Generation of particle number asymmetry in expanding universe

Work based on arXiv:1609:02990, arXiv:1709.08781

Apriadi Salim Adam*
In collaboration with: Takuya Morozumi*, Keiko Nagao†, Hiroyuki Takata‡

*Graduate School of Science, Hiroshima University
†National Institute of Technology, Niihama College
‡Tomsk State Pedagogical University, Russia

1st workshop on Phenomenology for Particle and Anti-Particle 2018 (PPAP2018)
6th-8th March, 2018
Hiroshima University, Japan

Outline

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The motivation and idea

- There are more matter than anti-matter at the present universe (Kneller and Steigman, 2004).
- Many scenarios: baryogenesis (Sakharov,1967;Yoshimura,1978), leptogenesis (Fukugita and Yanagida, 1986), etc.
- Describing how asymmetry which exists related to the past and how it develops from early universe to present.
- The origin of asymmetry of universe: “mass difference” and “interactions” of the scalar fields.
- We study a simple model which generates particle number asymmetry through “interactions” and develop formulation which is applicable to various type of expanding universe.
- We compute time evolution of asymmetry by using quantum field theory with density operator.

Outline

The Model

N: Neutral scalar, ϕ: Complex scalar

$$\begin{aligned} S &= \int d^4x\,\sqrt{-g}\,(\mathcal{L}_{\rm free}+\mathcal{L}_{\rm int.}+\xi(R-2\Lambda))\,, \\ \mathcal{L}_{\rm free} &= g^{\mu\nu}\nabla_\mu\phi^\dagger\nabla_\nu\phi - m_\phi^2|\phi|^2 + \frac{1}{2}\nabla_\mu N\nabla^\mu N \\ &\quad -\frac{M_N^2}{2}N^2 + \underline{\frac{B^2}{2}(\phi^2+\phi^{\dagger 2}) + \left(\frac{\alpha_2}{2}\phi^2 + h.c.\right)} R + \alpha_3|\phi|^2 R \\ \mathcal{L}_{\rm int.} &= \underline{A\phi^2 N + A^*\phi^{\dagger 2}N} + A_0|\phi|^2 N \end{aligned}$$

A: interaction (vertex) coupling, B: giving the mass difference of fields, α_2: matter-curvature coupling

The interactions among them are CP violating and particle number violating. With this Lagrangian, we aim to produce the PNA through the soft-breaking terms of U(1) symmetry whose coefficients are denoted by A and B^2.

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\mathcal{L}_{\text{free}} &= g^{\mu\nu}\nabla_\mu \phi^\dagger \nabla_\nu \phi - m_\phi^2 |\phi|^2 + \frac{1}{2}\nabla_\mu N \nabla^\mu N \\
&\quad -\frac{M_N^2}{2}N^2 + \underline{\frac{B^2}{2}(\phi^2 + \phi^{\dagger 2}) + \left(\frac{\alpha_2}{2}\phi^2 + h.c.\right)} R + \alpha_3 |\phi|^2 R \\
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U(1) transformation and initial condition

The particle number is related to U(1) transformation

- U(1) transformation of the complex scalar field $\phi'(x) \to \phi(x)e^{i\theta}$
- U(1) charge: particle-anti particle number represented by particle number operator N [Affleck and Dine, 1985],

$$N(x^0) = \int \sqrt{-g(x)} j_0(x) d^3\mathbf{x}$$

$$j_\mu(x) = i(\phi^\dagger \partial_\mu \phi - \partial_\mu \phi^\dagger \phi)$$

Initial condition: The state is given by density matrix

$$\rho(t_0) = \frac{e^{-\beta H_0}}{\mathrm{tr} e^{-\beta H_0}}, \quad \beta = \frac{1}{T}$$

The initial expectation value of scalar fields: $\bar{\phi}_i(t_0)$

The metric and Einstein equations

Space time: Friedmann Robertson Walker with scale factor $a(x^0)$,

$$g_{\mu\nu} = (1, -a^2(x^0), -a^2(x^0), -a^2(x^0))$$

$$R = 6\left[\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2\right], \quad H(x^0) = \frac{\dot{a}}{a}$$

