Mass vs flavour eigenstates in neutrino asymmetries (or when coherence matters)

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どうもありがとう

NEUTRINO OSCILLATIONS:

Two Flavors

flavor eigenstates $\neq$ mass eigenestates

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

W's produce ν_μ and/or ν_τ's

but ν_1 and ν_2 are the states

that change by a phase over time, mass eigenstates.

$$|\nu_j\rangle \to e^{-ip_j\cdot x}|\nu_j\rangle \qquad p_j^2 = m_j^2$$

$\alpha, \beta \ldots$ flavor index $\qquad$ $i, j \ldots$ mass index

Production:

$$|\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

Propogation:

$$\cos\theta e^{-ip_1\cdot x}|\nu_1\rangle + \sin\theta e^{-ip_2\cdot x}|\nu_2\rangle$$

Detection:

$$|\nu_1\rangle = \cos\theta|\nu_\mu\rangle - \sin\theta|\nu_\tau\rangle$$

$$|\nu_2\rangle = \sin\theta|\nu_\mu\rangle + \cos\theta|\nu_\tau\rangle$$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

Same E, therefore $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$

$$e^{-ip_j\cdot x} = e^{-iEt}e^{-ip_jL} \approx e^{-i(Et-EL)} \quad e^{-im_j^2L/2E}$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2\theta\cos^2\theta|e^{-im_2^2L/2E} - e^{-im_1^2L/2E}|^2$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$\delta m^2 = m_2^2 - m_1^2$ and $\frac{\delta m^2 L}{4E} \equiv \Delta$ kinematic phase:

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

Same E, therefore $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$

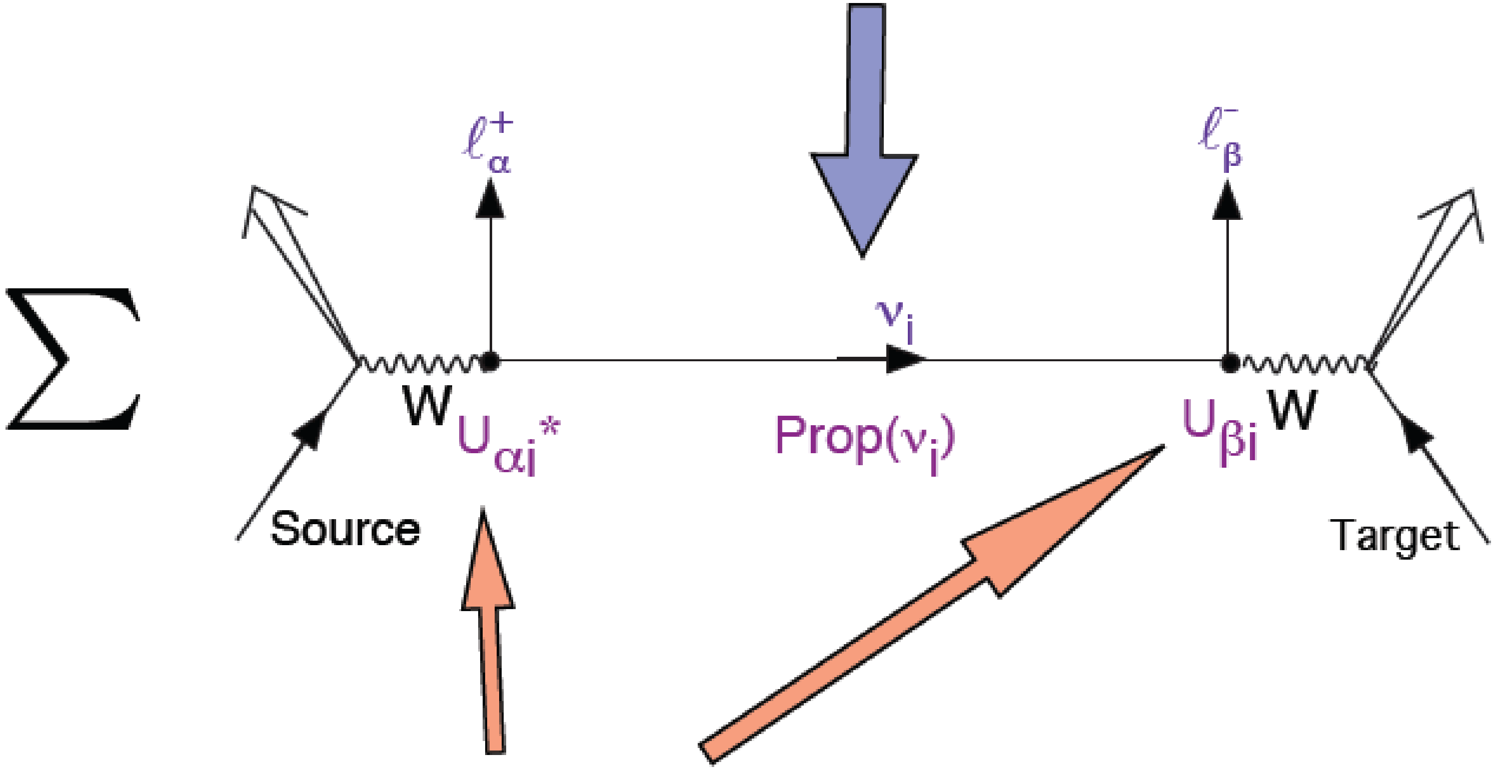
$$e^{-ip_j\cdot x} = e^{-iEt}e^{-ip_jL} \approx e^{-i(Et-EL)} \quad e^{-im_j^2L/2E}$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2\theta\cos^2\theta|e^{-im_2^2L/2E} - e^{-im_1^2L/2E}|^2$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2\left(\frac{\delta m^2 L}{4E}\,\frac{c^4}{hc}\right)$$

Amplitude

$$e^{-im_j^2 L/2E}$$



$$U_{\alpha j} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Appearance:

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

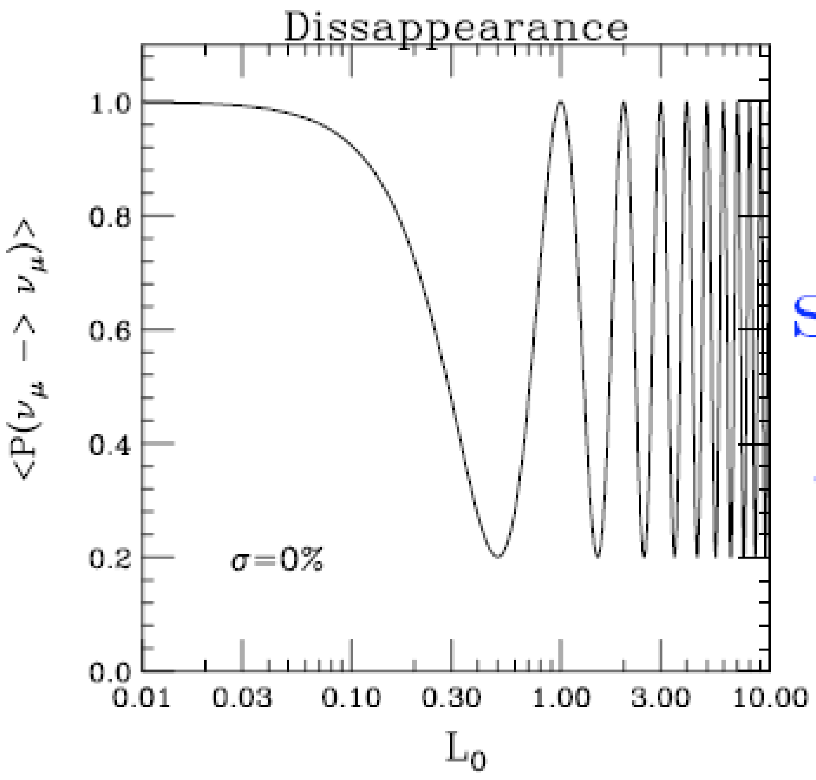
Disappearance:

