B \to D$ system and light meson, evaluated by light-cone QCD sum rule.

— Estimations of the corrections do *not* exceed $\mathcal{O}(1\%)$. Bordone *et al*. [2007.10338]

— ✔Final state interaction induced as elastic rescattering (Next slide) does not lead to an entirely consistent explanation. Endo, Iguro and Mishima[2109.10811]

✔(2) Implication of NP?

Final state interactions

$$B \to D\Pi \to D'\Pi' \qquad \Pi^{(\prime)} = \text{ pion octet}$$

quasi-elastic rescattering   Chua, Hou and Yang [01131248]

Example: $\bar{B}^0 \to D^+K^-$ vs . $\bar{B}^0 \to D^0\bar{K}^0$ $\begin{cases} S = -1 \\ I_z = 0 \end{cases}$

After rescattering   Before rescattering

$$\begin{pmatrix} A[\bar{B}^0 \to D^+K^-] \\ A[\bar{B}^0 \to D^0\bar{K}^0] \end{pmatrix} = S^{1/2}_{2\times 2} \begin{pmatrix} A^{\text{QCDF}}[\bar{B}^0 \to D^+K^-] \\ A^{\text{QCDF}}[\bar{B}^0 \to D^0\bar{K}^0] \end{pmatrix}$$

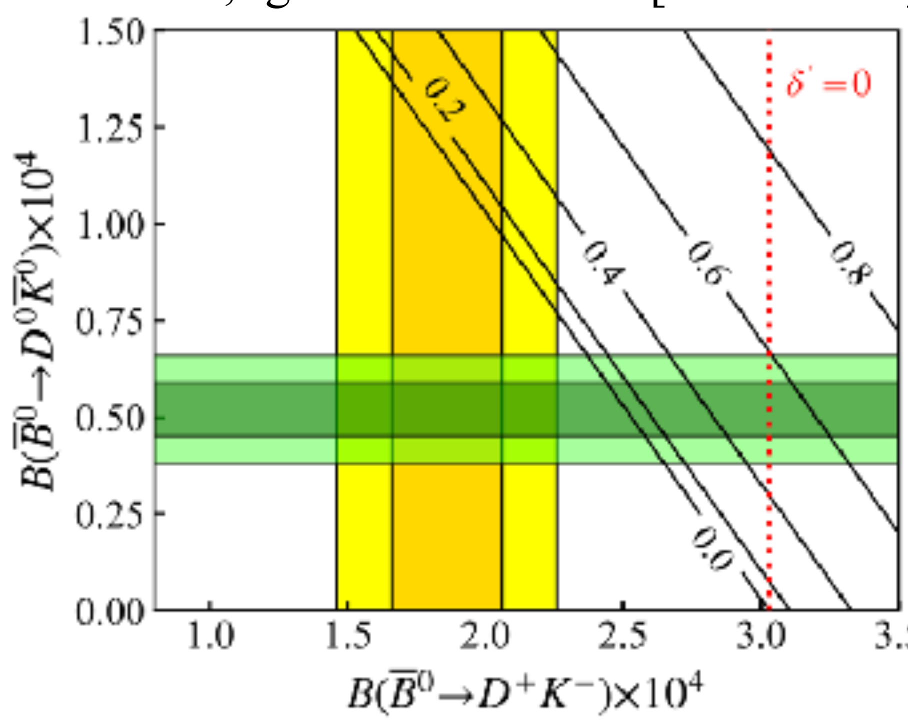
← Predictable

Restrictions $\begin{cases} \text{Unitarity of } S \text{ matrix} \\ \text{SU(3) flavor symmetry} \end{cases}$

$$S^{1/2}_{2\times 2} = \frac{e^{i\delta_{\overline{\mathbf{15}}}}}{2} \begin{pmatrix} 1 + e^{i\delta'} & 1 - e^{i\delta'} \\ 1 - e^{i\delta'} & 1 + e^{i\delta'} \end{pmatrix}$$

$$\delta' \equiv \delta_{\mathbf{6}} - \delta_{\overline{\mathbf{15}}}$$

Endo, Iguro and Mishima [2109.10811]



Simultaneous explanation does *not* work.

NP explanation

$$A^{\rm QCDF}[\bar{B}^0 \to D^{*+}h^-] = \frac{G_F}{\sqrt{2}} a_1(h) |V_{cb}V_{uq}|^2 (m_B^2 - m_{D^*}^2) f_h A_0(m_h^2)$$

$$a_1 = c_1 + \frac{c_2}{3} + \mathcal{O}(\alpha_s)$$

A_0: form factor for $B \to D^*$

$$\mathscr{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* (c_1 Q_1 + c_2 Q_2)$$

$$\begin{cases} Q_1 = (\bar{c}_\alpha b_\alpha)_{\rm V-A} (\bar{q}_\beta u_\beta)_{\rm V-A} \\ Q_2 = (\bar{c}_\alpha b_\beta)_{\rm V-A} (\bar{q}_\beta u_\alpha)_{\rm V-A} \end{cases}$$

$$|a_1(\pi)| = \begin{cases} 0.884 \pm 0.004 \pm 0.003 \pm 0.016 & \text{(Belle [1])} \\ 1.071^{+0.013}_{-0.014} & \text{(QCDF@NNLO [2])} \end{cases}$$

$$|a_1(K)| = \begin{cases} 0.913 \pm 0.019 \pm 0.008 \pm 0.013 & \text{(Belle [1])} \\ 1.069^{+0.010}_{-0.013} & \text{(QCDF@NNLO [2])} \end{cases}$$

[1] Phys. Rev. D **107,** 012003 (2023). [2] JHEP **09** (2016) 112.

◉ The data imply that a_1 is smaller than the QCDF prediction by $\approx 15\,\%$.

Other non-leptonic process are also affected by the changes in c_1, c_2.

