Flavor versus mass eigenstates in neutrino asymmetries (a question of coherence)

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We show that, if they exist, lepton number asymmetries (L_α) of neutrino flavors should be distinguished from the ones (L_i) of mass eigenstates as cosmological (BBN) bounds on the latters (formers) cannot be directly applied to the formers (latters). Due to the difference of mass and flavor eigenstates, the cosmological constraint on the asymmetries of neutrino flavors can be much stronger than conventional expectation.

INTRODUCTION

A large lepton number asymmetry of neutrinos is an intriguing possibility in regard of its capability of resolving several non-trivial issues of cosmology (see for example [1–3]), but has been known to be constrained tightly by BigBang nucleosynthesis (BBN) [4, 5]. However[6–8], even if BBN constrains the lepton number asymmetry of electron-neutrino very tightly such as $L_e \lesssim \mathcal{O}(10^{-3})$, much larger muon- and tau-neutrino asymmetries of $\mathcal{O}(0.1-1)$ are still allowed as long as the total lepton number asymmetry is sizable. Such large asymmetries are expected to be constrained mainly by cosmic microwave background (CMB) via the extra neutrino species $\Delta N_{\rm eff}$ [9].

If asymmetric neutrinos have a thermal distribution, their contributions to $\Delta N_{\rm eff}$ is expressed as

$$\Delta N_{\rm eff} = \frac{15}{7} \sum_\alpha \left(\frac{\xi_\alpha}{\pi}\right)^2 \left[2 + \left(\frac{\xi_\alpha}{\pi}\right)^2\right] \qquad (1)$$

where $\xi_\alpha \equiv \mu_\alpha/T$ is the neutrino degeneracy parameter. Conventionally, the summation in Eq. (1) has been done with neutrino flavors ($\nu_{e,\mu,\tau}$ in case of only three active neutrinos). An implicit assumption here is that the extra radiation energy coming from asymmetric neutrinos are solely from flavor-eigenstates. However, due to neutrino flavor oscillations, the equilibrium density matrix is not diagonal in the flavour basis (as one naively expects, not being the flavour eigestates the asymptotic states of the Hamiltonian) and their description in terms of only diagonal components (a more or less hidden asspumtion when assuming thermal distribution for flavors) cannot capture all the contributions to the extra radiation energy density. On the other hand, well after their decoupling from thermal bath, free-streaming neutrinos should be described as incoherent mass-eigenstates only. Hence, the appropriate estimation of $\Delta N_{\rm eff}$ could be done exclusively with neutrino's mass-eigenstates instead of flavor-eigenstates in Eq. (1).

Let's see why. A standard neutrino flavour transition, or "oscillation", can be understood as follows. A neutrino is produced by a source together with a charged lepton $\overline{\ell_\alpha}$ of flavour α. Therefore, at the production point, the neutrino is a ν_α. Then, after birth, the neutrino travels a distance L until it is detected. There, it is where it reaches a target with which it interacts and produces another charged lepton ℓ_β of flavour β. Thus, at the interaction point, the neutrino is a ν_β. If $\beta \neq \alpha$ (for example, if ℓ_α is a μ but ℓ_β is a τ), then, during its trip from the source to the detection point, the neutrino has transitioned from a ν_α into a ν_β.

This morphing of neutrino flavour, $\nu_\alpha \longrightarrow \nu_\beta$, is a text-book example of a quantum-mechanical effect.

Because, a ν_α is really a coherent superposition of mass eigenstates ν_i,

$$|\nu_\alpha> = \sum_i U^*_{\alpha i}\, |\nu_i> \quad . \qquad (2)$$

the neutrino that propagates since it is created until it interacts, can be any one of the ν_i's, therefore we must add the contributions of all the different ν_i coherently. Then, the transition amplitude, $\mathrm{Amp}(\nu_\alpha \longrightarrow \nu_\beta)$ contains a share of each ν_i and it is a product of three factors. The first one is the amplitude for the neutrino born at the production point in combination with an $\overline{\ell_\alpha}$ to be, specifically, a ν_i. This amplitude is given by $U^*_{\alpha i}$. The second factor is the amplitude for the ν_i created by the source to propagate until it reaches the detector. We will call this factor $\mathrm{Prop}(\nu_i)$. It is not difficult to see that

$$\mathrm{Prop}(\nu_i) = \exp[-i m_i^2 \frac{L}{2E}] \quad . \qquad (3)$$

The third factor is the amplitude for the charged lepton produced by the interaction of the ν_i with the detector to be, specifically, an ℓ_β, which is $U_{\beta i}$. Therefore the amplitude for a neutrino born as a ν_α to be detected as a ν_β after covering a distance L through vacuum with energy E yields

$$\mathrm{Amp}(\nu_\alpha \longrightarrow \nu_\beta) = \sum_i U^*_{\alpha i}\, e^{-i m_i^2 \frac{L}{2E}}\, U_{\beta i} \quad . \qquad (4)$$