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

Oscillation Length $L_0 = 4\pi E/\delta m^2$

Fixed E_ν



Amplitude of Oscillation

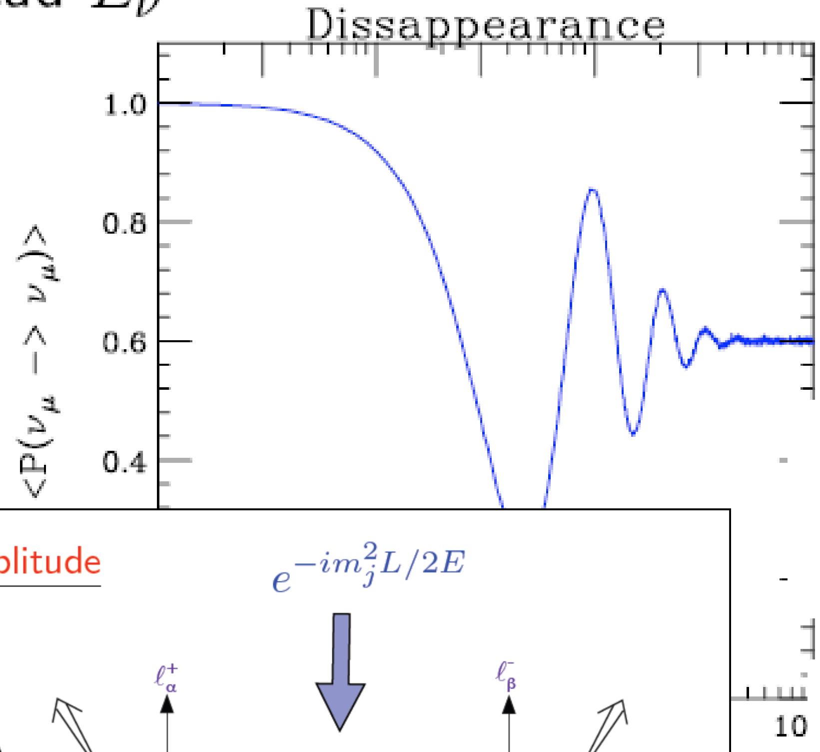
$\Uparrow$

$\sin^2 2\theta$

$\Downarrow$

$$\langle P(\nu_\mu \to \nu_\mu)\rangle = 1 - \sin^2 2\theta \left\langle \sin^2 \frac{\delta m^2 L}{4E} \right\rangle$$

Spread E_ν

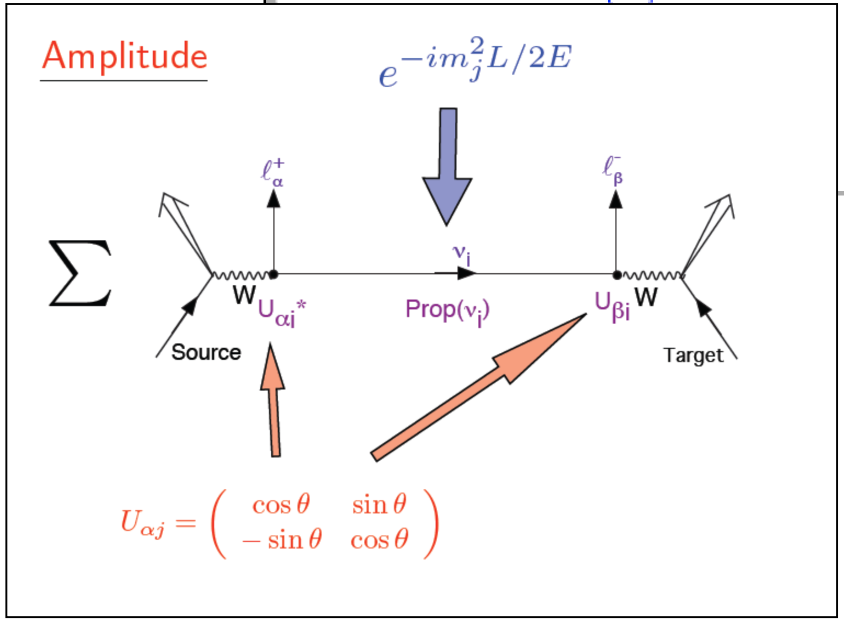


effectively incoherent mass eigenstates

$$1 - \sin^2 2\theta(\tfrac{1}{2}) = \cos^4\theta + \sin^4\theta$$

$W^+ \to \mu^+ + \nu_1$ probability $\cos^2\theta$

$W^+ \to \mu^+ + \nu_2$ probability $\sin^2\theta$



flavour fractions $|\nu_1\rangle$ and $|\nu_2\rangle$ during propagation remain unchanged

probability ν_1 contains ν_μ is $\cos^2\theta$

probability ν_2 contains ν_μ is $\sin^2\theta$

What about ξ ?

One of the open questions in cosmology is the possibility of admitting a large relic neutrino-antineutrino asymmetry

The trick is there are no direct observations of the cosmic neutrino background…

BBN CMB

Two flavors

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

where the same parameterisation applies to the antineutrino system.

For each momentum p, we write down the one-body reduced density matrices ρ and express them in terms of the function P_0 and a "polarization" vector **P**

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix} = \frac{1}{2}\left[P_0 + \mathbf{P}\cdot\sigma\right]$$

$$f_{\nu_e} = \frac{1}{2}\left[P_0 + P_z\right] f_{eq}(0)$$

$$f_{\nu_x} = \frac{1}{2}\left[P_0 - P_z\right] f_{eq}(0)$$

$$f_{eq}(\xi) \equiv \frac{1}{1+e^{p/T-\xi}}$$

$$\dot{\bar{\mathbf{P}}} = -\left[\frac{\Delta m^2}{2p}\mathbf{B} - \frac{8\sqrt{2}G_F p}{3m_W^2}E_{ee}\mathbf{z}\right]\times \bar{\mathbf{P}} + \sqrt{2}G_F(\mathbf{J}-\bar{\mathbf{J}})\times \bar{\mathbf{P}}$$

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electron-positrón energy density

$\mathbf{B} = (\sin 2\theta\,, 0, \cos 2\theta)$

$$\dot{\mathbf{P}} = +\left[\frac{\Delta m^2}{2p}\mathbf{B} - \frac{8\sqrt{2}G_F p}{3m_W^2}E_{ee}\mathbf{z}\right]\times\mathbf{P} + \sqrt{2}G_F(\mathbf{J}-\bar{\mathbf{J}})\times\mathbf{P}$$

electron-positrón energy density

$$\mathbf{B} = (\sin 2\theta\,, 0, \cos 2\theta)$$

self-interaction term

$$\mathbf{J} = \frac{1}{2\pi^2}\int \mathbf{P}\; f_{\rm eq}(0)\; p^2 dp$$

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$$\mathbf{J} = \frac{1}{2\pi^2}\int \mathbf{P}\ f_{\rm eq}(0)\ p^2 dp$$

where we are ignoring (so far) the collision term which may or may not be relevant

$$\Gamma^{-1} \sim (G_F^2 p\ T^4)^{-1}$$

$$L \sim 2\pi(V_x^2 + V_z^2)^{-1/2}$$

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$\mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)$