➡ Lifetime ratio

Lifetimes of B-mesons

Lifetime: $\tau(B) \propto 1/\Gamma(B)$

- This is one of the primary observables for testing weak and strong interactions.

- Experimental data (HFLAV) are precise, of $\mathcal{O}(0.1)\,\%$ uncertainty.

$$\tau_{B^+} = 1.638 \pm 0.004 \text{ ps} \qquad \tau_{B_d} = 1.519 \pm 0.004 \text{ ps} \qquad \tau_{B_s} = 1.516 \pm 0.006 \text{ ps}$$

- Theoretical method: heavy quark expansion (HQE)

$$\Gamma \propto m_b^5 \left(1 + \frac{c_2}{m_b^2} + \frac{c_3}{m_b^3} + \frac{c_4}{m_b^4} + \cdots \right)$$

$1/m_b$: expansion parameter

- Precision in theory is limited, especially by QCD uncertanties.

Errors are at most, $\begin{cases} \text{19\% for total width.} \\ \text{2\% for lifetime ratio.} \end{cases}$ Lenz, Piscopo and Rusov [2208.02643]

Total width

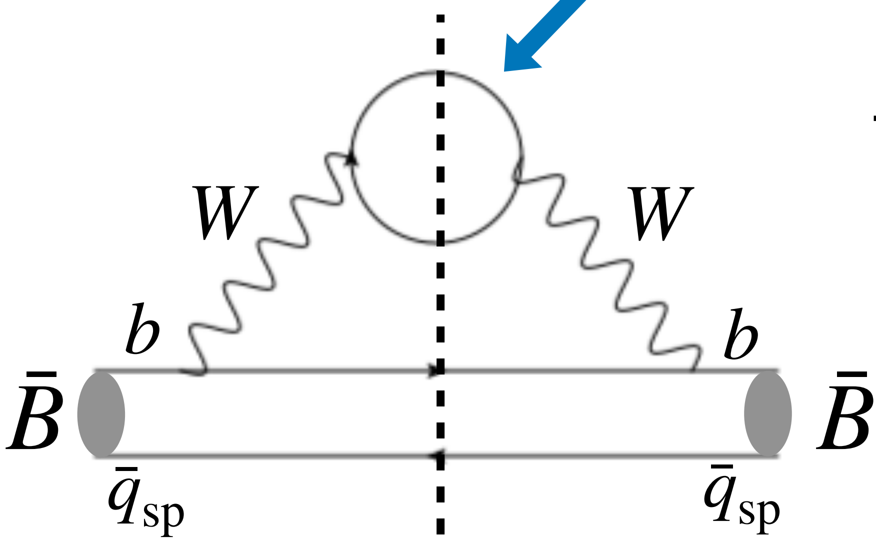
$1/m_Q$ expansion $\qquad (Q = b, c)$

$$\Gamma = \frac{G_F^2 m_Q^5}{192\pi^3}|V_{\rm CKM}|^2 \left[c_3 - c_3 \frac{\mu_\pi^2}{2m_Q^2} + c_G \frac{\mu_G^2}{2m_Q^2} + \frac{c_6}{m_Q^3} \frac{< H_Q|(\bar{Q}q)(\bar{q}Q)|H_Q >}{M_{H_Q}} + \cdots \right]$$

LO

$1/m_Q^2$ corrections $\qquad \mathcal{O}(1/m_Q^3)$ correction

vanish for $m_Q \to \infty$



The spectator does not join interaction.

LO

Lifetimes are common for

$B^+(\bar{b}u), B_d(\bar{b}d), B_s(\bar{b}s)$ and $D^0(c\bar{u}), D^+(c\bar{d}), D_s^+(c\bar{s})$

if $\mathcal{O}(1/m_Q^2)$ corrections are negligible.

lifetime ratio:
$$\frac{\tau(H_1)}{\tau(H_2)} = 1 + \frac{G_F^2 m_b^3}{384\pi^3} V_{cb}^2 \tau_1 \left[c_3 \left(\mu_\pi^2(H_1) - \mu_\pi^2(H_2) \right) + c_G \left(\mu_G^2(H_2) - \mu_G^2(H_1) \right) \right]$$
$$+ \frac{G_F^2 m_b^2}{192\pi^3} V_{cb}^2 \tau_1 \left[\frac{c_6(H_2)\langle H_2|Q|H_2\rangle - c_6(H_1)\langle H_1|Q|H_1\rangle}{M_B} + \mathcal{O}\left(\frac{\Lambda}{m_b}\right) \right]$$

If the lifetime ratio ≈ 1, the $1/m_Q$ expansion is convergent.

