Charged lepton flavour violation decays in the A_4 neutrino mass model

Jai More

Indian Institute of Technology Bombay, India

in collaboration with S. Uma Sankar, Ushak Rahaman and Rambabu Korrapati

based on: arXiv:2009.00865

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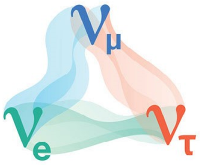
Neutrino

Neutrino (1/2, weak interaction, tiny mass)



Neutrino oscillation→Mass & Mixing

Charged lepton sector

- $\not{L}_e, \not{L}_\mu, \not{L}_\tau$
- FV is consequence of Neutrino oscillation
- FV decays becomes interesting

PMNS matrix

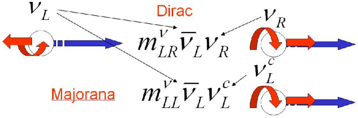
$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle \, (\alpha = e, \mu, \tau)$$

Parametrized in terms:

$\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}$

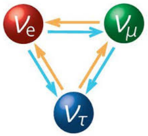
Mass

- Dirac Mass



- Majorana Mass

Neutrino Oscillation



- Neutrino oscillations arises through neutrino masses
- SM neutrinos are massless $\implies$ BSM
- Emitted/absorbed in flavour eigenstates but travel as mass eigenstates
- Flavour states mix to form three mass eigenstates
- The two bases are related by 3×3 mixing matrix
- Experimentally observed that mixing matrix has TBM form

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Particle content of SM particle

- Interactions are governed by $SU(2)_L \times U(1)_Y$
- Left chiral leptons

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \sim (2, -1)$$

- Right chiral leptons $e_R, \mu_R, \tau_R \sim (1, -2)$
- No $\nu_R \implies$ no neutrino mass
- Neutrino can acquire mass in two ways:
  - Dirac mass: additional RH components
  - Majorana mass: spinors of a single chirality
- Various mass model based on different discrete symmetries

A_4 based model [X. G. He, Y. Y. Keum and R. R. Volkas, JHEP **04** (2006), 039]

- Predicts tri-bimaximal mixing purely based on symmetry and symmetry breaking.
- Does not require fine tuning of parameters (VEVs or YCs).
- Fermion content is same as SM + multiple Higgs scalar.
- Higgs doublets in the model lead to charged lepton flavour violation.
- A small perturbation in the Majorana mass matrix of the heavy right-chiral neutrinos in this model, can lead to $\sin^2\theta_{13} \approx 0.02$ and maximal CP violation [A. Dev, P. Ramadevi and S. U. Sankar, JHEP **11** (2015), 034].

The gauge and the A_4 quantum numbers of all the fermions

$$Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \sim \left(3, 2, \tfrac{1}{3}\right)\left(\underline{\mathbf{3}}\right) \qquad D_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix} \sim (1, 2, -1)\left(\underline{\mathbf{3}}\right)$$

$$d_{1R} \oplus d_{2R} \oplus d_{3R} \sim \left(3, 1, -\tfrac{2}{3}\right)\left(\underline{\mathbf{1}} \oplus \underline{\mathbf{1'}} \oplus \underline{\mathbf{1''}}\right) \quad \ell_{1R} \oplus \ell_{2R} \oplus \ell_{3R} \sim (1, 1, -2)\left(\underline{\mathbf{1}} \oplus \underline{\mathbf{1'}} \oplus \underline{\mathbf{1''}}\right)$$

$$u_{1R} \oplus u_{2R} \oplus u_{3R} \sim \left(3, 1, \tfrac{4}{3}\right)\left(\underline{\mathbf{1}} \oplus \underline{\mathbf{1'}} \oplus \underline{\mathbf{1''}}\right) \qquad \nu_R \sim (1, 1, 0)\left(\underline{\mathbf{3}}\right).$$

The gauge and A_4 quantum numbers of three Higgs fields

$$\phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \sim (1,2,-1)\left(\underline{\mathbf{3}}\right), \quad \phi_0 = \begin{pmatrix} \phi_0^+ \\ \phi_0^0 \end{pmatrix} \sim (1,2,-1)\left(\underline{\mathbf{1}}\right), \quad \chi_i^0 \sim (1,1,0)\left(\underline{\mathbf{3}}\right).$$

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VEV of Higgs fields

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_3\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \ \langle\phi_0\rangle = \begin{pmatrix} 0 \\ v_0 \end{pmatrix}, \ \langle\chi_i^0\rangle = (0, w_2, 0).$$