The expression above is valid for an arbitrary number of neutrino flavours and mass eigenstates. The probability $\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta)$ for $\nu_\alpha \longrightarrow \nu_\beta$ can be found by squaring it, giving

$$
\begin{aligned}
&\mathrm{P}(\nu_\alpha \longrightarrow \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \longrightarrow \nu_\beta)|^2 \\
&= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) \sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right) \\
&\qquad + 2\sum_{i>j} \Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) \sin\left(\Delta m^2_{ij}\frac{L}{2E}\right) \quad , \quad (5)
\end{aligned}
$$

with

$$\Delta m^2_{ij} \equiv m^2_i - m^2_j \quad . \tag{6}$$

In order to get Eq. (5) we have used that the mixing matrix U is unitary.

So far, we have been working in natural units, if we return now the $\hbar$'s and c factor (we have happily left out) into the oscillation probability we find that

$$\sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right) \quad \longrightarrow \quad \sin^2\left(\Delta m^2_{ij}c^4\frac{L}{4\hbar c E}\right) \tag{7}$$

Having done that, it is easy and instructive to explore the semi-classical limit, $\hbar \longrightarrow 0$. In this limit the oscillation length goes to zero (the oscillation phase goes to infinity) and the oscillations are averaged out. Neutrino propagate as effectively incoherent mass eigenstates. The interference pattern is lost. We no longer talk about oscillations but about flavour transitions. The same happens if we let the mass difference Δm^2 become large or when the distance traveled or the time ellapsed is huge, as it is the case with the cosmic neutrino background after neutrino decoupling.

LEPTON NUMBER ASYMMETRIES OF NEUTRINO FLAVOR VS. MASS EIGENSTATES

The lepton number asymmetries of neutrinos in flavor basis can be defined as a matrix such as

$$\mathbf{L}_\mathrm{f} = \frac{\rho - \bar{\rho}}{n_\gamma} \tag{8}$$

where $\rho/\bar{\rho}$ and n_γ are the density matrices of neutrinos/anti-neutrinos and the photon number density. In the very early universe, it is natural to assume that neutrinos are in interaction eigenstates (i.e., flavor eigenstates), since their kinematic phases are very small and collisional interactions to thermal bath are large enough to block flavor oscillations. Hence, if it were generated by certain mechanism at very high energy, $\mathbf{L}_\mathrm{f}$ is likely to be diagonal and to remain constant. After all while oscillations are blocked, individual flavor lepton numbers are conserved. However, due to the fact that neutrino are not massless and mix, according to the values of the mixing parameters and mass differences measured by a variety of experiments [10] , as the temperature of the radiation dominated universe drops below around $T \sim 15\,\mathrm{MeV}$, flavor oscillations becomes active. $\mathbf{L}_\mathrm{f}$ starts evolving at this epoch, and settles down to an equilibrium state finally at $T \sim 2\,\mathrm{MeV}$ before BBN starts [4, 11–15].

Once it reaches its final equilibrium value, $\mathbf{L}_\mathrm{f}$ becomes time-independent. The shape of $\mathbf{L}_\mathrm{f}$ at the final equilibrium is determined by various effects including vacuum oscillation, MSW-like effect coming from charged lepton backgrounds, neutrinos self-interaction and collisional scattering. So, it is difficult to be predicted analytically, and in practice, only accesible via numerical methods. However, all these effects except vacuum oscillations are active during particular windows in temperature and eventually disappear. Hence, the final shape of $\mathbf{L}_\mathrm{f}$ should be determined by vacuum oscillation parameters only. Note that the flavor states mixed by vacuum oscillation parameters are nothing but mass-eigenstates in flavor-basis. Therefore, the statistical equilibrium state of $\mathbf{L}_\mathrm{f}$ should be that of mass-eigenstates expressed in the flavor-basis.