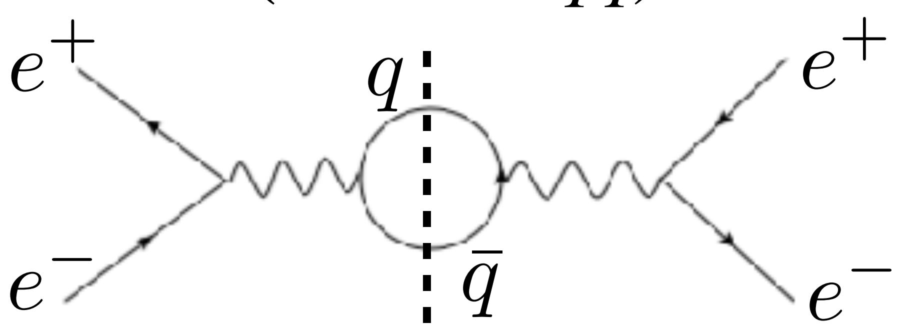
Quark-hadron duality

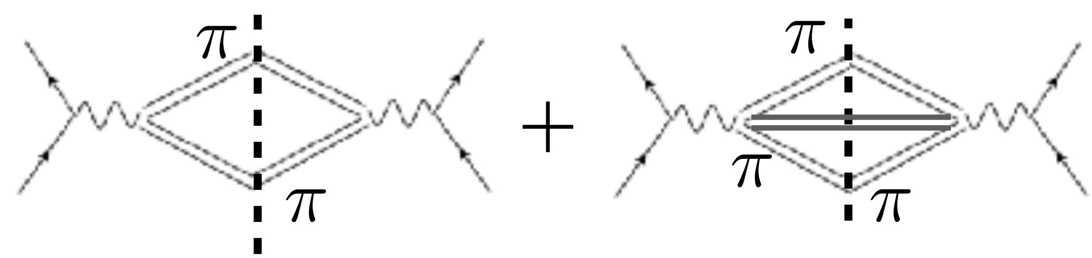
Quark | **Hadron**

$\sigma(e^+e^- \to q\bar{q})$

$e^+e^- \to \text{hadrons}$

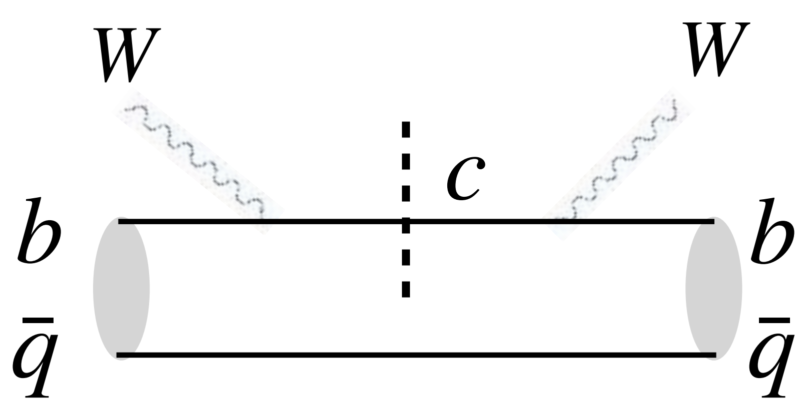
$\sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 3\pi) + \cdots$

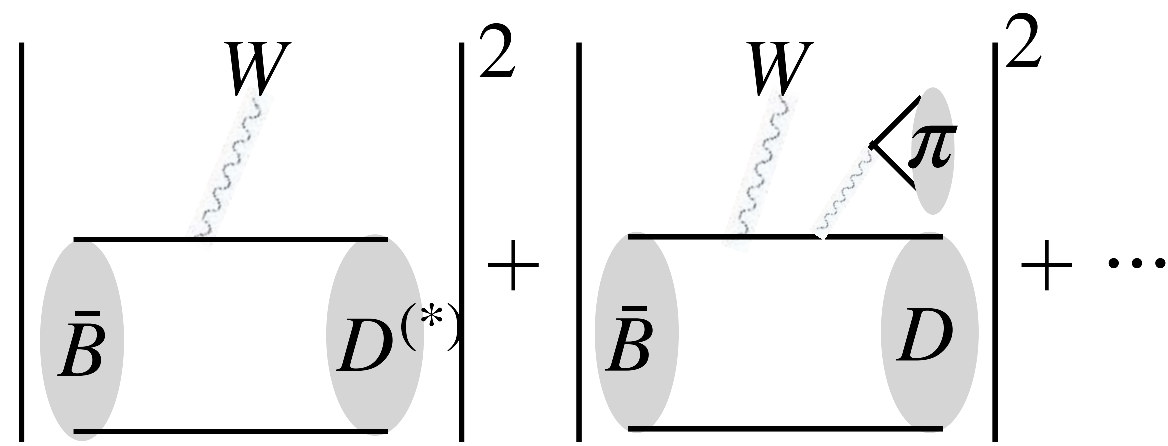




+ (condensates)

$\bar{B} \to X_c l \bar{\nu}$





+ QCD corrections, power corrections

Is duality violation possble?

- Ultimate accuracy of the OPE is limited due to divergences in perturbative series.
  (1) Proliferation of Feynman diagrams (2) Renormalons ✔(3) Power series

Approaches to duality violation

(1) Resonance-based model

[9510366, 9705390, 9708396, 9805404, 9805241, 9902315, 9903258, 0006346, 0106205, 0112323, 0605248, 2106.06215, 2111.01401]

- The large-N_c limit and linear Regge tragectory are considered as a starting point of discussion.

Pros: Theoretical predictions are rigorous.

Cons: This is mostly a toy model of QCD.

(2) Instanton-based model [9605465, 2208.11896]

- This method takes account of a (fixed-sized) background instanton, more or less as an orientational direction to capture contributions related to the factorial divergence in power corrections.

Pros: Some realistic contributions could be examined (not a toy model).

Cons: Detail of the QCD vacuum is involved in a non-trivial way.

(3) Lattice QCD [1703.01881, 2005.13730, 2203.11762]

- Reconstruction of the spectral function is considered for inclusive B-decays.

Pros: It is based on *ab initio* evaluation of the hadronic correlator.

Cons: Specific duality-violating components are suppressed in the Euclid space.