*Vacuum alignment is key to obtain TBM form of PMNS matrix

Yukawa Lagrangian of this model [He 2006, Grimus 2012]

$$\mathcal{L}_{Yuk} = -\Big[h_{1d}\,(\overline{Q}_{1L}\,\phi_1 + \overline{Q}_{2L}\,\phi_2 + \overline{Q}_{3L}\,\phi_3)\,d_{1R} + h_{2d}\,(\overline{Q}_{1L}\,\phi_1 + \omega^2\overline{Q}_{2L}\,\phi_2 + \omega\,\overline{Q}_{3L}\,\phi_3)\,d_{2R}$$
$$+h_{3d}\,(\overline{Q}_{1L}\,\phi_1 + \omega\,\overline{Q}_{2L}\,\phi_2 + \omega^2\,\overline{Q}_{3L}\,\phi_3)\,d_{3R} + h_{1u}\,(\overline{Q}_{1L}\,\tilde{\phi}_1 + \overline{Q}_{2L}\,\tilde{\phi}_2 + \overline{Q}_{3L}\,\tilde{\phi}_3)\,u_{1R}$$
$$+h_{2u}\,(\overline{Q}_{1L}\,\tilde{\phi}_1 + \omega^2\,\overline{Q}_{2L}\,\tilde{\phi}_2 + \omega\,\overline{Q}_{3L}\,\tilde{\phi}_3)\,u_{2R} + h_{3u}\,(\overline{Q}_{1L}\,\tilde{\phi}_1 + \omega\,\overline{Q}_{2L}\,\tilde{\phi}_2 + \omega^2\,\overline{Q}_{3L}\,\tilde{\phi}_3)\,u_{3R} + h.c$$
$$-\Big[h_{1\ell}(\overline{D}_{1L}\phi_1 + \overline{D}_{2L}\phi_2 + \overline{D}_{3L}\phi_3)\ell_{1R} + h_{2\ell}(\overline{D}_{1L}\phi_1 + \omega^2\overline{D}_{2L}\phi_2 + \omega\overline{D}_{3L}\phi_3)\ell_{2R}$$
$$+h_{3\ell}(\overline{D}_{1L}\phi_1 + \omega\overline{D}_{2L}\phi_2 + \omega^2\overline{D}_{3L}\phi_3)\ell_{3R} + h_0(\overline{D}_{1L}\,\nu_{1R} + \overline{D}_{2L}\,\nu_{2R} + \overline{D}_{3L}\,\nu_{3R})\tilde{\phi}_0 + h.c.\Big]$$
$$+\frac{1}{2}\Big[M(\nu_{1R}^T\,C^{-1}\,\nu_{1R} + \nu_{2R}^T\,C^{-1}\,\nu_{2R} + \nu_{3R}^T\,C^{-1}\,\nu_{3R}) + h.c.\Big]$$
$$+\frac{1}{2}\Big[h_\chi\,((\chi_1(\nu_{2R}^T\,C^{-1}\,\nu_{3R} + \nu_{3R}^T\,C^{-1}\,\nu_{2R}) + \chi_2(\nu_{3R}^T\,C^{-1}\,\nu_{1R} + \nu_{1R}^T\,C^{-1}\,\nu_{3R})$$
$$+\chi_3(\nu_{1R}^T\,C^{-1}\,\nu_{2R} + \nu_{2R}^T\,C^{-1}\,\nu_{1R})) + h.c.\Big],$$

where $\tilde{\phi}_i = i\sigma_2\phi_i^*$ and $\tilde{\phi}_0 = i\sigma_2\phi_0^*$

Yukawa Lagrangian

$$-\bar{f}_L\, M_f\, f_R - \bar{\nu}_L\, M_D\, \nu_R + \frac{1}{2}\nu_R^T\, C^{-1}\, M_R\, \nu_R + h.c.$$

Higgs VEVs, we have $M_D = h_0 v_0 \mathbb{I}$

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Mass matrices for charged fermions and the neutrinos

$$M_f = \sqrt{3}\, v\, U_\omega^\dagger \begin{pmatrix} h_{1f} & 0 & 0 \\ 0 & h_{2f} & 0 \\ 0 & 0 & h_{3f} \end{pmatrix} \mathbb{I}, \; M_R = \begin{pmatrix} M & 0 & h_\chi w_2 \\ 0 & M & 0 \\ h_\chi w_2 & 0 & M \end{pmatrix},$$

where $f = (u, d, \ell)$.

The matrix U_ω

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

ω is cube root of unity.