Since in vacuum mass- and flavor-eigenstates are related to each other by PMNS matrix, U_PMNS [16, 17], our argument implies that for a diagonalization matrix D, $\mathbf{L}_\mathrm{m}$ the matrix of asymmetries in mass basis is given by

$$\mathbf{L}_\mathrm{m} = D\mathbf{L}_\mathrm{f}D^{-1} = U^{-1}_\mathrm{PMNS}\mathbf{L}_\mathrm{f}U_\mathrm{PMNS} \tag{9}$$

implying $D = U^{-1}_\mathrm{PMNS}$.

On general grounds, at late times we do not expect $\mathbf{L}_\mathrm{f}$ to be diagonal. The operator responsible for the evolution of the density matrix is not diagonal, so that a diagonal density matrix will not be the asymptotic solution of those equations unless it is proportional to the identity matrix, which is clearly not the case. Hence, generically the asymmetries of neutrino mass eigenstates differ from those of flavor, and this fact should be taken into account when observational constraints on lepton number asymmetries are considered.

In order to verify our argument, we solved numerically the quantum kinetic equations of neutrino/anti-neutrino density matrices in a simplified way as done in Ref. [6]. An example is shown in Fig. 1 where one finds the evolutions of $L_{\alpha\beta}$, the entries of $\mathbf{Re}\,[\mathbf{L}_\mathrm{f}]$. The change across e^+e^--annihilation around $T \sim 2\,\mathrm{MeV}$ (or $x \sim 0.5$) was taken into account as a global suppression factor $4/11$ for simplicity. As shown in the right panel of the figure, the off-diagonal entries of $\mathbf{Re}\,[\mathbf{L}_\mathrm{f}]$ do not disappear, making $\mathbf{L}_\mathrm{m}$ be different from $\mathbf{L}_\mathrm{f}$. Also, we found that the numerical simulation reproduces the relation $D = U^{-1}_\mathrm{PMNS}$ quite precisely within errors of $\mathcal{O}(0.1)\%$ even at $x = 1$.

The differences between diagonal entries of $\mathbf{L}_\mathrm{f}$ and $\mathbf{L}_\mathrm{m}$ can be seen by expressing the former in terms of the latter. At first, L_e is given by

$$L_e = c^2_{13}\left(c^2_{12}L_1 + s^2_{12}L_2\right) + s^2_{13}L_3 \tag{10}$$

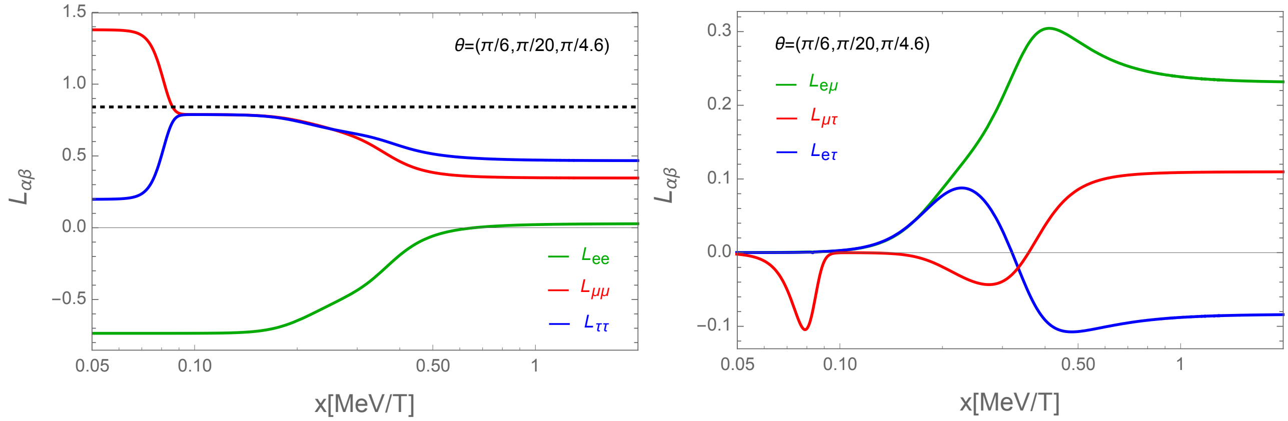

FIG. 1. Evolutions of $\mathbf{L}_\mathrm{f}$ for $\theta = (\theta_{12}, \theta_{13}, \theta_{23})$ with $(\xi_e, \xi_\mu, \xi_\tau) = (-1, 1.6, 0.3)$. *Left/Right*: Diagonal/off-diagonal entries.