Comparison between theory and experiment

	Lifetime ratio	HQE	HFLAV	Theory detail
beauty mesons	$\tau(B^+)/\tau(B_d)$	$1.0851^{+0.0230}_{-0.0217}$ [1]	1.076 ± 0.004	Darwin term consistent with HQET/VIA
	$\tau(B_s)/\tau(B_d)$	$1.0032^{+0.0063}_{-0.0063}$ [1]	0.998 ± 0.005	Darwin term consistent with HQET/VIA
charmed mesons	$\tau(D^+)/\tau(D^0)$	$2.89^{+0.66+0.42}_{-0.78-0.35}$ [2]	2.54 ± 0.02	MSR scheme
	$\bar{\tau}(D_s^+)/\tau(D^0)$	$1.00^{+0.23+0.01}_{-0.21-0.01}$ [2]	1.30 ± 0.01	MSR scheme
beauty baryons	$\tau(\Lambda_b^0)/\tau(B_d)$	0.955 ± 0.014 [3]	0.969 ± 0.006	Darwin term consistent with HQET/VIA
	$\tau(\Xi_b^0)/\tau(B_d)$	0.956 ± 0.023 [3]	0.974 ± 0.020	Darwin term consistent with HQET/VIA
	$\tau(\Xi_b^-)/\tau(B_d)$	1.029 ± 0.015 [3]	1.035 ± 0.027	Darwin term consistent with HQET/VIA
	$\tau(\Omega_b^-)/\tau(B_d)$	1.081 ± 0.042 [3]	$1.080^{+0.118}_{-0.112}$	Darwin term consistent with HQET/VIA
charmed baryons	$\tau(\Xi_c^+)/\tau(\Lambda_c^+)$	$1.20^{+0.22+0.12}_{-0.13-0.07}$ [2]	2.25 ± 0.04	MSR scheme
	$\tau(\Xi_c^0)/\tau(\Lambda_c^+)$	$0.81^{+0.29+0.03}_{-0.17-0.03}$ [2]	0.75 ± 0.02	MSR scheme
	$\tau(\Omega_c^0)/\tau(\Lambda_c^+)$	$0.83^{+0.30+0.05}_{-0.18-0.04}$ [2]	1.36 ± 0.02	MSR scheme

[1] Lenz, Piscopo and Rusov [2208.02643], [2] Gratrex, Melic and Nisandzic [2204.11935], [3] Gratex, Lenz, Melic, Nisandzic, Piscopo and Rusov [2301.07698].

- *b*-hadrons: The lifetime ratios are close to unity. Data are consistently reproduced by the HQE.
  → $1/m_b$ expansion successfully works. No implications of duality violation are seen.

- *c*-hadrons: Some lifetime ratios show large deviations from unity.
  Tensions are found for $\tau(\Xi_c^+)/\tau(\Lambda_c^+)$ as well as for $\tau(\Omega_c^0)/\tau(\Lambda_c^+)$.

Input parameters from non-perturbative QCD adopted in this work

2-quark operators

$$\begin{cases} Q_\pi = -\bar{b}_v(iD_\mu)(iD^\mu)b_v, \quad \langle B|Q_\pi|B\rangle = 2m_B\mu_\pi^2 \\ Q_G = \bar{b}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})b_v, \langle B|Q_G|B\rangle = 2m_B\mu_G^2 \end{cases}$$

Bordone, Capdevila and Gambino [2107.00604]

Fit to the moments of semi-leptonic differential width @ three-loop QCD

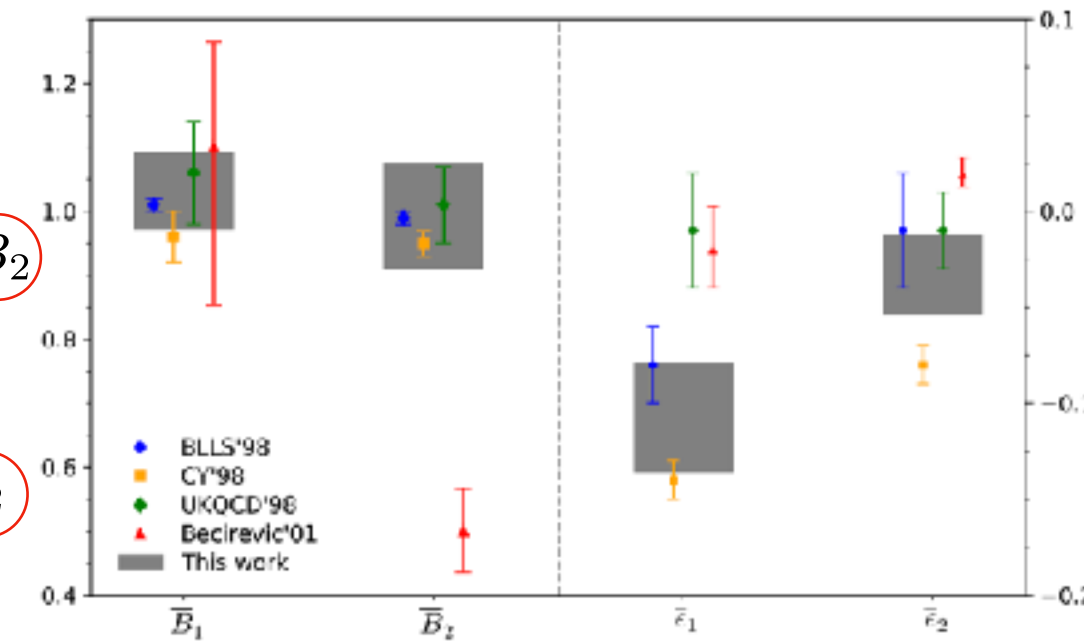
$$\begin{cases} \mu_\pi^2 = (0.477 \pm 0.056)\ \mathrm{GeV}^2 \\ \mu_G^2 = (0.306 \pm 0.050)\ \mathrm{GeV}^2 \\ m_b^{\mathrm{kin}} = (4.573 \pm 0.012)\ \mathrm{GeV} \end{cases}$$

4-quark operators

$$\begin{cases} Q_1 = \bar{b}\gamma_\mu(1-\gamma^5)q\bar{q}\gamma^\mu(1-\gamma^5)b, \ \langle B|Q_1|B\rangle = f_B^2M_B^2B_1 \\ Q_2 = \bar{b}(1-\gamma^5)q\bar{q}(1-\gamma^5)b, \quad \langle B|Q_2|B\rangle = f_B^2M_B^2\frac{M_B^2}{(m_b+m_q)^2}B_2 \\ T_1 = \bar{b}(1-\gamma^5)T^aq\bar{q}(1-\gamma^5)T^ab, \ \langle B|T_1|B\rangle = f_B^2M_B^2\epsilon_1 \\ T_2 = \bar{b}(1-\gamma^5)T^aq\bar{q}(1-\gamma^5)T^ab, \ \langle B|T_2|B\rangle = f_B^2M_B^2\frac{M_B^2}{(m_b+m_q)^2}\epsilon_2 \end{cases}$$

$q\bar{q} \equiv u\bar{u} - d\bar{d}$

Kirk, Lenz and Rauh [1711.02100]



- ◉ The results of the HQET sum rules are adopted in this work.
- ◉ HQET sum rules predict $\bar{\epsilon}_1(m_b) \approx -0.1$ whereas lattice QCD leads to $\bar{\epsilon}_1(m_b) \approx -0.01$, for the central values.