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The diagonalizing matrix of M_R

$$U_\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

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PMNS matrix

$$U = U_\omega\, U_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & -\frac{\omega}{\sqrt{3}} & -\frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & -\frac{\omega^2}{\sqrt{3}} & \frac{e^{i\pi/6}}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L}_{Yuk} = \mathcal{L}^{\ell}_{Yuk} + \mathcal{L}^{u}_{Yuk} + \mathcal{L}^{d}_{Yuk} + \mathcal{L}^{\nu}_{Yuk}$$

where

$$\begin{aligned}
\mathcal{L}^{\ell}_{Yuk} &= -\frac{h_{1\ell}}{\sqrt{3}}\Big[(\bar{e}_L + \bar{\mu}_L + \bar{\tau}_L)\, \phi^0_1 + \left(\bar{e}_L + \omega\bar{\mu}_L + \omega^2\bar{\tau}_L\right) \phi^0_2 + \left(\bar{e}_L + \omega^2\bar{\mu}_L + \omega\bar{\tau}_L\right) \phi^0_3\Big] e_R \\
&\quad -h_{1\ell}\Big[\bar{\nu}_{1L}\phi^+_1 + \bar{\nu}_{2L}\phi^+_2 + \bar{\nu}_{3L}\phi^+_3\Big] e_R \\
&- \frac{h_{2\ell}}{\sqrt{3}}\Big[........\Big]\mu_R + -\frac{h_{3\ell}}{\sqrt{3}}\Big[.......\Big]\tau_R \\
&\quad + \frac{h_0}{\sqrt{3}}\Big[(\bar{e}_L + \bar{\mu}_L + \bar{\tau}_L)\nu_{1R} + (\bar{e}_L + \omega\bar{\mu}_L + \omega^2\bar{\tau}_L)\nu_{2R} + (\bar{e}_L + \omega^2\bar{\mu}_L + \omega\bar{\tau}_L)\nu_{3R}\Big]\phi^-_0 \\
&+ h.c.
\end{aligned}$$

Note

1. Light neutrino mass nearly degenerate (upper limit 1.1 eV)
2. $M_D = 1$ MeV and $M_R = 1$ TeV
3. $h_0, h_{1\ell} \ll h_{2\ell}, h_{3\ell}$

Higgs Potential

$$V = V(\phi_i) + V(\chi) + V(\phi_0) + V(\phi_i, \chi) + V(\phi_i, \phi_0) + V(\phi_0, \chi) + V(\phi_i, \chi, \phi_0).$$

Simplification

- FCN interactions of charged leptons are only due to scalar (Higgs doublets).
- χ terms are dropped as they do not take part in CLFV.
- Neglect admixture of ϕ_i and ϕ_0.
- Higgs potential is CP conserving.

Higgs potential becomes

$$\begin{aligned} V(\phi_\alpha) &= \mu_1^2(\phi_1^2 + \phi_2^2 + \phi_3^2) + \lambda_1 (\phi_1^2 + \phi_2^2 + \phi_3^2)^2 \\ &+ \mu_2^2 \phi_0^2 + \lambda_3 \phi_0^4 + \lambda_4(\phi_1^2 + \phi_2^2 + \phi_3^2) \phi_0^2, \end{aligned}$$

where $\phi_\alpha^2 = \phi_\alpha^\dagger \phi_\alpha$ $(\alpha = 0, 1, 2, 3)$.

Steps to obtain mass eigenstates of neutral scalar

- Obtain mass square matrix from the potential
- Obtain the diagonalizing matrix U_H of the mass square matrix

$$\phi^0_\alpha = (U_H)_{\alpha\beta}\,\Phi^0_\beta\,,$$

where $\Phi^0_\beta\,(\beta = 0, 1, 2, 3)$ is mass eigenstates of neutral scalar.
It can be shown that

- $\mathfrak{Im}(\Phi^0_0)$ becomes Goldstone boson coupling to Z^0
- $\mathfrak{Re}(\Phi^0_0)$ becomes SM Higgs boson

Finally

- Write the Lagrangian in mass eigenbasis of the scalar fields
- Couplings of Φ_2 and Φ_3 to charged fermions are FV and that of Φ_0 and Φ_1 are flavour conserving.