ergy density

$$V_x = \frac{\Delta m^2}{2p}\sin 2\theta$$

on term

$$V_z = -\frac{\Delta m^2}{2p}\cos 2\theta - \frac{8\sqrt{2}G_F p}{3m_W^2}E_{ee}$$

where we are ignoring (so far) the collision term which may or may not be re vant

$$\Gamma^{-1} \sim (G_F^2 p\, T^4)^{-1}$$

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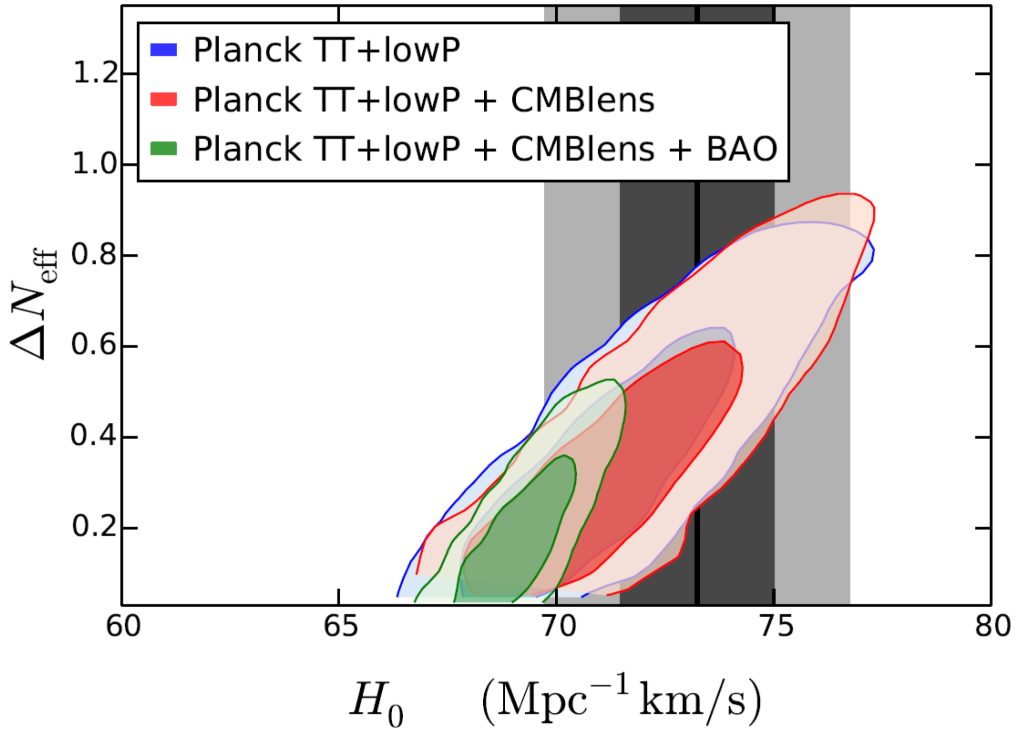
Riess et al (2016) vs CMB fit

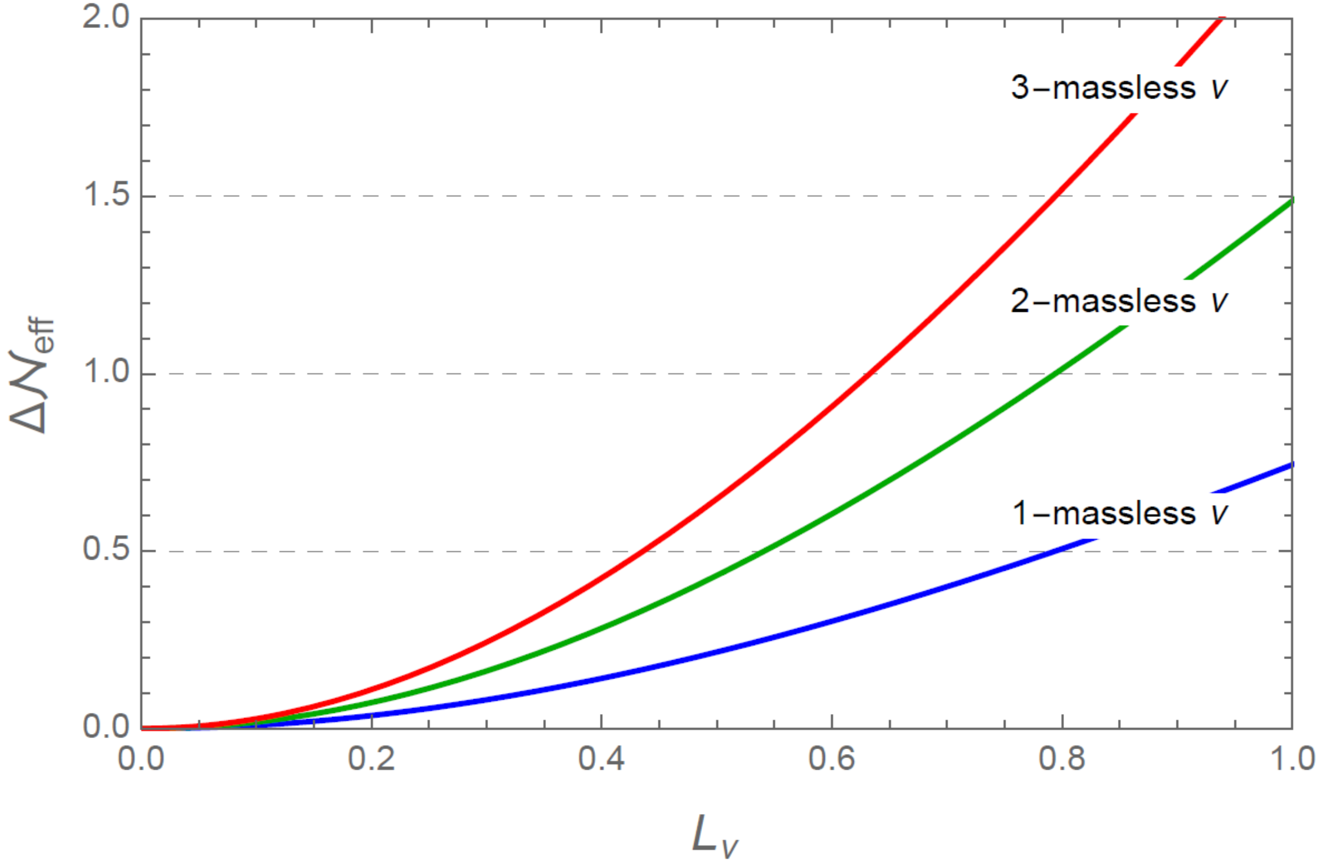
H_0 = 73.24 ± 1.74 km/s/Mpc

H_0 = 67.8 ± 0.9 km/s/Mpc



Under the assumption of thermal distribution,

$$L_\alpha = \frac{\pi^3}{12\zeta(3)}\left(\frac{\xi_\alpha}{\pi}\right)\left[1+\left(\frac{\xi_\alpha}{\pi}\right)^2\right], \quad \Delta N_{\rm eff} = \frac{15}{7}\sum_\alpha\left(\frac{\xi_\alpha}{\pi}\right)^2\left[2+\left(\frac{\xi_\alpha}{\pi}\right)^2\right]$$



BBN constrains L_α

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$$Y_p \equiv \frac{\rho(^4He)}{\rho_{\text{baryon}}} = \frac{2(n/p)}{1+(n/p)}$$

the ratio of neutron-proton number densities
(depends on weak interaction rate)