Einstein equations (EE): We consider Einstein equation for scale factor coupled with scalar fields

$$\begin{aligned} \text{00 component:} \ -3(1-8\pi G\beta_i\phi_i^2)\left(\frac{\dot{a}}{a}\right)^2 + \Lambda &= -8\pi G T_{00} \\ ii \text{ component:} \ (1-8\pi G\beta_i\phi_i^2)(2a\ddot{a}+\dot{a}^2) - a^2\Lambda &= -8\pi G T_{ii} \\ \text{off diagonal component:} \ 0 &= -8\pi G T_{\mu\nu(\neq\mu)} \end{aligned}$$

$$T_{\mu\nu} = \partial_\mu\phi_i\partial_\nu\phi_i - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi_i\partial_\beta\phi_i - \frac{1}{2}m_i^2\phi_i^2 + \frac{1}{3}A_{ijk}\phi_i\phi_j\phi_k\right)$$

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Einstein equations (EE): We consider Einstein equation for scale factor coupled with scalar fields

We have not solved EE for the scale factor!

$$\begin{aligned} \text{00 component:} \; -3(1 - 8\pi G\beta_i\phi_i^2)\left(\frac{\dot{a}}{a}\right)^2 + \Lambda &= -8\pi G T_{00} \\ ii \text{ component:} \; (1 - 8\pi G\beta_i\phi_i^2)(2a\ddot{a} + \dot{a}^2) - a^2\Lambda &= -8\pi G T_{ii} \\ \text{off diagonal component:} \; 0 &= -8\pi G T_{\mu\nu(\neq\mu)} \end{aligned}$$

At present, we work for the case that the time dependence of the scale factor is given!

$$T_{\mu\nu} = \partial_\mu\phi_i\partial_\nu\phi_i - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi_i\partial_\beta\phi_i - \frac{1}{2}m_i^2\phi_i^2 + \frac{1}{3}A_{ijk}\phi_i\phi_j\phi_k\right)$$

Complex scalar in terms of real fields

One can decompose complex scalar → real and imaginary.

$$\phi \equiv \frac{\phi_1 + i\phi_2}{\sqrt{2}}, \quad \phi_3 \equiv N$$

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{3} \sqrt{-g}\left[\nabla_\mu \phi_i \nabla^\mu \phi_i - \tilde{m}_i^2(x^0)\phi_i^2\right] + \phi_3 \sum_{i,j=1}^{3} \phi_i A_{ij}$$

- The mass terms, B^2 and α_2, break U(1) symmetry, so that one complex scalar field splits into the two mass eigenstates of real scalars

$$\tilde{m}_1^2(x^0) = m_\phi^2 - B^2 - (\alpha_2 + \alpha_3)R(x^0),$$
$$\tilde{m}_2^2(x^0) = m_\phi^2 + B^2 + (\alpha_2 - \alpha_3)R(x^0).$$

Current expectation value in terms of real fields

- The current:

$$j_\mu = \frac{1}{2}\phi_2 \overleftrightarrow{\partial}_\mu \phi_1 - \frac{1}{2}\phi_1 \overleftrightarrow{\partial}_\mu \phi_2$$

- Current expectation value with initial density matrix:

$$\begin{aligned}\langle j_\mu(x)\rangle &= \mathrm{tr}(j_\mu(x)\rho(t_0)) \\ &= \mathrm{Re.}\left(\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial y^\mu}\right) G_{12}(x,y)\Big|_{y\to x} + \mathrm{Re.}\left\{\bar{\phi}_2^*(x)\overleftrightarrow{\partial}_\mu \bar{\phi}_1(x)\right\}\end{aligned}$$

- $G_{ij}(x,y)$ and $\bar{\phi}_i$ are obtained from 2PI CTP EA.

Outline

2PI formalism (a brief review)

[E. Calzetta and B.L. Hu, Nonequilibrium Quantum Field Theory, Cambridge University, 2008]

The generating functional with source J and non-local source K is defined as,

$$e^{iW[J,K]} = \int d\Phi \text{Exp}\left\{ i \left[S + \int d^4x J_A(x)\Phi^A(x) + \frac{1}{2}\int d^4x d^4y \Phi^A(x) K_{AB}(x,y)\Phi^B(y) \right] \right\}$$

The sources are connected to the mean fields and Green's function through

$$\frac{\delta W}{\delta J_A} = \phi^A; \quad \frac{\delta W}{\delta K_{AB}} = \frac{1}{2}[\phi^A\phi^B + G^{AB}]$$

2PI CTP effective action (EA), Γ_2, is given by Legendre transform W

$$\Gamma_2[\phi, G] \quad = \quad W[J,K] - J_A\phi^A - \frac{1}{2}K_{AB}[\phi^A\phi^B + G^{AB}]$$

The equations of motion, i.e. Schwinger-Dyson equations, is given by

$$\frac{\delta \Gamma_2}{\delta \phi^A} = -J_A - K_{AB}\phi^B; \quad \frac{\delta \Gamma_2}{\delta G^{AB}} = -\frac{1}{2}K_{AB}$$

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These are our main equation must be solved!