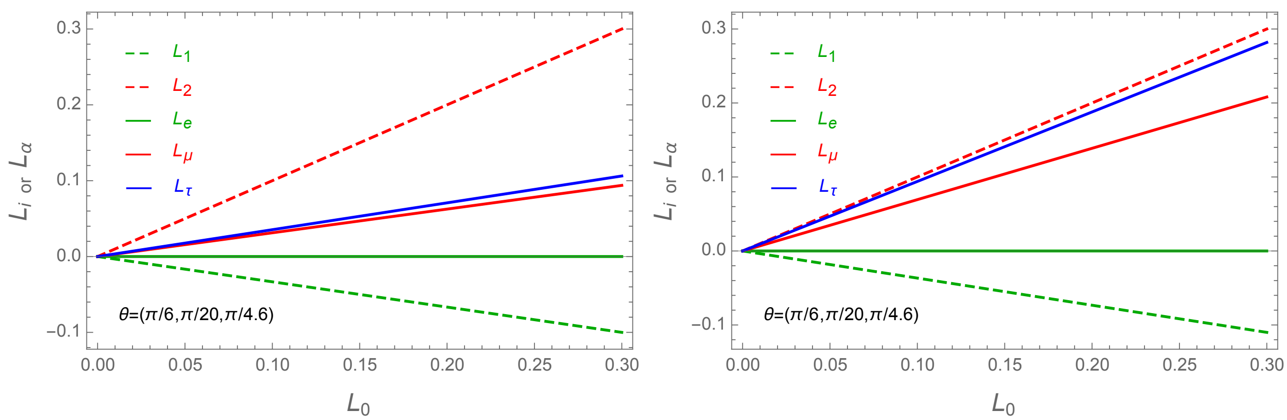

FIG. 2. Comparisons of lepton number asymmetries of both mass-eigenstates (L_i; $i = 1, 2, 3$) and flavor-eigenstates (L_α; $\alpha = e, \mu, \tau$) for $\theta = (\theta_{12}, \theta_{13}, \theta_{23})$ with θ_{ij} being the mixing angles in PMNS matrix. Dashed lines are the asymmetries of mass eigenstates. Solid line are for flavors eigenstates. Left and right panels are showing two examples of $\mathbf{L}_\mathrm{m}$ leading to $L_e = 0$ satisfying BBN constraint. *Left*: $\mathbf{L}_\mathrm{m} = \mathrm{diag}(L_1, L_2, L_3) = (-t_{12}^2 L_0, L_0, 0)$. *Right*: $\mathbf{L}_\mathrm{m} = \mathrm{diag}(-(t_{12}^2 + t_{13}^2/c_{12}^2)L_0, L_0, L_0)$.

where $c_{ij}/s_{ij}/t_{ij} = \cos\theta_{ij}/\sin\theta_{ij}/\tan\theta_{ij}$ with θ_{ij} being the mixing angle in PMNS matrix. Since BBN requires $L_e \lesssim \mathcal{O}(10^{-3})$, we may set $L_e = 0$ for an illustration. In this case, L_μ and L_τ are given by

$$\begin{aligned} L_\mu = {} & c_{23}\left[(1-t_{12}^2)c_{23} - 2s_{13}s_{23}t_{12}\right] L_2 \\ & + \left[(1-t_{13}^2)s_{23}^2 - t_{12}t_{13}^2 c_{23}(2s_{13}s_{23} + t_{12}c_{23})\right] L_3 \quad (11) \\ L_\tau = {} & s_{23}\left[(1-t_{12}^2)s_{23} + 2s_{13}c_{23}t_{12}\right] L_2 \\ & + \left[(1-t_{13}^2)c_{23}^2 - t_{12}t_{13}^2 s_{23}(2s_{13}c_{23} - t_{12}s_{23})\right] L_3 \quad (12) \end{aligned}$$

From Eqs. (11) and (12) with measured values of mixing angles [10], we find that $L_\mu \sim L_\tau$ for $|L_3| \lesssim |L_2|$, as shown in Fig. 2.