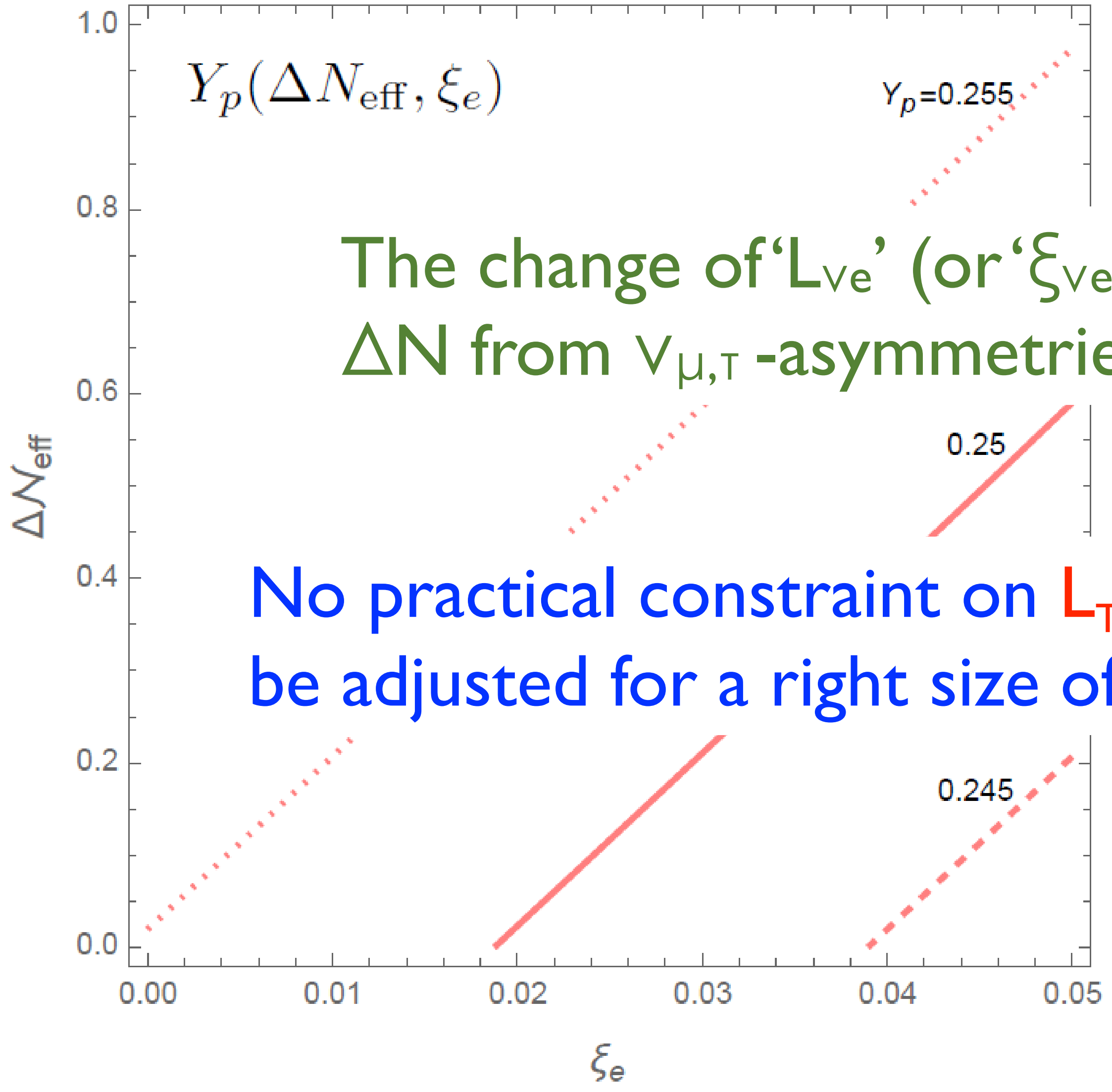
Helium abundance: affected by expansion rate and ν_e-asymmetry

$$Y_p = [a(B) + b(B)\Delta N]\, e^{-\xi_{\nu_e}}$$

[JHEP 11 (1999) 015]

$$\text{c.f} \quad Y_p^{\text{obs}} \sim 0.245 - 0.255$$

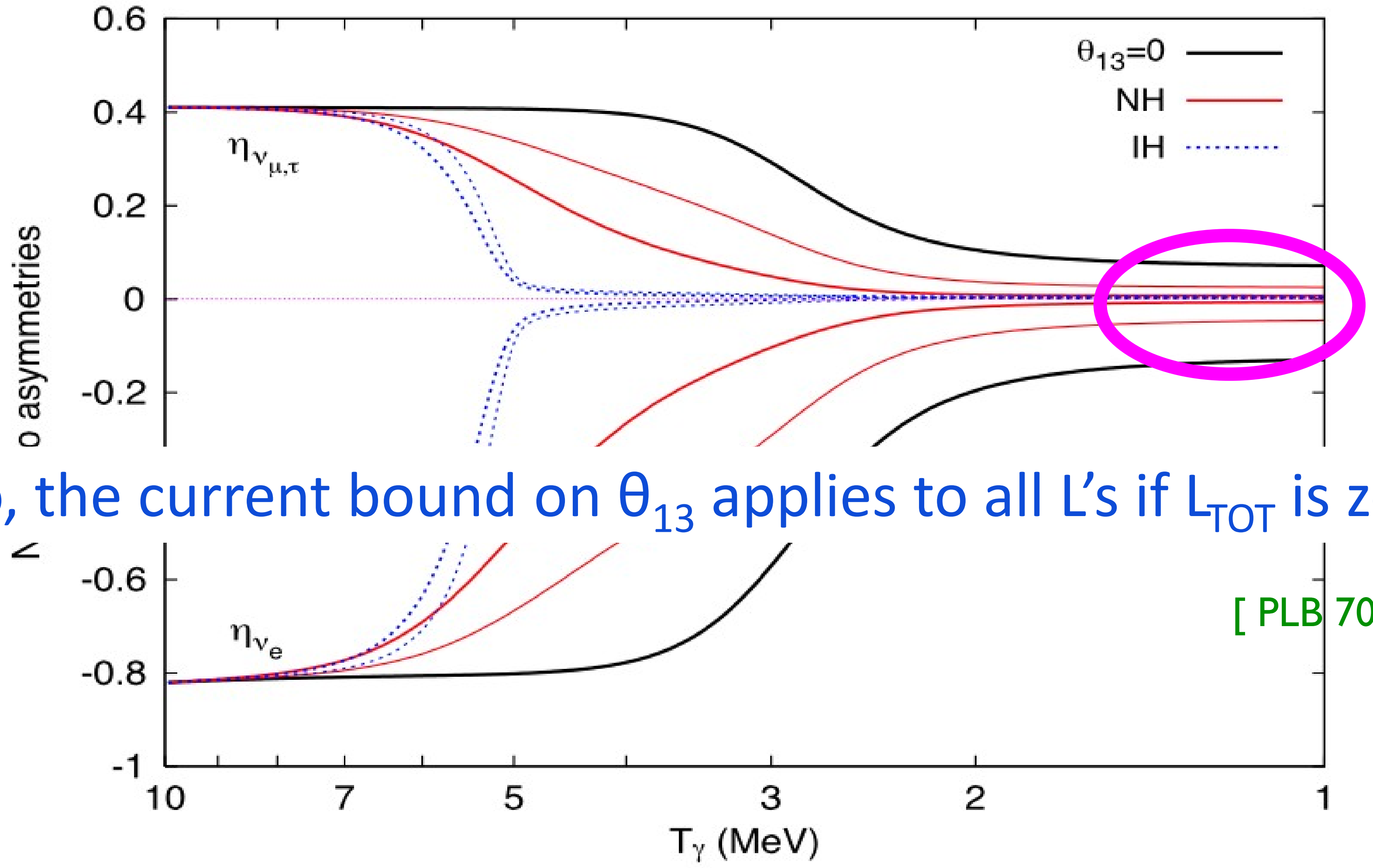
See also 'Mon.Not.Roy.Astron.Soc. 445 (2014) no.1,778; JCAP 07,011 (2015)')



The change of '$L_{\nu e}$' (or '$\xi_{\nu e}$') can be compensated by ΔN from $\nu_{\mu,\tau}$ -asymmetries

No practical constraint on L_{TOT} as long as the L_α can be adjusted for a right size of $L_{\nu e}$.