2PI CTP EA in curved space

[Ramsey and Hu, 1997]

$$e^{iW[J,K]} = \int d\phi \exp\left(i\left[S + \int \sqrt{-g(x)} d^4x J_i^a c^{ab} \phi_i^b + \frac{1}{2}\int d^4x d^4y \sqrt{-g(x)} \phi_i^a(x) \times c^{ab} K_{ij}^{bc}(x,y) c^{cd} \phi_j^d(y) \sqrt{-g(y)}\right]\right)$$

$$\Gamma_2[G, \bar{\phi}, g] = S[\bar{\phi}, g] + \frac{i}{2}\mathrm{TrLn}\, G^{-1} + \Gamma_Q - \frac{i}{2}\mathrm{Tr}\, \mathbf{1} + \frac{1}{2}\int d^4x \int d^4y \frac{\delta^2 S[\bar{\phi}, g]}{\delta\bar{\phi}_i^a(x)\delta\bar{\phi}_j^b(y)} G_{ij}^{ab}(x,y),$$

- Solving Schwinger-Dyson equations (SDE) for field and GF

$$\frac{\delta\Gamma_2}{\delta\bar{\phi}^A} \text{ and } \frac{\delta\Gamma_2}{\delta G^{AB}}$$

- Inputting the solution to the current expression, one obtains

$$\langle j_\mu(x)\rangle = \mathrm{Re.}\left(\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial y^\mu}\right) G_{12}(x,y)\Big|_{y\to x} + \mathrm{Re.}\left\{\bar{\phi}_2^*(x)\overleftrightarrow{\partial}_\mu \bar{\phi}_1(x)\right\} \neq 0$$

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2PI CTP EA in curved space

[Ramsey and Hu, 1997]

$$e^{iW[J,K]} = \int d\phi \exp\left(i\left[S + \int \sqrt{-g(x)}d^4x J_i^a c^{ab}\phi_i^b + \frac{1}{2}\int d^4x d^4y \sqrt{-g(x)}\phi_i^a(x) \times c^{ab}K_{ij}^{bc}(x,y)c^{cd}\phi_j^d(y)\sqrt{-g(y)}\right]\right)$$

$$\Gamma_2[G,\bar{\phi},g] = S[\bar{\phi},g] + \frac{i}{2}\mathrm{TrLn}\, G^{-1} + \Gamma_Q - \frac{i}{2}\mathrm{Tr}\,\mathbf{1} + \frac{1}{2}\int d^4x \int d^4y \frac{\delta^2 S[\bar{\phi},g]}{\delta\bar{\phi}_i^a(x)\delta\bar{\phi}_j^b(y)} G_{ij}^{ab}(x,y),$$

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Non-zero current ⇔ particle number violation (asymmetry)!

SDE for GF and field, and rescaling

It is more convenient to rescale field, GF and interaction coupling A as,

$$
\begin{aligned}
\bar{\phi}(x^0) &= \left(\frac{a(t_0)}{a(x^0)}\right)^{3/2} \hat{\varphi}(x^0) \\
G(x^0, y^0, \mathbf{k}) &= \left(\frac{a(t_0)}{a(x^0)}\right)^{3/2} \hat{G}(x^0, y^0, \mathbf{k}) \left(\frac{a(t_0)}{a(y^0)}\right)^{3/2} \\
\hat{A}(x^0) &= \left(\frac{a(t_0)}{a(x^0)}\right)^{3/2} A
\end{aligned}
$$

Schwinger-Dyson equations (GF) :

$$
\begin{aligned}
&\left[\frac{\partial^2}{\partial x^{02}} + \frac{\mathbf{k}^2}{a(x^0)^2} + \bar{m}_i^2(x^0)\right] \hat{G}^{ab}_{ij,x^0y^0}(\mathbf{k}) \\
= \quad & 2(c \cdot ((D \otimes \hat{A}) \cdot \hat{\varphi}))^{ac}_{ik,x^0} \hat{G}^{cb}_{kj,x^0y^0}(\mathbf{k}) - i\delta_{ij}\delta_{x^0y^0} \frac{c^{ab}}{a_0^3} \\
& -i\delta_{t_0x^0} \kappa^{ae}_{ik}(\mathbf{k}) a_0^3 c^{ef} \hat{G}^{fb}_{kj,t_0y^0}(\mathbf{k}) + O(A^2)
\end{aligned}
$$