Recent study (lattice 2022)
Lin, Detmold and Meinel [2212.09275]

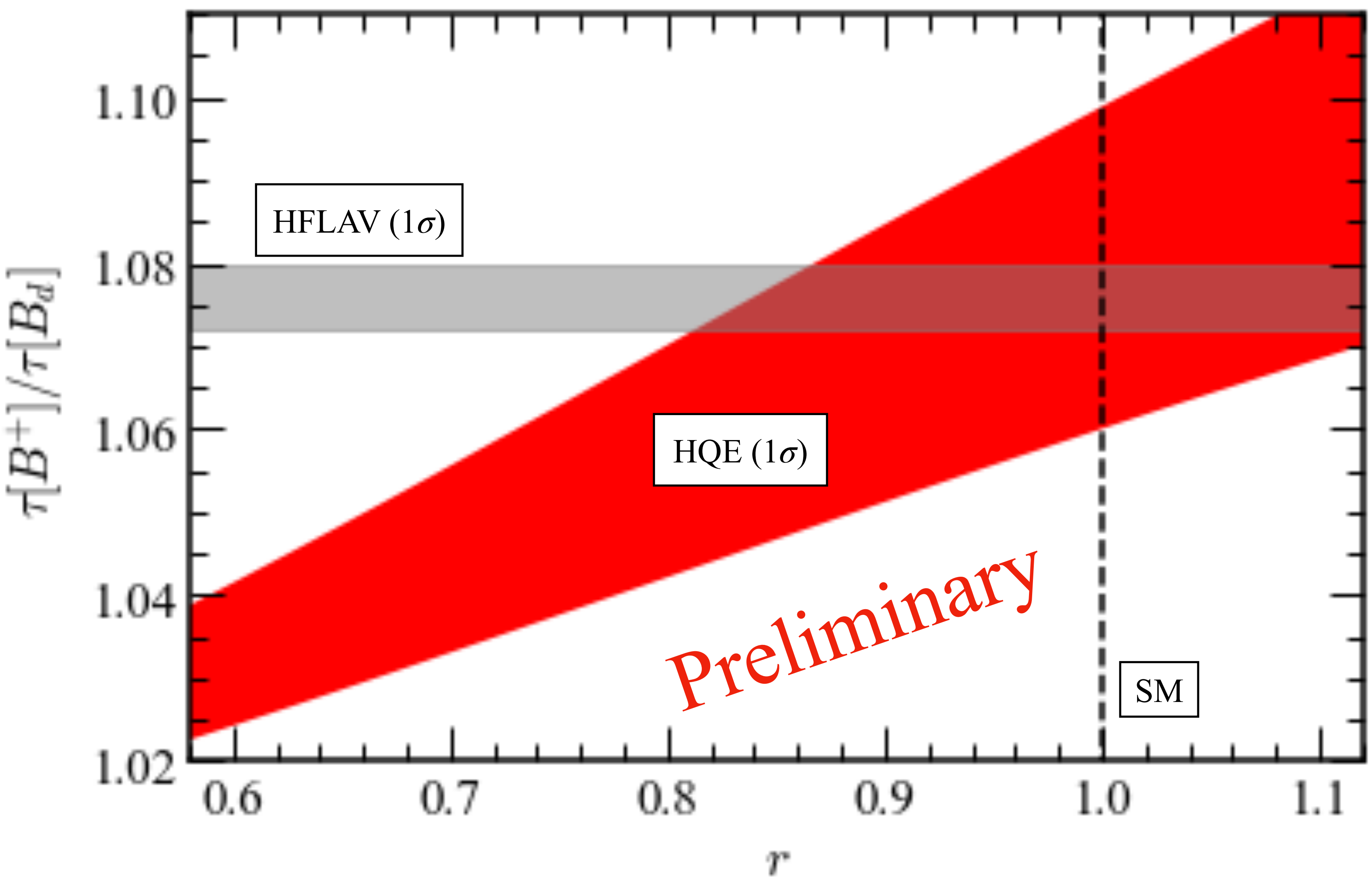
Numerical results

Numerical results: lifetimes at leading order in QCD

$$\frac{\tau(B_u)}{\tau(B_d)} = 1 + \frac{\Gamma_d^{(4)} - \Gamma_u^{(4)}}{\Gamma(B_u)}$$

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* (c_1 Q_1 + c_2 Q_2)$$

$b \to c\bar{u}q$ decays
$(q = d, s)$



$$\begin{cases} Q_1 = (\bar{c}_\alpha b_\alpha)_{\rm V-A} (\bar{q}_\beta u_\beta)_{\rm V-A} \\ Q_2 = (\bar{c}_\alpha b_\beta)_{\rm V-A} (\bar{q}_\beta u_\alpha)_{\rm V-A} \end{cases}$$

$$\begin{cases} c_1 = +\frac{9}{8} a_1^{\rm SM} r - \frac{3}{8} a_2^{\rm SM} \\ c_2 = -\frac{3}{8} a_1^{\rm SM} r + \frac{9}{8} a_2^{\rm SM} \end{cases}$$

$r = 1$: SM

$r \neq 1$: NP

$a_2^{\rm SM}$ is fixed.