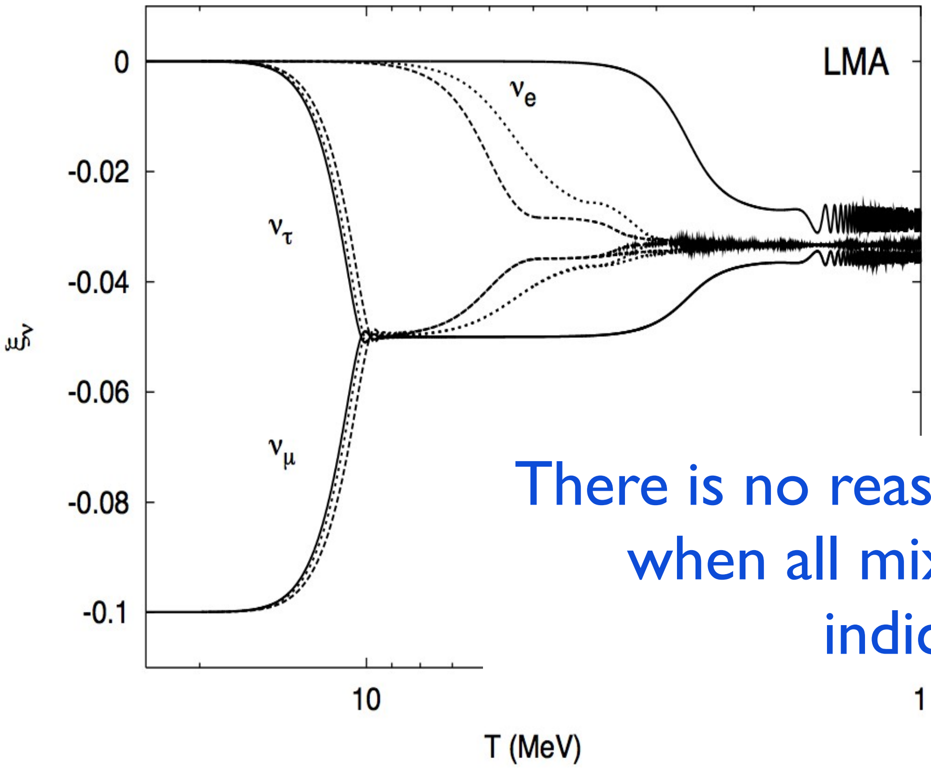
Neutrino oscillations: In the early universe before BBN, oscillations mix asymmetries of all neutrino flavors before BBN starts.

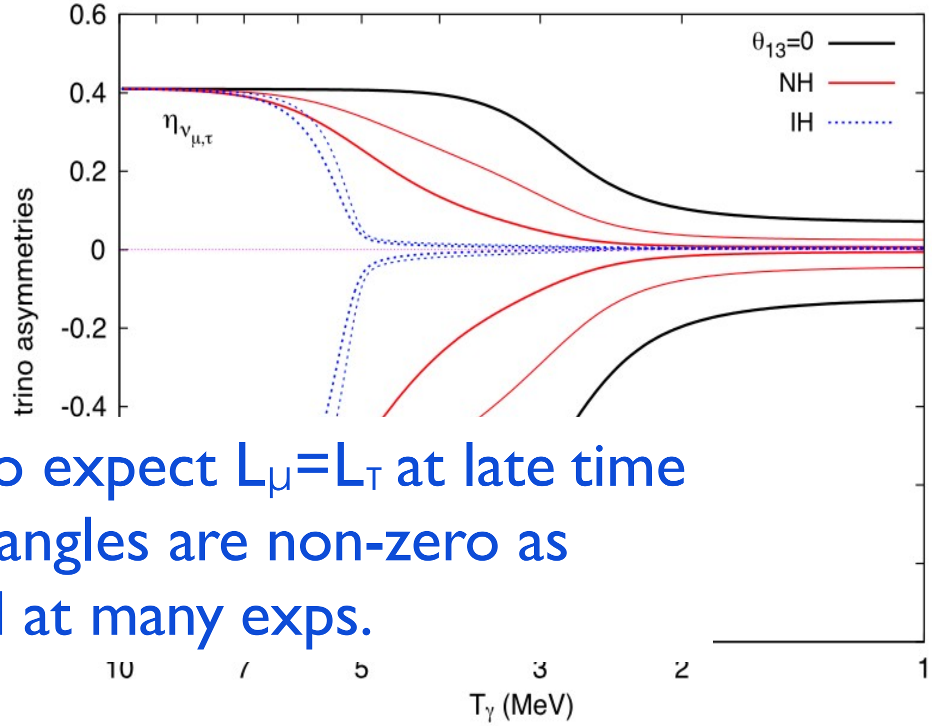


So, the current bound on θ_{13} applies to all L's if L_{TOT} is zero

[PLB 708 (2012) 1-5]

Equalization of L_μ & L_τ: once equalized, never separated afterward.





There is no reason to expect $L_\mu = L_\tau$ at late time when all mixing angles are non-zero as indicated at many exps.

$$4m_\mu(m_\mu T/2\pi)^{3/2}\exp(-m_\mu/T)$$

$$\frac{1}{2p} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger - \frac{8\sqrt{2}G_F p}{3m_W^2} \begin{pmatrix} E_{ee}+E_{\mu\mu} & 0 & 0 \\ 0 & E_{\mu\mu} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Very simple to decipher !!!

The evolution of the three flavor system can be broken down into three separate two flavor systems

$$\begin{pmatrix} 1 & 0 & 0 \\ c_{23} & s_{23} & 0 \\ -s_{23} & c_{23} & 0 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Delta m^2_{\rm atm}/2\widetilde{p} \simeq (8\sqrt{2}G_F\widetilde{p}/3m_W^2)E_{\mu\mu}$$

(i) at T=12 MeV when $\nu_\mu \rightarrow \nu_\tau$ oscillations cease to be matter suppresed

$$\nu_\mu \longrightarrow \nu_x \equiv \frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau), \qquad \nu_\tau \longrightarrow \nu_y \equiv \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau);$$