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Schwinger-Dyson equations (GF) :

Inhomogeneous Diff. Eq. (IDE)

$$\left[\frac{\partial^2}{\partial x^{02}} + \frac{\mathbf{k}^2}{a(x^0)^2} + \bar{m}_i^2(x^0)\right] \hat{G}^{ab}_{ij,x^0y^0}(\mathbf{k})$$

$$= 2(c \cdot ((D \otimes \hat{A}) \cdot \hat{\varphi}))^{ac}_{ik,x^0} \hat{G}^{cb}_{kj,x^0y^0}(\mathbf{k}) - i\delta_{ij}\delta_{x^0y^0} \frac{c^{ab}}{a_0^3}$$

$$-i\delta_{t_0x^0} \kappa^{ae}_{ik}(\mathbf{k}) a_0^3 c^{ef} \hat{G}^{fb}_{kj,t_0y^0}(\mathbf{k}) + O(A^2)$$

The solutions

We are interested the solutions up to first order of interaction coupling A.

$$\begin{aligned} \hat{G}_{12}(x^0, y^0, \mathbf{k}) &= \hat{G}_{12}^{o(A)}(x^0, y^0, \mathbf{k}), \\ \hat{\varphi}_i(x) &= \hat{\varphi}_{i,\text{free}}(x) + \hat{\varphi}_i^{o(A)}(x) \end{aligned}$$

The solutions are written in terms of integral equations which are iteratively solved by treating interaction coupling A is small.

$$\begin{aligned} \hat{G}_{x^0y^0}^{o(A)} &= \int_{t_0}^{y^0} R_{x^0t}^{o(A)} \cdot \bar{K}_{y^0t} dt - \int_{t_0}^{x^0} \bar{K}_{x^0t} \cdot \Big[Q_{tt_0}^{o(A)} \cdot \bar{K}'_{y^0t_0} \\ & \quad - Q_{tt_0}^{o(A)} \cdot E^T \cdot \bar{K}_{y^0t_0} \Big] dt, \end{aligned}$$

where

$$\begin{aligned} Q_{ts}^{o(A)} &= 2c \cdot \{(D \otimes \hat{A}) \cdot \hat{\varphi}_{\text{free}}\}_t \cdot \hat{G}_{\text{free},ts} \\ R_{ts}^{o(A)} &= 2\hat{G}_{\text{free},ts} \cdot \{(D \otimes \hat{A}) \cdot \hat{\varphi}_{\text{free}}\}_s \cdot c \end{aligned}$$

The solutions

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$$\hat{G}_{12}(x^0, y^0, \mathbf{k}) = \hat{G}^{o(A)}_{12}(x^0, y^0, \mathbf{k}),$$
$$\hat{\varphi}_i(x) = \hat{\varphi}_{i,\text{free}}(x) + \hat{\varphi}^{o(A)}_i(x)$$

The solutions are written in terms of integral equations which are iteratively solved by treating interaction coupling A is small.

$$\hat{G}^{o(A)}_{x^0y^0} = \int_{t_0}^{y^0} R^{o(A)}_{x^0t} \cdot \bar{K}_{y^0t} dt - \int_{t_0}^{x^0} \bar{K}_{x^0t} \cdot \left[Q^{o(A)}_{tt_0} \cdot \bar{K}'_{y^0t_0} - Q^{o(A)}_{tt_0} \cdot E^T \cdot \bar{K}_{y^0t_0} \right] dt,$$

where

Functions as solution IDE

$$Q^{o(A)}_{ts} = 2c \cdot \{(D \otimes \hat{A}) \cdot \hat{\varphi}_{\text{free}}\}_t \cdot \hat{G}_{\text{free},ts}$$
$$R^{o(A)}_{ts} = 2\hat{G}_{\text{free},ts} \cdot \{(D \otimes \hat{A}) \cdot \hat{\varphi}_{\text{free}}\}_s \cdot c$$