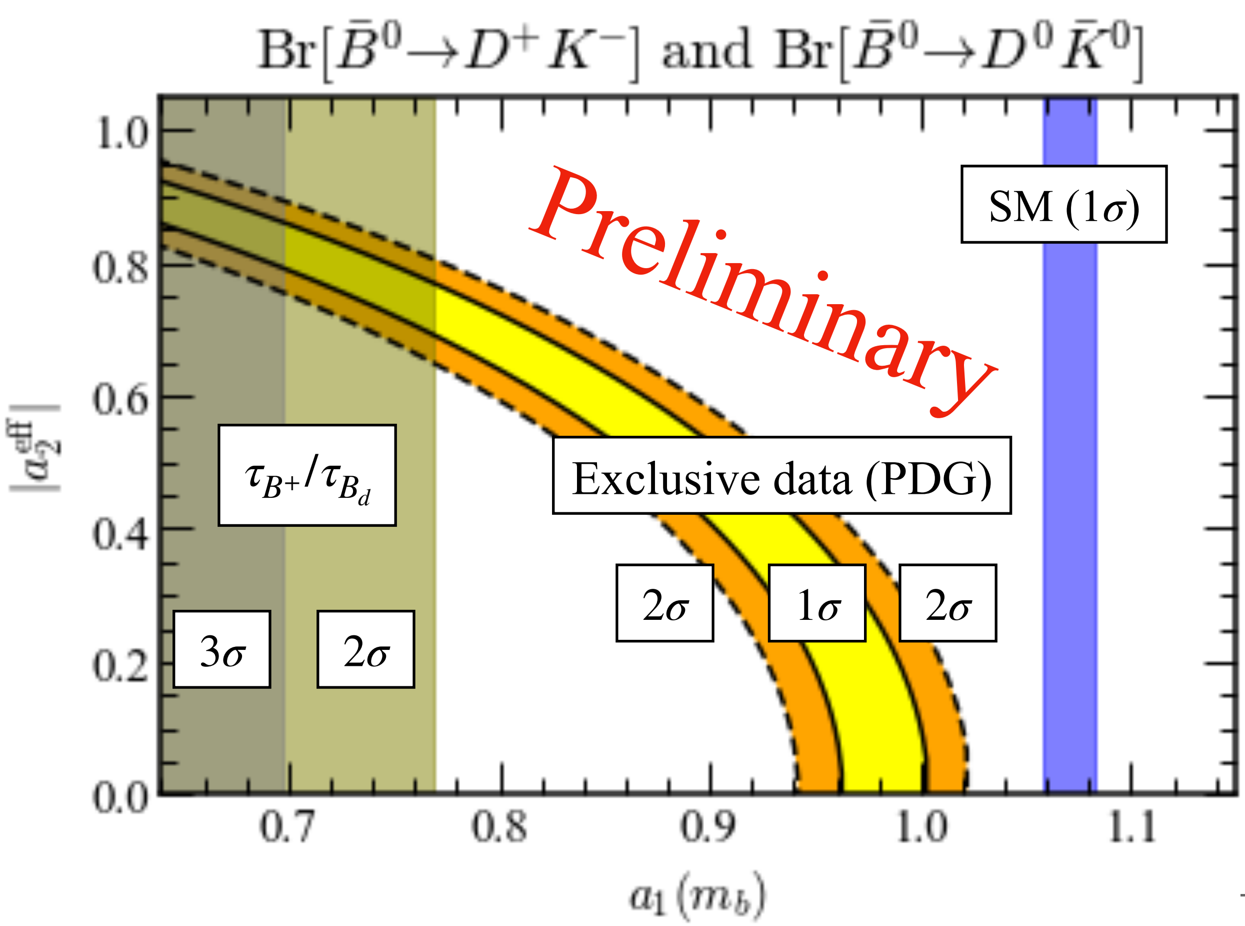
Numerical results: constraints on exclusive parameters

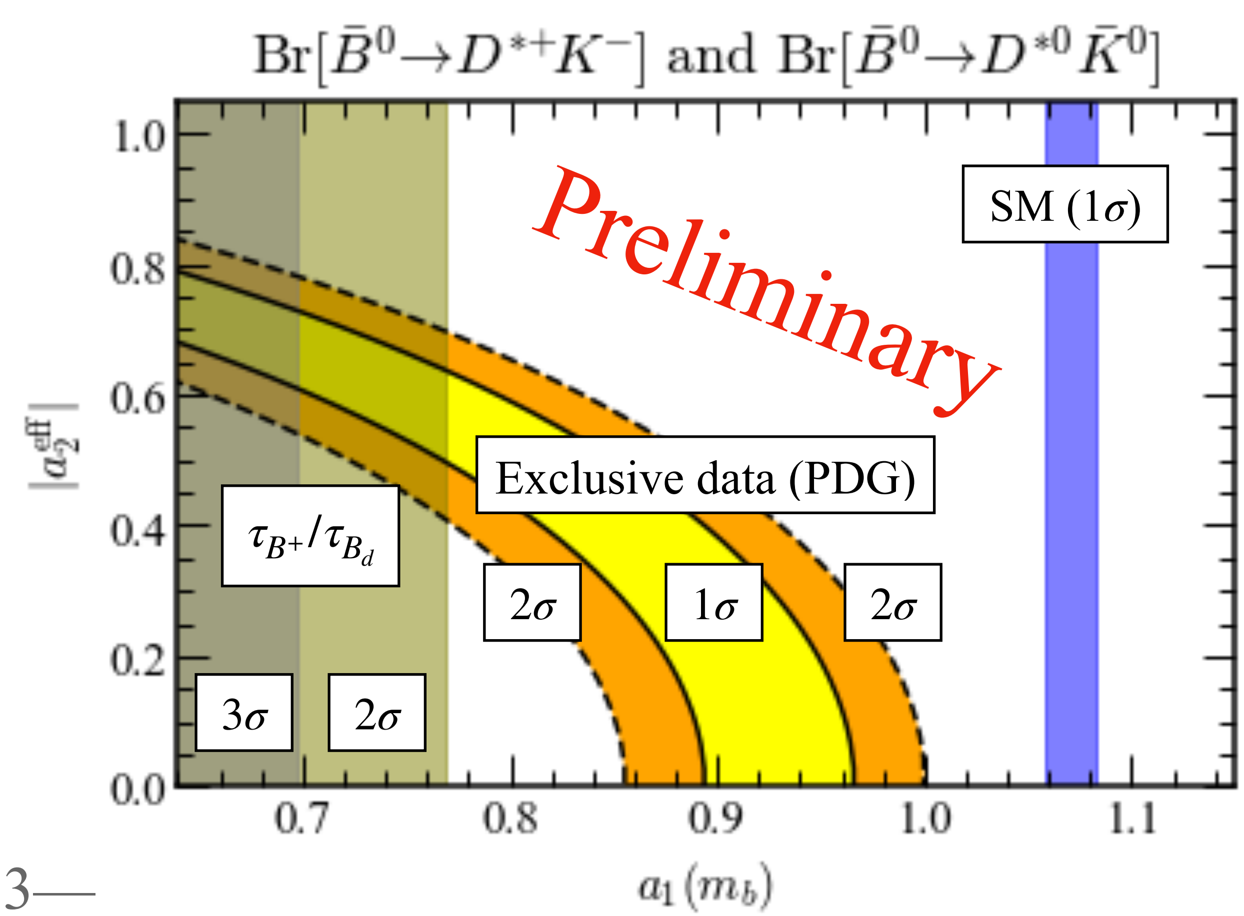
See also Endo, Iguro and Mishima [2109.10811]