Highlights

- SM Higgs no FV couplings
- Multiple Higgs, FV couplings
- $f - \phi$ couplings dictated by A_4 symmetry

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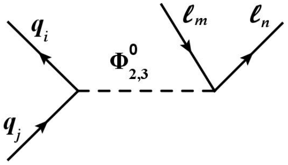
LFV decays of neutral mesons



$\bar{q}_i \, q_j \to \ell_m^+ \, \ell_n^-$

Generic form

$$g^{ij}\bar{f}_{iL}f_{jR}\Phi_2^0 + \tilde{g}^{ij}\bar{f}_{iL}f_{jR}\Phi_3^0 + \left(g^{ji}\right)^* \bar{f}_{iR}f_{jL}\left(\Phi_2^0\right)^* + \left(\tilde{g}^{ji}\right)^* \bar{f}_{iR}f_{jL}\left(\Phi_3^0\right)^*,$$

where $f = (u, d, \ell)$

$$g^{ij} = \frac{h_j}{\sqrt{6}}(-1+\omega) \qquad \tilde{g}^{ij} = -\frac{h_j}{\sqrt{2}}\omega^2,$$

$(ij) = (21), (32), (13)$ [odd]

$$g^{ij} = \frac{h_j}{\sqrt{6}}(-1+\omega^2) \qquad \tilde{g}^{ij} = -\frac{h_j}{\sqrt{2}}\omega,$$

$(ij) = (12), (23), (31)$ [even] .

Neutral meson decays

- $K^0(\bar{s}d) \to \mu^+ e^-$
  ($\leq 4.7 \times 10^{-12} \to m_\Phi \geq 380$ GeV)
  [Ambrose *et al.* [BNL], PRL 98]
- $B_d^0(\bar{b}d) \to e^+ \mu^-$
- $B_d^0(\bar{b}d) \to \mu^+ \tau^-$
- $B_d^0(\bar{b}d) \to \tau^+ e^-$
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BR for m_Φ = 380 GeV

- $BR(B_d^0 \to \tau^+ e^-) = 6 \times 10^{-9} (3 \times 10^{-5})$

  [Aubert, *et al.*, BaBar, 2008]
- $BR(B_s^0 \to \mu^+ e^-) = 2{\times}10^{-11} (5.4{\times}10^{-9})$

  [Aaji, *et al.*, LHCb 2018]
- $BR(B_s^0 \to \tau^+ \mu^-) = 6{\times}10^{-9} (4.2{\times}10^{-5})$

  [Aaji, *et al.*, LHCb 2019]

Tree level decays

- $\mu \not\to e\,\bar{e}\,e$,
- $\tau \not\to \mu\,\bar{\mu}\,\mu$,
- $\tau \not\to e\,\bar{e}\,e$,
- $\{K_L, B_d, B_s\} \not\to \mu^+\,\mu^-$.

τ decays

- $\tau^- \to e^- e^- \mu^+$
- $\tau^- \to \mu^- \mu^- e^+$ BR $\approx 10^{-12}$
- $\tau^- \to \mu^+ \mu^- e^-$
- $\tau^- \to \mu^- e^+ e^-$

Tree level decays

- $\mu \not\to e\,\bar{e}\,e$,
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Top decays

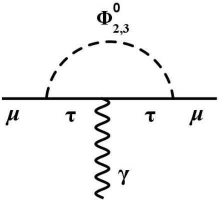
- $t \to c\tau^+\mu^-$ BR $\sim 10^{-8}$
- $t \to u\tau^+e^-$
- $t \to c\mu^+e^-$ BR $\sim 3 \times 10^{-11}$
- $t \to u\mu^+\tau^-$

BR $\sim (2 \times 10^{-5})$
[Gottardo[ATLAS] arXiv:1809.09048]

τ decays

- $\tau^- \to e^-e^-\mu^+$
- $\tau^- \to \mu^-\mu^-e^+$ BR $\approx 10^{-12}$
- $\tau^- \to \mu^+\mu^-e^-$
- $\tau^- \to \mu^-e^+e^-$

Muon $g-2$



$\propto h^2_{3\ell} \implies 6 \times 10^{-14}$

Conclusions

- Model predicts the tri-bi-maximal form of the neutrino mixing matrix purely from the symmetry considerations.
- The Yukawa couplings of the fermions to the multiple Higgs doublets of this model are governed by the A_4 symmetry.
- The FV decays, mediated by heavy neutral scalars of this model, carry signatures of the A_4 symmetry of the Yukawa couplings.
- BR$(K_L \to \mu^+ e^-)$ gives lower bound $m_\Phi = 380$ GeV, the mass of the heavy neutral scalars.
- CLF selection is a signature of the A_4 symmetry.
- No neutral meson mixing and radiative CLFV.
- Few CLFV decay modes of neutral mesons and the top quark can be measured in the near future at LHC.

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Thank you!