Outline

$O(A)$ contribution to the current and the initial conditions

Up to $o(A)$, the current reads

$$\left(\frac{a(x^0)}{a(t_0)}\right)^3 \langle j_0(x^0)\rangle_{o(A)} = \int \frac{d^3k}{(2\pi)^3}\left(\frac{\partial}{\partial x^0}-\frac{\partial}{\partial y^0}\right)[\mathrm{Re}.\hat{G}^{o(A)}_{12}(x^0,y^0,\mathbf{k})]\big|_{y^0\to x^0}$$
$$+\mathrm{Re}\{\hat{\varphi}^*_{2,\mathrm{free}}(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}_{1,\mathrm{free}}(x^0)\}+\mathrm{Re}\{\hat{\varphi}^*_{2,\mathrm{free}}(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}^{O(A)}_1(x^0)\}$$
$$+\mathrm{Re}\{\hat{\varphi}^{*,O(A)}_2(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}_{1,\mathrm{free}}(x^0)\}$$

- The initial conditions for the field and Green's function:

$$\hat{G}^{ab}_{ij,t_0t_0}(\mathbf{k}) = \delta_{ij}\frac{1}{2\omega_i(\mathbf{k})a^3_{t_0}}\left[\frac{\sinh\beta\omega_i(\mathbf{k})}{\cosh\beta\omega_i(\mathbf{k})-1}\right]; \qquad \begin{pmatrix}\hat{\varphi}_1(t_0)\\ \hat{\varphi}_2(t_0)\\ \hat{\varphi}_3(t_0)\end{pmatrix} = \begin{pmatrix}0\\0\\v_3\end{pmatrix}$$

- The non-zero asymmetry comes from $O(A)$ contribution to the Green function.

$$\langle j_0(x^0)\rangle_{O(A)} = \int \frac{d^3k}{(2\pi)^3}\left(\frac{\partial}{\partial x^0}-\frac{\partial}{\partial y^0}\right)[\mathrm{Re}.\hat{G}^{O(A)}_{12}(x^0,y^0,\mathbf{k})]\big|_{y^0\to x^0}$$

The time dependence of scale factor

One expands scale factor around t_0 as follows,

$$\begin{aligned} a(x^0) &= a(t_0) + (x^0 - t_0)\dot{a}(t_0) + \ldots, (0 < t_0 \leqslant x^0) \\ &= a^{(0)} + a^{(1)}(x^0) + \ldots \end{aligned}$$

First we assume that $a^{(n+1)}(x^0) < a^{(n)}(x^0)$ when x^0 is near t_0. Then one can keep only the following terms,

$$a(x^0) \simeq a^{(0)} + a^{(1)}(x^0)$$

and $a^{(n)}(x^0)$ for $(n \geqslant 2)$ are set to be zero. Thus it can be written as,

$$\frac{a(x^0)}{a(t_0)} = 1 + (x^0 - t_0)H(t_0)$$

where $H(t_0) = \frac{\dot{a}(t_0)}{a(t_0)}$, and t_0 is definite time.

Linear $H(t_0)$ and $O(A)$ contributions to PNA

$$\langle j_0(x^0)\rangle_{O(A)} = \text{constant} + \text{dilution} + \text{freezing int.} + \text{redshift}$$

$$\langle j_0(x^0)\rangle_{1\text{st}} = \frac{2\hat{\varphi}_{3,t_0}A_{123}}{a_{t_0}^3}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int_{t_0}^{x^0}\left\{1-3(x^0-t_0)H(t_0)-\frac{3}{2}(t-t_0)H(t_0)\right\}$$
$$\times\left[\left\{\frac{(-\bar{K}^{(0)\prime}_{3,tt_0,\mathbf{0}})}{2\omega_{2,\mathbf{k}}(t_0)}\coth\frac{\beta\omega_{2,\mathbf{k}}(t_0)}{2}\left[\left(\bar{K}^{(0)\prime}_{2,x^0t_0,\mathbf{k}}\overleftrightarrow{\partial}\cdot\bar{K}^{(0)}_{1,x^0t,\mathbf{k}}\right)\bar{K}^{(0)\prime}_{2,tt_0,\mathbf{k}}\right.\right.\right.$$
$$\left.\left.\left.+\omega^2_{2,\mathbf{k}}(t_0)\left(\bar{K}^{(0)}_{2,x^0t_0,\mathbf{k}}\overleftrightarrow{\partial}\cdot\bar{K}^{(0)}_{1,x^0t,\mathbf{k}}\right)\bar{K}^{(0)}_{2,tt_0,\mathbf{k}}\right]\right\}-\{1\leftrightarrow 2\}\right]dt,$$