$$\begin{pmatrix} A[B \to D^+K^-] \\ A[B \to D^0\bar{K}^0] \end{pmatrix} = \mathcal{S}^{1/2} \begin{pmatrix} A^{\mathrm{QCDF}}[B \to D^+K^-] \\ A^{\mathrm{QCDF}}[B \to D^0\bar{K}^0] \end{pmatrix}$$ ← Predictable

Rescattering: $$\mathcal{S}^{1/2} = \frac{e^{i\delta_{\overline{15}}}}{2} \begin{pmatrix} 1 + e^{i\delta'} & 1 - e^{i\delta'} \\ 1 - e^{i\delta'} & 1 + e^{i\delta'} \end{pmatrix}$$ $\delta' \equiv \delta_{\mathbf{6}} - \delta_{\overline{\mathbf{15}}}$

$$\begin{cases} A^{\mathrm{QCDF}}[B \to D^-K^+] = (G_F/\sqrt{2})\, a_1(m_b)\, |V_{cb}V_{us}|^2 (m_B^2 - m_D^2) f_K f_0^{B\to D}(m_K^2) \\ A^{\mathrm{QCDF}}[B \to D^0\bar{K}^0] = (G_F/\sqrt{2})\, a_2^{\mathrm{eff}}\, |V_{cb}V_{us}|^2 (m_B^2 - m_K^2) f_D f_0^{B\to K}(m_D^2) \end{cases}$$





Summary

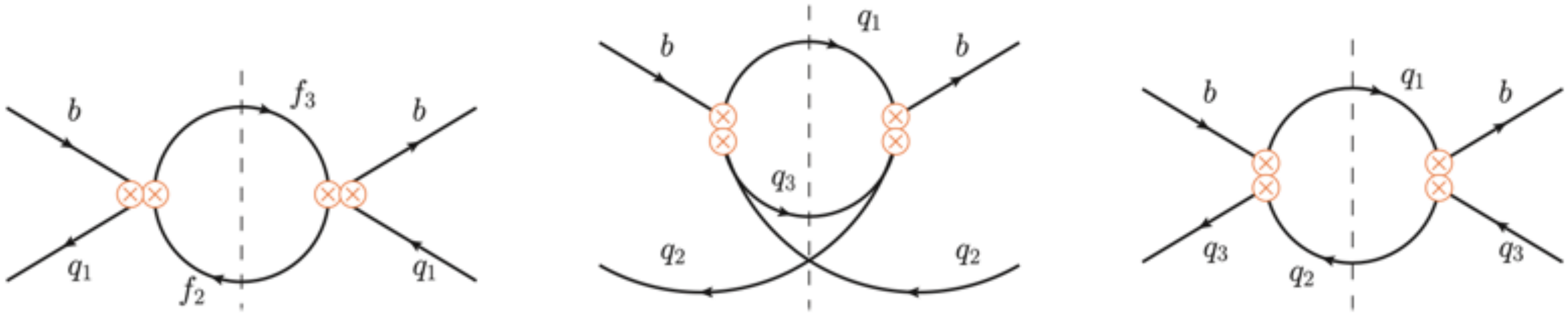
- In light of the $\bar{B} \to D^{(*)}M$ puzzle, we investigated the lifetimes of B-mesons based on the HQE.

- In the presence of the quasi-elastic rescattering, we find that some of exclusive parameter regions are constrained by the lifetime-ratio, $\tau(B^+)/\tau(B_d)$.

- The numerical results will be consistently updated with the $1/m_b^3$ power correction (Darwin term), as well as next-to-leading order QCD corrections to (a) partonic rates, (b) short-distance coefficients of 4-quark operators, and (c) Wilson coefficients of $\Delta B = 1$ operators.