COSMOLOGICAL CONSTRAINTS

A large lepton number asymmetry in one or more neutrino species creates an extra radiation density in the universe relative to the standard contributions of photons and CP-symmetric active neutrinos, a form of so-called "dark radiation". Extra relativistic degrees of freedom in cosmology have attracted considerable recent attention as a way to resolve the apparent discrepancy in measurement of the Hubble parameter from CMB data and type-Ia supernovae [9, 18]. In this section, we investigate the possibility that a primordial lepton asymmetry may provide a dark radiation density which can reconcile CMB and SNIa values for the Hubble parameter.

We consider an eight-parameter ΛCDM+ξ cosmology without contribution from primordial tensor fluctuations and assume a normal mass hierarchy for neutrinos, with one massive neutrino with mass $m_\nu = 0.06$ eV. Since the BBN constraint on L_e should be satisfy, we are not free to choose $|L_i| \gg |L_e|$ in an arbitrary way, but constrained to satisfy approximately

$$c_{12}^2 L_1 + s_{12}^2 L_2 + t_{13}^2 L_3 = 0 \quad (13)$$

coming from $L_e = 0$. As the simplest possibility, we may set $L_3 = 0$ leading to $L_1 = -t_{12}^2 L_2$. Then, for thermal distributions of two light mass eigenstates,

$$\begin{aligned}\Delta N_{\rm eff} &= \frac{15}{7} \sum_{i=1,2} \left(\frac{\xi_i}{\pi}\right)^2 \left[2 + \left(\frac{\xi_i}{\pi}\right)^2\right] \\ &\approx \frac{15}{7} \left(\frac{\xi_2}{\pi}\right)^2 \times \\ &\quad \left\{(1+t_{12}^4)2 + \left[1 + (4+t_{12}^4)t_{12}^4\right] \left(\frac{\xi_2}{\pi}\right)^2\right\} \end{aligned} \quad (14)$$

where ξ_is are degeneracy parameters of each mass eigenstate, and $|\xi_i| \lesssim 1$ and $t_{12}^2 \ll 1$ were assumed. Strictly speaking, the late-time free-streaming neutrino mass-eigenstates are not in thermal distribution since they are linear combinations of thermal distributions of flavor-eigenstates. Hence, ξ_is in Eq. (14) should be understood as effective degeneracy parameters. The error in $\Delta N_{\rm eff}$ depends on the initial configuration of the lepton number asymmetries in flavor-basis, but it is expected to be of or small than $\mathcal{O}(10)\%$ for $\xi \lesssim 1$.

Figure 3 shows constraints on H_0 and ξ for the case of the eight-parameter ΛCDM+ξ fit. We plot constraints from Planck+BICEP/Keck only (filled contours), and Planck+BICEP/Keck+Riess *et al.* (dotted contours). The CMB data alone show no evidence for nonzero neutrino chemical potential, with a 95%-confidence upper bound of $\xi < 0.53$. For combined CMB and supernova data, there is weak evidence for a nonzero chemical potential, with $\xi = 0.50 \pm 0.19$ at 68% confidence. The combined CMB+supernova data, however, should be interpreted with caution: as the filled contours illustrate, the CMB data and supernova data taken separately are barely compatible, with only a small overlap in the 95% confidence regions, even when dark radiation from a neutrino asymmetry is included as a parameter. Combining two fundamentally incompatible data sets in a Bayesian analysis is likely to give a biased fit, which is reflected in the best-fit values for the two cases, with the best-fit to CMB alone having $-\ln(\mathcal{L}) = 6794.87$, while the best-fit for the the combined CMB+supernova data is measurably worse, with $-\ln(\mathcal{L}) = 6798.47$. For the CMB data alone, including lepton asymmetry, the best-fit Hubble parameter is $H_0 = 67.7 \pm 0.9$, with a 95%-confidence upper bound of $H_0 < 69.7$. This can be compared with a 95%-confidence *lower* bound from Type-Ia supernovae of $H_0 > 69.8$. We therefore conclude, that inclusion of dark radiation from a neutrino asymmetry does not fully reconcile the discrepancy between CMB and supernova data but may be a step in the direction of doing it.