$$\langle j_0(x^0)\rangle_{2\text{nd}} = \frac{2\hat{\varphi}_{3,t_0}A_{123}}{a_{t_0}^3}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int_{t_0}^{x^0}\left[\left\{\frac{(-\bar{K}^{(0)\prime}_{3,tt_0,\mathbf{0}})}{2\omega_{2,\mathbf{k}}(t_0)}\coth\frac{\beta\omega_{2,\mathbf{k}}(t_0)}{2}\right.\right.$$
$$\times\left[\left(\bar{K}^{(0)\prime}_{2,x^0t_0,\mathbf{k}}\overleftrightarrow{\partial}\cdot\bar{K}^{(0)}_{1,x^0t,\mathbf{k}}\right)\bar{K}^{(1)\prime}_{2,tt_0,\mathbf{k}}+\left(\bar{K}^{(1)\prime}_{2,x^0t_0,\mathbf{k}}\overleftrightarrow{\partial}\cdot\bar{K}^{(0)}_{1,x^0t,\mathbf{k}}+\bar{K}^{(0)\prime}_{2,x^0t_0,\mathbf{k}}\overleftrightarrow{\partial}\cdot\bar{K}^{(1)}_{1,x^0t,\mathbf{k}}\right)\bar{K}^{(0)\prime}_{2,tt_0,\mathbf{k}}\right.$$
$$+\omega^2_{2,\mathbf{k}}(t_0)\left[\left(\bar{K}^{(0)}_{2,x^0t_0,\mathbf{k}}\overleftrightarrow{\partial}\cdot\bar{K}^{(0)}_{1,x^0t,\mathbf{k}}\right)\bar{K}^{(1)}_{2,tt_0,\mathbf{k}}\right.$$
$$\left.\left.\left.\left.+\left(\bar{K}^{(1)}_{2,x^0t_0,\mathbf{k}}\overleftrightarrow{\partial}\cdot\bar{K}^{(0)}_{1,x^0t,\mathbf{k}}+\bar{K}^{(0)}_{2,x^0t_0,\mathbf{k}}\overleftrightarrow{\partial}\cdot\bar{K}^{(1)}_{1,x^0t,\mathbf{k}}\right)\bar{K}^{(0)}_{2,tt_0,\mathbf{k}}\right]\right]\right\}-\{1\leftrightarrow 2\}\right]dt$$

Linear $H(t_0)$ and $O(A)$ contributions to PNA

Table 1: The classification of $o(H_{t_0})$ contributions to the PNA

The effect	The origin
Dilution	The increase of volume of the universe due to expansion, $\frac{1}{a(x^0)^3} - \frac{1}{a_{t_0}^3}$
Freezing interaction	The decrease of the strength of the cubic interaction $\hat{A}$ as $\hat{A}_{123} - A_{123}$.
Redshift	The effective energy of particle, $\frac{\mathbf{k}^2}{a(x^0)^2} + \bar{m}_i^2(x^0)$.

Outline

Dependence on temperature T-1

Amplitude of the oscillation decreases due to dilution, freezing interaction effect and redshift effect.

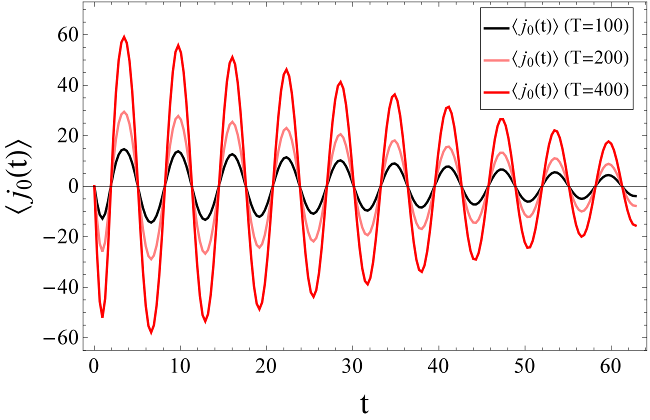

Figure 1: $(\tilde{m}_1, \tilde{m}_2, B, H_{t_0}, \omega_{3,\mathbf{0}})$=$(0.04, 0.05, 0.021, 10^{-3}, 0.0035)$, $t = 0.35(x^0$

Dependence on temperature T-2

We are interested for the case that the oscillation period is shorter than the age of the Universe.