(ii) at T=5 MeV only if θ_{13} is non zero

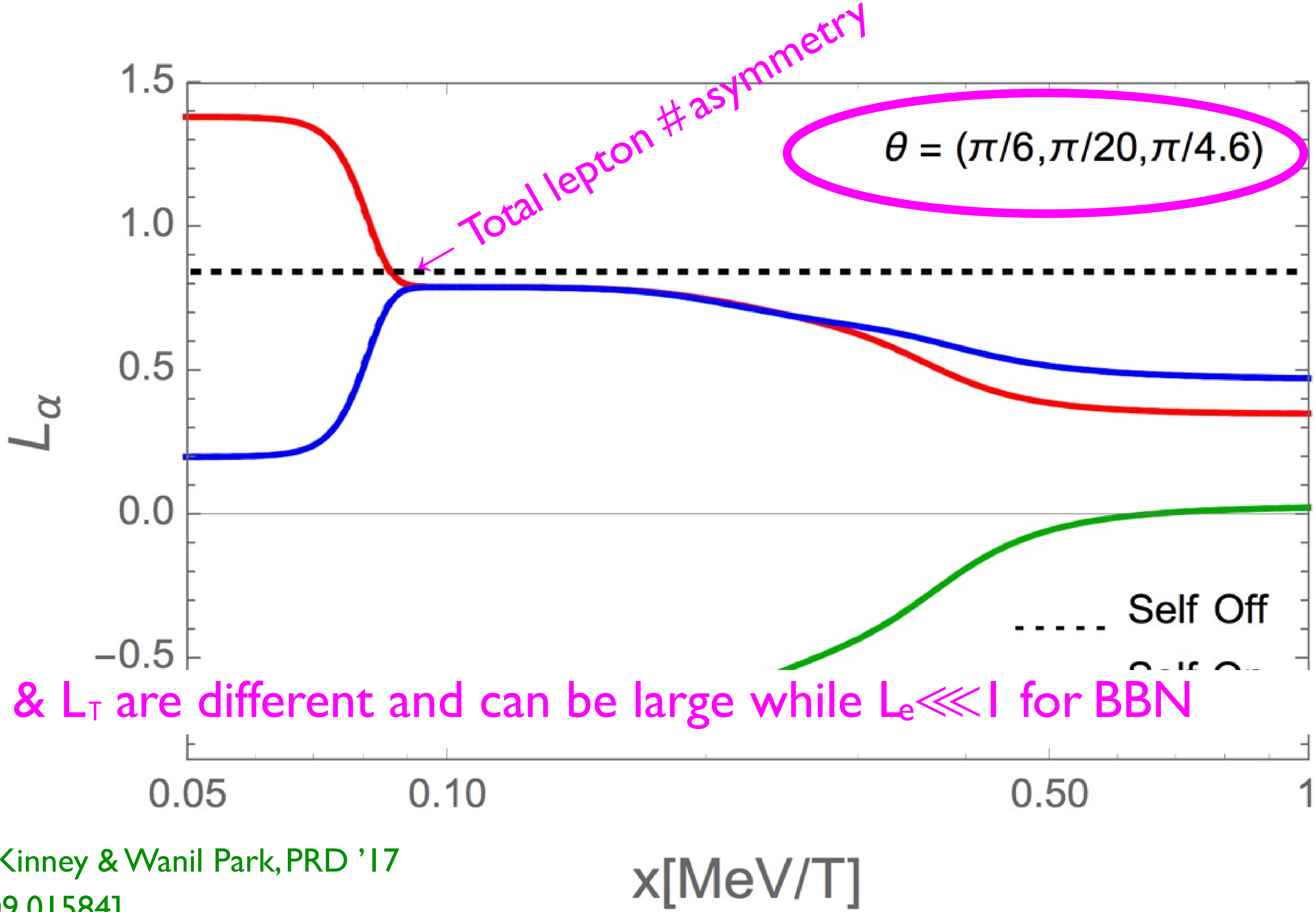
$$(\Delta m^2_{\rm atm} + c_{12}^2 \Delta m^2_{\rm sun})/2\widetilde{p} \simeq (8\sqrt{2}G_F\widetilde{p}/3m_W^2)(E_{ee} + E_{\mu\mu}/2)$$

$$\nu_e \longrightarrow c_{13}\nu_e - s_{13}\nu_y, \qquad \nu_y \longrightarrow s_{13}\nu_e + c_{13}\nu_y,$$

(iii) at T=2 MeV

$$c_{13}\nu_e - s_{13}\nu_y \longrightarrow c_{12}\, c_{13}\, \nu_e - \left(\frac{c_{12}\, s_{13}}{\sqrt{2}} + \frac{s_{12}}{\sqrt{2}}\right)\nu_\mu + \left(\frac{s_{12}}{\sqrt{2}} - \frac{c_{12}\, s_{13}}{\sqrt{2}}\right)\nu_\tau$$

$$\nu_x \longrightarrow s_{12}\, c_{13}\, \nu_e + \left(\frac{c_{12}}{\sqrt{2}} - \frac{s_{12}\, s_{13}}{\sqrt{2}}\right)\nu_\mu - \left(\frac{c_{12}}{\sqrt{2}} + \frac{s_{12}\, s_{13}}{\sqrt{2}}\right)\nu_\tau$$



L_μ & L_τ are different and can be large while $L_e \lll 1$ for BBN

G. B, W. H. Kinney & Wanil Park, PRD '17
[arXiv:1609.01584]

$$L_\alpha = \frac{\pi^3}{12\zeta(3)}\left(\frac{\xi_\alpha}{\pi}\right)\left[1+\left(\frac{\xi_\alpha}{\pi}\right)^2\right] \Rightarrow \Delta N_{\rm eff} = \frac{15}{7}\sum_\alpha\left(\frac{\xi_\alpha}{\pi}\right)^2\left[2+\left(\frac{\xi_\alpha}{\pi}\right)^2\right]$$

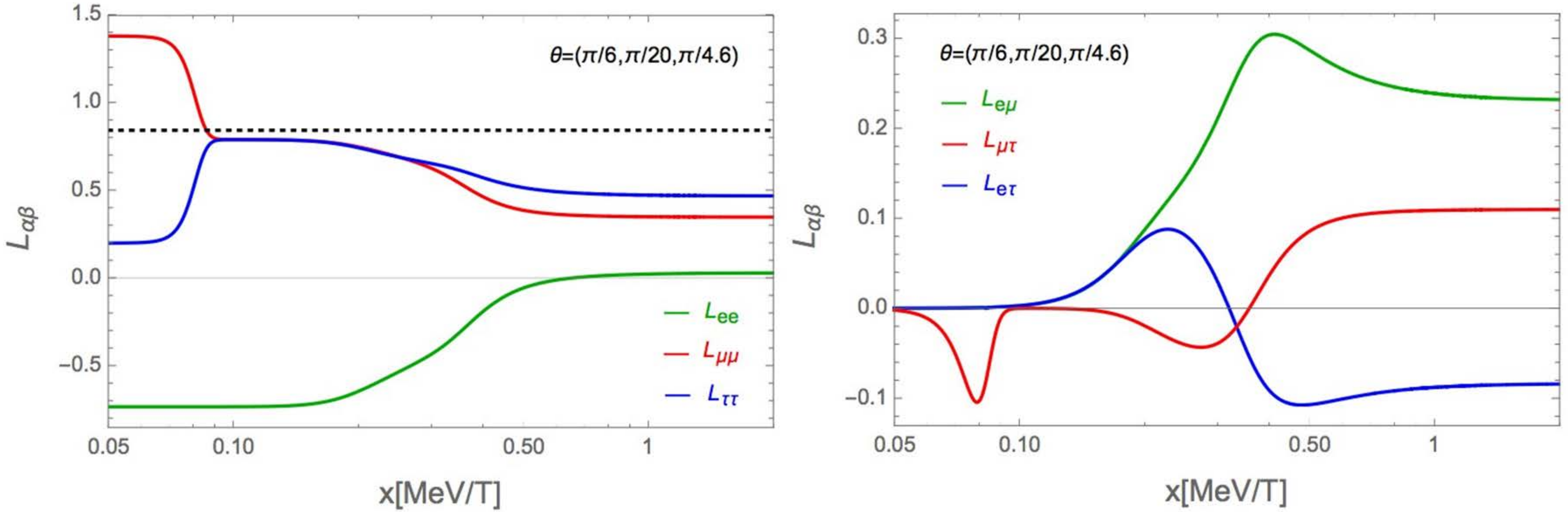
It is valid only if neutrino flavor-mixings are absent.