CONCLUSIONS

In this talk, I argued that, when lepton number asymmetries of neutrinos in flavor basis are mixed among themselves due to neutrino oscillation in the early universe before BBN, the asymmetries at the final equilibrium are well described in the basis of mass eigenstates which is related to flavor eigenstates by PMNS matrix. That is, the matrices of lepton number asymmetries in mass- and flavor-basis ($\mathbf{L}_{\rm m}$ and $\mathbf{L}_{\rm f}$, repectively) are related as

$$\mathbf{L}_{\rm m} = U_{\rm PMNS}^{-1} \mathbf{L}_{\rm f} U_{\rm PMNS} \quad (15)$$

where $U_{\rm PMNS}$ is the PMNS matrix, and $\mathbf{L}_{\rm m}$ appears to be diagonal. We demonstrated this argument by a numerical simulation, and showed analytically that the asymmetries of mass-eigenstate can be even larger than those of flavor-eigenstates.

Conventionally, the constraint on the lepton number asymmetries of neutrino flavors has been associated with neutrino flavor-eigenstates, counting their contributions to the extra radiation energy density $\Delta N_{\rm eff}$. However, our argument and finding showed that a proper estimation should be done with neutrino mass-eigenstates in order not to miss the contributions of flavor-mixed states in flavor-basis, and the resulting $\Delta N_{\rm eff}$ can be larger than the one estimated with flavor-eigenstates only.

As shown in Ref. [6–8] and realized in [19] in principle $\Delta N_{\rm eff}$ can be of $\mathcal{O}(0.1-1)$ just from asymmetric neutrinos without resorting to an unknown "dark radiation". Such a large $\Delta N_{\rm eff}$ has been considered in literature as a possible solution to the discrepancy of the measured expansion rate H_0 in CMB and SNIa data. In analyses of cosmological data, typically, if $\Delta N_{\rm eff}$ is from asymmetric neutrinos, the neutrino degeneracy parameters have been taken in an arbitrary way without distinguishing mass- and flavor-eigenstates, although implicitly the lepton number asymmetry (L_e) of electron-neutrinos might be assumed to be small to satisfy BBN constraint. We showed that this approach is inconsistent unless the lepton number asymmetries (L_i) of mass-eigenstates which are relevant for CMB data for example are constrained to satisfy

$$L_e = c_{12}^2 L_1 + s_{12}^2 L_2 + t_{13}^2 L_3 \approx 0 \quad (16)$$

for $|L_e| \lll |L_i|$. Also, analyzing cosmological data (CMB only or CMB+SNIa), we found that CMB data alone show no evidence for nonzero neutrino lepton number asymmetries, with 95% CL upper bound of $|\xi| \lesssim 0.5 - 0.6$ at 95% CL as the degeneracy parameters of two light neutrinos only. For combined CMB and SNIa data, there is weak evidence for nonzero lepton number asymmetries, with $\xi \approx 0.50 \pm 0.19$ at 68% CL, but the fit became worse relative to the case of CMB data alone. So, even if large lepton number asymmetries may fit to the data, it does not look preferred.

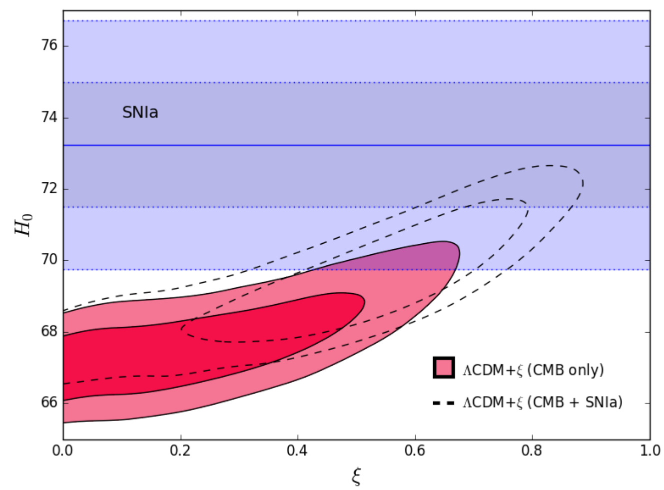

FIG. 3. Constraints on H_0 and ξ for the eight-parameter ΛCDM+ξ case. Filled contours show the 68% (dark red) and 95% (light red) constraints from Planck+BICEP/Keck alone. Dashed contours show the corresponding constraints with the addition of the Riess *et al.* supernova data. The constraint on H_0 from the supernova data alone, $H_0 = 73.24 \pm 1.74$ [18] is shown by the grey filled regions, with 1σ limits in dark grey, and 2σ limits in light grey.

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