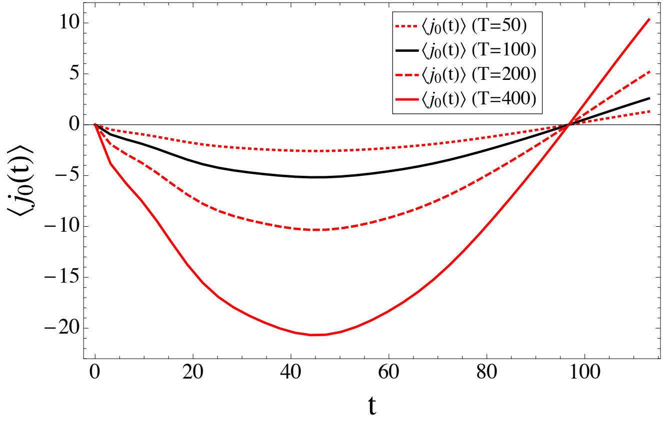

Figure 2: $(\tilde{m}_1, \tilde{m}_2, B, \omega_3, H_{t_0}) = (2, 3, 1.58, 0.35, 10^{-3})$

Dependence on parameter B

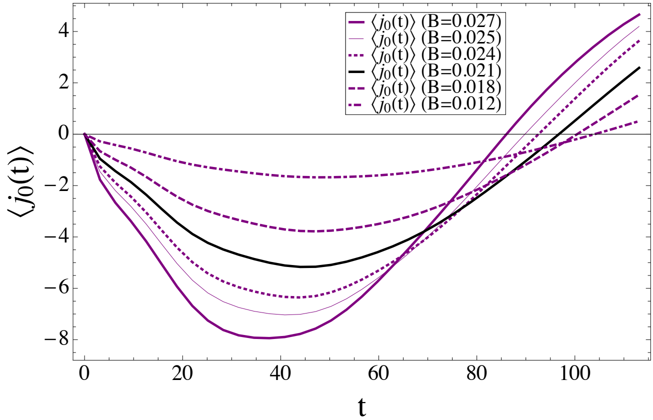

Figure 3: $(\tilde{m}_2, T, H_{t_0}, \omega_{3,\mathbf{0}})$=(0.05, 100, 10^{-3}, 0.0035), $2B^2 = \tilde{m}_2^2 - \tilde{m}_1^2$

$\omega_{3,0}$ dependence

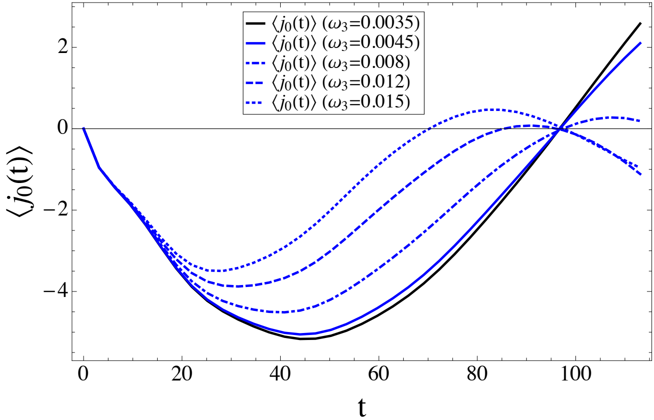

Figure 4: $(\tilde{m}_1, \tilde{m}_2, B, T, H_{t_0})$=$(0.04, 0.05, 0.021, 100, 10^{-3})$.

Dependence on the expansion rate H_{t_0}

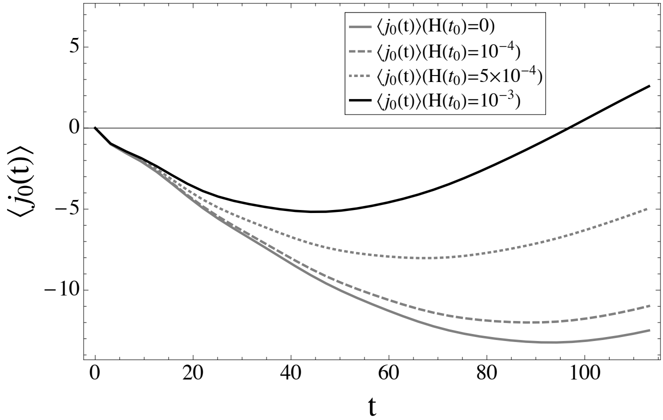

Figure 5: $(\tilde{m}_1, \tilde{m}_2, B, T, \omega_{3,\mathbf{0}})$=(0.04, 0.05, 0.021, 100, 0.0035).