It has been conventional even if flavor-mixings were confirmed several decades ago. What's wrong with it?

L_α under flavor mixings: There are mixed states contributing to energy density

$$L_\alpha \equiv \frac{\rho_\alpha - \rho_{\bar\alpha}}{n_\gamma} \;;\quad \rho_\alpha/\rho_{\bar\alpha} = \text{density matrix of } \nu_\alpha/\bar\nu_\alpha$$

This is a matrix having off-diagonal entries as well as diagonal ones.



from flavor eigenstates

$\Delta N(\xi_\alpha)$ is missing off-diagonal contributions!

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Asymmetries (L_i) in mass basis:

Neutrinos free-stream after decoupling at T~2 MeV before BBN starts.
Free-streaming neutrinos are in mass-eigenstates.
The late-time asymmetries should be diagonal in mass-basis.

from mass eigenstates

$\Delta N(\xi_i)$ contains all contributions!

Then, manifestly,

$$\mathbf{L}_{\mathrm{m}} = D^{-1}\mathbf{L}_{\mathrm{f}}D$$

diagonalization matrix

Asymmetries in mass-basis (diagonal)

Then, manifestly,

diagonalization matrix

well-known neutrino mixing matrix

$$\mathbf{L}_{\rm m} = D^{-1}\mathbf{L}_{\rm f}D = U_{\rm PMNS}^{-1}\mathbf{L}_{\rm f}U_{\rm PMNS}$$

Asymmetries in mass-basis (diagonal)

Asymmetries in flavor-basis (non-diagonal)

We numerically confirmed

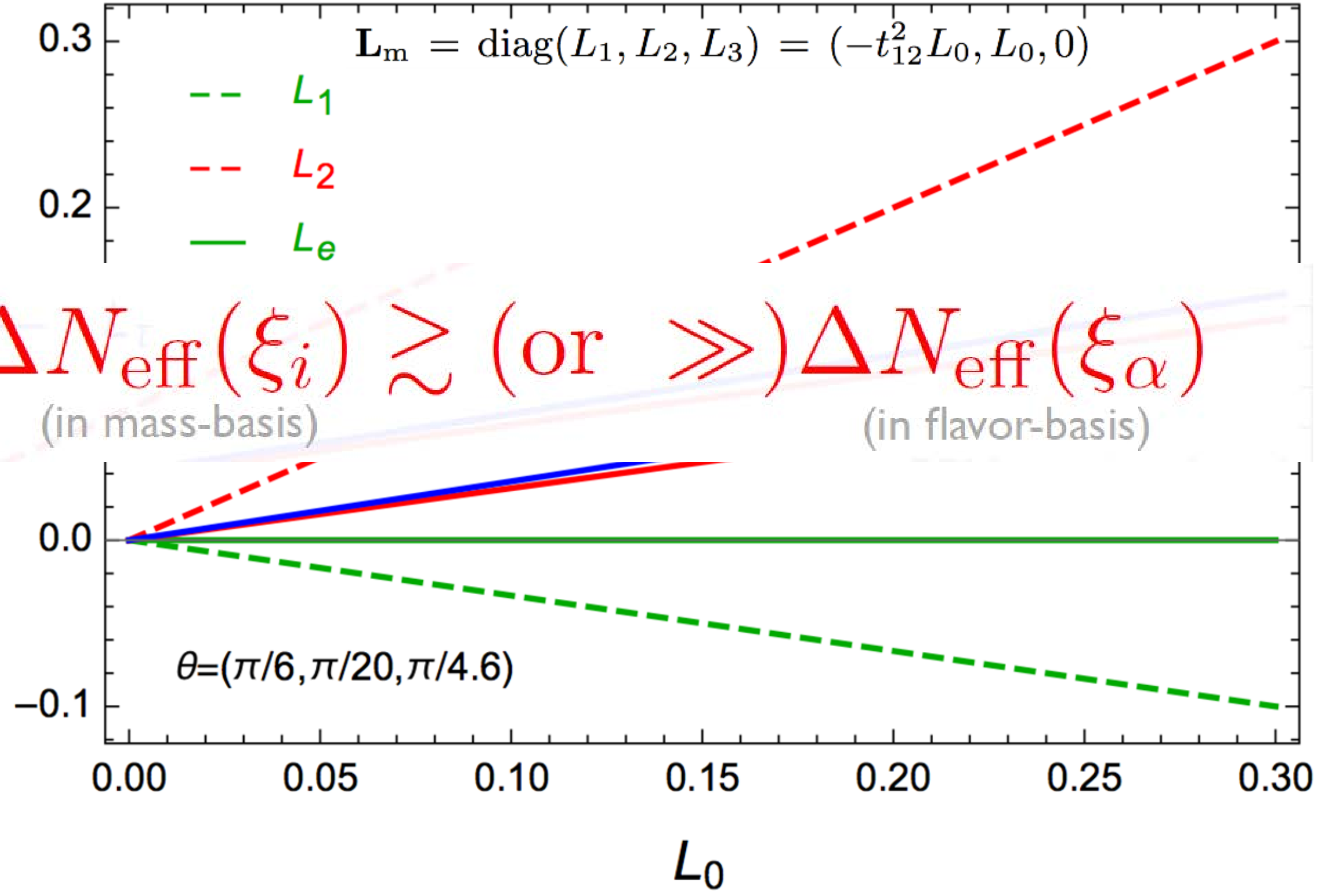
$$D = U_{\rm PMNS}$$

with sub-% level errors.

L_α vs L_i (numerical analysis):

L_α can be expressed in terms of L_i (diagonal entries of L_m).

BBN constraint: $L_e = c_{13}^2\left(c_{12}^2 L_1 + s_{12}^2 L_2\right) + s_{13}^2 L_3$