Backup

Pauli interference (PI) and weak annihilation (WA)



$$\Gamma_u^{\rm int} = \frac{G_F^2 m_b^2}{12\pi}|V_{cb}V_{ud}^*|^2 f_{B^+}^2 m_{B^+}(1-x)^2 \Bigg\{[6c_1^{ud}c_2^{ud} + (c_1^{ud})^2 + (c_2^{ud})^2]\left[B_1 - \left(\frac{1+x}{1-x}+\frac{1}{2}\right)\right.$$
$$\left.\times\left(\frac{m_{B^+}^2}{m_b^2}-1\right)\right] + 6[(c_1^{ud})^2 + (c_2^{ud})^2]\epsilon_1\Bigg\},$$

$$\Gamma_d^{\rm ann} = -\frac{G_F^2 m_b^2}{12\pi}|V_{cb}V_{ud}^*|^2 f_{B^0}^2 m_{B^0}(1-x)^2 \Bigg\{\left[\frac{1}{3}(c_1^{ud})^2 + 2c_1^{ud}c_2^{ud} + 3(c_2^{ud})^2\right]\left[\left(1+\frac{x}{2}\right)B_1\right.$$
$$-(1+2x)B_2 + \left[\frac{1+x+x^2}{1-x} + \frac{6x^2}{1-x} - \frac{1}{2}\left(1+\frac{x}{2}\right) - \frac{1}{2}(1+2x)\right]\left.\left(\frac{m_{B^0}^2}{m_b^2}-1\right)\right]$$
$$+2(c_1^{ud})^2\left[\left(1+\frac{x}{2}\right)\epsilon_1 - (1+2x)\epsilon_2\right]\Bigg\},$$

Cheng [1807.00916] and references therein

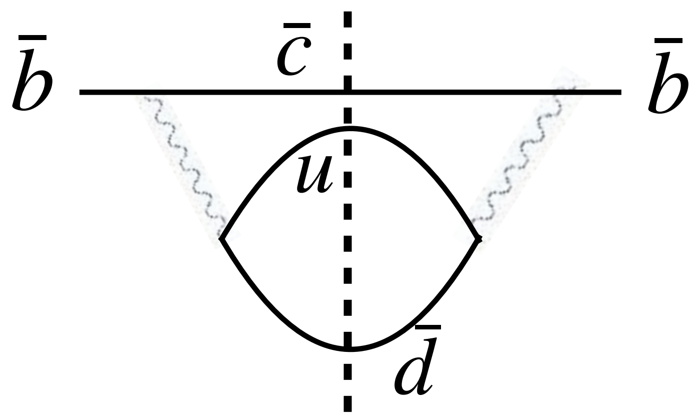
Pattern of lifetimes for B^+ and B_d

HFLAV data: $\dfrac{\tau[B^+]}{\tau[B_d]} = 1.076 \pm 0.004$

2-quark | **4-quark**

✔ Pauli interference (PI)

B^+



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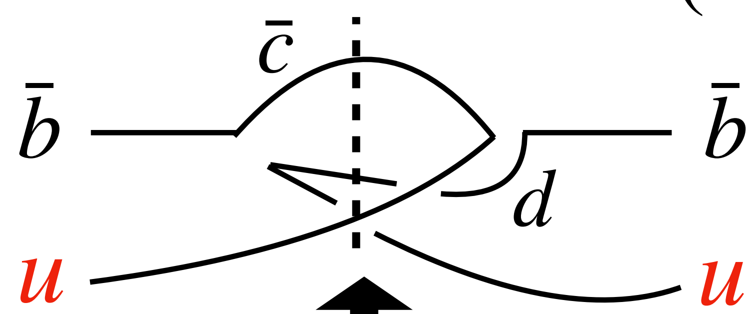
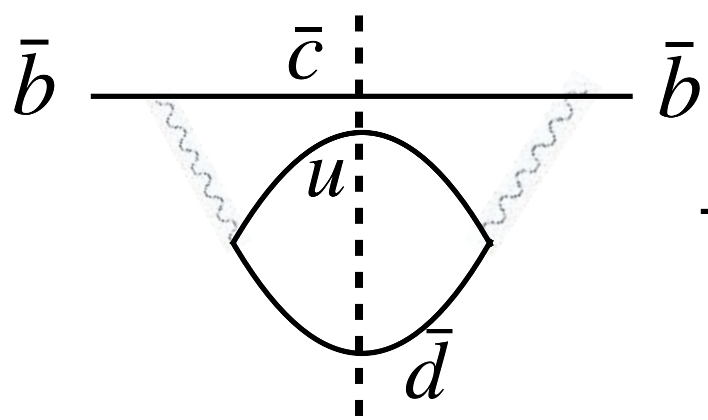


$\Gamma[B^+] = \Gamma^{(2)} + \Gamma^{(4)}[\mathrm{PI}]$

$\mathcal{O}(1/m_b^3)$ $+\mathcal{O}(N_c^0, g_s)$ +other flavors

Weak annihilation (WA)

B_d



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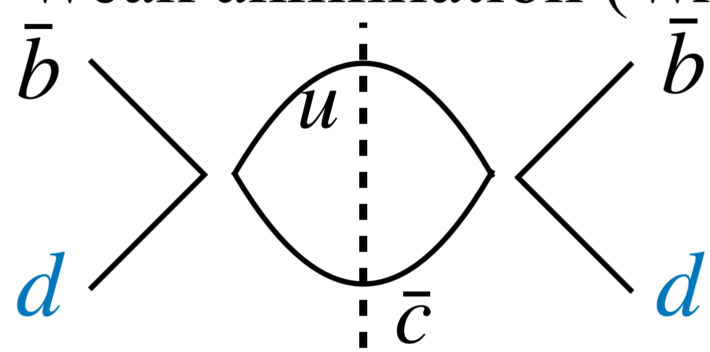

$\Gamma[B_d] = \Gamma^{(2)} + \Gamma^{(4)}[\mathrm{WA}]$

$+\mathcal{O}(N_c^0, g_s)$+other flavors
(Helicity-suppressed, analogous to $B^+ \to \tau^+ \nu_\tau$)

$$\Gamma[B^+] - \Gamma[B_d] \approx \Gamma^{(4)}[\mathrm{PI}]$$

Lenz, Müller, Piscopo and Rusov [2211.02724]