Comparison two different periods of the time evolution PNA

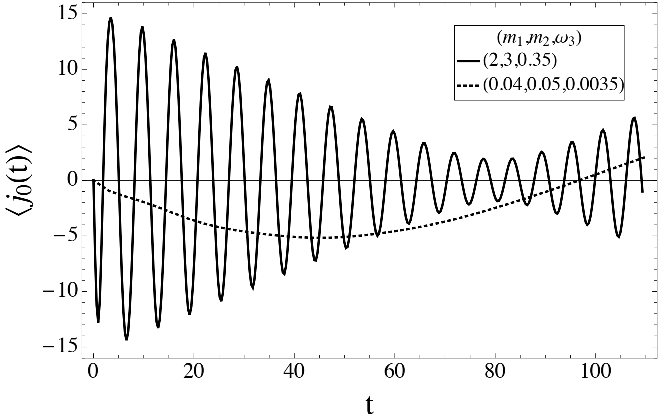

Figure 6: $(T, H_{t_0}) = (100, 10^{-3})$. The black (shorter period) and black dotted (longer period) lines show the parameter B, 1.58 and 0.021, respectively.

Outline

Summary

- We study an interacting model which particle number asymmetry is generated through interactions of scalar fields.
- The current for the particle and anti-particle asymmetry is given up to the first order of A and linear $H(t_0)$.
- Time evolution of the particle number asymmetry and its parameter dependence is investigated.

THANK YOU...

BACK UP SLIDE

Parameters dependence for non-zero current

$$\left(\frac{a(x^0)}{a_0}\right)^3 \langle j_0(x^0)\rangle_{o(A)} = \int \frac{d^3k}{(2\pi)^3}\left(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial y^0}\right)[\mathrm{Re}.\hat{G}_{12,\mathrm{int}}(x^0, y^0, \mathbf{k})]\Big|_{y^0\to x^0}$$

$$+\mathrm{Re}\{\hat{\varphi}^*_{2,\mathrm{free}}(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}_{1,\mathrm{free}}(x^0)\} + \mathrm{Re}\{\hat{\varphi}^*_{2,\mathrm{free}}(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}_{1,\mathrm{int}}(x^0)\}$$

$$+\mathrm{Re}\{\hat{\varphi}^*_{2,\mathrm{int}}(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}_{1,\mathrm{free}}(x^0)\}$$

$$B \to 0 \to \omega_{1,\mathbf{0}} = \omega_{2,\mathbf{0}}, \bar{K}_2 = \bar{K}_1; \quad \bar{K}_{i,x^0y^0,\mathbf{k}} = \frac{\sin\omega_{i,\mathbf{k}}(x^0-y^0)}{\omega_{i,\mathbf{k}}}$$

$$B \neq 0 \to \omega_{1,\mathbf{0}} \neq \omega_{2,\mathbf{0}}, \bar{K}_2 \neq \bar{K}_1$$

Parameters	j_0^{free}	j_0^{GF}	$j_0^{\text{Free.Int}}$	$j_0^{\text{Free.Int}}$
B	$B \neq 0$	$B \neq 0$	$B \neq 0$ or $B = 0$	$B \neq 0$ or $B = 0$
A	-	$A_{123} \neq 0$	$A_{123} \neq 0$	$A_{113} \neq A_{223}$
$\hat{\varphi}_{i,0}$	$\hat{\varphi}_{1,0} \neq 0$, $\hat{\varphi}_{2,0} \neq 0$	$\hat{\varphi}_{3,0} \neq 0$	$\hat{\varphi}_{3,0} \neq 0$, $\hat{\varphi}^2_{1,0} \neq \hat{\varphi}^2_{2,0}$	$\hat{\varphi}_{1,0}\hat{\varphi}_{2,0}\hat{\varphi}_{3,0} \neq 0$

Table 2: All requirements for non zero current

The cubic interactions and their property

Table 3: The cubic interactions and their property

$A_{113} = \frac{A_0}{2} + \text{Re.}(A)$	-
$A_{223} = \frac{A_0}{2} - \text{Re.}(A)$	-
$A_{113} - A_{223} = 2\text{Re.}(A)$	U(1) violation
$A_{123} = -\text{Im.}(A)$	U(1), CP violation