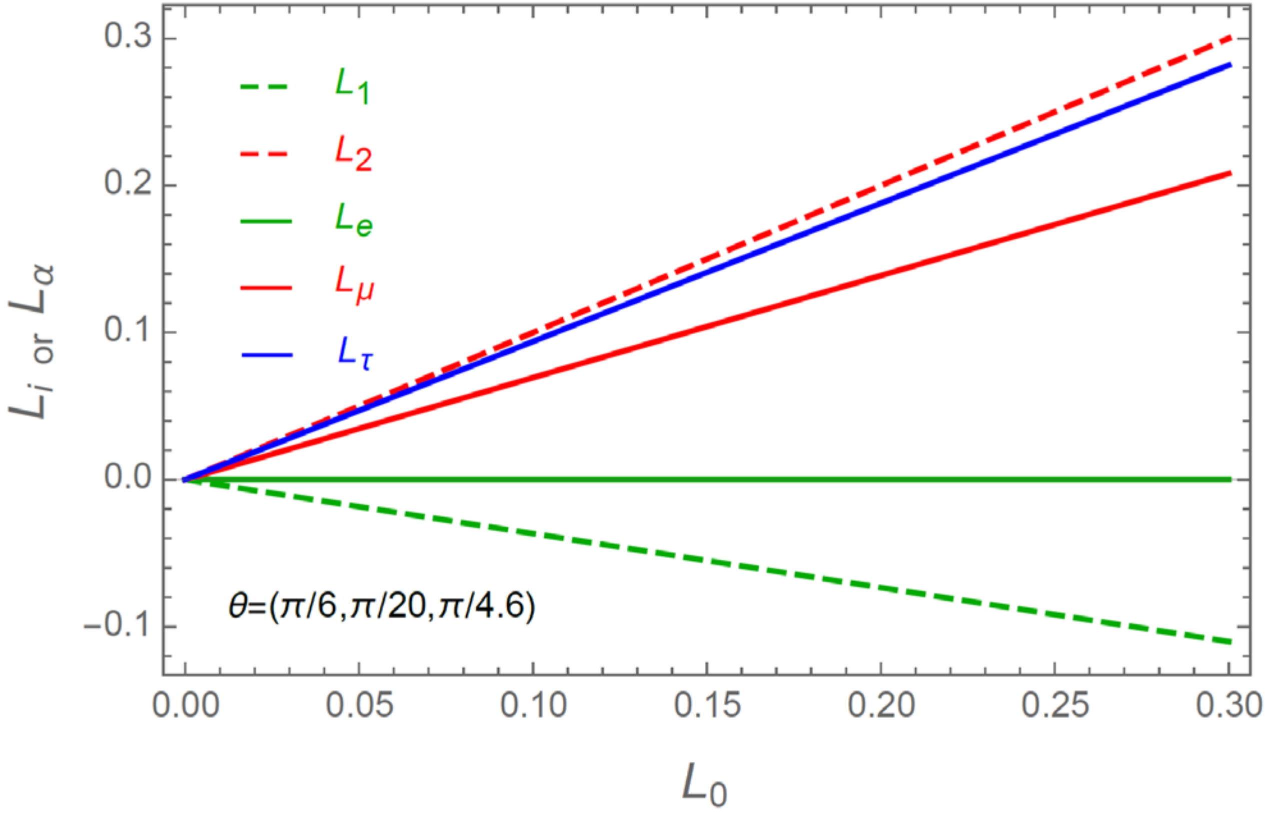


$\Delta N_\mathrm{eff}(\xi_i) \gtrsim (\text{or} \gg) \Delta N_\mathrm{eff}(\xi_\alpha)$

(in mass-basis) (in flavor-basis)

$$L_e = c_{13}^2 \left(c_{12}^2 L_1 + s_{12}^2 L_2\right) + s_{13}^2 L_3$$

$$\mathbf{L}_\mathrm{m} = \mathrm{diag}(-(t_{12}^2 + t_{13}^2/c_{12}^2)L_0, L_0, L_0)$$



Set L_1 such that

$$L_e = 0$$

$$\begin{aligned} L_\mu &= c_{23}\left[(1-t_{12}^2)c_{23} - 2s_{13}s_{23}t_{12}\right]L_2 \\ &+ \left[(1-t_{13}^2)s_{23}^2 - t_{12}t_{13}^2c_{23}(2s_{13}s_{23}+t_{12}c_{23})\right]L_3 \\ L_\tau &= s_{23}\left[(1-t_{12}^2)s_{23} + 2s_{13}c_{23}t_{12}\right]L_2 \\ &+ \left[(1-t_{13}^2)c_{23}^2 + t_{12}t_{13}^2s_{23}(2s_{13}c_{23} - t_{12}s_{23})\right]L_3 \end{aligned}$$

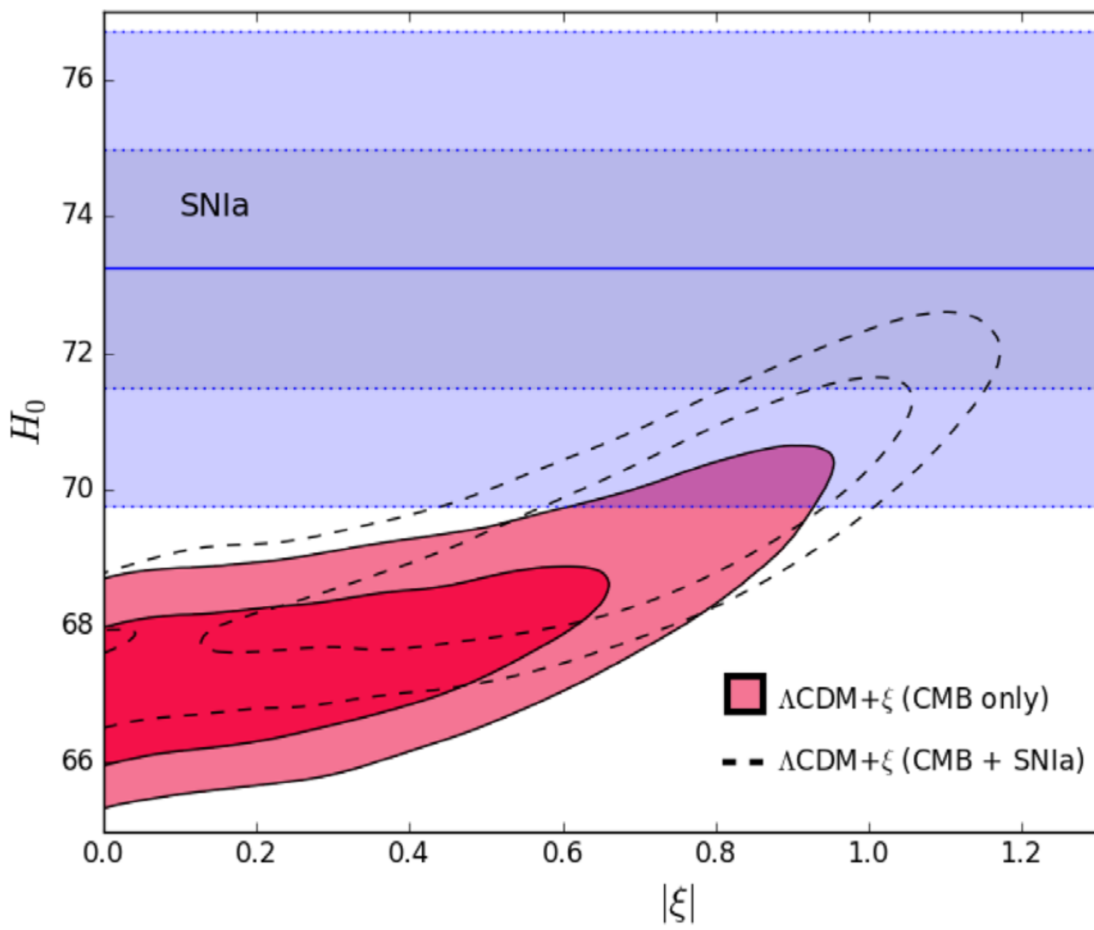
two variables to play with !!!

or

$$L_3 = 0$$

$$L_1 \approx -t_{12}^2 L_2$$

$$\begin{aligned} \Delta N_{\rm eff} &= \frac{15}{7}\sum_{i=1,2}\left(\frac{\xi_i}{\pi}\right)^2\left[2+\left(\frac{\xi_i}{\pi}\right)^2\right] \\ &\approx \frac{15}{7}\left(\frac{\xi_2}{\pi}\right)^2 \times \\ &\left\{(1+t_{12}^4)2 + \left[1+(4+t_{12}^4)t_{12}^4\right]\left(\frac{\xi_2}{\pi}\right)^2\right\} \end{aligned}$$


SNIa
ΛCDM+ξ (CMB only)
ΛCDM+ξ (CMB + SNIa)
H_0
$|\xi|$
76
74
72
70
68
66
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Conclusions

A large extra radiation contribution (or ΔN) can arise from neutrino asymmetries without disturbing BBN.

The ΔN coming from neutrino asymmetries should be estimated not from flavor-eigenstates but from mass-eigenstates.

Unfortunately the SIMPLEST version (only one degree of freedom) is not the final answer.